Risky funding: a unified framework for counterparty and liquidity charges

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Abstract

Standard techniques for incorporating liquidity costs into the fair value of derivatives produce counter-intuitive results when credit risk of the counterparty (CVA) and of the investor (DVA) are added to the picture. Here, Massimo Morini and Andrea Prampolini show that a consistent framework can only be achieved by giving an explicit representation to the funding strategy, including associated default risks.

1 Introduction

The pricing of funding liquidity and the pricing of counterparty credit risk are closely related. Companies usually compute a spread for funding costs that includes a compensation for their own risk of default. However, interactions between the two are still poorly understood, while banks are in need of a sound framework to underpin consistent policies for charging funding and credit costs. In this work we try and provide some cornerstones of a unified consistent framework for liquidity and credit risk adjustments that can help banks in this process. We first argue that a naive application of the standard approach to including funding costs by modifying the discounting rate, when it is put in place together with the standard approach for the computation of CVA (credit value adjustment) and DVA (debt value adjustment) leads to double-counting of assets that can be realized only once. Here we show how this issue can be avoided. We devise a practical and general approach to the problem, by taking explicitly

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into account, in the valuation of a derivative, also the funding strategy that needs to be put in place to manage the liquidity absorbed or generated by the derivative, and we study the effect of a default on this strategy. We arrive at an important finding: the consistent incorporation of funding costs and credit risk implies that the crucial variable determining the cost of liquidity is not the bond spread or the CDS spread, but the bond-CDS basis, or more generally the difference between the funding spread of a bank and the spread measuring its risk of default. This generates an asymmetric market where different parties give different prices to the same derivative, but does not preclude the possibility to reach an agreement, that may be struck at a price which generates day-one profits for both counterparties. This reconciles the theory with the reality of the market. Our analysis necessarily covers DVA, an 'asset' generated by the inclusion of a bank's own risk of default, the value of which increases the more the bank approaches default. This characteristic appears to lead to a distortion of financial choices and of financial communications (as they put it in Algorithmics [1], "can you profit now from your own future default?"). The issue is: can DVA be hedged and realized? In this work we show some steps towards a solution of this DVA puzzle, because we display analytically how DVA can be evaluated as a funding benefit. We show that even if a company does not take its own risk of default explicitly into account, a full accounting of the funding benefit generates an asset equal to DVA. If funding benefits can be realized before the default of the company, then DVA becomes a realized financial benefit.

2 The setting

We consider a deal in which one entity, that we call B (borrower), commits to pay a fixed amount K at time T to a party L (lender). This is a very simple payoff that allows us to focus on liquidity and credit costs without unnecessary complications. The simple payoff has several advantages. It is the derivative equivalent of a zero-coupon bond issued by B or a loan from L to B, so that we will be able to compare the results of our analysis with the well-established market practice for such products. It is also a payoff where it is always clear who is a net borrower and who is a net lender, thus we always have a premium when the deal is struck, where we can incorporate a liquidity charge, similarly to the real-world bank practice. The same would apply to stochastic option payoffs. Instead, in case of bilateral stochastic payoffs like swaps, the deal is struck at equilibrium but stochastic market movements will generate then a net lender and a net borrower, analogously to the simple payoff analyzed here, so that the extension to bilateral payoffs should be done in line with the results shown here.

We keep also the modelling assumptions simple. We assume that party X, with $X \in \{B, L\}$, has a recovery rate R_X and that the risk free interest rate that applies to maturity T has a deterministic value r. As usual, r represents the time-value of money on which the market agrees, excluding effects of liquidity or credit risk (it is an approximation for the OIS rate). A party X makes funding in the market, that we call for simplicity *the bond market.* Party X is also reference entity in the CDS market. We have therefore the following information:

1. the CDS spread π_X. We take this spread to be deterministic and paid instantaneously, and following the standard market model for credit risk in the CDS market, called reduced-form or intensity model (see for example [6]), the spread π_X can be written

$$\pi_X = \lambda_X \, \mathsf{LGD}_X \tag{1}$$

where λ_X is the deterministic default intensity and $\mathsf{LGD}_X = 1 - \mathsf{R}_X$ is the loss given default of entity X. If recovery is null, we have $\mathsf{LGD}_X = 1$ and the CDS spread coincides with λ_X, so that $\Pr(\tau_X > T) = e^{-\pi_X T}$. Therefore π_X must be intended as the best estimate of the risk-adjusted instantaneous default probability of X, taking expected recovery into account like in (1). Notice that when the CDS market is illiquid or strongly affected by the default risk of protection sellers, CDS spreads may not fully satisfy this requirement. Clearly $\pi_X \geq 0$.

