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LIBOR vs. OIS: The Derivatives Discounting Dilemma*

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ABSTRACT

Traditionally practitioners have used LIBOR and LIBOR-swap rates as proxies for risk-free rates when valuing derivatives. This practice has been called into question by the credit crisis that started in 2007. Many banks now consider that overnight indexed swap (OIS) rates should be used as the risk-free rate when collateralized portfolios are valued and that LIBOR should be used for this purpose when portfolios are not collateralized. This paper examines this practice and concludes that OIS rates should be used in all situations.

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Introduction

The "risk-free" term structure of interest rates is a key input to the pricing of derivatives. It is used for defining the expected growth rates of asset prices in a risk-neutral world and for determining the discount rate for expected payoffs in this world. Before 2007, derivatives dealers used LIBOR, the short-term borrowing rate of AA-rated financial institutions, as a proxy for the risk-free rate. The most widely traded derivative is a swap where LIBOR is exchanged for a fixed rate. One of the attractions of using LIBOR as the risk-free rate was that the valuation of this product was straightforward because the reference interest rate was the same as the discount rate.

Collin-Dufresne and Solnik (2001) show that LIBOR-swap rates carry the same risk as a series of short-term loans to financial institutions that are rated AA at the start of each loan. For this reason, swap rates are sometimes referred to as "continually-refreshed" AA rates and are used to bootstrap the LIBOR curve. The resultant zero rates are those that are applicable to low-risk, but not zero-risk, expected cash flows.

The use of LIBOR to value derivatives was called into question by the credit crisis that started in mid-2007. Banks became increasingly reluctant to lend to each other because of credit concerns. As a result, LIBOR quotes started to rise. The TED spread, which is the spread between three-month U.S. dollar LIBOR and the three-month U.S. Treasury rate, is less than 50 basis points in normal market conditions. Between October 2007 and May 2009, it was rarely lower than 100 basis points and peaked at over 450 basis points in October 2008.

Most derivatives dealers now use interest rates based on overnight indexed swap (OIS) rates rather than LIBOR when valuing collateralized derivatives. LCH.Clearnet, a central clearing party, which was clearing over $300 trillion notional of interest rates swaps at the end of 2012, has also switched to using OIS rates. The reason often given for using OIS rates for valuing a collateralized derivative is that the derivative is funded by the collateral and the federal funds rate (which, as we will explain, is linked to the OIS rate) is the interest rate most commonly paid on collateral. For non-collateralized transactions, most dealers continue to use LIBOR rates for valuation. Here the most commonly used argument is that LIBOR is a better estimate of the dealer's cost of funding than the OIS rate. However, the arguments used for both the collateralized and non-collateralized transactions are questionable because it is a long-established principle in finance that the evaluation of an investment should depend on the risk of the investment and not on the way it is funded.[1]

The determination of the value of a derivative must be considered in conjunction with credit risk and collateral agreements. Credit risk is now a significant issue for derivatives dealers when they trade in the non-centrally-cleared over-the-counter market. Also, the interest rate paid on cash collateral can influence pricing and this is likely to become a more important consideration because recent regulatory proposals are expected to lead to an increase in collateral requirements.[2] The usual approach to dealing with counterparty credit risk is to first calculate a "no-default value." This value is then reduced by the expected loss due to the possibility of a default by the counterparty and increased by the expected gain due to the possibility of a default by the dealer. The expected loss due to a possible default by the counterparty is referred to as the credit value adjustment (CVA) and the expected gain due to a possible default by the dealer is referred to as the debit (or debt) value adjustment (DVA).[3] A further adjustment for the interest paid on cash collateral may be necessary. We will refer to this as the collateral rate adjustment (CRA).

In this paper, we argue that the OIS rate is the most appropriate rate for calculating the no-default value of both collateralized and non-collateralized transactions. The OIS rate should be used as the interest rate because it is the best proxy for the risk-free rate. Using a higher interest rate such as LIBOR leads to an incorrect no-default value and, when used in conjunction with CVA and

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[1] In spite of this many derivatives dealers choose to make what is termed a funding value adjustment to reflect differences between their average borrowing costs and their discount rate. For more discussion of this see Burgard and Kjaer (2011, 2012) and Hull and White (2012b, 2013).

[2] See Basel Committee on Banking Supervision (2012).

[3] The calculation of CVA and DVA is discussed by, for example, Canabarro and Duffie (2003), Picault (2005), Hull and White (2012a) and Gregory (2012)

DVA credit adjustments, is liable to lead to double counting for credit risk. We find that it is possible to use the discount rate to take account of credit risk or the impact of the rate of interest on the collateral only in very special circumstances.

In Sections I and II, we present some preliminary institutional material on overnight rates. Section I explains how overnight money markets work and why the fed funds rate is a good proxy for the one-day risk-free rate. Section II then explains how the OIS rate is calculated and why a zero curve calculated from OIS rates provides a reasonable proxy for the risk-free zero curve. In Section III, we review the way counterparty credit risk affects the economic values of derivatives. Section IV discusses the impact of collateralization. Section V illustrates the magnitude of the errors that can arise as a result of using the wrong discount rate. Conclusions are in Section VI.

