Hedge Fund Management using Cointegration:
The Latest Evidence

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1. Introduction

- This paper presents a classic 'market neutral' hedge fund strategy: the key objective is to minimize investment risk in an attempt to deliver profits under all market circumstances.
- The hedge is designed to have minimal correlation with the market and, irrespective of market direction, the fund seeks to generate positive alpha.
- A significant difference between this model and more traditional hedge fund strategies is that portfolio optimization is based upon the cointegration of prices rather than just the correlation of returns.

Traditional Models

- Models that are based on mean-variance analysis seek portfolio weights to minimise the variance of the portfolio for a given level of return.
- The portfolio variance is measured using a covariance matrix and these matrices are notoriously difficult to estimate.
- Moreover the mean-variance criterion has nothing to ensure that tracking errors are stationary. Although the portfolios will be efficient, the tracking errors will in all probability be random walks.
- Therefore the replicating portfolio can drift very far from the benchmark unless it is frequently re-balanced.

Minimum Variance Portfolios

The capital allocation problem of finding minimum variance portfolios having a given minimum level of return μ becomes:

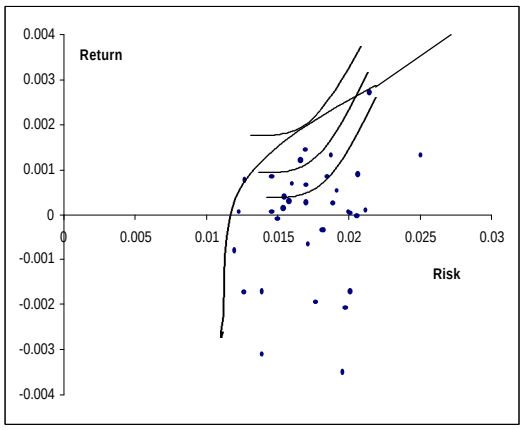
$$\text{Min}_{\mathbf{w}} \quad \mathbf{w'Vw} \quad \text{such that} \quad \Sigma w_i = 1 \text{ and } \mathbf{w'r} \geq \mu$$

where **V** is the covariance matrix. In this case the optimal weights $\mathbf{w}^{\dagger} = (w_1^{\dagger}, \ldots, w_n^{\dagger})'$ are given by

$$w_i^{\dagger} = ((a\psi_i - b\xi_i) + \mu(V^*\xi_i - b\psi_i)) / (V^*b - a^2)$$

where V^* is the variance of the global minimum variance portfolio; ψ_i is the sum of the elements in the ith column of $\mathbf{V}^{-1}$; ξ_i is the returns weighted sum of the ith column of $\mathbf{V}^{-1}$ a = $\boldsymbol{\psi}'\mathbf{r}$ and b = $\mathbf{r'V}^{-1}\mathbf{r}$.

Problems with Efficient Frontier



Lacks robustness unless long term averages used for covariance matrix

Shorter term averages give frontiers that are unstable and often dominated by a few stocks

Remedies

- Frequent rebalancing
- Smoothing allocations
- Rebalancing limits
- Priors for mean, variance and correlation of returns
- Limited rebalancing in direction only

- **Root of the Problem:** the basic data for mean variance analysis consists of asset returns.

Cointegration

- Cointegration refers not to co-movements in returns, but to co-movements in asset prices, exchange rates or yields.
- If spreads are mean-reverting, asset prices are tied together in the long-term by a common stochastic trend, and we say that the prices are 'cointegrated'
- Engle and Granger (1987):

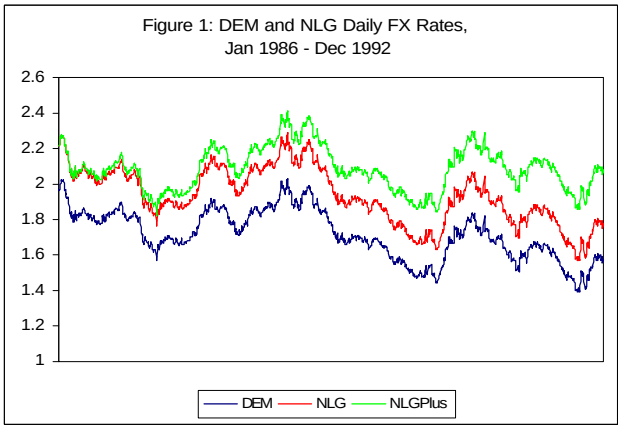
  x and y are cointegrated if x,y ~ I(1)

  but there exists a *such that* $z = x - ay \sim I(0)$.