2. the cost of funding s_X. For most issuers this is measured in the secondary bond market and represents the best estimate of the spread over a risk-free rate that a party pays on his funding. Normally banks maintain a well defined funding curve, which is published internally by the treasury department to provide guidance for the price of new funding or for other purposes in connection with their liquidity policy. Notice however that banks usually measure a spread over Libor, but since in these days the basis between Libor and OIS in non-negligible, OIS rates are better estimate of risk-free rates. Thus s_X must be intended as a spread over OIS. We take s_X to be instantaneous and deterministic too, so that we can compute by difference a liquidity basis γ_X with the same properties such that

$$s_X = \pi_X + \gamma_X$$

A proxy for the liquidity basis can be found in the bond-CDS basis, or more precisely its opposite, since in the market jargon the CDS-bond basis is defined as the difference between the CDS spread and a bond spread, thus its sign is opposite to γ_X. The bond-CDS basis has generally been negative and particularly high in absolute terms during the credit crunch. Positive values of the bond-CDS basis are possible, but they are now mainly observed for certain sovereign issuers, therefore we assume for now $\gamma_X \geq 0$. The name "liquidity basis" is justified by the fact that it is usually associated with the cost of the liquidity provision that the buyer makes to the issuer when buying the bond. This is dependent on the greater or lower ease with which the bonds of X can be sold in the secondary market, thus

γ_X is related to both funding liquidity costs and market liquidity risk. In particular, in the following we will shed more light on the role of the liquidity spread in the valuation of funding costs and benefits.

Our aim is to describe the net present value V_X (at time zero) of all cashflows generated by the transaction for the party, by consistently accounting for liquidity and counterparty risk. We proceed as follows: we first consider the current standard approach to DVA, then we attempt to introduce liquidity costs by adjusting the discount rate, and show that this path would lead to double accounting of the funding benefit associated with the DVA. Then we introduce our approach that includes risky funding and discuss some interesting implications. We start with the assumption $\mathsf{R}_X = 0$, but in Section 5.3 we extend the results to the case of positive recovery.

3 Standard DVA: Something is missing?

Let's start from the market standard for CVA and DVA. Following for example [3], the above deal has a premium paid by the lender L that equals the risk-free price minus the CVA,

$$P = e^{-rT} K - \mathsf{CVA}_L,$$

since the net present value of all cashflows for party L is

$$V_L = e^{-rT} K - \mathsf{CVA}_L - P, \tag{2}$$

then

$$V_L = 0 \quad \Rightarrow \quad P = e^{-rT} K - \mathsf{CVA}_L$$

The CVA takes into account the probability that the borrower B defaults before maturity

$$\begin{aligned} \mathsf{CVA}_L &= \mathbb{E}[e^{-rT} K 1_{\{\tau_B \le T\}}] = e^{-rT} K\, \mathbb{Q}[\tau_B \le T] \\ &= e^{-rT} K \left[1 - e^{-\pi_B T}\right] \end{aligned}$$

At the same time party B sees a value

$$\begin{aligned} V_B &= -e^{-rT} K + \mathsf{DVA}_B + P \qquad (3) \\ V_B &= 0 \quad \Rightarrow \quad P = e^{-rT} K - \mathsf{DVA}_B \end{aligned}$$

with $\mathsf{CVA}_L = \mathsf{DVA}_B$.