I. The Overnight Risk-Free Rate

Banks can borrow money in the overnight market on a secured or unsecured basis. Overnight U.S. dollar secured debt can be raised in the form of an overnight repurchase agreement (a repo) or at the Federal Reserve's Discount Window.[4] Unsecured U.S. dollar overnight financing comes in the form of federal funds and Eurodollars.

A large fraction of the federal funds loans are brokered. Major brokers report the dollar amount loaned at each interest rate to the Federal Reserve Bank of New York (FRBNY) daily.[5] The statistics reported by the brokers are used by FRBNY to calculate a weighted average interest rate paid on federal funds loans each day (with weights being proportional to transaction size). This average is called the "effective federal funds rate." The FRBNY controls the level of the effective federal funds rate through open market transactions.

[4] For a discussion of the repo market see Stigum (1990) or Demiralp *et al* (2004). For a discussion of the Discount Window see Furfine (2004).

[5] See Demiralp *et al* (2004). Most federal funds transactions take place at interest rates that are an integral multiple of either one basis point or one-thirty-second of one per cent.

Money market brokers do not report statistics for Eurodollar financing in the way that they do for federal funds. As a result, the only easily available measure of the cost of overnight borrowing in the Eurodollar market is the level of overnight LIBOR reported by the British Bankers Association. On average, overnight LIBOR has been about 6 basis points higher than the effective federal funds rate except for the tumultuous period from August 2007 to December 2008.

Given the substitutability of Eurodollar and federal funds financing in the overnight market, the apparent difference between the rates seems difficult to explain. This issue is addressed by Bartolini *et al* (2008). These authors attribute the observed differences to timing effects, the composition of the pool of borrowers in London as compared to New York, market microstructure differences between the dominant settlement mechanisms in London (CHIPS) and New York (Fedwire),[6] and the difference between transaction prices (the brokered trades) and quotes which just provide the starting point for a negotiation.

The overnight rate, whether federal funds or overnight LIBOR, is a rate on unsecured borrowing and as such is not totally risk-free. Longstaff (2000) and other authors argue that the overnight repo rate is a better indicator of the risk-free rate since the borrowing is collateralized. Certainly a secured loan is subject to less credit risk than an unsecured loan. However there is substantial cross-sectional variation in repo rates related to the type of collateral posted. Up to mid-2007, rates for repos secured by U.S. federal government securities were 5 to 10 basis points below the federal funds rate while, for repos secured by U.S. agency debt, the rates were about one basis point below the federal funds rate. During the crisis, the rate for repos secured by federal government securities fell relative to the federal funds rate, but for other repos the rate rose relative to this rate. These cross-sectional variations suggest that market microstructure issues may play a larger role in explaining the difference between repo and federal funds rates than credit risk does.[7] Given these points and the point that there does not appear to be any way to determine a complete term structure for repos, the effective federal funds rate and the longer

[6] CHIPS is the Clearing House Inter-Bank Payment System and Fedwire is the real time wire transfer system run by the Federal Reserve

[7] For example, it is possible that lenders in the repo market rather than making secured loans are using the market to acquire ownership of securities that are otherwise difficult to acquire. See Bank for International Settlements (2008).

term rates based on it (see Section II) provide more useful and more robust proxies for the risk-free rate than repo rates.

Ultimately, the magnitude of the credit spread in the effective federal funds rate is an empirical question. A very rough indication of the size of the credit spread can be obtained by looking at the term structure of credit spreads calculated from the difference between USD LIBOR rates and the Federal Reserve's estimate of constant maturity Treasury bill rates for 1-, 3-, 6- and 12-month maturities. In the 2009 to 2012 period, the shape of the term structure of credit spreads was consistent. The yearly average term structures of spreads are shown in Figure 1. Averaging over all four years, the average difference between USD LIBOR and T-bill rates is about 80 basis points at one year declining to 20 basis points at one month. Extrapolation suggests a spread of about 11 basis points for the over-night LIBOR rate. In the same period, the average spread between overnight LIBOR and the effective federal funds rate was about 6 basis points. This suggests that the credit spread for the effective federal funds rate in this period is about 5 basis points, much of which may be attributable to non-credit elements.[8]

Our discussion of overnight rates has been couched in terms of U.S. dollar interest rates but it can be extended to interest rates in other currencies. An overnight market with similar characteristics to that in the U.S. exists for the Euro, the pound sterling, and other major currencies.

II. Overnight Indexed Swaps

Overnight indexed swaps are interest rate swaps in which a fixed rate of interest is exchanged for a floating rate that is the geometric mean of a daily overnight rate. The calculation of the payment on the floating side is designed to replicate the aggregate interest that would be earned from rolling over a sequence daily loans at the overnight rate. In U.S. dollars, the overnight rate

[8] In the United States, Treasury rates tend to be low compared with the rates offered on other very-low-credit-risk instruments. Elton *et al* (2001) show that one reason for this is the tax treatment of Treasury instruments.

used is the effective federal funds rate. In Euros, it is the Euro Overnight Index Average (EONIA) and, in sterling, it is the Sterling Overnight Index Average (SONIA).

