

ISMA CENTRE THE BUSINESS SCHOOL FOR FINANCIAL MARKETS RMOR MSc

## MARKET RISK MEASUREMENT

Lecture 1  
Value-at-Risk

Professor Carol Alexander  
Spring Term 2000

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## 1.1 New Regulations for Market Risk Capital

- Market risk is the risk of financial loss as a direct result of adverse movements in market prices.
- Market risk exposure increases with trading volume and volatility
- The 1998 regulatory framework for capital adequacy has engendered a move from the standardized rules approach towards the use of internal models for measuring market risk capital
- Prior to this, capital regulation consisted only of minimum capital standards that ignored off-balance sheet positions

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## The Standardized Rules

- Regulators have traditionally used a 'building block' approach to determine risk capital, where capital charges are determined for each category of asset separately and then added
- Within each category (e.g. equities) there are both 'general market risk' and 'specific risk' components
- Capital charges are calculated as a percentage of net exposure (e.g. 8% for equities)
- These rules have been criticized for many reasons, particularly since they do not allow for portfolio effects

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## Calculating Risk Capital with Internal Models

- Internal models for calculating risk capital can take either of two approaches:
  - Value-at-Risk (VaR)
  - Scenario Analysis
- For internal purposes normally both approaches are used
  - VaR models produce simple numbers that are easily compared and understood, for example in setting trading limits
  - Scenario analysis focuses on the real quantity of interest (maximum loss)

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## Granularity Of Calculations

- VaR or maximum loss could be estimated at the transaction, portfolio or trading unit level
- From the firms capital viewpoint, increasing granularity is desirable
- All that regulators require is that
  - scenarios for maximum loss must be country specific
  - VaR measures must be disaggregated according to category of risk (e.g. equity, interest rate, commodity etc)

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## VaR Models: Capital Charges

- The regulatory capital charge is  $k$  times the average of the last 60 days 1% 10-day VaR measures, when netted across the whole firm
- The multiplier  $k$  depends on the 'quality' of the VaR model: see section 1.9
- Or it is the previous day VaR figure, if this is greater

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## Scenario Analysis: Capital Charges

- The regulator determines a library of scenarios to be employed according to risk category and country
- For example, up to 25% changes in underlying volatilities, at the same time as up to 8% changes in underlyings
- The capital charge is the maximum loss that is obtained using these scenarios
- These charges are then added to obtain the market risk capital charge for the whole institution

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## 1.2 Value-at-Risk

- VaR is the expected loss from an adverse market movement with a specified probability ( $\alpha$ ) over a particular period of time ( $h$ )
- The change in portfolio value over the next  $h$  days is