This guarantees the symmetry $V_B = V_L = 0$ and the possibility for the parties to agree on the premium of the deal,

$$P = e^{-rT} e^{-\pi_B T} K. \tag{4}$$

This approach does not consider explicitly the value of liquidity. In fact, in exchange for the claim, at time 0 party B receives a cash flow from party L

equal to P, so while party L has to finance the amount until the maturity of the deal at its funding spread s_L, party B can reduce its funding by P. So party B should see a funding benefit, and party L should see the fair value of its claim reduced by the financing costs. How come that these funding components do not appear in the above valuation? Can we justify it by assuming that the two companies have negligible funding costs? Not completely. In fact the absence of the funding term for L can indeed be justified by assuming $s_L = 0$. This implies $\pi_L = 0$. However the same assumption cannot be made for B without changing completely the nature of the deal. In fact assuming $s_B = 0$ would imply $\pi_B = 0$, which would cancel the DVA and CVA term. Thus when B is a party with non-negligible risk of default he must have a funding cost given at least by $s_B = \pi_B > 0$. The effect of this funding costs seems to be missing in the above formula. In the next sections we analyze if it is really missing.

4 Standard DVA plus liquidity: Something is duplicated?

We introduce liquidity costs by adjusting the discounting term, along the lines of [8], but we also introduce defaultability of the payoff along the lines of [3], getting for the lender

$$
\begin{aligned}
V_L &= \mathbb{E}\left[e^{-(r+s_L)T} K 1_{\{\tau_B > T\}}\right] - P \qquad (5)\\
&= \mathbb{E}\left[e^{-rT} e^{-\gamma_L T} e^{-\pi_L T} K 1_{\{\tau_B > T\}}\right] - P\\
&= e^{-rT} e^{-\gamma_L T} e^{-\pi_L T} K e^{-\pi_B T} - P,
\end{aligned}
$$

and analogously for the borrower

$$
\begin{aligned}
V_B &= -\mathbb{E}\left[e^{-(r+s_B)T} K 1_{\{\tau_B > T\}}\right] + P \qquad (6)\\
&= -\mathbb{E}\left[e^{-rT} e^{-\pi_B T} e^{-\gamma_B T} K 1_{\{\tau_B > T\}}\right] + P\\
&= -e^{-rT} e^{-\pi_B T} e^{-\gamma_B T} K e^{-\pi_B T} + P\\
&= -e^{-rT} e^{-2\pi_B T} e^{-\gamma_B T} K + P
\end{aligned}
$$

To compare this result, including CVA, DVA and liquidity from discounting, with results on DVA obtained in the previous section 3, it is convenient to reduce ourselves to the simplest situation where L is default free and with no liquidity spread, while B is defaultable and has the minimum liquidity spread allowed in this case: $s_L = 0$, $s_B = \pi_B > 0$. We have

$$
\begin{aligned}
V_L &= e^{-rT} K e^{-\pi_B T} - P\\
V_B &= -e^{-rT} e^{-2\pi_B T} K + P
\end{aligned}
$$

There are two bizarre aspects in this representation. First, even in a situation where we have assumed no liquidity spread, two counterparties do not agree on the simplest transaction with default risk. A day-one profit should be accounted by borrowers in all transactions with CVA. This belies years of market reality. Secondly, the explicit inclusion of the DVA term results in the duplication of the funding benefit for the party that assumes the liability. The formula implies against all evidence that the funding benefit is remunerated twice. If this were correct then a consistent accounting of liabilities at fair value would require pricing zero-coupon bonds by multiplying *twice* their risk-free present value by their survival probabilities. This also belies years of market reality.

5 Solving the puzzle

In order to solve the puzzle, we do not compute liquidity by the adjusted discounting of (5) and (6), but generate liquidity costs and benefits by modelling explicitly the funding strategy. The approach we take is that companies capitalize and discount money with the risk-free rate r, and then they add or subtract the actual credit and funding costs that arise in the management of the deal. This allows us to introduce explicitly in the picture both credit and liquidity, which is an approach not pursued by [3] and [8], and to investigate more precisely where credit/liquidity gains and losses are financially generated. We now take into account that the above deal has two legs. If we consider for example the lender L, one leg is the "deal leg", with net present value

$$\mathbb{E}\left[-P + e^{-rT}\,\Pi\right]$$

where Π is the payoff at T, including a potential default indicator; the other leg is the "funding leg" with net present value

$$\mathbb{E}\left[+P - e^{-rT}\,F\right]$$

where F is the funding payback at T, including a potential default indicator. When there is no default risk or liquidity cost involved, this funding leg can be overlooked because it has a value

$$\mathbb{E}\left[+P - e^{-rT}\,e^{rT}P\right] = 0.$$

Instead, in the general case the total net present value is

$$\begin{aligned} V_L &= \mathbb{E}\left[-P + e^{-rT}\,\Pi + P - e^{-rT}\,F\right] \\ &= \mathbb{E}\left[e^{-rT}\,\Pi - e^{-rT}\,F\right]. \end{aligned}$$