OIS swaps tend to have relatively short lives (often three months or less). However, transactions that last as long as five to ten years are becoming more common. For swaps of one-year or less there is only a single payment at the maturity of the swap equal to the difference between the fixed swap rate and the compounded floating rate multiplied by the notional and the accrual fraction. If the fixed rate is greater than the compounded floating rate, it is a payment from the fixed rate payer to the floating rate payer; otherwise it is a payment from the floating rate payer to the fixed rate payer. Similarly to LIBOR swaps, longer term OIS swaps are divided into 3-month sub-periods and a payment is made at the end of each sub-period.

Earlier we mentioned the continually-refreshed argument of Collin-Dufresne and Solnik (2001). This shows that the 5-year swap rate for a transaction where payments are exchanged quarterly is equivalent to the rate on 20 consecutive 3-month loans where the counterparty's credit rating is AA at the beginning of each loan. A similar argument applies to an OIS swap rate. This rate is the interest that would be paid on continually-refreshed overnight loans to borrowers in the overnight market.

There are two sources of credit risk in an OIS. The first is the credit risk in overnight federal funds borrowing which we have argued is very small. The second is the credit risk arising from a possible default by one of the swap counterparties. This possibility of counterparty default is liable to lead to an adjustment to the fixed rate. The size of the adjustment depends on the slope of the term structure, the probability of default by a counterparty, the volatility of interest rates, the life of the swap, and whether the transaction is collateralized. The size of the adjustment is generally very small for at-the-money transactions where the two sides are equally creditworthy and the term structure is flat. It can also reasonably be assumed to be zero in collateralized transactions.[9] Based on these arguments, we conclude that the OIS swap rate is a good proxy for a longer term risk-free rate.

The three-month LIBOR-OIS spread is the spread between three-month LIBOR and the three-month OIS swap rate. This spread reflects the difference between the credit risk in a three-month loan to a bank that is considered to be of acceptable credit quality and the credit risk in continually-refreshed one-day loans to banks that are considered to be of acceptable credit quality. In normal market conditions it is about 10 basis points. However, it rose to a record 364 basis points in October 2008. By a year later, it had returned to more normal levels, but it rose to about 30 basis points in June 2010 and to 50 basis points at the end of 2011 as a result of European sovereign debt concerns. These statistics emphasize that LIBOR is a poor proxy for the risk-free rate in stressed market conditions.

The OIS zero curve can be bootstrapped similarly to the LIBOR/swap zero curve. If the zero curve is required for maturities longer than the maturity of the longest OIS a natural approach is to assume that the spread between the OIS zero curve and the LIBOR/swap zero curve is the same at the long end as it is at the longest OIS maturity for which there is reliable data. Subtracting this spread from the LIBOR zero curve allows it to be spliced seamlessly onto the end of the OIS zero curve. In this fashion, a risk-free term structure of interest rates can be created. An alternative approach for extending the OIS zero curve is to use basis swaps where three-month LIBOR is exchanged for the average federal funds rate plus a spread. These swaps have maturities as long as 30 years in the U.S.[10]

[9] Legislation requiring standard swaps to be cleared centrally means that swap quotes are likely to reflect collateralized transactions in the future.

[10] A swap of the federal funds rate for LIBOR involves the arithmetic average (not geometric average) of the effective federal funds rate for the period being considered. A "convexity adjustment" is in theory necessary to adjust for this. See, for example, Takada (2011).

III. Can the Use of LIBOR be Justified for Non-Collateralized Portfolios?

Many derivatives dealers continue to use LIBOR interest rates for valuing non-collateralized portfolios. We have argued that the OIS rate is the best proxy for the risk-free rate. Risk-neutral valuation shows that risk-neutral expected cash flows should be calculated using the risk-free rate and discounted at the risk-free rate. Why then do many market participants continue to use LIBOR interest rates when no collateral in posted? In this section, we explore whether there are any arguments in favor of this.

An argument often made is that non-collateralized transactions are funded at the bank's borrowing cost and LIBOR is a good estimate of this. However, the evaluation of an investment should not depend on the way it is funded. The correct discount rate for an investment, whether in a hedged derivatives position or anything else, should depend on the risk of the investment not on the bank's average funding costs. This point is discussed further in Hull and White (2012b and 2012c).

Another potential argument is that LIBOR is often a reflection of the credit risk of the two parties in a derivatives transaction. This overlooks the fact that the purpose of the valuation is to calculate the no-default value of a derivative or derivatives portfolio. The credit risk of the two sides is in practice taken into account by a credit valuation adjustment (CVA) and debit (or debt) value adjustment (DVA). CVA is the reduction in the value of a derivatives portfolio to allow for a possible default by counterparty. DVA is the increase in the value of the portfolio to allow for a default by the dealer. The value of a derivatives portfolio after credit risk adjustments is given by

$$f = f_{\mathrm{nd}} - \mathrm{CVA} + \mathrm{DVA} \tag{1}$$

where f_{nd} is the no-default value of the portfolio.

Using LIBOR discounting has the effect of incorporating an adjustment for a least some of the credit risk into the discount rate. It cannot then be correct to calculate CVA and DVA using credit spreads for the counterparty and the dealer that reflect their total credit risk. If this were done, there would be an element of double counting for credit risk.