Examples

- Spot and future prices
- Term structures
- International equity markets
- Equities within an index

Cointegration vs Correlation



High correlation

but no cointegration

Common Trends

- Stock and Watson (1988): High cointegration implies a common stochastic trend

$$x_t = w_t + \varepsilon_{xt}$$
$$y_t = w_t + \varepsilon_{yt}$$
$$w_t = w_{t-1} + \varepsilon_t$$

- Cointegration can occur without high correlation

2. Fund Management Methodology

- A linear regression of log prices is employed for benchmarking or index tracking models that are based on cointegration.
- The dependent variable is the log index price or some other benchmark, such as LIBOR, that is used to evaluate the performance of the portfolio.
- In the case of tracking an index 'plus' alpha percent per annum, the dependent variable will be index returns plus a small increment that amounts to alpha percent over the year.
- The explanatory variables are the log prices of the assets in the tracking portfolio, and the residuals are the tracking errors.
- **The portfolio is cointegrated with the benckmark if, and only if, the tracking error is stationary**

Model Parameters

- The basic cointegration index tracking model is defined in terms of certain parameters:
  - Any **alpha** return over and above the index;
  - The time-span of daily data that is used in the cointegrating regression. This is reported as the number of months for **training** the model ;
  - The **number of assets** in the portfolio; In fact the number of non-zero allocations need not be specified. Instead the number of assets chosen can depend on a bound that is set for the tracking error variance;
  - Any **constraints** on allocations that are defined by the preferences of the investor.

In-Sample Diagnostics

- *Tracking error stationarity*:

  The standard ADF test is used to test the level of cointegration between the portfolio the benchmark on the historic data: the larger and more negative the ADF statistic, the greater the level of cointegration and the more stationary the tracking error

- *Standard error of the regression*:

  The in-sample tracking error will be stationary if the portfolio is cointegrated with the benchmark, but this does not imply that the short-term deviations between the portfolio and the benchmark are necessarily small. It is also important to choose a portfolio for which the in-sample tracking error has a low volatility, and this is measured by the standard error of the regression.

Post-Sample Diagnostics

- *Tracking error variance*:

  This is the variance of the daily tracking errors during the testing period (1, 2 or 3 months). The tracking error variance is equivalent to the root mean square forecast error if it is measured as an equally weighted average;

- *Differential return:*

  The difference between the portfolio return and the benchmark return over the testing period;

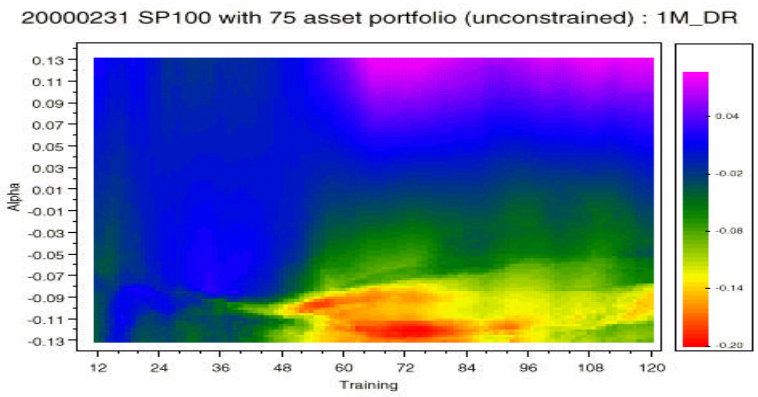
- *Information ratio:*

  The ratio between the mean daily tracking error and the standard deviation of the daily tracking error over the testing period. Alternative measure the excess return over a risk free rate with the Sharpe ratio

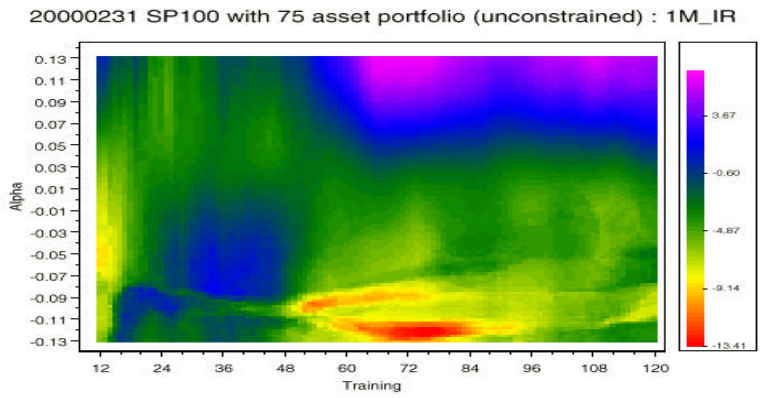
Long – Short Hedge

- The long leg of hedge is a basket of positions that is cointegrated with the index plus alpha_long%
- The short leg of hedge is a basket of positions that is NOT cointegrated with the index plus alpha_short% because it significantly underperforms the benchmark
- The long basket may contain both long and short psoitions; similarly for the short basket. The net position for each asset is the difference between the allocation in the long and the short baskets.
- Usually alpha_long > 0 and alpha_short < 0; but either could be positive or negative:

