Expanded View: Pricing and Conventions for Fed Funds Instruments

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In this paper, we look at the conventions and formulae that are used in the Numerix implementation of fed funds overnight index swap and 30-day fed funds future instruments.

1 Introduction

This paper covers the trading conventions and pricing formulae for fed funds overnight index swaps and 30-day fed funds futures. This document was created using the Numerix internal specifications for these instruments ([1], [2]).

2 Fed Funds Overnight Index Swap

The fed funds overnight index swap (OIS) is a USD fixed-for-floating interest rate swap. The parties agree to exchange the difference between an agreed fixed rate and the interest accrued from the geometric average of the USD overnight fed funds effective rate on the agreed notional amount at periodical payment dates.

The fed funds effective rate for a given day is published in the Fed's H.15 release at 7:30 AM New York time the following business day.

2.1 Conventions

2.1.1 Timeline

Fed funds OIS generates the following schedule for both fixed and floating legs:

Trade Date ⟶ Settlement Date ⟶ Coupon Payment Date 1 ⟶ Trade Payment Date 2 ⟶ … ⟶ Trade Payment Date k

- The settlement date is two business days after the trading date and is the accrual start date of the first payment period.
- Both fixed and floating rate payments occur on the payment dates.
- If the tenor of the fed funds OIS does not exceed one year, there is normally only one coupon payment date. If the tenor exceeds a year, coupons a normally paid annually.
- Each coupon payment date is two business days after the last accrual end date of the coupon period.
- The New York calendar is used for holidays.

2.1.2 Fixed Rate Payment

Fixed rate payment is calculated based on the ACT/360 day count basis. The fixed rate payment is

$$C_{\text{fix}} = N r_{\text{fix}} \frac{a}{360}, \tag{2.1}$$

where N is the notional, r_{fix} is the fixed rate, and a is the actual number of days in the coupon payment period to which the fixed rate is applied.

2.1.3 Floating Rate Payment

Floating rate payment is calculated based on the ACT/360 day count basis. Each payment period is composed of accrual periods with compounding. Normally, the length of an accrual period is one day. However, if an accrual period ends on a non-business day, the length of the accrual period will be adjusted. Each accrual period has its own fixing. The rate for the daily accrual period is fixed on the same day that the accrual starts, but is reported on the next day.

The floating rate payment is

$$C_{\text{float}} = N \left[\prod_{i=1}^{n} (1 + r_i + \text{DCF}_i) - 1 \right], \tag{2.2}$$

where N is the notional, n is the number of accrual periods, r_i is the fed funds effective rate for the ith accrual period, and DCF_i is the day count fraction for the ith accrual period.

2.1.4 Odd Coupon Periods

For a fed funds OIS with tenor longer than one year, there can be an odd coupon at the beginning of the swap. For example, if the tenor of a fed funds OIS is 18 months, one coupon will be paid at six months and one at 18 months. The formulae for calculating coupon payments on both the fixed and floating sides do not change.

2.1.5 Netting

The payments of a fed funds OIS are made on a net basis. The formula for a net payment for swap buyer is

$$C_{\text{net}} = C_{\text{float}} - C_{\text{fix}}. \tag{2.3}$$

2.2 Calculations and Valuation

2.2.1 Accrued Interest

The calculation of accrued interest follows the same steps as that of coupon payment. The start of the accrual is the accrual start date of the current coupon payment period. The end of the accrual is the settlement date. The accrued interest for fixed rate payment follows Eq. (2.1). The accrued interest for floating rate payment follows Eq. (2.2). For past fixings, we need to track a vector of historical dates and corresponding rates. The combined accrued interest is netted following Eq. (2.3).

2.2.2 PV

The present value of a fed funds OIS is

$$PV = \sum_{i=1}^{k} \text{DF}_i C_{\text{net},i}, \tag{2.4}$$

where DF_i is the discount factor from the financing curve at the ith payment date and $C_{\mathsf{net},i}$ is the net payment projected from the index curve at the last accrual end date of the ith payment period.

2.3 Curve Stripping

The fed funds OIS curve is constructed in the same manor as a conventional interest rate swap curve. Standard bootstrapping methods are used on the instruments to strip the curve.