The appendix shows that if

a) The LIBOR/swap curve is used for discounting the risk-neutral expected payoffs to obtain the no-default value of a derivatives portfolio, f_{nd};

b) CVA is calculated as $\text{CVA}_1-\text{CVA}_2$ where CVA_1 is the actual expected loss to the dealer from counterparty defaults and CVA_2 is the expected loss that would be calculated if the LIBOR/swap curve defined the counterparty's borrowing rates.; and

c) DVA is calculated as $\text{DVA}_1-\text{DVA}_2$ where DVA_1 is the actual expected loss to the counterparty from a default by the dealer and DVA_2 is the counterparty's expected loss that would be calculated if the LIBOR/swap curve defined the dealer's borrowing rates.

then all three calculated values are incorrect but the net value calculated using equation (1) is correct. It is interesting to note that there is nothing special about the role of the LIBOR/swap curve in this result. Any yield curve can be used for discounting the expected derivative payoffs providing it is also used instead of the LIBOR/swap curve in b) and c) above.

This result may provide some solace to those dealers using LIBOR discounting. Credit spreads for the counterparty and the dealer are an input to CVA and DVA calculations. If the credit spreads are calculated as the excess of borrowing rates over swap rates, rather than as the excess of borrowing rates over OIS rates, the CVA and DVA that are calculated are likely an approximation to the CVA and DVA in b) and c). This is because CVA and DVA are usually approximately linear functions of the underlying credit spreads. However, using LIBOR as the benchmark risk-free rate for calculating credit spreads in some circumstances and OIS in others is liable to cause confusion. Furthermore, LIBOR is used only for discounting in a), b), and c). We assume that the risk-neutral expected payoffs are determined using the risk-free (OIS) rates.

Another point to consider is that just as the risk-free (OIS) rate must be used to determine the risk-neutral expected payoff, it must also be used to determine the expected exposure used to calculate CVA and DVA. (These quantities are estimated in the same way as the no-default value of a derivative.)

A natural question is whether it is necessary to allow for credit risk by calculating CVA and DVA. Can we adjust for credit risk by adjusting the discount rate? The appendix shows that this is possible in only three special cases:

1. If a derivatives portfolio will always have a positive value to the dealer, the correct value, f_{nd} – CVA, is calculated using a discount curve determined from the counterparty's borrowing rates.
2. If a derivatives portfolio will always have a negative value to the dealer, the correct value, f_{nd} + DVA, is calculated using a discount curve determined from the dealer's borrowing rates.
3. If the counterparty and dealer have identical credit spreads, the correct value, f_{nd}– CVA+DVA, for any derivatives portfolio can calculated using a discount curve determined from their common borrowing rates.

For each result, it is assumed that no collateral is posted.

IV. Is the Use of OIS Always Correct for Collateralized Portfolios?

Dealers usually use OIS discounting when valuing collateralized portfolios. The reason most often given for this is that the derivatives are funded by the collateral and the interest rate most commonly paid on collateral is the effective federal funds rate, which as we explained in Section II is closely linked to the OIS rate. As already mentioned, a dealer's funding costs should not play a role in the valuation of derivatives and so this argument is questionable. The correct argument in favor of using the OIS rate is simply that it is the best estimate of the risk-free rate for the purposes of applying risk-neutral valuation and calculating no-default derivative values.

In this section we examine whether there are any circumstances where discount rates other than the OIS rate can be justified for collateralized transactions. In particular we consider whether the interest rate paid on cash collateral can affect the discount rate.

First we note that, if the rate of return on the collateral is not economic (i.e., is not commensurate with the risk of the collateral), a collateral requirement in a derivatives agreement will have a non-zero value which leads to another adjustment to the value of a derivatives portfolio. We will refer to the cost of a collateral agreement as the Collateral Rate Adjustment (CRA). Equation (1) then becomes

$$f = f_{\text{nd}} - \text{CVA} + \text{DVA} - \text{CRA} \tag{2}$$

If a market participant expects to be a net payer of collateral and the rate paid on the collateral is less than the economic rate, CRA is positive; if the rate is greater than the economic rate, the CRA is negative.

Often securities are posted as collateral. Usually the firm posting the collateral is the beneficiary of the market return (income and capital gains) on the collateral and this by definition constitutes an economic return. As a result, in this case the CRA is zero. This is true no matter how large a haircut is applied to the posted collateral.[11]

To examine the possible impact of cash collateral consider an idealized situation where the following is true

a) There is a two-way zero-threshold collateral agreement. This means that, when the value of the outstanding derivatives portfolio to one side is X, the other side is required to post $\max(X, 0)$ as collateral.
b) There is no minimum transfer amount and collateral is transferred continuously.
c) All collateral must be in the form of cash.
d) Collateral is always posted right up to the time of a default.
e) Transactions can be replaced at their mid-market value at the time of a default.

In this situation, the collateral is a perfect hedge for losses due to default so that CVA and DVA are both zero. Because the credit risk has been hedged, the collateral investment is risk-free and should earn the risk-free rate of interest. If the interest paid on cash collateral is the risk-free rate, the CRA is zero.

[11] However, the haircut can affect CVA and DVA.