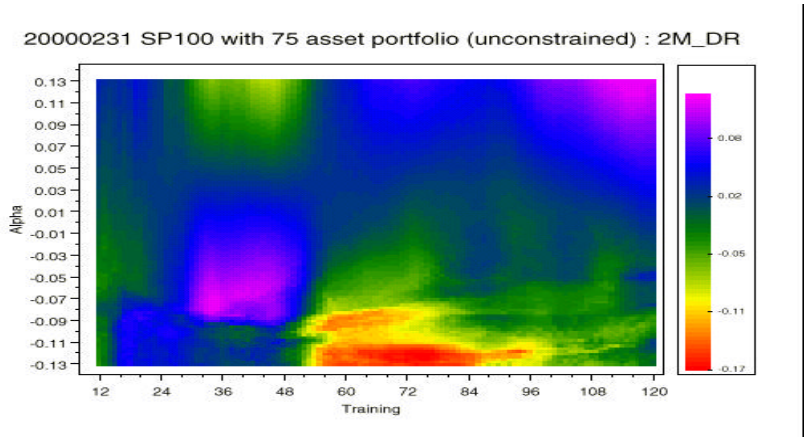
1 Month Differential Return



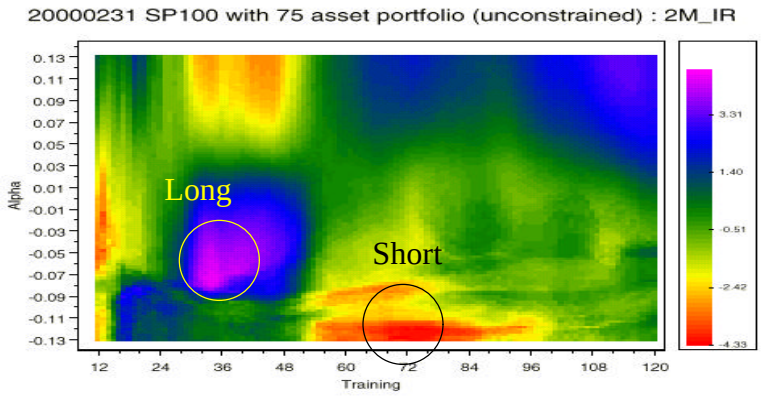
1 Month Information Ratio



2 Month Differential Return



2 Month Information Ratio



Optimal Parameter Choice

Long and Short Portfolio Parameter Choices: February 2000, Unconstrained Allocations, 75 Asset Portfolio in SP100

	Alpha	Training Months
Long Leg	-5%	36
Short Leg	-12%	72

3. Results

Optimal Parameter Choices (with Constrained Allocations):

	Long		Short	
	Alpha	Training	Alpha	Training
Jan-00	0.12	16	-0.12	88
Feb-00	0.12	6	-0.11	12
Mar-00	0.12	64	-0.12	92
Apr-00	0.12	64	-0.1	60
May-00	0.12	16	-0.08	40
Jun-00	0.06	12	-0.08	28
Jul-00	0.12	48	-0.1	88
Aug-00	0.12	120	-0.13	20
Sep-00	0.12	120	-0.12	20
Oct -00	0.11	108	-0.13	88
Nov-00	-0.11	90	-0.12	24
Dec-00	-0.09	20	-0.09	26