Thus the premium at time 0 cancels out with its funding, and we are left with the discounting of a total payoff including the deal's payoff and the liquidity payback. An analogous relationship applies to the borrower, as detailed in the next section. In the following we work under the hypothesis that all liquidity management happens in the cash market, so that funding is made by issuing bonds and excess funds are used to reduce or to avoid increasing the stock of bonds. This is the most natural assumption since it is similar to the assumption that banks make in their internal liquidity management, namely what the treasury desk assumes in charging or rewarding trading desks.

5.1 Risky Funding with DVA for the borrower

The borrower B has a liquidity advantage from receiving the premium P at time zero, as it allows him to reduce its funding requirement by an equivalent amount P. The amount P of funding would have generated a negative cashflow at T, when funding must be paid back, equal to

$$-P\,e^{rT}\,e^{s_B T}\,1_{\{\tau_B>T\}} \tag{7}$$

The outflow equals P capitalized at the cost of funding, times a default indicator $1_{\{\tau_B>T\}}$. Why do we need to include a default indicator $1_{\{\tau_B>T\}}$? Because in case of default, under the assumption of zero recovery, the borrower does not pay back the borrowed funding and there is no outflow. Thus reducing the funding by P corresponds to receiving at T a positive amount equal to (7) in absolute value,

$$\begin{aligned} & P\,e^{rT}\,e^{s_B T}\,1_{\{\tau_B>T\}} \\ = \; & P\,e^{rT}\,e^{\pi_B T}\,e^{\gamma_B T}\,1_{\{\tau_B>T\}} \end{aligned} \tag{8}$$

to be added to what B has to pay in the deal: $-K\,1_{\{\tau_B>T\}}$.

Thus the total payoff at T is

$$1_{\{\tau_B>T\}}P\,e^{rT}\,e^{\pi_B T}\,e^{\gamma_B T} - 1_{\{\tau_B>T\}}K \tag{9}$$

Taking discounted expectation,

$$\begin{aligned} V_B &= e^{-\pi_B T}\,P\,e^{\pi_B T}\,e^{\gamma_B T} - K\,e^{-\pi_B T}\,e^{-rT} \\ &= P\,e^{\gamma_B T} - K\,e^{-\pi_B T}\,e^{-rT} \end{aligned} \tag{10}$$

Compare with (6). Now we have no unrealistic double accounting of default probability. Notice that

$$V_B = 0 \quad \Rightarrow \quad P_B = K\,e^{-\pi_B T}e^{-\gamma_B T}\,e^{-rT} \tag{11}$$

where P_B is the breakeven premium for the borrower, in the sense that the borrower will find this deal convenient as long as

$$V_B \geq 0 \quad \Rightarrow \quad P \geq P_B.$$

Assume, as in (3), that $\gamma_B = 0$ so that in this case

$$P_B = K\,e^{-\pi_B T}\,e^{-rT}. \tag{12}$$

and compare with (4). We can conclude that in this case the standard computation from Section 3 is correct, as taking into account the probability of default in the valuation of the funding benefit removes any liquidity advantage for the borrower. Our formula shows what happens when there is also a 'pure liquidity basis' component in the funding cost, $\gamma_B > 0$. On the other hand, charging liquidity costs by an adjusted funding spread as in Section 4 cannot be naturally extended to the case where we want to observe explicitly the possibility of default events in our derivatives; for it to be consistent we need, as in [8], to take the default events out of the picture. We will discuss this further in sections 5.4 and 5.5. In writing the payoff for the borrower we have not explicitly considered the case in which the deal is interrupted by the default of the lender, since, following standard derivative documentation such as [5], page 15, the closeout amount at default of the lender should allow the borrower to replace the transaction with an identical one with a new counterparty. This keeps V_B independent of the default time of the lender, consistently with the reality of bond and deposit markets.