Suppose next that the rate paid on cash collateral is different from the risk-free rate. Define the risk-free rate as r_f and the daily rate paid on the collateral as r_c. With the collateral arrangements we are considering, the situation is equivalent to one where the rate paid on the collateral is r_f and the derivatives portfolio provides a daily yield equal to $r_c - r_f$. This follows from the fact that, from the point of view of each party, the value of the collateral posted is always equal to the value of the derivative and in determining the economic value of the derivative we act as if the derivative is expected to earn r_f. As a result, the CRA is equal to the change in the value of the derivative that occurs when it is assumed to provide a yield of $r_c - r_f$.

This change in the value of the derivative can be determined by changing the discount rate from the risk-free curve to one corresponding to r_c.[12] For example, if the rate paid on collateral is x basis points below the risk-free rate, the discount curve should be x basis points below the risk-free curve. If a constant interest rate, y, is paid on collateral the discount curve should be flat and equal to y. The CRA is $f_{\text{nd}}(r_c) - f_{\text{nd}}(r_f)$ where $f_{\text{nd}}(k)$ is the no-default value of the derivative when the discount rate is k.

With the assumptions in a) to e) above, CVA and DVA are zero and the value of the portfolio in equation (2) simplifies to $f_{\text{nd}}(r_c)$. We now consider the effect of relaxing assumptions a) to e) given above. If b) is relaxed so that there is a not-too-large minimum transfer amount and collateral is transferred at the end of each day rather than continuously, the analysis should still be approximately true. If collateral can be provided in the form of marketable securities or cash and the interest on cash collateral, r_c, is different from the risk-free rate, r_f, a reasonable assumption (at least when the counterparty is a financial institution) is that cash collateral will be provided when $r_c > r_f$ and marketable securities will be provided when $r_f > r_c$. It follows that CRA = 0 in the latter case and can be calculated as described above in the former case.

We now consider the situation where assumption a) is relaxed. Many different collateral arrangements are observed in practice. For example, the threshold is sometimes non-zero; collateral arrangements are sometimes one-way rather than two-way; sometimes an independent

[12] Note that only the curve used for discounting is changed. The risk-neutral expected payoffs are still determined using the risk-free discount rate.

amount is required. The CRA arising from the difference between r_c and r_f can then no longer be calculated by adjusting the discount rate. Typically, Monte Carlo simulation must be used to estimate CRA as the present value of the expected excess of payments at r_c over payments at r_f.[13]

In practice, assumptions d) and e) do not hold. This means that CVA and DVA must be estimated carefully even when there is a two-way zero threshold agreement. However, the impact of assumptions d) and e) on CRA is likely to be very small.

To summarize, the discount rate used to determine the no-default value of a derivative or derivatives portfolio should always be the risk-free rate. The cost or benefit of a non-economic rate paid on collateral should then be calculated separately. In the circumstances covered by assumptions (a) and (c), these two calculations can be merged (to a good approximation) by changing the discount rate to the rate paid on cash collateral.

V. Numerical Results

In this section we will use a simple example to show that the choice of a risk-free rate has a material effect on valuation and that current market practice may result in significant errors.

Suppose that the dealer's portfolio with the counterparty consists of a non-collateralized forward contract to buy a non-dividend-paying stock in one year. The stock price and delivery price are both \$100, the volatility of the stock price is 30% per annum, and the risk-free (OIS) rate is 3% for all maturities. We consider two cases. Case 1 corresponds to what might be termed normal market conditions. LIBOR is 3.1% for all maturities. The dealer's and the counterparty's credit spreads are 0.5% and 2.0%, respectively, for all maturities. Case 2 corresponds to stressed market conditions. LIBOR is 4.5% for all maturities. The dealer's and the counterparty's credit

[13] One difference between CRA and CVA/DVA is that CRA is additive across transactions whereas CVA and DVA are not. The CRA for a transaction can be calculated by simulating that transaction in isolation from the rest of the portfolio with the counterparty. The same is not true of CVA and DVA. In the case of CVA and DVA, when a new transaction with a counterparty is contemplated dealers must calculate the incremental effect of the new transaction on CVA and DVA. It is usually assumed that the new transaction will not affect the probability of either the dealer or the counterparty defaulting. If that assumption is relaxed for the dealer, it can be necessary to look at all counterparties when considering the incremental DVA.

spreads are 2% and 3%, respectively, for all maturities. (The spreads are assumed to have been adjusted for any differences between recovery rates on derivatives and bonds, as mentioned in the appendix.) All rates are continuously compounded. The integrals necessary to calculate CVA and DVA are evaluated by dividing the one-year life of the derivative into 200 equal time steps. Equations (A6), (A8) and (A10) in the appendix are used to calculate expected losses between two times from credit spreads.

Tables 1 and 2 show that, if the calculation of CVA and DVA is adjusted for LIBOR discounting as described in Section III, the same values are found whether we use the risk-free rate or LIBOR for discounting. (As discussed in Section III, the OIS rate is used to define the risk-neutral growth rate even when LIBOR is used as the discount rate.) However, if this adjustment to the calculation of CVA and DVA is not made, the no-default value in the LIBOR discounting column is combined with the CVA and DVA estimates in the OIS discounting column and the total price is incorrect.

Table 3 shows that, when the derivative price is relatively low, using LIBOR instead of OIS for both risk-neutral growth rates and discount rates can result in large errors. However, in many circumstances, derivatives such as forward contracts and options are priced in terms of forward prices observed in the market, not in terms of spot prices. The risk-neutral growth rate (LIBOR or OIS) is then not an input to the pricing model and errors may not be as great.