Back Testing: October 2000 Parameters

	Long					Short				
	ADF	Turnover	1mth IR	2mth IR	3mth IR	ADF	Turnover	1mth IR	2mth IR	3mth IR
Jan-00	-12.03	0.35	2.65	0.95	1.07	-11.82	1.04	4.99	-0.32	-0.83
Feb-00	-12.15	0.42	-5.15	-1.31	-1.15	-11.31	0.89	-2.00	1.88	-0.82
Mar-00	-13.12	0.50	-1.55	-3.04	-1.41	-11.48	0.95	1.40	0.13	1.69
Apr-00	-13.33	0.32	2.57	0.63	-0.79	-10.94	1.39	2.67	2.01	1.02
May-00	-12.65	0.27	1.77	2.18	0.88	-11.01	1.06	3.37	3.04	2.45
Jun-00	-12.07	0.44	-0.52	0.60	1.42	-11.68	1.45	-5.59	-0.97	0.25
Jul-00	-12.48	0.45	-1.18	-0.88	-0.08	-11.68	1.06	0.65	-2.79	-0.53
Aug-00	-12.01	0.36	-3.85	-1.94	-1.46	-10.97	1.23	-0.79	-0.12	-2.13
Sep-00	-11.80	0.25	4.05	1.64	0.73	-11.40	1.13	2.28	0.68	0.67

Turnover

- Only consider portfolios that show realistic turnover projections when the model parameters are fixed and allocations are optimized monthly over a long back test period.
- When parameters as well as allocations are re-optimized each month the turnover from cointegration based strategies will typically be much lower than those based on mean-variance analysis.
- In practice there is a daily turnover of approximately 2% with a leverage of about 1.5 on each leg of the hedge

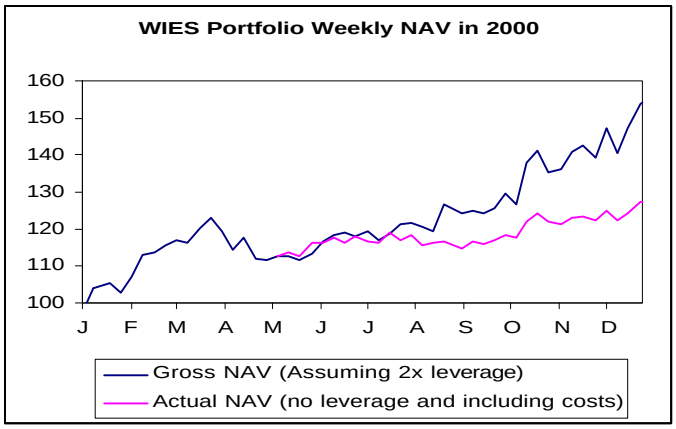
Annual Returns

	1987	1990	1993	1995	1996	1997	1998	1999	2000
Compound Rtn									
SP100	**8.9%**	**-4.4%**	**8.7%**	**37.2%**	**24.1%**	**29.8%**	**34.3%**	**33.7%**	**-11.2%**
L_SHEDGE	**14.3%**	**7.3%**	**15.8%**	**18.3%**	**13.6%**	**11.6%**	**46.4%**	**82.5%**	**58.4%**
Fund Daily Returns:									
Maximum	4.6%	2.5%	2.9%	2.2%	2.0%	3.2%	3.8%	5.9%	4.3%
Minimum	-6.0%	-3.0%	-1.5%	-2.0%	-1.9%	-3.0%	-4.3%	-4.3%	-4.1%
Average	0.1%	0.0%	0.1%	0.1%	0.1%	0.0%	0.2%	0.2%	0.2%
Median	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%	0.2%	0.1%	0.0%
Volatility (St Dev)	1.0%	0.7%	0.6%	0.7%	0.6%	0.9%	1.1%	1.5%	1.5%
Max 30-Day Peak-to-Trough	7.8%	10.2%	4.9%	7.3%	5.0%	12.6%	14.4%	10.1%	7.4%
Max 30-Day Trough-to-Peak	9.7%	9.8%	6.9%	8.7%	5.4%	13.9%	22.9%	24.0%	13.8%