5.2 Risky funding with CVA for the lender, and the conditions for market agreement

If the lender pays P at time 0, he incurs a liquidity cost. In fact he needs to finance (borrow) P until T. At T, L will give back the borrowed money with interest, but only if he has not defaulted. Otherwise he gives back nothing, so the outflow is

$$\begin{aligned} & P\, e^{rT}\, e^{s_L T}\, 1_{\{\tau_L > T\}} \\ = \;& P\, e^{rT}\, e^{\gamma_L T}\, e^{\pi_L T}\, 1_{\{\tau_L > T\}} \end{aligned} \tag{13}$$

while he receives in the deal: $K\, 1_{\{\tau_B > T\}}$. The total payoff at T is therefore

$$-\, P\, e^{rT}\, e^{\gamma_L T}\, e^{\pi_L T}\, 1_{\{\tau_L > T\}} + K\, 1_{\{\tau_B > T\}}. \tag{14}$$

Taking discounted expectation

$$\begin{aligned} V_L &= -P\, e^{\gamma_L T}\, e^{-\pi_L T}\, e^{\pi_L T} + K\, e^{-rT}\, e^{-\pi_B T} \\ &= -P\, e^{\gamma_L T} + K\, e^{-rT}\, e^{-\pi_B T} \end{aligned} \tag{15}$$

The condition that makes the deal convenient for the lender is

$$\begin{aligned} V_L &\geq 0 \quad \Rightarrow \quad P \leq P_L, \\ P_L &= K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_B T} \end{aligned} \tag{16}$$

where P_L is the breakeven premium for the lender. It is interesting to note that the lender, when he computes the value of the deal taking into account all future cashflows as they are seen from the counterparties, does not include a charge to the borrower for that component π_L of the cost of funding which is associated with his own risk of default. This is canceled by the fact that funding is not given back in case of default. In terms of relative valuation of a deal this fact about the lender is exactly symmetric to the fact that for the borrower the inclusion of the DVA eliminates the liquidity advantage associated with π_B. In terms of managing cashflows, instead, there is an important difference between borrower and lender, which is discussed in Section 5.5. For reaching an agreement in the market we need

$$V_L \geq 0,\ V_B \geq 0$$

which, recalling (11) and (16), implies

$$\begin{aligned} P_L &\geq P \geq P_B \\ K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_B T} &\geq P \geq K e^{-rT}\, e^{-\pi_B T}\ e^{-\gamma_B T} \end{aligned} \tag{17}$$

Thus an agreement can be found whenever

$$\gamma_B \geq \gamma_L$$

This solves the puzzle, and shows that, if we only want to guarantee a positive *expected* return from the deal, the liquidity cost that needs to be charged to the counterparty of an uncollateralized derivative transaction is just the liquidity basis, rather than the bond spread or the CDS spread. This is in line with what happened during the liquidity crisis in 2007-2009, when the bond-CDS basis exploded. The results of the last two sections go beyond [8] in showing that only the bond-CDS basis is a proper liquidity spread, while the CDS spread associated with the default intensity is a component of the funding cost offset by the probability of defaulting in the funding strategy. In Section 5.5 we show how the picture changes when we look at the possible *realized* cashflows (as opposed to the expected cashflow), and we explore further the connections between this work and [8].

5.3 Positive recovery extension

In this section we look at what happens if we relax the assumption of zero recovery. The discounted payoff for the borrower is now

$$\begin{aligned} &1_{\{\tau_B>T\}} e^{-rT}\, P\, e^{\pi_B T}\, e^{\gamma_B T}\, e^{rT} \\ &+1_{\{\tau_B\leq T\}} e^{-r\,\tau_B}\, \mathsf{R}_B\, e^{-r\,(T-\tau_B)}\, P\, e^{\pi_B T}\, e^{\gamma_B T} e^{rT} \\ &-1_{\{\tau_B>T\}} e^{-rT}\, K \\ &-1_{\{\tau_B\leq T\}} e^{-r\,\tau_B}\, \mathsf{R}_B\, e^{-r\,(T-\tau_B)}\, K \end{aligned}$$

