THE BUSINESS SCHOOL
FOR FINANCIAL MARKETS

Exploring Bayesian Methods for the Measurement of Operational Risk

ICBI Global Derivatives

Paris, April 2000

Professor Carol Alexander
Chair of Risk Management,
ISMA Centre, Reading University, UK
www.ismacentre.reading.ac.uk

THE BUSINESS SCHOOL
FOR FINANCIAL MARKETS

Definitions?

"The risk of direct or indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events"

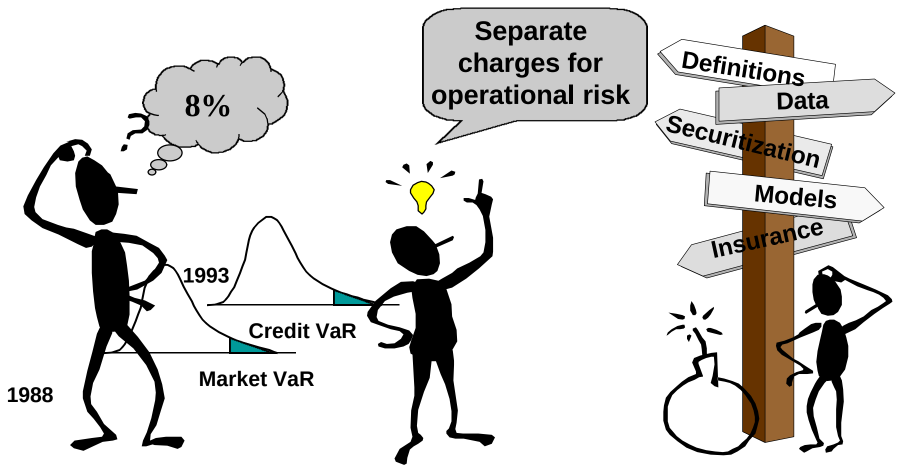
Operations Risks

Transaction Processing
Unauthorized Activities
System

Other Risks

Reputation
Information
Human
Legal

Policy?



Options?

- **Qualitative Risk Assessment:**
  - Possibly building on existing qualitative risk assessment methods (e.g. CAMEL, RATE) where different levels of supervision apply depending on the risk rating of the firm. But it may be very difficult to maintain a 'level playing field'
- **Quantitative Risk Assessment:**
  - Proportional charges or internal models for operational risk capital
- **Insurance or Securitisation of Operational Risks:**
  - Alternative Risk Transfer, OR Bonds etc. But then, how should regulators regulate the insurers?

Data?

Qualitative

Self-Appraisal

Risk Maps

Quantitative

Scores

Ratings

Loss Events

ISMA CENTRE
THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Methods?