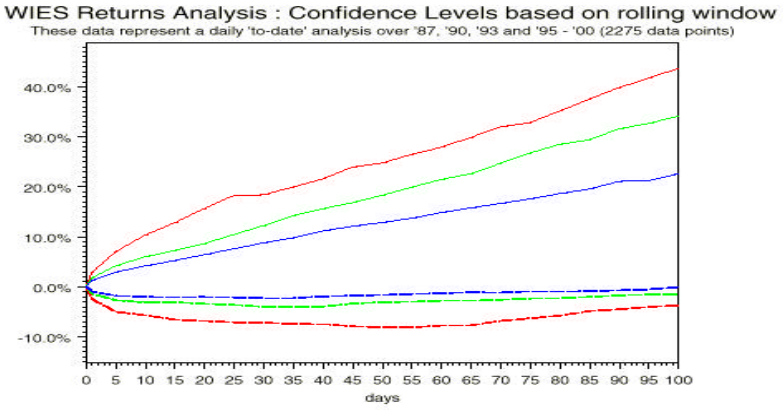
Monthly Returns in 2000

	SP100	LS HEDGE
Jan-00	-4.74%	3.22%
Feb-00	-1.69%	11.92%
Mar-00	11.16%	0.84%
Apr-00	-3.57%	-1.89%
May-00	-2.18%	0.34%
Jun-00	3.85%	4.51%
Jul-00	-0.86%	2.14%
Aug-00	3.85%	4.75%
Sep-00	-8.43%	-0.59%
Oct-00	-0.86%	7.37%
Nov-00	-7.22%	4.21%
Dec-00	-1.19%	9.17%
Simple Rtn	**-11.88%**	**45.98%**
Compound Rtn	**-11.20%**	**58.38%**
Standard Dev	**5.09%**	**3.92%**
Excess Return (Cmpd)		**53.38%**
Sharpe Ratio (over 5%)		**3.93**
Mean		**3.83%**
Correlation	**6.87%**	

Weekly NAV in 2000



Confidence Intervals



Monthly Returns: 1987, 1993, 1998

1987	SP100	L-S HEDGE	1993	SP100	L-S HEDGE	1998	SP100	L-S HEDGE
Jan-87	13.06%	1.72%	Jan-93	1.26%	-0.33%	Jan-98	2.04%	6.61%
Feb-87	3.89%	-0.37%	Feb-93	1.56%	4.54%	Feb-98	6.88%	-0.23%
Mar-87	3.31%	3.08%	Mar-93	1.69%	4.17%	Mar-98	5.30%	2.89%
Apr-87	0.68%	0.02%	Apr-93	-1.72%	-0.78%	Apr-98	1.49%	4.95%
May-87	0.68%	5.64%	May-93	2.59%	0.16%	May-98	-1.34%	3.33%
Jun-87	4.86%	0.30%	Jun-93	-0.17%	1.33%	Jun-98	4.83%	13.79%
Jul-87	4.26%	0.57%	Jul-93	-0.52%	1.84%	Jul-98	-0.54%	4.94%
Aug-87	4.50%	0.33%	Aug-93	3.31%	-0.89%	Aug-98	-15.16%	-8.83%
Sep-87	-3.09%	-1.91%	Sep-93	-1.53%	4.67%	Sep-98	5.10%	2.91%
Oct-87	-19.77%	2.27%	Oct-93	1.54%	-2.14%	Oct-98	8.78%	6.53%
Nov-87	-9.87%	-0.75%	Nov-93	-0.55%	0.42%	Nov-98	7.33%	-2.27%
Dec-87	6.00%	2.54%	Dec-93	0.88%	1.63%	Dec-98	4.73%	3.54%
Simple Return	8.51%	13.41%		8.34%	14.63%		29.46%	38.14%
Compound Return	8.88%	14.35%		8.70%	15.75%		34.26%	46.43%
Standard Deviation	8.10%	1.95%		1.52%	2.16%		6.08%	5.23%
Excess Return (Compound vs Index)		5.47%			7.05%			1.7%
Sharpe Ratio (Over Risk Free Rate 5%)		1.38			1.44			2.29

Summary of Results

- Over the entire period [1987, 1990, 1993, and 1995 – 2000]:
  - the correlation between the hedge strategy and the S&P100 was only –15.2%;
  - the average annual Sharpe Ratio was 1.51;
  - the average leverage was approximately 1.5 on both the long and the short legs of the hedge;
  - the average annual returns were 27.2%
  - the average annualized volatility from the monthly standard deviation of returns was 11.4% .

BARRA Analysis

- BARRA have performed a verification analysis of the 1999 returns.
- The strategy derives its excess returns primarily from risk assessment items such as earnings yield, earnings variation, momentum, size and, as one would expect, some leverage.
- Both "Value" and "Growth" assessment are actually negative contributors to the return, which somewhat distinguishes the strategy from the status quo.

Summary

- This paper has described a long short hedge strategy that is based on cointegration between asset prices.
- Traditional strategies will not guarantee that the tracking error is stationary and will therefore require frequent rebalancing for the hedge to remain tied to the benchmark.
- The cointegration strategy, on the other hand, is based on the criterion that the hedge is mean-reverting to the benchmark; tracking errors are designed to be stationary and this may be achieved with relatively few stocks and with much lower turnover rates.

Summary

- The strategy will accommodate investor's preferences for the alpha as well as flexible constraints on allocations.
- A sophisticated training and testing methodology has been described for the selection of model parameters.
- Extensive back testing results were reported and these have demonstrated the ability of the model to capture market upswings whilst not compromising the downside protection.