where the recovery is a fraction R_X of the present value of the claims at the time of default of the borrower, consistently with standard derivative documentation. Notice that B acts as a borrower both in the deal and in the funding leg, since we represented the latter as a reduction of the existing funding of B. Simplifying the terms and taking the expectation at 0 we obtain

$$\begin{aligned} V_B &= \mathbb{Q}\{\tau_B > T\}\, P\, e^{\pi_B T}\, e^{\gamma_B T} + \mathbb{Q}\{\tau_B \le T\}\, e^{-r\,T}\, \mathsf{R}_B\, P\, e^{\pi_B T}\, e^{\gamma_B T} e^{rT} \\ &\quad -\mathbb{Q}\{\tau_B > T\}\, e^{-rT}\, K - \mathbb{Q}\{\tau_B \le T\}\, \mathsf{R}_B\, e^{-r\,T}\, K \\ &= P\, e^{\pi_B T}\, e^{\gamma_B T}\, \left[S_B(T) + \mathsf{R}_B\,(1 - S_B(T))\right] \\ &\quad -e^{-r\,T}\, K\, \left[S_B(T) + \mathsf{R}_B\,(1 - S_B(T))\right] \\ &= \left[1 - \mathsf{LGD}_B\,(1 - S_B(T))\right] \left(P\, e^{\pi_B T}\, e^{\gamma_B T} - e^{-r\,T}\, K\right) \end{aligned} \tag{18}$$

where $S_B(T) = \mathbb{Q}\{\tau_B > T\}$ is the survival probability of the borrower. Using (1), we can write the first order approximation

$$1 - e^{-\pi_B\,T} \approx \mathsf{LGD}_B\, \left(1 - e^{-\lambda_B\,T}\right)$$

which allows us to approximate (18) as

$$\begin{aligned} V_B &\approx e^{-\pi_B T}\, \left(P\, e^{\pi_B T}\, e^{\gamma_B T} - e^{-r\,T}\, K\right) \\ &= P\, e^{\gamma_B T} - e^{-\pi_B T}\, e^{-r\,T}\, K \end{aligned}$$

We have thus shown that (10) is recovered as a first order approximation in the general case of positive recovery rate. Similar arguments apply to the value of the claim for L, that acts as a lender in the deal and as a borrower in the funding leg. For L, (15) is recovered as a first order approximation of

$$\begin{aligned} V_L &= -\left[1 - \mathsf{LGD}_L\,(1 - S_L(T))\right]\, P\, e^{\pi_L T}\, e^{\gamma_L T} \\ &\quad +\left[1 - \mathsf{LGD}_B\,(1 - S_B(T))\right]\, e^{-r\,T}\, K \end{aligned}$$

In the following we show for simplicity a few more results under the assumption $\mathsf{R}_X = 0$. The extension to the general case can be performed along the lines of this section.

5.4 The accounting view for the borrower: DVA as a funding benefit

One of the most controversial aspects of DVA relates to its consequences in the accounting of liabilities in the balance sheet of a company. In fact the DVA allows a borrower to condition future liabilities on survival, and this may create a distorted perspective in which our default is our lucky day. However, liabilities are already reduced by risk of default in the case of bonds when banks use the *fair value option* according to international accounting standard, and even when banks mark the bond liabilities at historical cost. What is the meaning of DVA? Are we really taking into account a benefit that will be concretely observed just in case of our default? In this section we show what happens if the borrower does not condition its liabilities upon survival, namely he pretends to be default-free thereby ignoring DVA and avoiding a possibly distorted view where default is a positive event. The borrower can perform valuation for accounting purposes using an accounting credit spread π_B that can be different from the market spread and an accounting liquidity basis γ_B possibly different from the market one, with the constraint that their sum s_B must match the market funding spread. In particular, when the party pretends to be default free, we have $\pi_B = 0$ and $\gamma_B = s_B$, and there are no more indicators for our own default in the payoffs.