Top Down

Income or Expense Volatility

Proportional Charge

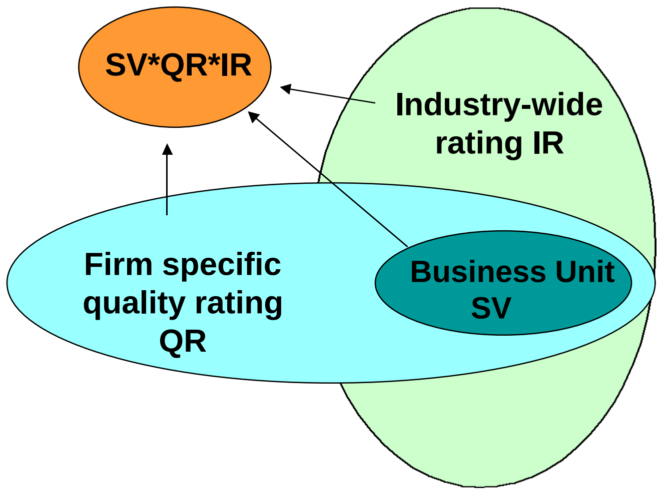
Bottom Up

Internal or External Risk Ratings

Loss Distribution

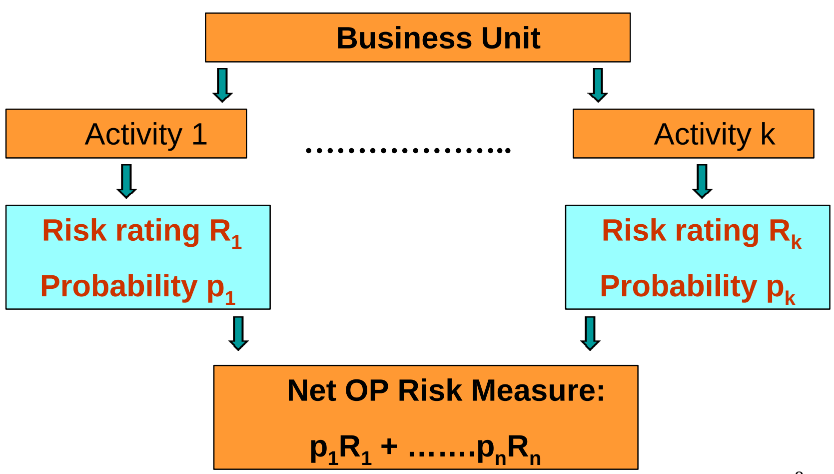
ISMA CENTRE THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

External Risk Ratings



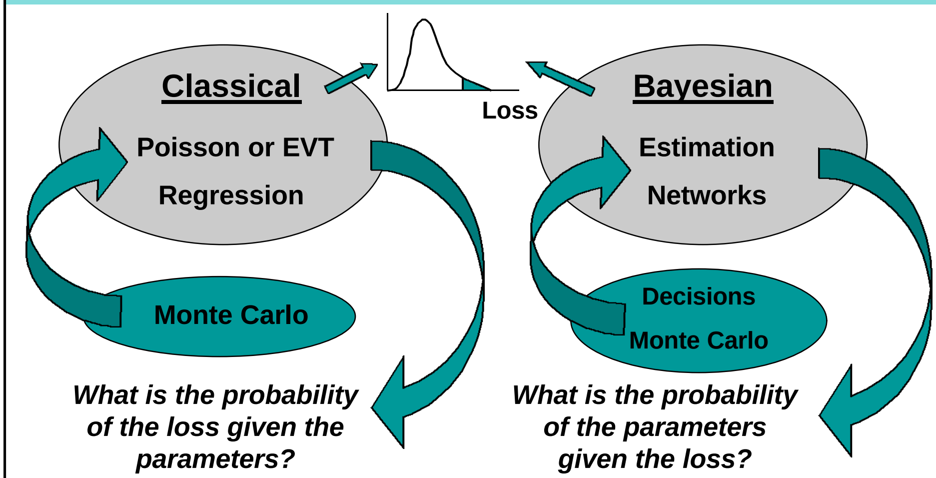
ISMA CENTRE THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Internal Risk Ratings



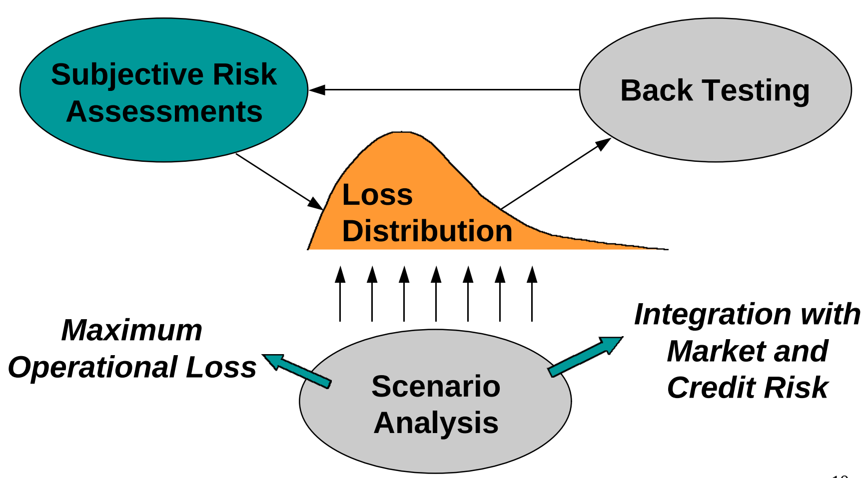
ISMA CENTRE THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Statistical Models?



ISMA CENTRE THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Model Requirements



Bayesian Methods

THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

The Reverend Thomas Bayes was born in London (1702) and died in Kent (1761). His "*Essay Towards Solving a Problem in the Doctrine of Chances*", published posthumously in 1763, laid the foundations for modern statistical inference.