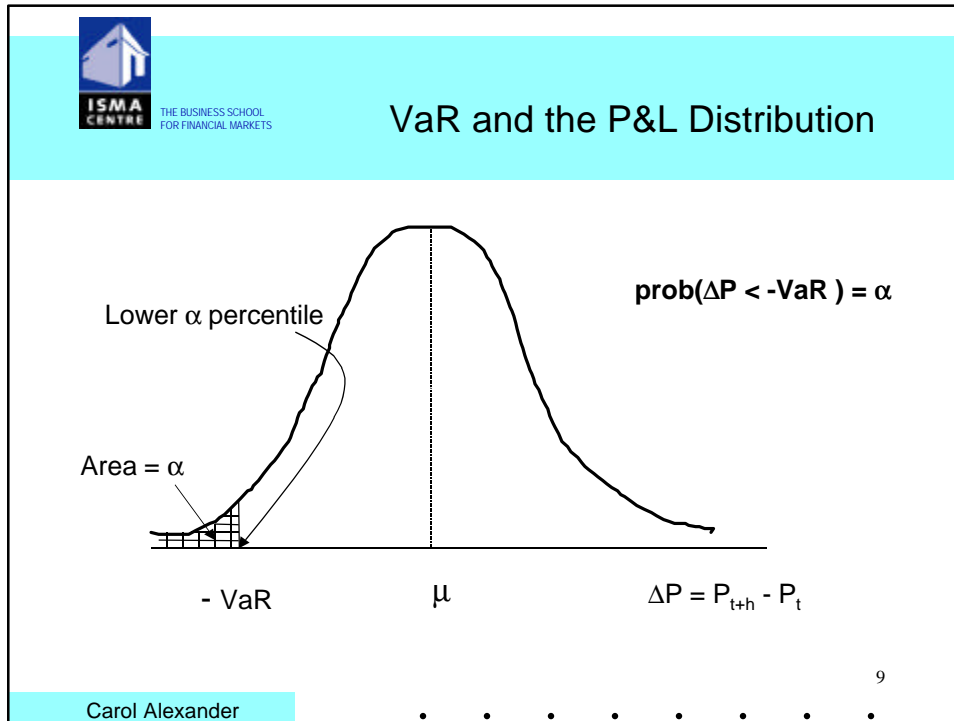
$$\Delta P_t = P_{t+h} - P_t$$

- So a  $100\alpha\%$   $h$ -day VaR number is that amount  $x$  such that

$$\text{Prob}(\Delta P_t < -x) = \alpha$$

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- ### VaR vs Volatility
- The VaR of linear portfolios may be calculated by the Covariance method, where the P&L distribution is assumed to be normal.
  - In this case the lower  $\alpha$  percentile is a simple function of the standard deviation of the P&L (e.g. the lower 5% percentile is  $\mu - 1.645 \sigma$ )
  - So 'Covariance' VaR behaves very much like a standard deviation: it is really just the volatility of P&L.
  - Therefore one should not add VaR numbers!
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## Advantages and Limitations of VaR

- Why should we use VaR?
  - VaR focuses on daily trading P&Ls
  - It is widely applicable across different trading activities
  - And easily understood .....
  - So VaR encourages the adoption of efficient trading and management strategies
- Why should we not use VaR?
  - VaR estimation is extremely problematic
  - VaR is not a good measure of risk

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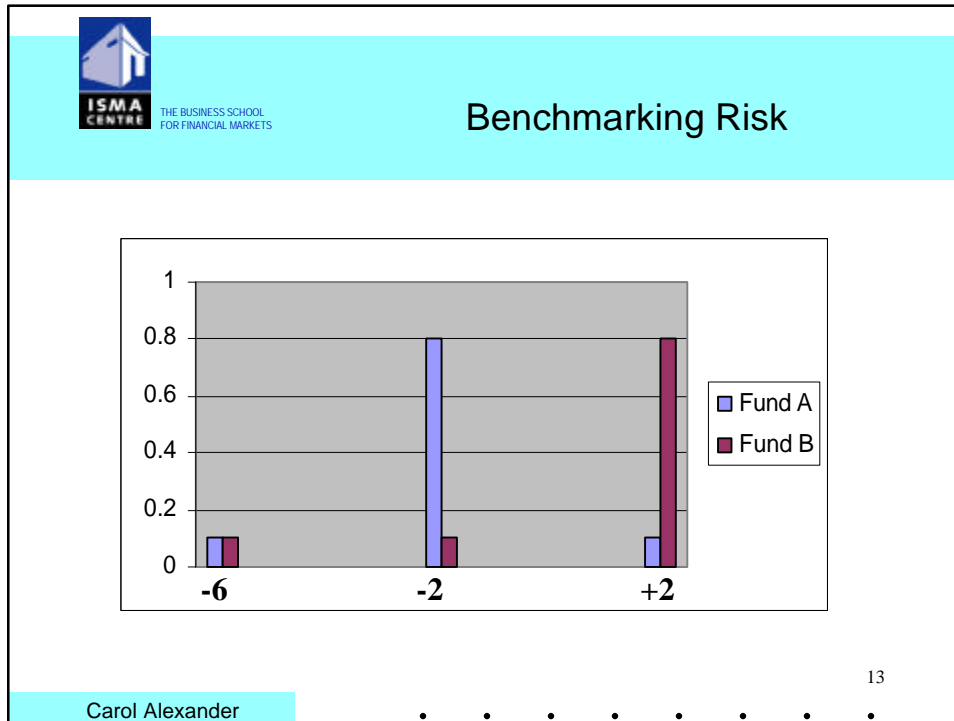
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## Why is VaR Not a Good Risk Measure?

- VaR is like volatility: it weights upside & downside equally so that positive returns are considered as unattractive as negative returns
- Standard VaR models do not measure risk relative to any benchmark return
- Also standard VaR is not a 'coherent' risk measure (Artzner et. al., 1997)

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### Regret

- Regret is the probability weighted sum of all returns less than the benchmark return
- In the previous example both funds have the same  $VaR_{0.1}$  and both have the same variance, but for fund A:
 
$$\text{regret} = 0.8 \cdot 2 + 0.1 \cdot 6 = 2.2$$
 and for fund B:
 
$$\text{regret} = 0.1 \cdot 2 + 0.1 \cdot 6 = 0.8$$

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## Measures of Downside Risk

- Markovitz (1959) introduced the concept of *semi-variance*

$$SV = E((\min(0, R - E(R)))^2)$$

so  $\min = 0$  if returns are greater than the average

- A more general definition of 'downside risk' is measured relative to a benchmark B

$$BV = E((\min(0, R - B))^n)$$

- The case  $n=1$  corresponds to Regret. So Regret is like a put option, the cost of insuring risk on the downside.