VI. Conclusions

There is no "perfect" risk-free rate. We have argued that the OIS rate is the best proxy currently available.

The application of risk-neutral valuation to calculate the no-default value of a derivative portfolio requires the interest rate to be the best proxy available for the risk-free rate. The advantage of consistently using the risk-free (OIS) rates is that the following three aspects of the valuation of a derivatives portfolio are clearly separated

1. The calculation of the no-default value of the portfolio;
2. The impact of the credit risk of the dealer and the counterparty; and
3. The impact of a non-economic the interest rate paid on cash collateral.

We agree that the current practice of using the rate paid on collateral as the discount rate when a zero-threshold two-way collateral agreement applies results (at least approximately) in the correct economic value for the portfolio. However, this approach does not produce the correct value if the terms of the collateral agreement are not two-way and zero threshold.

It is tempting to try and handle credit risk in a derivative portfolio by changing the discount curve. However, this is only possible in three special cases. These are when the derivatives portfolio is always an asset to the dealer, when the derivatives portfolio is always a liability to the dealer, and when the borrowing rates are identical for the dealer and the counterparty.

An interesting result is that it is possible to use the LIBOR/swap zero curve (or any other zero curve) to define discount rates (but not risk-neutral growth rates) when the no-default value of non-collateralized portfolios is calculated providing CVA and DVA are defined appropriately. However, this makes a dealer's systems unnecessarily complicated and is liable to cause confusion.

The best and simplest approach is to use OIS interest rates when calculating the no-default value of all derivatives making credit risk and collateral rate adjustments as appropriate. To use LIBOR for non-collateralized transactions and OIS for collateralized transactions makes no sense and would seem to be a violation of the law of one price. From an economic perspective, the no-default value of a collateralized transaction should be the same as the no-default value of a non-collateralized transaction.

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Appendix

Consider a non-collateralized portfolio of derivatives between a dealer and a counterparty. The value today of the derivatives position to the dealer is

$$f = f_{\text{nd}} + \text{DVA} - \text{CVA} \tag{A1}$$

where f_{nd} is the no-default value of the position and CVA and DVA are defined as follows:[14]

$$\text{CVA} = \int_0^T \left(1 - R_c\left(t\right)\right) f^{+}\left(t\right) q_c\left(t\right) dt \tag{A2}$$

$$\text{DVA} = \int_0^T \left(1 - R_d\left(t\right)\right) f^{-}\left(t\right) q_d\left(t\right) dt \tag{A3}$$

In these equations, T is the longest maturity of the derivatives in the portfolio, $f^{+}\left(t\right)$ is the value today of a derivative that pays off the dealer's exposure to the counterparty at time t, $f^{-}\left(t\right)$ is the value today of a derivative that pays off the counterparty's exposure to the dealer at time t, $q_d\left(t\right)\Delta t$ is the probability of the dealer defaulting between times t and $t+\Delta t$, and $q_c\left(t\right)\Delta t$ is the probability of the counterparty defaulting between times t and $t+\Delta t$.[15] The variables $R_d\left(t\right)$ and $R_c\left(t\right)$ are the dealer's and the counterparty's expected recovery rate from a default at time t. The amount claimed on an uncollateralized derivative exposure in the event of a default is the no-default value. The recovery rates, $R_d\left(t\right)$ and $R_c\left(t\right)$, can therefore be more precisely defined as the percentage of no-default value of the derivatives portfolio that is recovered in the event of a default.

The probabilities q_c and q_d can be estimated from the credit spreads on bonds issued by the counterparty and the dealer. A complication is that the claim in the event of a default for a

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[14] We assume that the recovery rate, default rate, and value of the derivative are mutually independent.

[15] Some adjustment for the possibility that both parties will default during the life of the derivatives portfolio may be necessary. See, for example, Brigo and Morini (2011).

derivatives portfolio is in many jurisdictions different from that for a bond. Hull and White (2012c) show how borrowing rates can be adjusted to allow for this.

For convenience, we define the loss rate for the dealer as $L_d(t) = q_d(t)(1 - R_d(t))$ and the loss rate for the counterparty as $L_c(t) = q_c(t)(1 - R_c(t))$ so that equations (A1), (A2), and (A3) become

$$f = f_{\text{nd}} + \int_0^T f^-(t) L_d(t)\, dt - \int_0^T f^+(t) L_c(t)\, dt \tag{A4}$$

As a first application of equation (A4) suppose that the dealer has a portfolio consisting only of a zero-coupon bond issued by the counterparty and promising a payoff of $1 at time *T*. The zero-coupon is treated like a derivative in the event of a default. In this case, $f^-(t) = 0$ and $f^+(t) = f_{\text{nd}}$. Furthermore

$$f = f_{\text{nd}} \exp(-s_c(T)T)$$

where $s_c(t)$ is the adjusted credit spread for a zero coupon bond with maturity *t* issued by the counterparty. It follows that

$$1 - \int_0^T L_c(t)\, dt = \exp(-s_c(T)T) \tag{A5}$$

and

$$\int_{t_1}^{t_2} L_c(t)\, dt = \exp(-s_c(t_1)t_1) - \exp(-s_c(t_2)t_2) \tag{A6}$$