Bayes' Rule

THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

- The cornerstone of Bayesian methods is the theorem of conditional probability of events X and Y:

$$\text{prob}(X \text{ and } Y) = \text{prob}(X|Y)\ \text{prob}(Y)$$

So $$\text{prob}(X|Y)\ \text{prob}(Y) = \text{prob}(Y|X)\ \text{prob}(X)$$

- This can be written in a form that is referred to as Bayes' rule, which allows prior information about Y to revise the probability of X:

$$\mathbf{prob(X|Y) = prob(X)\ [\ prob(Y|X) / prob(Y)]}$$

Example of Bayes' Rule

- You are in charge of client services, and your team in the UK has not been very reliable.
- You believe that one fifth of the time they provide an unsatisfactory service, and that when this occurs the probability of losing the client rises from 0.2 to 0.65.
- If a client in the UK is lost, what is the probability that they have received unsatisfactory service from the UK team?

Example of Bayes' Rule

- Let X be the event 'unsatisfactory service' and Y be the event 'lose the client'. Your prior belief is that prob(X) = 0.2

$$\text{prob}(Y) = \text{prob}(Y \text{ and } X) + \text{prob}(Y \text{ and not } X)$$

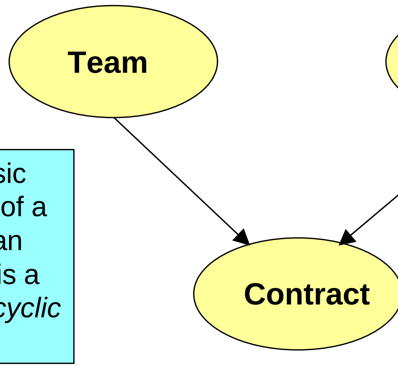
$$\text{prob}(Y) = \text{prob}(Y|X)\ \text{prob}(X) + \text{prob}\ (Y|\text{not } X)\ \text{prob}\ (\text{not } X)$$

$$= 0.65 * 0.2 + 0.2 * 0.8 = 0.29.$$

- Now Bayes' Rule gives the posterior probability of unsatisfactory service given that a client has been lost as:

$$\text{prob}(X|Y) = \text{prob}(Y|X)\ \text{prob}(X)/\text{prob}(Y) = 0.65 * 0.2 / 0.29 = 0.448.$$

Bayesian Networks

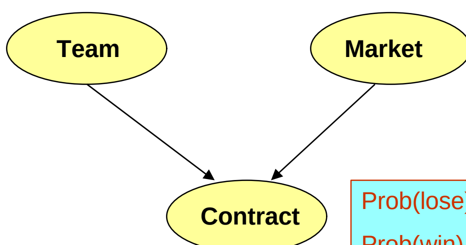


The basic structure of a Bayesian network is a *directed acyclic graph*

Nodes represent *random variables*
Edges represent *causal links*

Assigning Probabilities

Prob(good) = 0.8
Prob(bad) = 0.2

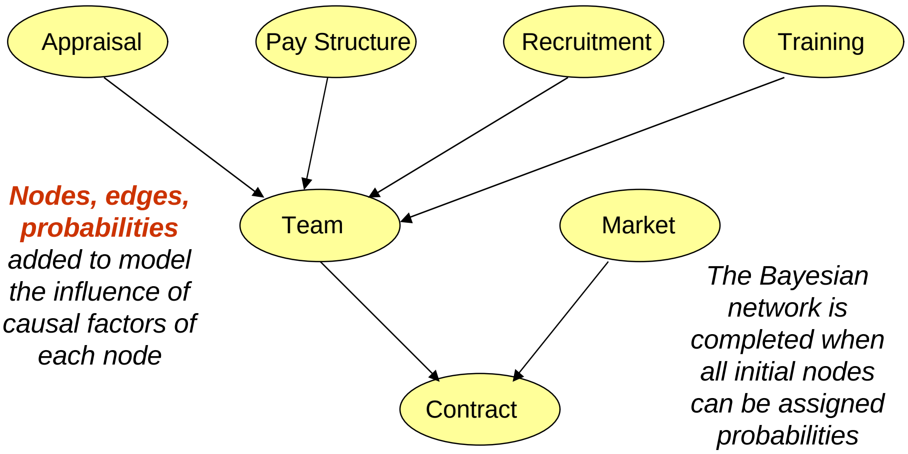


Prob(good) = 0.5
Prob(bad) = 0.5

Prob(lose) = 0.29
Prob(win) = 0.71

Prob(lose) = prob(lose | T=good and M=good) prob(T=good and M=good)
+ prob(lose | T=good and M=bad) prob(T=good and M=bad)
+ prob(lose | T=good and M=bad) prob(T=good and M=bad)
+ prob(lose | T=good and M=bad) prob(T=good and M=bad)