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## Coherent Risk Measures

A 'coherent' risk measure  $\rho$  assigns to each loss  $X$  a risk measure  $\rho(X)$  such that,

- Gains are irrelevant:  $\rho(X) = \rho(X^+)$
- Total risk is no more than the sum of the risks of individual positions:  $\rho(X+Y) \leq \rho(X) + \rho(Y)$
- Risk is measured in the same units as losses
- Risk increases linearly with position size:

$$\rho(a + bX) = a + b \rho(X)$$

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## Conditional VaR

- Standard VaR is not a coherent risk measure because generally
  - Covariance VaR does not satisfy  $\rho(X) = \rho(X^+)$
  - Percentiles do not satisfy  $\rho(X+Y) \leq \rho(X) + \rho(Y)$
- A coherent risk measure proposed by Artzner et. al. (1997) is *conditional VaR*:

$$E(X \mid X \geq \text{VaR}_\alpha(X))$$

- Conditional VaR is the average of all losses that are worst than the  $\alpha$  percentile.

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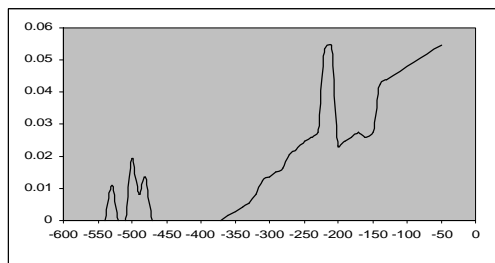


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## Example

- For example, in 1,000 simulations the  $\text{VaR}_{0.05}$  is the smallest of the 50 largest losses, but conditional  $\text{VaR}_{0.05}$  is the average of these 50 largest losses.

Tail Probability Density




VaR = 50m\$

Conditional VaR = 175.2m\$

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## 1.3 Setting the Parameters

VaR assumes the risk and pricing model parameters such as volatilities, correlations and risk factor sensitivities, are fixed.

		Significance Level						
		0.01	0.02	0.03	0.04	0.05	$\alpha$	
Holding Period	1						State parameter values	
	2	←				VaR increases		
	3							↓

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## Holding Period

- Regulators require a holding period  $h$  of 10 days for capital charges, and 1 day for model back testing.
- For internal purposes the appropriate holding period corresponds to the optimal liquidation period. These can be determined from traders knowledge or, perhaps, an economic model.
- Calculations are often based on 1-day P&L's and then transformed, commonly using the square root of time rule

$$(h\text{-day VaR} = \sqrt{h} * 1\text{-day VaR})$$

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## Significance Level

- Regulators require capital charges and back tests based on 1% VaR numbers
- For internal purposes the choice of significance level should reflect the manager's degree of risk aversion
- For example:
  - Losses that occur in normal market circumstances 1 day in 20 are measured by a 5% 1-day VaR
  - Losses occurring 1 week in 100 are measured by a 1% 5-day VaR

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## 1.4 Types of VaR Model

- **Linear portfolios**
  - Covariance method or historical simulation
- **Options portfolios**
  - Simulation methods (historical or Monte Carlo)
  - Delta-Gamma methods

Different VaR models give different results, even when the risk and pricing model parameters are the same!

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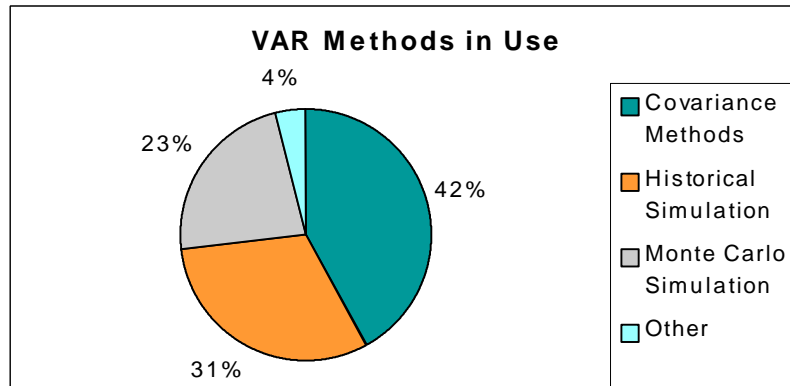
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## SFA Survey of Internal Models (1997)



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## 1.5 The Covariance Method

Assume

$$\Delta P \sim N(\mu, \sigma^2)$$

where  $\Delta P$  denotes the h-day unrealised P&L

Then

$$\text{Covariance VaR} = Z_{\alpha} \sigma - \mu$$

where  $Z_{\alpha}$  denotes the critical value of  $N(0,1)$

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## Volatility of P&L in the Covariance Method

- Often we assume that P&L has zero mean, so the main determinant of VaR in the covariance method is the P&L volatility,  $\sigma$
- In the Covariance VaR model this P&L volatility is the square root of  $\mathbf{p}'\mathbf{V}\mathbf{p}$  where
  - $\mathbf{V}$  is a covariance matrix
  - $\mathbf{p}$  is a vector of MtM values
- The precise definition of  $\mathbf{V}$  and  $\mathbf{p}$  depends on the linear model used