Let a party B pretend, for accounting purposes, to be default free. The premium P paid by the lender gives B a reduction of the funding payback at T corresponding to a cashflow at T

$$Pe^{rT}e^{s_B T},$$

where there is no default indicator because B is treating itself as default-free. This cashflow must be compared with the payout of the deal at T, which is

$$-K$$

again without indicator, *i*e without DVA. Thus the total payoff at T is

$$e^{rT}e^{s_B T} - K \tag{19}$$

By discounting to zero we obtain an accounting value V_B such that

$$V_B = P\, e^{s_B T} - K\, e^{-rT}$$

which yields an accounting breakeven premium P_B for the borrower equal to the breakeven of (11),

$$P_B = K\, e^{-rT}e^{-\pi_B T}e^{-\gamma_B T}, \tag{20}$$

where now π_B and γ_B are those provided in the market. So also in this case the borrower B recognizes on its liability a funding benefit that actually takes into account its own market risk of default π_B, plus additional liquidity basis γ_B, thereby matching the premium computed by the lender that includes the CVA/ DVA term. But now this term is accounted for as a funding benefit and not as a benefit coming from the reduction of future expected liabilities thanks to default. The results of these sections give an indication on how the DVA term can be realized. When a bank enters a deal in a borrower position, it is making funding for an amount as large as the premium. If this premium is used to reduce existing funding which is equally or more expensive, that in our setting means buying bonds or avoiding some issuance that would be necessary otherwise, this provides a tangible financial benefit that is enjoyed in survival by a reduction of the payments at maturity.[1] The reduction is given by the difference in (19).

[1]With reference to some considerations in [4], we point out that a bank can buy back its own bonds. In fact, this is actually 'selling protection on yourself', but it is fully *funded.* When a sale of protection is funded, there is no counterparty risk and therefore no limit to whom can sell protection, differently from the case of an unfunded CDS. In fact buying their own bonds is a standard and important activity of banks.

For most banks the quantity of outstanding bonds is sufficiently high to allow the implementation of such a strategy, although it involves difficulties that vary from bank to bank. In any case we have shown how the DVA term can be seen not as a 'default benefit' moral hazard, but rather a natural component of fair value whenever fair value mark-to-market takes into account counterparty risk and funding costs.

5.5 The accounting view for the lender: expected value vs carry

The above results show that the borrower's valuation does not change if he considers himself default free by using an accounting credit spread $\pi_B = 0$ and treating all the funding cost s_B he sees in the market as a pure liquidity spread $\gamma_B = s_B$. Do we have a similar property also for the lender? Not at all. If the lender computes the breakeven premium using an accounting credit spread π_L and an accounting liquidity spread $\gamma_L = s_L - \pi_L$ different from those provided by the market, he gets a different breakeven premium, because, following Section 5.2,

$$P_L = K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_B T}$$

thus the breakeven premium and the agreement that will be reached in the market depend crucially on γ_L. In Figure 1, for a sample deal, we show how P_L varies when, holding s_L fixed, we vary $\kappa_L = \frac{\gamma_L}{s_L}$, that we call the *liquidity ratio* of the lender. This is not the only difference between the situation of the borrower and the lender. Notice that the borrower's net payout at maturity T is given in (9) and is non-negative in all states of the world if we keep $P \geq P_B$, although the latter condition was designed only to guarantee that the *expected* payout is non-negative. For the lender instead the payout at maturity is given by (14). The condition (16) for the non-negativity of the expected payout of the lender does not imply the non-negativity of (14), in particular we can have a negative carry even if we assume that both counterparties will survive until maturity. If we want to guarantee a non-negative carry at least when nobody defaults, in addition to (16) we need the following condition to be satisfied

$$\pi_L \leq \pi_B. \tag{21}$$

Otherwise the lender, differently from the borrower, is exposed to liquidity shortage and negative carry even if the deal is, on average, convenient to him. Liquidity shortages when no one defaults can be excluded by imposing for each deal (21), or, with a solution working for whatever deal with whatever counterparty, by working as if the lender was default-free. Only if the lender pretends for accounting purposes to be default-free the condition for the convenience of the deal based on expected cashflows becomes

$$P \;\leq\; K\, e^{-rT}\, e^{-s_L T}\, e^{-\pi_B T} = K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_L T} e^{-\pi_B T}$$