Similarly when the portfolio consists of a zero-coupon bond issued by the dealer, $f^-(t) = -f_{\text{nd}}$ and $f^+(t) = 0$ and

$$f = f_{\text{nd}} \exp(-s_d(T)T)$$

where $s_d(t)$ is the adjusted credit spread for a zero coupon bond with maturity *t* issued by the dealer. It follows that

$$1-\int_0^T L_d(t)\,dt = \exp\left(-s_d(T)T\right) \tag{A7}$$

and

$$\int_{t_1}^{t_2} L_d(t)\,dt = \exp\left(-s_d(t_1)t_1\right) - \exp\left(-s_d(t_2)t_2\right) \tag{A8}$$

There are three special cases where it is possible to use the discount rate to adjust for default risk

1. The portfolio promises a single positive payoff to the dealer (and negative payoff to the counterparty) at time *T*. This is analogous to the case in which the dealer buys a discount bond issued by the counterparty. In this case, $f^-(t)=0$ and $f^+(t)=f_{\text{nd}}$ so that from equation (A4)

   $$f = f_{\text{nd}}\left[1-\int_0^T L_c(t)\,dt\right]$$

   Using equation (A5) we obtain

   $$f = f_{\text{nd}}\exp\left(-s_c(T)T\right)$$

   This result shows that the derivative can be valued by using a discount rate equal to the risk-free rate for maturity *T* plus $s_c(T)$.

2. The portfolio promises a single negative payoff to the dealer (and positive payoff to the counterparty) at time *T*. This is analogous to the case in which the counterparty buys a discount bond issued by the dealer. In this case, $f^-(t)=-f_{\text{nd}}$ and $f^+(t)=0$ so that from equation (A4)

   $$f = f_{\text{nd}}\left[1-\int_0^T L_d(t)\,dt\right]$$

Using equation (A7) we obtain

$$f = f_{\text{nd}} \exp\left(-s_d\left(T\right)T\right)$$

This result shows that the derivative can be valued by using a discount rate equal to the risk-free rate for maturity T plus $s_d(T)$.

3. The derivatives portfolio promises a single payoff, which can be positive or negative at time T, and the two sides have identical loss rates: $L_c(t) = L_d(t) = L(t)$ for all t. Because $f^+ - f^- = f_{\text{nd}}$ equation (A4) becomes

$$f = f_{\text{nd}}\left[1 - \int_0^T L\left(t\right)dt\right]$$

From equation (A5) or (A7) we obtain

$$f = f_{\text{nd}} \exp\left(-s_d\left(T\right)T\right)$$

where $s(T)$ is the common adjusted credit spread for the dealer and the counterparty. This result shows that the derivative can be valued by using a discount rate equal to the risk-free rate for maturity T plus $s(T)$.

These three cases can be generalized. Consider a derivative that produces a series of expected payoffs $c_1, c_2, \ldots, c_n$ at times $t_1, t_2, \ldots, t_n$, with $t_1 > 0$, $t_i > t_{i-1}$, and $t_n = T$. The expected payoffs may be positive or negative. Define the no default value of the ith payoff as

$$f_{nd}^{i}\left(t\right) = \begin{cases} c_i \exp\left(-r\left(t_i - t\right)\right) & t_i \geq t \\ 0 & t_i < t \end{cases}$$

The no default value of the derivative at time t is then

$$f_{\text{nd}}\left(t\right) = \sum_{i=1}^{n} f_{\text{nd}}^{i}\left(t\right)$$

If $f_{\mathrm{nd}}(t)$ is non-negative for all t then equation (A4) becomes

$$\begin{aligned} f(0) &= f_{\mathrm{nd}}(0) - \int_0^T \exp(-rt)\, f_{\mathrm{nd}}(t)\, L_c(t)\, dt \\ &= \sum_{i=1}^{n} f_{nd}^{i}(0)\left(1 - \int_0^{t_i} L_c(t)\, dt\right) \\ &= \sum_{i=1}^{n} c_i \exp\left(-\left[r + s_c(t_i)\right] t_i\right) \end{aligned}$$

This shows that for any derivative whose value is always non-negative the adjustment for counterparty credit risk can be achieved by adjusting the discount rate. A similar argument shows that for any derivative whose value is always non-positive the adjustment for counterparty credit risk can be achieved by adjusting the discount rate. Finally, an extension of case 3 shows that when the two sides have identical credit risks, any derivative can be valued by using discount rates that reflect their common adjusted borrowing rates.

We now return to considering the general situation. We define $L_\ell(t)$ as the loss rate at time t for a company whose adjusted credit spread is the LIBOR-OIS credit spread. This means that, similarly to equations (A5) to (A8)

$$1 - \int_0^T L_\ell(t)\, dt = \exp\left(-s_\ell(T)\, T\right) \tag{A9}$$

and

$$\int_{t_1}^{T_2} L_\ell(t)\, dt = \exp\left(-s_\ell(t_1)\, t_1\right) - \exp\left(-s_\ell(t_2)\, t_2\right) \tag{A10}$$

where $s_\ell(t)$ is the zero-coupon LIBOR-OIS spread for maturity t.