2.4 Deriving Fed Funds Effective Rate from Fed Funds OIS Curve

Given the fed funds OIS curve, we may compute the fed funds effective rate that starts on a settlement date (the date the rate is applied) and ends on maturity date (the next business calendar day following the settlement date). The fed funds effective rate is calculated by

$$r_{s,m} = \frac{\frac{\mathsf{DF}_s}{\mathsf{DF}_m} - 1}{\mathsf{DCF}(s,m)}, \tag{2.5}$$

where s is the settlement date (or start date) of the fed funds rate, m is the maturity date of the fed funds rate, DF_i is the discount factor from the index curve at date i, and $\mathsf{DCF}(i,j)$ is the day count fraction from date i to date j using ACT/360 basis.

3 30-Day Fed Funds Futures

30-day fed funds futures is an exchange-traded and cash-settled futures contract based on the weighted average of the daily fed funds effective rates for the contract month.

The fed funds effective rate for a given day is published in the Fed's H.15 release at 7:30 AM New York time the following business day.

3.1 Conventions

The 30-day fed funds futures uses the conventions described below.

3.1.1 Trading Unit

The trading unit is the interest on fed funds that has a notional of $5,000,000 for one month. The interest is calculated at a rate equal to the weighted average of the overnight fed funds effective rate for the contract month.

3.1.2 Contract Months

Contract months are the first 24 consecutive months.

3.1.3 Last Trading Day

The last trading day is the last business day of the contract month.

3.1.4 Final Settlement Day

The final settlement day is the business day that the fed funds effective rate for the last trading day is announced. The final settlement price of the contract is determined on the final settlement day. The contract is also cash settled on the same day.

3.2 Calculations and Valuation

3.2.1 Final Settlement Price

The final settlement price of one contract is 100 minus the weighted average of the daily fed funds effective rates for the contract month. This gives

$$\text{FSP} = 100 - \text{ROUND}\left(\frac{\sum_{i=1}^{n} \text{DCF}_i r_i}{\sum_{i=1}^{n} \text{DCF}_i}\right), \tag{3.1}$$

where FSP is the final settlement price, n is the total number of fund funds effective rates in the contract month, r_i is the ith fed funds effective rate in the contract month, expressed as a percentage, DCF_i is the day-count fraction for r_i, and ROUND takes its argument as a percentage and rounds to the nearest tenth of a basis point, rounding up in the case of a tie.

Note that the rate for weekend or holiday assumes the rate of the previous business day. For example, if r_i is the fed funds effective rate for a Friday, then r_i will be assigned to Friday, Saturday, and Sunday.

3.2.2 Model Price

Current Month Contract The model price of the current month contract has two key components:

- Weighted average of realized fed funds effective rates from the first calendar day of the month to the current date.
- Weighted average of expected fed funds effective rates from the current date to the last calendar day of the month.

The model price of the current month contract is

$$P_m = 100 - \frac{\sum_{i=1}^{k} \text{DCF}_i r_i + \sum_{i=k+1}^{n} \text{DCF}_i u_i}{\sum_{i=1}^{n} \text{DCF}_i}, \tag{3.2}$$

where P_m is the model price, k is the total number of realized fed funds effective rates in the contract month, r_i is the ith realized fed funds effective rate in the contract month, expressed as a percentage, DCF_i is the day count fraction for r_i or u_i, n is the total number of fed funds effective rates in the contract, u_i is the expected ith fed funds effective rate, expressed as a percentage. (See Section 3.4 for more explanation of u_i.)

No rounding rule is applied to the fed funds futures model price calculation.

Deferred Month Contract For deferred month contracts, the model price is calculated based on the weighted average of expected fed funds effective rates from the first calendar day until the last calendar day of every month. This gives

$$P_m = 100 - \frac{\sum_{i=1}^{k} \text{DCF}_i u_i}{\sum_{i=1}^{n} \text{DCF}_i}, \tag{3.3}$$

where P_m is the model price, n is the total number of fed funds effective rates in the contract, u_i is the expected ith fed funds effective rate, expressed as a percentage (see Section <ref>), and DCF_i is the day count fraction u_i.

3.2.3 Model Price Adjustment

Model price adjustment is the difference between the quoted price and the model price:

$$\text{Model Price Adj} = P_q - P_m, \tag{3.4}$$

where P_q is the quoted price and P_m ist he model price.

3.2.4 Adjusted Model Price

Adjusted model price is the sum of model price and the model price adjustment:

$$\text{Adjusted Model Price} = P'_m + \text{Model Price Adj}, \tag{3.5}$$

where P'_m is the model price and Model Price Adj is given by Eq. (3.4). This output is useful when a risk report is applied to the analytic. If no risk report is applied, the adjusted model price equals to the quoted price.