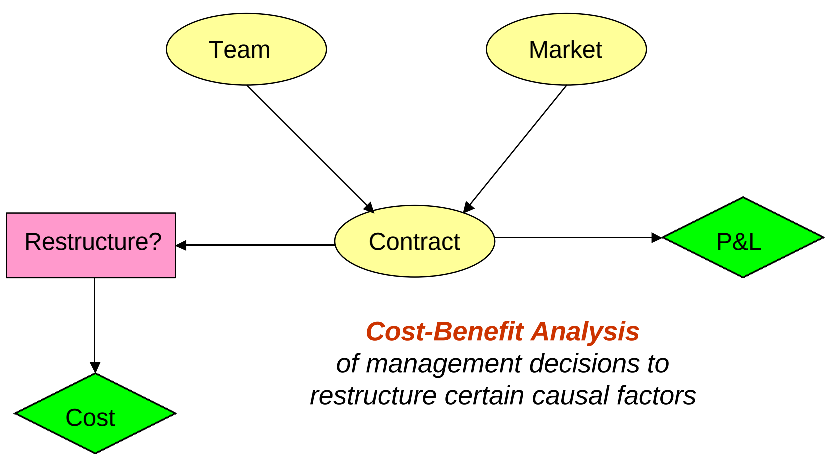
Adding More Causal Links



Nodes, edges, probabilities *added to model the influence of causal factors of each node*

The Bayesian network is completed when all initial nodes can be assigned probabilities

Adding Decision Nodes and Utilities

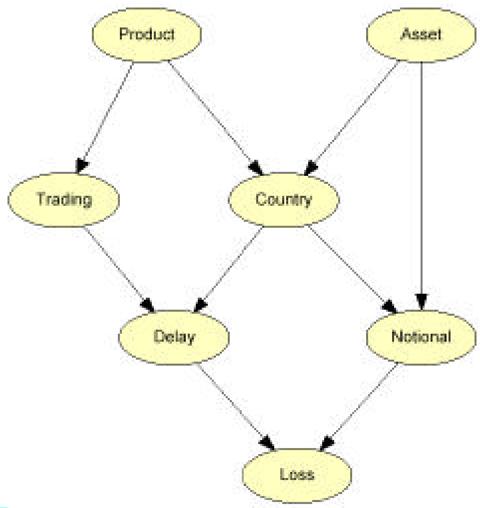


Cost-Benefit Analysis *of management decisions to restructure certain causal factors*

THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Settlement Loss

Operational (as opposed to credit) settlement loss is "***the interest lost and the fines imposed as a result of incorrect settlement***"



THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Initial Probabilities

Asset	
80.00	FX
20.00	Security

Country	
47.20	Europe
52.80	Asia

Product	
30.00	Underlying
70.00	Derivative

Trading	
17.00	OTC
83.00	Exchange

Delay	
85.65	None
6.18	1 day
3.63	2 days
1.94	3 days
0.88	4 days
1.72	> 4 days

Notional	
13.96	<10
10.00	10-20
10.36	20-30
17.64	30-40
26.36	40-50
21.68	>50

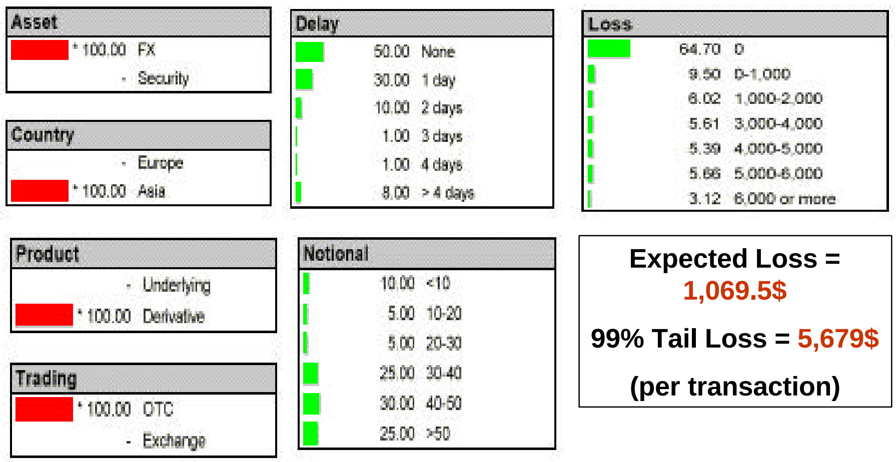
Loss	
90.22	0
3.11	0-1,000
1.77	1,000-2,000
1.50	3,000-4,000
1.40	4,000-5,000
1.31	5,000-6,000
0.69	6,000 or more

Expected Loss = 274.4$

99% Tail Loss = 4,763$

(per transaction)

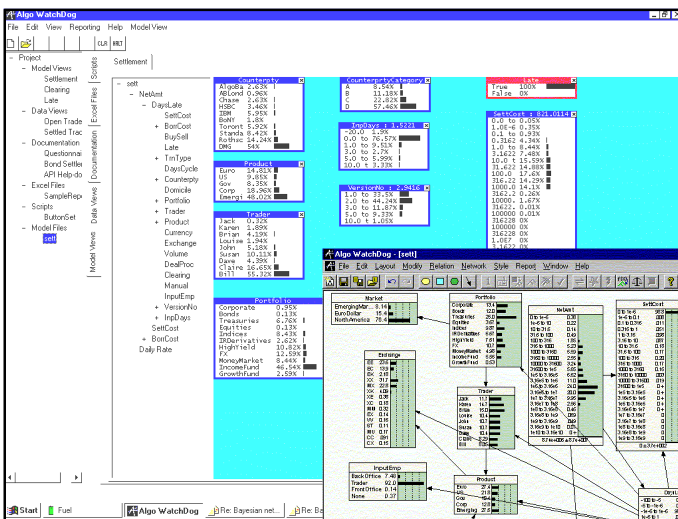
Scenario Analysis:
Maximum Operational Loss

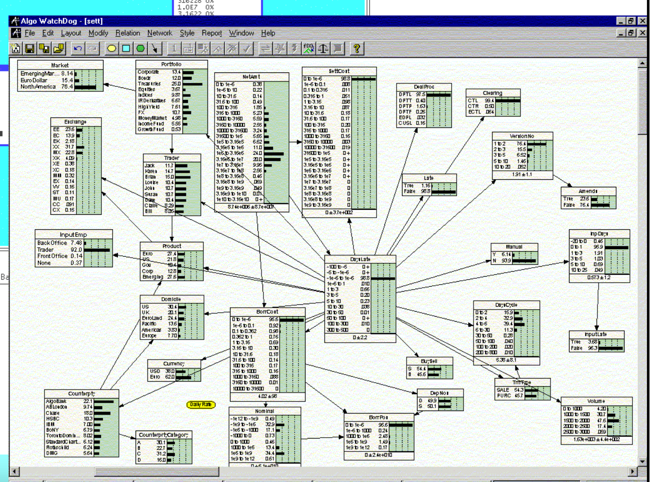


Expected Loss =
1,069.5$

99% Tail Loss = 5,679$

(per transaction)





Algorithmics WatchDog™

THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Bayesian Estimation

Bayes rule is: prob(X | Y) = prob(Y | X) prob(X) / prob(Y)

Applying this to distributions about model parameters:

prob(parameters | data) = prob(data | parameters)*prob(parameters) / prob(data)

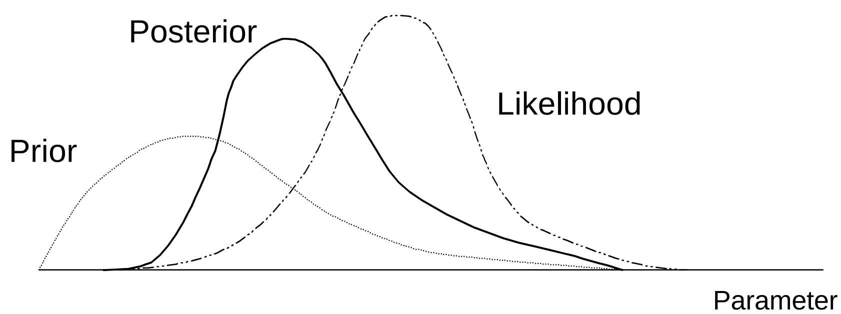
Or:

prob(parameters | data) ∝ prob(data | parameters)*prob(parameters)

Posterior Density ∝ *Sample Likelihood* * *Prior Density*

THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Priors, Likelihoods and Posteriors



The posterior is a mixture of prior and current information.