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## Simple Cash Portfolios

- Suppose the portfolio return  $R$  can be represented as a simple sum of individual asset returns
 
$$R = a_1 X_1 + \dots + a_n X_n$$
- For example, consider a simple portfolio with 1m\$ in asset 1 and 2m\$ in asset 2, so  $\mathbf{p} = (1,2)'$
- If asset 1 has 1-day variance of 0.01, asset 2 has 1-day variance of 0.005 and their 1-day covariance is 0.002, the P&L volatility is  $\sqrt{\mathbf{p}'\mathbf{V}\mathbf{p}} = \sqrt{0.038} = 0.195$  m\$ .
- So the 5% 1-day VaR is  $1.645 \cdot 0.195 = 0.32$  m\$

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## Equity Factor Models

- $V$  is the covariance matrix of factor returns
- $p$  is the vector of net betas for each risk factor
- For example, consider an equity portfolio with net FTSE beta of £6m and net S&P beta of £1m.
- Suppose FTSE and S&P 1-day returns variance are 0.002 and 0.004 resp., and their 1-day covariance is 0.001.
- Then 1-day P&L variance is £0.1m, so 1-day 1% VaR is  $2.33 \cdot \sqrt{0.1} = £0.737m$

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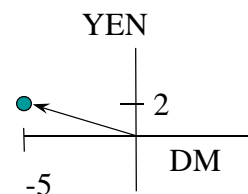


## Cash-Flows

- Fix vertices for cash-flow map
  - currency (or commodity)
  - payment dates
- Represent portfolios in a fixed cash-flow space as a vector  $p$
- The P&L volatility is  $\sqrt{p'Vp}$  where  $V$  is the covariance matrix of the risk factors

3mth futures,  
short £5m in  
DM and long  
£2m in YEN

The risk factors are DM/£ and  
YEN/£ and  $p = (-5, 2)$



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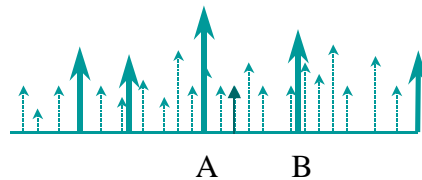


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## VaR Constant Cash-Flow Maps

- Simple linear interpolation between the adjacent vertices is not a VaR invariant cash-flow map
- Instead use quadratic interpolation based on

$$V(X) = p^2V(A) + (1-p)^2 V(B) + 2p(1-p)COV(A,B)$$



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## Limitations of the Covariance Method

- The covariance method is only applicable to linear portfolios.
- Often a covariance VaR is based on a factor model. But many different factor models could be chosen and who is to say which is the most appropriate?
- The sensitivity estimates in factor models are very unreliable. And even when no factor model is used, the covariance matrix itself will be full of inaccuracies
- Risk factor returns are assumed to be normal, but they are not. Typically they are fat-tailed and/or asymmetric

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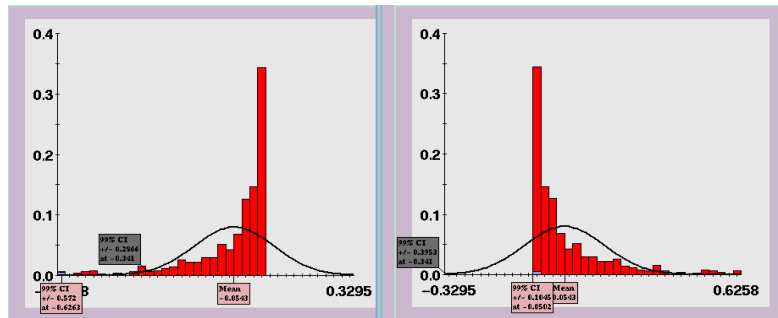


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## Limitations of the Covariance Method

Covariance VaR = \$0.26  
Same for long & short

With 500 Monte Carlo scenarios:  
 $VaR_{short} = \$0.05$   $VaR_{long} = \$0.63$



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## Aggregating Covariance VaR Measures

- Summing of VaR measures is not generally appropriate
- Covariance VaR is calculated as a standard deviation so the rule for variance of a sum gives:  

$$(Total\ VaR)^2 = (Sum\ of\ VaR)^2 - 2*(1-p)\ VaR1 * VaR2$$
- To add covariance VaR measures assumes perfect correlation
- When aggregating VaR numbers one needs to account for the appropriate correlations

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## 1.6 Historical Simulation

- Using the *current* values of the price function parameters, risk factor market prices over a long history are used to create a price history of the portfolio
- From the simulated portfolio price history, an empirical distribution of n-day P&L is generated.
- For options portfolios, delta or delta-gamma or analytic approximations are often employed because the time taken to run full pricing functions for each option can be too demanding