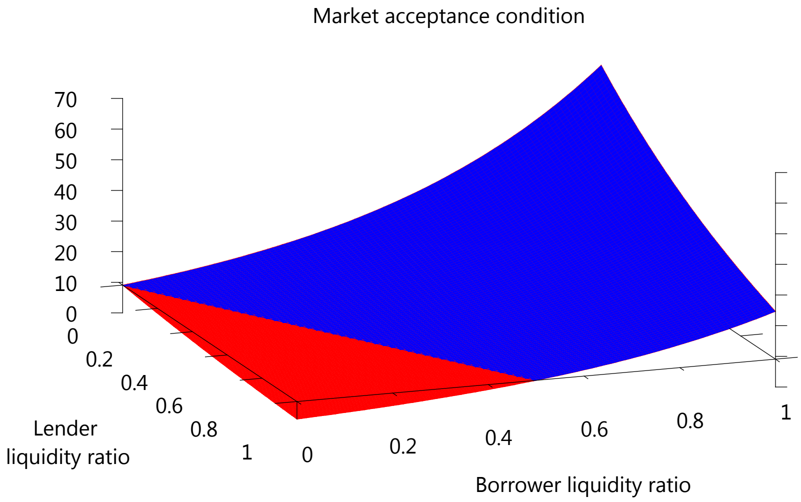

Figure 1: Breakeven premium for the Lender P_L as a function of the liquidity cost ratios κ_L and κ_B when $s_L = 0.05$, $s_B = 0.1$, $T = 20$, $K = 100$, $r = 0.02$; xy plane crosses the z axis at the breakeven premium for the Borrower P_B. A deal is possible only in the blue region.

that clearly implies the non-negativity of (14). On the other hand, the lender's assumption to be default-free makes a market agreement more difficult, since

$$K\, e^{-rT}\, e^{-\gamma_B T}\, e^{-\pi_B T} \le P \le K\, e^{-rT}\, e^{-\gamma_L T}\, e^{-\pi_L T} e^{-\pi_B T}$$

implies

$$\gamma_B \ge \gamma_L + \pi_L$$

rather than $\gamma_B \ge \gamma_L$. We finally notice that this assumption leads to results equivalent to [8]. In fact, under this assumption, uncollateralized payoffs should be discounted at the full funding also in our simple setting. Let's consider a bank X that pretends to be default-free and thus works under $\kappa_B = \kappa_X = 1$ when the bank X is a net borrower and $\kappa_L = \kappa_X = 1$ when X is in a lender position. When the bank is in the borrower position we have

$$P_B = P_X = e^{-s_X T} e^{-rT} K$$

while when it is in a lender position with respect to a counterparty (with no risk of default as in the example of [8]) the breakeven premium will be given by

$$P_L = e^{-s_X T} e^{-rT} K = P_B = P_X.$$

and the discounting at the funding rate $r + s_X$ is recovered for both positive and negative exposures.

6 Conclusions

In this article we have laid the groundwork of a consistent framework for the joint pricing of liquidity costs and counterparty risk. By explicitly modelling the funding components of a simplified derivative where both counterparties can default, we have shown how bilateral counterparty risk adjustments (CVA and DVA) can be combined with liquidity/funding costs without unrealistic double counting effects. We have shown that DVA has a meaningful representation in terms of funding benefit for the borrower, so that a bank can take into account DVA and find an agreement with lenders computing CVA even when it neglects its own probability of default. On the other hand, the lender's cost of funding includes a component that is associated with his own risk of default, but this component cancels out with his default probability, so that only his liquidity spread (or equivalently his bond-CDS basis) contributes as a net funding cost to the value of a transaction. We have shown that the comparison between the liquidity spread of lender and borrower is crucial to assess if a trade is convenient for both counterparties. In the end we have also discussed how the situation of the borrower and that of the lender are different, in particular the lender can have negative carry upon no default even if the value of the deal is positive to him. Thus, while the debate appears to be focussed on the impact of accounting choices on the valuation of liabilities, according to our results it is rather on the valuation of assets that such choices make a difference. The extension of these results to more general derivative payoffs, where a counterparty can shift between a net borrower position and a net lender position depending on market movements, is a crucial topic for future research.

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