Bayesian Estimation as a Decision

- Bayesians view an estimate b of a parameter β as an optimal choice, where the decision criterion is to:

minimize the expected loss

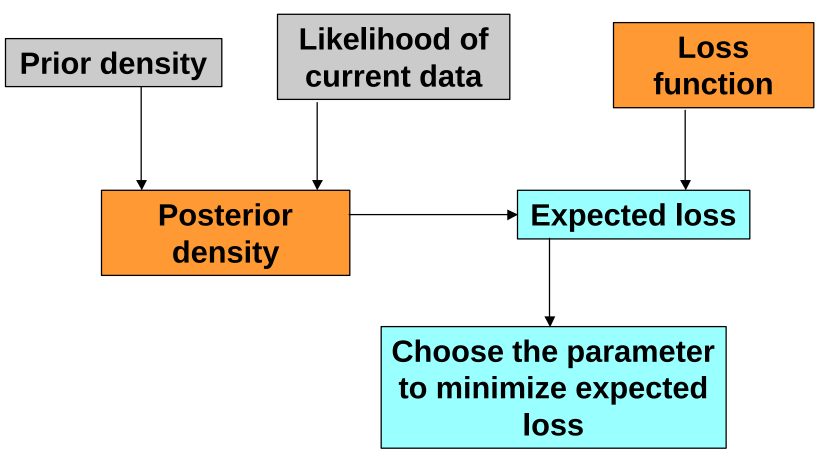
- Expected loss is defined by

loss function

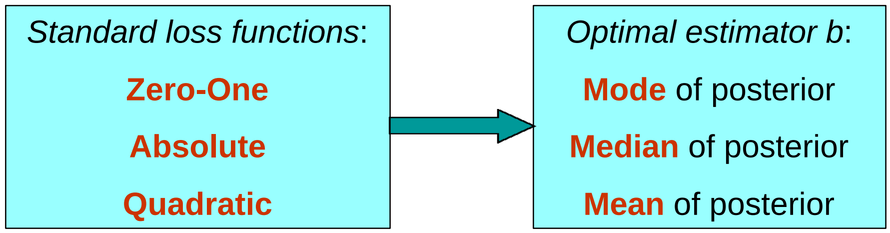
probability distribution

- In Bayesian estimation the probabilities are given by the posterior distribution.

The Logic of Bayesian Decisions

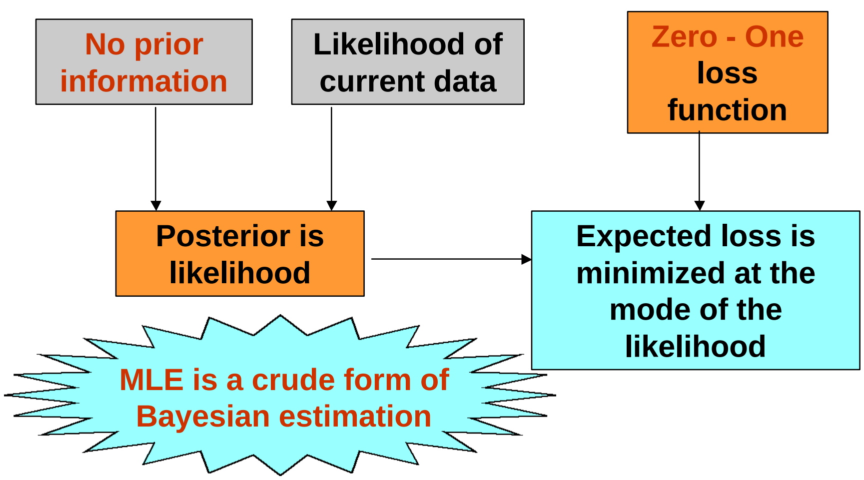


Loss Functions



So with a beta prior and posterior, and a quadratic loss function, the Bayesian estimator is the **mean of a beta distribution**