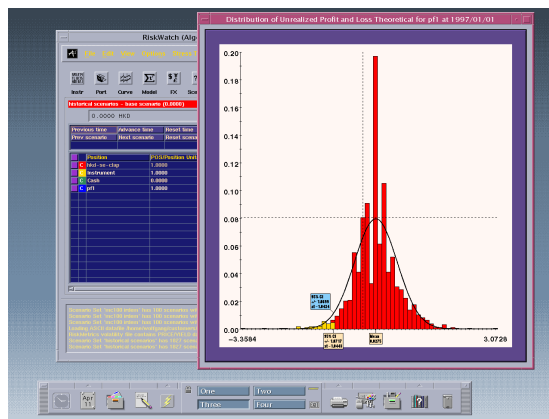
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## Simulating the P&L Distribution



Using linear interpolation cut off the lower  $\alpha$ -percentile of the n-day portfolio P&L histogram to get the n-day  $\alpha\%$  VaR

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## Choosing the Sample Size

- How long an historic data period should be used?
- BIS recommend 3-5 years and a minimum of 250 days
- With only 250 data points accuracy in the tails of the distribution is rather low
- But 'mirroring' upward and downward price changes doubles the size of the sample and avoids bias from trending markets

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## Choosing the Sample Size

- Using a very long historic period produces a greater degree of statistical accuracy in the tails but:
  - stress events form far in the past will have a big effect on current VaR measures
  - it becomes less and less appropriate to use the current values of the pricing model parameters
  - if complex numerical methods are being used to price the portfolio it may be very difficult to compute VaR figures in real time

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## Advantages and Limitations of Historical Simulation

- **Advantages**

- widely applicable
- doesn't assume risk factors are normal
- doesn't need a covariance matrix

- **Limitations**

- the current values of pricing model parameters may not be appropriate for MtM over an historic period
- there is no clear answer to length of look-back period, and data are 'equally weighted'
- stress events may be mixed into every-day VaR measures

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## The Effect of Stress

- A single stress event will cause one outlier in the lower tail of the 1-day histogram of P&L, and  $n$  outliers in the  $n$ -day P&L histogram
- So the longer the holding period, the bigger the effect of a single stress event on the VaR measure
- Some VaR measures (particularly conditional VaR) may be dramatically increased by a single stress event, even if the stress event occurred a long time ago.

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## Exponential Historical Simulation

- Boudoukh et. al. (1998) suggest weighting past returns exponentially before constructing the historical histogram:
  - weight past n-day returns by the factors  $1, \lambda, \lambda^2, \lambda^3, \dots$
  - construct the P&L histogram on weighted returns
  - take the lower  $\alpha$  percentile of this distribution
  - multiply it by the current MtM of the portfolio to obtain the n-day  $\alpha\%$  VaR

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## Advantages and Limitations of Exponential Historic Simulation

- **Advantages** (in addition to those of standard historical simulation)
  - Captures 'clustering' of events in financial markets
  - No need to define fixed 'look-back' period
  - Avoids the problem of lumpy tails from stress events far in the past
- **Limitations**
  - introduces another choice which is quite arbitrary, that is, the value of lambda
  - Probably not acceptable under current regulations

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## 1.7 Monte Carlo VaR

- Monte Carlo VaR is similar to historical VaR in that a set of scenarios on the risk factors are put into the option pricing functions to get a set of portfolio values, and then these portfolio values are used to generate a P&L distribution.
- However the scenarios are forward looking, over a pre-determined risk horizon (e.g. 10 days)
- They are generated using a covariance matrix of risk factor returns (only historical simulation does not use the covariance matrix)

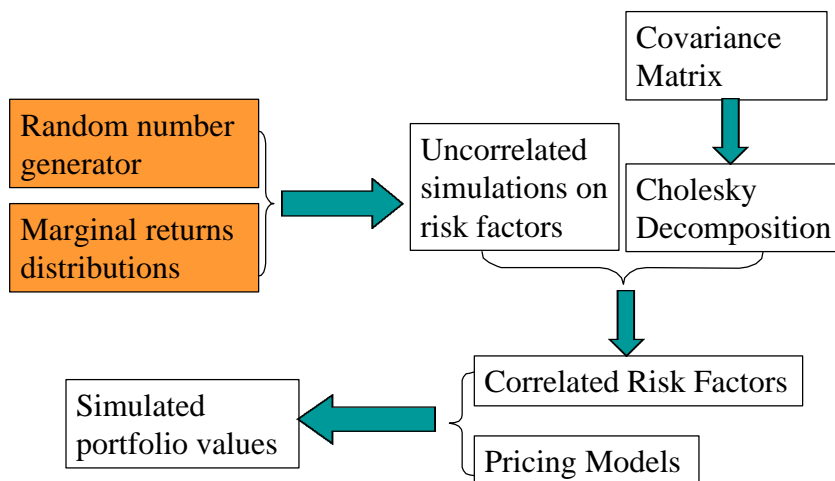
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## Monte Carlo Simulation: Step 1



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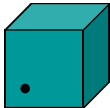
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## The Connection between Simulation and Sampling

- Suppose the portfolio depends on correlated risk factors, such as a swap curve, whose  $n$  - day returns are summarised in a vector  $\mathbf{x}$ . We assume this is multivariate normal
- Take an independent Monte Carlo sample from multivariate  $N(0,1)$ . This corresponds to sampling the hyper-cube .....