If the LIBOR/swap curve defined the loss rate for both the dealer and the counterparty, equation (A4) would show that the value of the portfolio to the dealer is given by

$$f_\ell = f_{\mathrm{nd}} + \int_0^T f^-(t)\, L_\ell(t)\, dt - \int_0^T f^+(t)\, L_\ell(t)\, dt \tag{A11}$$

From equations (A4) and (A11)

$$f = f_\ell + \int_0^T f^-(t)\left[L_d(t) - L_\ell(t)\right]dt - \int_0^T f^+(t)\left[L_c(t) - L_\ell(t)\right]dt \qquad \text{(A12)}$$

From the extension of the third result given above we know that f_ℓ is the value of the derivatives portfolio when LIBOR/swap curve is used for discounting. It follows that we obtain correct valuations of any derivatives portfolio if the LIBOR/swap curve is used for discounting and CVA and DVA are calculated using loss rates for the dealer and counterparty that are the excess of the actual loss rates over the loss rates that would apply for an entity able to borrow at LIBOR/swap rates.

This result can be generalized. There is nothing special about the LIBOR/swap zero curve in our analysis. Any yield curve can be used as the “risk-free” zero curve when a derivative is valued providing CVA and DVA are calculated using loss rates that are the excess of the actual loss rate over the loss rate implied by the yield curve.

Figure 1: Relationship between Credit Spreads and Maturity

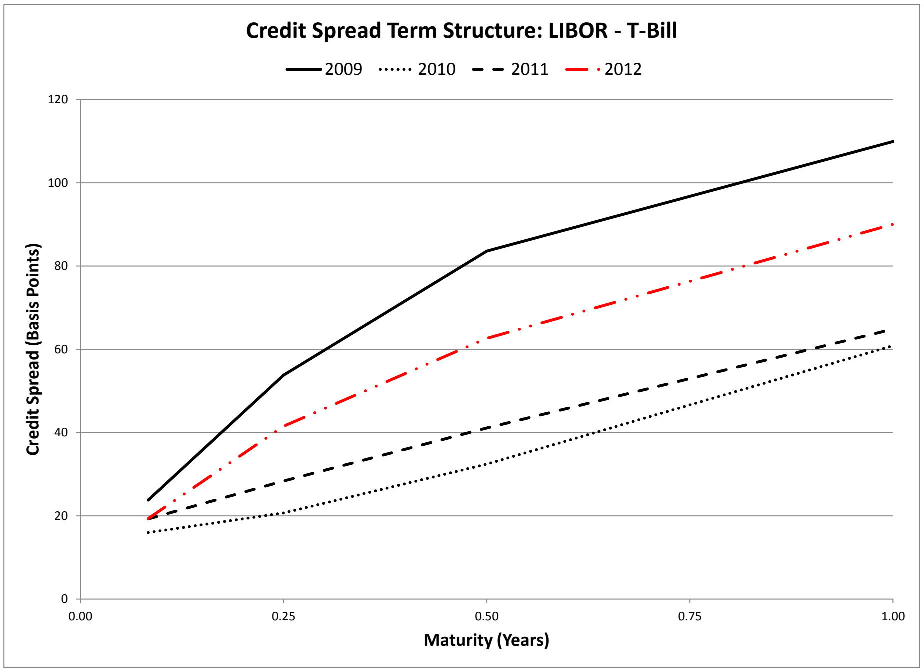

Table 1

Value to a dealer of a non-collateralized long position in a one-year forward contract to buy a non-dividend paying stock. Stock price = 100, delivery price = 100, volatility = 30%, OIS rate = 3%, LIBOR = 3.1%, dealer credit spread = 0.5%, counterparty credit spread = 2%. In the case of LIBOR discounting, the calculation of CVA and DVA is adjusted as described in Section III.

	OIS Discounting	LIBOR Discounting
No-Default Value	2.955	2.952
CVA	0.187	0.178
DVA	0.032	0.026
Total	2.801	2.801

Table 2

Value to a dealer of a non-collateralized long position in a one-year forward contract to buy a non-dividend paying stock. Stock price = 100, delivery price = 100, volatility = 30%, OIS rate = 3%, LIBOR = 4.5%, dealer credit spread = 2%, counterparty credit spread = 3%. In the case of LIBOR discounting, the calculation of CVA and DVA is adjusted as described in Section III.

	OIS Discounting	LIBOR Discounting
No-Default Value	2.955	2.911
CVA	0.279	0.138
DVA	0.128	0.032
Total	2.805	2.805

Table 3

Percentage price errors in the calculated value to dealer of a non-collateralized long position in a one-year forward contract to buy a non-dividend paying stock when LIBOR interest rates are used for both discounting and to define risk-neutral growth rates.

Case 1: Stock price = 100, delivery price = 100, volatility = 30%, OIS rate = 3%, LIBOR = 3.1%, dealer credit spread = 0.5%, counterparty credit spread = 2%.

Case 2: Stock price = 100, delivery price = 100, volatility = 30%, OIS rate = 3%, LIBOR = 4.5%, dealer credit spread = 2%, counterparty credit spread = 3%.

	Case 1	Case 2
No-Default Value	3.052	4.400
CVA	0.179	0.152
DVA	0.026	0.029
Total	2.899	4.277
Error	3.5%	52.5%