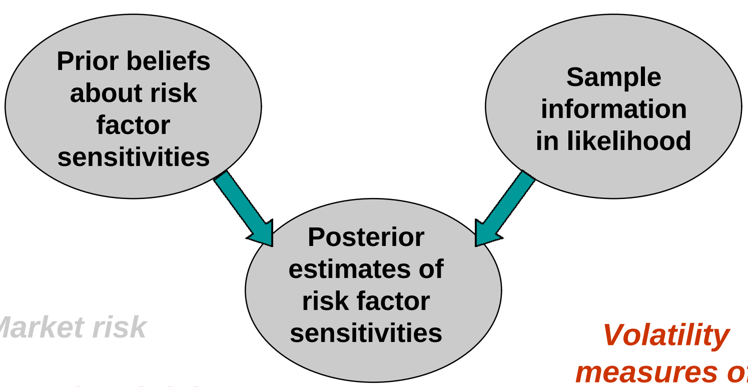
Maximum Likelihood Estimation



Example: Operational Risk Scores

- Suppose that in a sample of 10 booked trades, 1 was incorrectly marked:
  - The MLE of the percentage of mis-marked trades will be **10%.**
  - But the Bayesian estimate with no prior information (and a quadratic loss function) is **16.67%**
  - But if more information were available, say from a previous sample where 1 out of 36 trades were mis-marked, the Bayesian estimate will be is **6.25%**

Bayesian Betas



Market risk

Reputational risk

Volatility measures of operational risk

THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Example

	OLS	Bayesian with Prior $N(0.5,0.1^2)$	Bayesian with Prior $N(0.5, 0.01^2)$
Estimate of Market Beta	1.211	1.179	0.6256
Estimated Standard Error	0.021586	0.0211	0.0091

Bayesian estimates always have have lower standard errors, reflecting the value of prior information

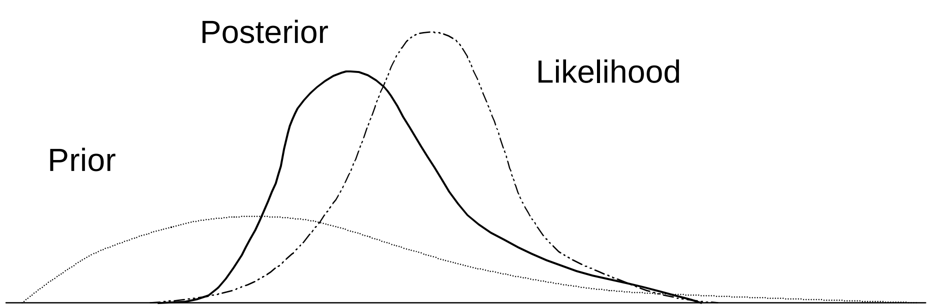
Uncertain prior ⇒ Bayesian and OLS estimates are similar

Confident prior ⇒ Bayesian and OLS estimates are different

Carol Alexander

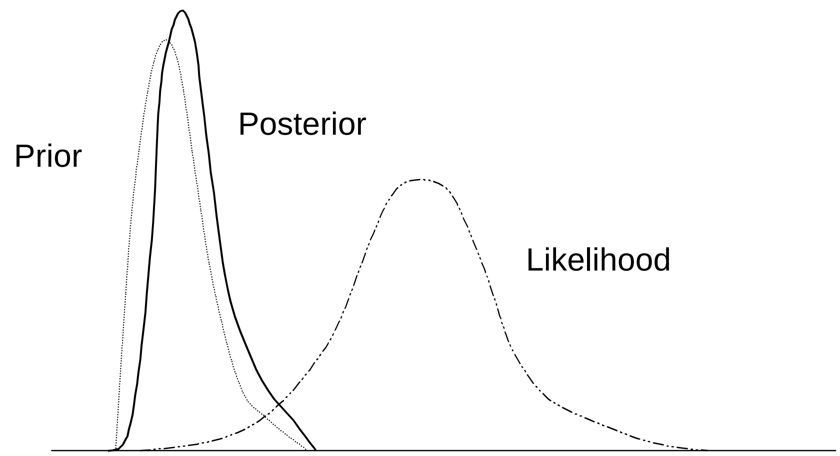
THE BUSINESS SCHOOL FOR FINANCIAL MARKETS