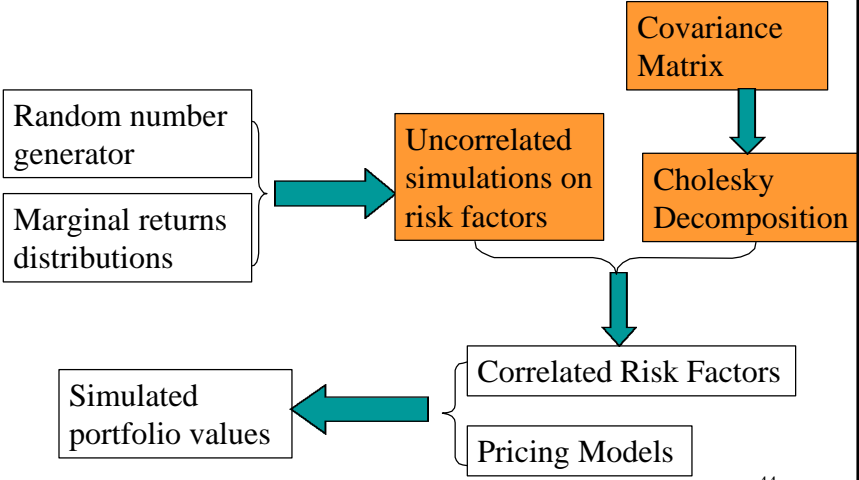
With 3 risk factors, a random sample in the cube gives a simulation of 3 independent risk factor returns



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## Monte Carlo Simulation: Step 2



```

graph TD
    A["Random number generator  
Marginal returns distributions"] --> B["Uncorrelated simulations on risk factors"]
    C["Covariance Matrix"] --> D["Cholesky Decomposition"]
    B --> E["Correlated Risk Factors"]
    D --> E
    E --> F["Pricing Models"]
    F --> G["Simulated portfolio values"]
    
```

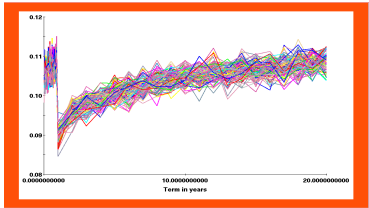
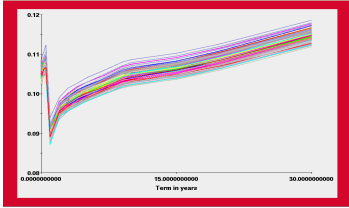
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## Generating Correlated Scenarios

- Estimate the n-day covariance matrix  $V$  and compute the Cholesky matrix  $C$
- Use Monte Carlo to generate  $r \sim N(0, I)$
- Then  $C \cdot r$  transforms this into a correlated n-day returns vector  $x$


→


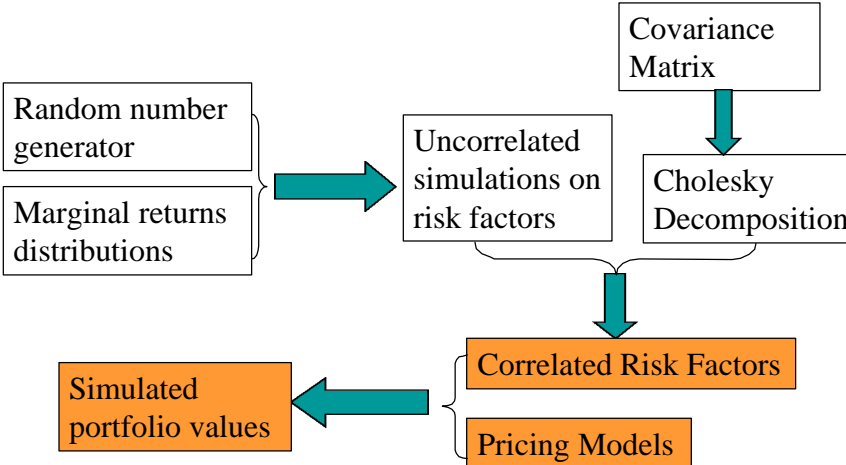
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## Monte Carlo Simulation: Step 3