3.2.5 Model-Implied Weighted-Average Fed Funds Rate

The model-implied weighted-average fed funds rate is

$$R_m = \frac{100 - P_m}{100}, \tag{3.6}$$

where P_m is the model price.

3.2.6 DV01 Per Contract

DV01 per contract is the change in the dollar value of one contract which results from one base point change in the expected weighted average fed funds rate.

Current Month Contract DV01 per contract of the current month is given by

$$\text{DV01}_{\text{contract}} = I \frac{\sum_{i=j+1^n} (0.0001) \text{DCF}_i}{\sum_{i=1^n} \text{DCF}_i}, \tag{3.7}$$

where j is the total number of realized fed funds effective rates in the contract month, n is the total number of fed funds effective rates in the contract month, DCF_i is the day count fraction for the $i$the fed funds effective rate, and I is the notional per contract.

Deferred Month Contract The DV01 per contract of the deferred month is

$$\text{DV01}_{\text{contract}} = (0.0001)N, \tag{3.8}$$

where N is the notional per contract.

3.2.7 DV01

DV01 is the change in the dollar value of the whole position which results from one base point change in the expected weighted average fed funds rate. DV01 is given by

$$\text{DV01} = M\text{DV01}_{\text{contract}}, \tag{3.9}$$

where M is the number of contracts and $\text{DV01}_{\text{contract}}$ is the DV01 per contract as defined before.

3.2.8 PV

The PV of a position is given by

$$\text{PV} = \frac{MN(\text{Adjusted Model Price} - P_s)}{100}, \tag{3.10}$$

where M is the number of contracts, N is the notional per contract, and P_s is the starting quoted price.

3.2.9 Marked-to-Market Cash Flow

The marked-to-market cash flow of the position is given by

$$\text{MTM Cashflow} = MN\frac{P_q - P_s}{100}, \tag{3.11}$$

where M is the number of contracts, N is the notional per contract, P_q is the quoted price, and P_s is the starting quoted price.

3.3 Constructing Fed Funds Curve

The fed funds curve is built from a set of fed funds futures contracts for various contract months. We use bootstrapping methods on the instruments to strip the fed funds curve.

The detailed steps to take to strip the curve are:

1. Start from the fed funds futures of the current month. The discount factor of the curve at now date is 1, which we call DF0. Suppose we know the discount factor of the curve at the first calendar day of the next month, which we call DF1.

2. Use Eq. (3.12) to get all the necessary expected fed funds rates. Here the discount factors in the formula are obtained linear interpolation of the logarithm of DF0 and the logarithm of DF1.

3. Use Eq. (3.2) for the model price of the current month contract. Set the model price equal to the market price and solve for DF1.

4. Consider now the fed funds futures of the next contract month. Assume the last solved discount factor is DF1. Suppose we know the discount factor of the curve at the first calendar day of the next month, which we call DF2.

5. Use Eq. (3.12) to get all the necessary expected fed funds rates. Here the discount factors in the formula are obtained either from the curve that has been set up or by doing linear interpolation of the logarithm of DF1 and the logarithm of DF2.

6. Use Eq. (3.3) for the current month to get the model price of the contract. Set the model price equal to the market price of the contract and solve for DF2.

7. Repeat steps 4 to 6 on contracts of future months to get the discount factors of the curve at all final settlement dates of the corresponding contracts. Here we use a method of linear interpolation of the logarithm of discount factors in the curve construction. Other methods, such as linear, cubic, flat right, and flat left, will also be supported.

3.4 Deriving Fed Funds Effective Rate from Fed Funds Curve

Given the fed funds curve, we use the following formula to project the fed funds effective rate that starts on a settlement date (which is the date to which the rate is applied) and ends on a maturity date(which is the next business calendar day following the settlement date). The expected fed funds effective rate is

$$u_{s,m} = \frac{\frac{\mathsf{DF}_s}{\mathsf{DF}_m} - 1}{\mathsf{DCF}(s,m)}, \tag{3.12}$$

where s is the settlement date, m is the maturity date, DF_i is the discount factor from the curve at date i, and DCF(i,j) is the day count fraction from date i to date j (using ACT/360 basis).

References

[1] Zhao, M. and Zucker, J. (2010), Fed Funds Overnight Indexed Swap (OIS), Numerix Internal Document

[2] Zhao, M. (2011), 30-Day Fed Funds Futures, Numerix Internal Document

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