Posterior with Uncertain Prior



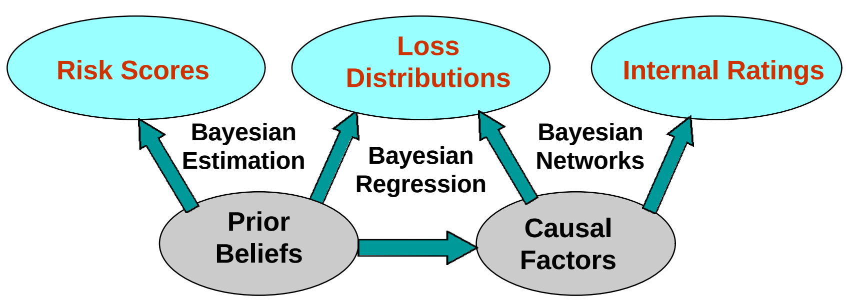
Carol Alexander

Posterior with Confident Prior



Summary

- Quantitative models for measuring operation risk are at an early stage of development
- Bayesian methods offer many advantages:



Further Reading

- Much information on operational risk and Bayesian networks is available on the internet. The most useful sites include:
  - www.idsa.org (survey report - not free)
  - www.bba.org.uk (useful (free) executive summary)
  - www.research.microsoft.com/research/dtg/msbn/default.htm (free non-commercial Excel compatible Bayesian network)
  - http.cs.berkeley.edu/~murphyk/Bayes/bnsoft.html (list of free Bayesian network software)

Beta Distributions

- More or less any prior that is observed in practice may be approximated by a beta density

$$f(x) \propto x^a (1-x)^b \qquad 0 < x < 1.$$

- Using the fact that

$$\int x^{a+1} (1-x)^b \, dx = (a+1)!\, b!\, /(a+b+2)!$$

it follows that the mean of a beta density is

$$(a+1)/(a+b+2).$$

Bayesian Estimators for Operational Risk Scores

- The likelihood is the probability of getting n 'sucessses' in m Bernoulli trials with probability of success p.
- So the likelihood is proportional to $p^n(1-p)^{m-n}$.
- Using a beta prior $p^a(1-p)^b$ gives the posterior density as another beta distribution $p^{n+a}(1-p)^{m-n+b}$.
- Suppose the loss function is quadratic, so the Bayesian estimator is the mean of the posterior, that is

$$(n+a+1)/((m+a+b+2)$$

Comparison with Standard Score Estimates

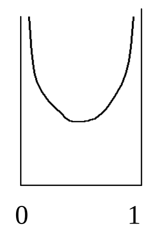
- The maximum likelihood estimate is just n/m.
- This is just a Bayesian estimator corresponding to the beta prior with a = b= -1 (see next slide).
- More natural beta priors have non-negative a and b.
- For example with the uniform prior (no information), a = b = 0 and the Bayesian estimator is (n+1)/(m+2).

MLE Score Estimate

- The MLE corresponds to a Bayesian estimate with the peculiar prior density

$$f(p) \propto 1/p(1-p).$$

- So the classical method is actually assuming that it is equally likely that almost all trades, or almost no trades, have been mis-marked.



$f(p) \propto 1/p(1-p)$

Bayesian Betas

- Factor Model: $\mathbf{y} = \mathbf{X}\beta + \varepsilon$ where $\varepsilon \sim N(0, \sigma^2\mathbf{I})$
- OLS estimates of betas: $\mathbf{b} = (\mathbf{X'X})^{-1}\mathbf{X'y}$
- Assume the conjugate prior: $\beta \sim N(\beta_0, \Sigma_0)$.
- Then the Bayesian estimates of the market betas are:

$$\mathbf{b}^* = \Sigma^{*\,-1}\,(\,\Sigma_0^{-1}\beta_0 + \Sigma_1^{-1}\mathbf{b})$$

with inverse covariance matrix $\Sigma^{*\,-1} = c\,(\Sigma_0^{-1} + \Sigma_1^{-1})$

where $\Sigma_1 = s^2(\mathbf{X'X})^{-1}$ and $c = m/(m-2)$, m being the degrees of freedom in the model.