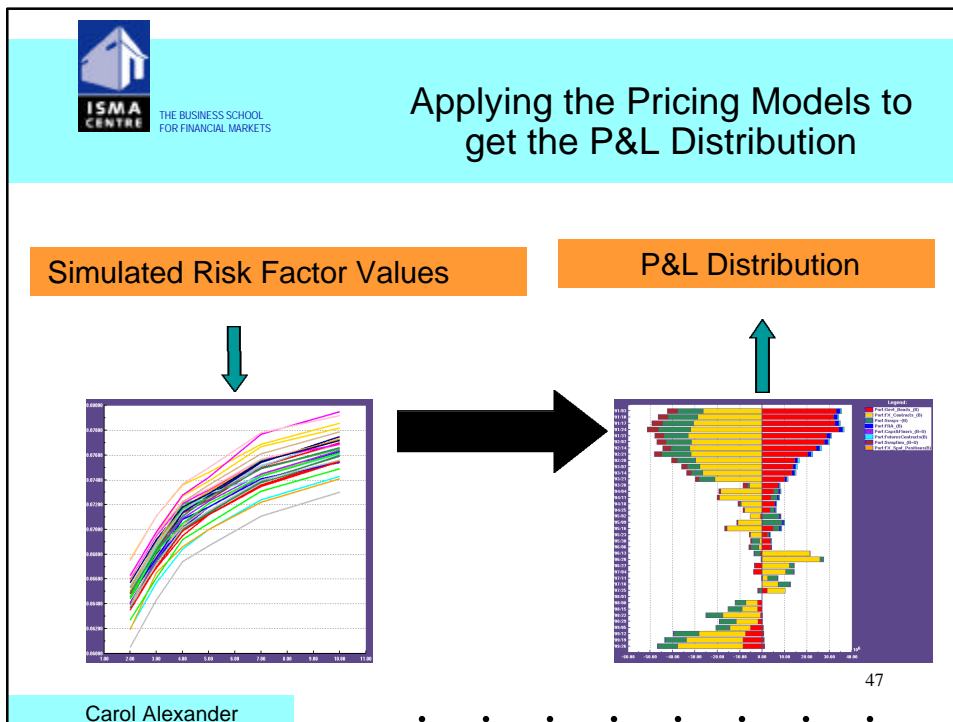


```

graph TD
    A[Random number generator] --> D[Uncorrelated simulations on risk factors]
    B[Marginal returns distributions] --> D
    C[Covariance Matrix] --> E[Cholesky Decomposition]
    D --> F[Correlated Risk Factors]
    E --> F
    F --> G[Simulated portfolio values]
    H[Pricing Models] --> G
    
```

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- Advantages and Limitations of Monte Carlo VaR
- **Advantages**
    - The MtM problems caused by using poor parameter estimates may not be as unrealistic as they are with historic simulation
    - Monte Carlo generation of risk factors easily captures path dependent behaviour (e.g. in barrier options)
  - **Limitations**
    - Covariance matrices may be inaccurate
    - The time taken for 10,000 portfolio re-valuations is just not feasible, so short-cuts are necessary
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## Methods for Speeding up Monte Carlo VaR Calculations

- It is common to use approximate pricing functions
  - net delta-gamma representation at the portfolio level
  - analytic approximations for exotic options
- Also advanced sampling techniques can reduce the number of re-valuations necessary
- Finally, principal components analysis can be used to reduce both the sampling dimension and the number of samples needed (Jamshidian and Zhu, 1997, Frye, 1998)

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## Extensions of Monte Carlo VaR

- Holton (1998) shows how weighted scenarios may be applied in Monte Carlo VaR
- Morokoff et. al. (1998) show how tolerance intervals may be used to provide confidence intervals for Monte Carlo VaR measures.
- So we can state that  

'we are 95% sure that the 1% 1-day VaR will be no less than x'

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## 1.8 Delta-Gamma VaR

- Sometimes VaRs are based on a delta approximation.
- Since this is linear the covariance method may be applied or, to avoid the assumptions of normality and inaccuracies in the covariance matrix, delta approximations can be used in historical simulations.
- But delta is only a local approximation, so the inaccuracy of the delta method becomes particularly pronounced
  - in the extremes of a distribution (i.e. just where VaR matters!)
  - when gamma effect are large (e.g. for short term options)

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## An Analytic Form for Options VaR

- The delta-gamma approximation to a portfolio P&L with a single underlying S is
 
$$\Delta P = \mu + \delta \Delta S + 0.5 \gamma \Delta S^2$$
- Completing the square gives:
 
$$\Delta P = (\mu - 0.5 (\delta^2/\gamma)) + 0.5 \gamma ((\delta/\gamma) + \Delta S)^2 = \mu^* + 0.5 \gamma \sigma^2 X$$
- $X = (((\delta/\gamma) + \Delta S) / \sigma)^2$  will be a  $\chi^2$  variable

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## An Analytic Form for Options VaR

- $\text{Prob}(\Delta P < \text{VaR}) = \text{prob}(\mu^* + 0.5 \gamma \sigma^2 X < \text{VaR})$   
 $= \text{prob}(X < (\text{VaR} - \mu^*) / 0.5 \gamma \sigma^2)$
- So  $(\text{VaR} - \mu^*) / 0.5 \gamma \sigma^2 = \chi^2_{\alpha}$   
where  $\chi^2_{\alpha}$  is the lower percentile of a  $\chi^2$  density
- Hence the  $\alpha$  % VaR is  
$$\mu^* + 0.5 \gamma \sigma^2 \chi^2_{\alpha}$$
- Rouvinez (1997)

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## 1.9 Testing the VaR Model

- 'Back testing' is one of a number of tools available for internal and external validation of VaR models
- It identifies any weakness of an internal model, but doesn't diagnose the cause
- It prevents the under estimation of VaR and therefore capital reserves which are too small to cover unexpected losses in 'normal' market circumstances

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## Counting Exceptions

- Regulators require each model to pass a back test of its 1% 1-day VaR measures against the daily P&L over the last 250 days
- Theoretical P&L should exceed a 1% VaR measure 1 day in 100. Therefore one should expect 2.5 exceptions in a sample size 250 (with a standard error of  $\sqrt{0.99 \cdot 0.01 \cdot 250} \approx 1.6$ )
  - Less than this number of exceptions and the VaR measure may be too high
  - Too many exceptions and the VaR measure may be too small - this is what regulators wish to guard against

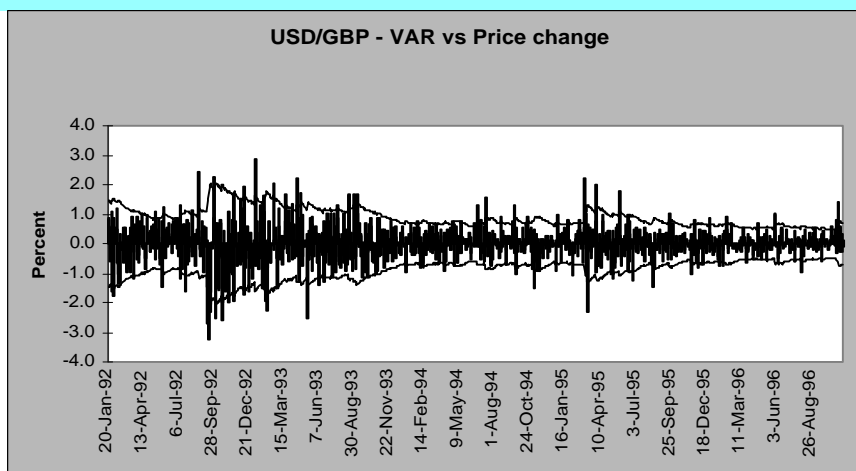
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## Example: USD/GBP Rate



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## Model Classification

- VaR models are classified according to the number of exceptions in the 250 day back testing period
  - less than 5 exceptions:* the model is in the 'green zone', multiplication factor  $k = 3$
  - 5-9 exceptions:* the model is classified as 'yellow zone', multiplication factor  $k = 3.4$  to  $3.8$
  - 10 or more exceptions:* the model is 'red zone', and has a multiplication factor  $k = 4$  (or model may even be disallowed)
- Market risk capital reserves are then set as  $k$  times the average of the last 60 days 1% 10-day VaR measures

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## Types of P&L Data

- Theoretical or 'unrealised' P&L is the change in MtM value assuming the portfolio remains unchanged during the holding period and pricing parameters are constant
- Actual P&L includes P&L from intra-day trades and other realised P&L (such as fees or funding costs)
- Cleaned P&L is actual P&L less funding costs, marketing profits, brokerage fees, reserves released etc
- The choice of P&L data for the back test may depend on which data are available for the portfolio in question.

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## Which P&L Data Should be Used for Testing the VaR Model?

- Since VaR measures are calculated on theoretical P&L it makes sense to test the model against this
- But it is actual or cleaned P&L that determines the extent of a firm's real losses
- There is no systematic reason why one type of P&L data should give better results in back tests:
  - speculative trading may induce more volatile actual or cleaned P&L
  - hedging may induce less volatile actual or cleaned P&L

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## 1.10 What if your VaR Model Fails?

- The follow up of any large unpredicted loss requires analysis tools to check the accuracy of:
  - covariance matrices
  - normality assumptions
  - factor models
  - specific risks
  - factor sensitivities
  - MtM prices over the risk horizon

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## Sensitivity Analysis

- *How robust the linear VaR measures are to*
  - errors in covariance matrices
  - errors in betas or deltas
  - changes in cash-flow maps or factor models
- *How robust option VaR measures are to*
  - pricing approximations
  - sampling methods

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## VaR Distributions

*Movements of risk factors over the risk horizon*      *MtM values of portfolio at the risk horizon*

Risk Model

→

Pricing Model

→

VaR

Statistical Volatilities


↑

Risk Model


Implied Volatilities

↑

Pricing Model



*VaR distribution due to uncertain volatilities*



*Parameter Uncertainty*

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