Choice of collateral currency

*Collateral agreements are becoming popular in the over-the-counter derivatives market. **Masaaki Fujii** and **Akihiko Takahashi** demonstrate its significant impact on derivatives pricing with a direct link to the cross-currency market. The importance of embedded cheapest-to-deliver options is also shown*

Collateralisation in the over-the-counter market has grown rapidly in the past decade. According to the International Swaps and Derivatives Association (2009 and 2010), about 70% of all OTC trades were collateralised at the end of 2009, compared with only 30% in 2003. Stringent collateral management will also be a crucial issue for successful installation of central clearing houses.

The role of collateralisation is mainly twofold: reducing counterparty credit risk and changing the funding costs of trades. The first is well recognised and has been studied extensively. The second, though not as obvious as the first, is also important. Recently, it has gained strong attention among practitioners, since they have experienced significant differences between Libors and the funding costs of collateralised trades. The work of Johannes & Sundaresan (2007) was the first to focus on the cost of collateralisation. It studied the effect on swap prices based on empirical analysis. More recently, Piterbarg (2010) discussed general option pricing using a similar formula to take the funding cost of collateral into account.

The impact of collateralisation is most significant in the interest rate and long-dated foreign exchange markets, where they affect various types of basis spread and also forex forwards. In two previous works, Fujii, Shimada & Takahashi (2009a and 2009b) extended the formula used in Johannes & Sundaresan (2007) and Piterbarg (2010) to the situation where the payment and collateral currencies are different, which is crucial for handling multi-currency products. Based on the result, we have presented systematic procedures of curve construction in the presence of collateral and multiple currencies, and also their no-arbitrage dynamics in a Heath-Jarrow-Morton (HJM) framework.

In this article, we construct the collateralised swap curves consistently with the actual market data and demonstrate the importance of collateralisation in derivatives pricing.[1] It is well known among market participants that the existence of large basis spreads in cross-currency swap (CCS) markets reflects differences in the funding costs among various currencies. Hence, it is natural to ask what the impact is on derivatives pricing from a different choice of collateral currency. In fact, by making use of information in CCS markets, we have found that the choice of the collateral currency has a non-negligible impact on derivatives prices. This finding gives rise to another interesting twist. When the relevant credit support annex (CSA), which specifies all the details of collateral agreement, allows multiple choices of collateral currency and free replacement among them, a payer of the collateral has the 'cheapest-to-deliver' (CTD) option. We have demonstrated the embedded option can significantly change the effective discounting factor and hence the fair value of the trade, especially when the CCS market is volatile.

Pricing under the collateralisation

Here, we review Fujii, Shimada & Takahashi (2009a), which includes results on pricing derivatives with collateralisation. Let us make the following simplifying assumptions about the collateral contract:

- Full collateralisation (zero threshold) by cash.
- The collateral is adjusted continuously with zero minimum transfer amount.

Actually, daily margin call is now quite popular in the market, which makes the above assumptions a reasonable proxy. Since the assumptions allow us to neglect the loss given default of the counterparty, we can treat each trade/payment separately without worrying about the non-linearity arising from the netting effects and the asymmetric handling of exposure.

We consider a derivative whose payout at time T is given by $h^{(i)}(T)$ in terms of currency i. We suppose that currency j is used as the collateral for the contract. Note that the instantaneous return (or cost when it is negative) of holding the cash collateral at time t is given by:

$$y^{(j)}(t) = r^{(j)}(t) - c^{(j)}(t) \quad (1)$$

where $r^{(j)}$ and $c^{(j)}$ denote the risk-free interest rate and the collateral rate of the currency j, respectively. A common practice in the market is to set $c^{(j)}$ as the overnight rate of currency j. A distinction between the theoretical risk-free rate and the market overnight rate is required for the unified treatment of different collateral and also for calibration to a cross-currency basis, which will become clearer in later discussions. If we denote the present value of the derivative at time t by $h^{(i)}(t)$ (in terms of currency i), the collateral amount posted from the counterparty is given by $(h^{(i)}(t)/f_x^{(i,j)}(t))$, where $f_x^{(i,j)}(t)$ is the forex rate at time t representing the price of the unit amount of currency j in terms of currency i. These considerations lead to the following calculation for the collateralised derivative price:

[1] *All the market data used in this article was taken from Bloomberg*

$$h^{(i)}(t) = E_t^{Q^i}\left[e^{-\int_t^T r^{(i)}(s)ds} h^{(i)}(T)\right]$$
$$+ f_x^{(i,j)}(t) E_t^{Q^i}\left[\int_t^T e^{-\int_t^s r^{(j)}(u)du} y^{(j)}(s)\left(\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}\right)ds\right]$$

where $E_t^{Q^i}[\cdot]$ is the time t conditional expectation under the risk-neutral measure of currency i, where the money-market account of currency i is used as the numeraire. By aligning the measure in the above formula, it is easy to see that:

$$X(t) := e^{-\int_0^t r^{(i)}(s)ds} h^{(i)}(t) + \int_0^t e^{-\int_0^s r^{(i)}(u)du} y^{(j)}(s) h^{(i)}(s) ds \quad (2)$$

is a Q^i-martingale under appropriate integrability conditions. This tells us that the process of the option price can be written as:

$$dh^{(i)}(t) = \left(r^{(i)}(t) - y^{(j)}(t)\right) h^{(i)}(t) dt + dM(t) \quad (3)$$

with some Q^i-martingale M.

As a result, we have the following theorem[2]: suppose that $h^{(i)}(T)$ is a derivative's payout at time T in terms of currency i and that currency j is used as the collateral for the contract. Then, the value of the derivative at time t, $h^{(i)}(t)$, is given by:

$$h^{(i)}(t) = E_t^{Q^i}\left[e^{-\int_t^T r^{(i)}(s)ds}\left(e^{\int_t^T y^{(j)}(s)ds}\right) h^{(i)}(T)\right] \quad (4)$$

$$= D^{(i)}(t,T) E_t^{T^i}\left[e^{-\int_t^T y^{(i,j)}(s)ds} h^{(i)}(T)\right] \quad (5)$$

where:

$$y^{(i,j)}(s) = y^{(i)}(s) - y^{(j)}(s) \quad (6)$$

with $y^{(i)}(s) = r^{(i)}(s) - c^{(i)}(s)$ and $y^{(j)}(s) = r^{(j)}(s) - c^{(j)}(s)$. Here, we have defined the collateralised zero-coupon bond of currency i as:

$$D^{(i)}(t,T) = E_t^{Q^i}\left[e^{-\int_t^T c^{(i)}(s)ds}\right] \quad (7)$$

We have also defined the collateralised forward measure T^i of currency i, for which $E_t^{T^i}[\cdot]$ denotes the time t conditional expectation where $D^{(i)}(t, T)$ is used as its numeraire.[3]

As a corollary of the theorem, we have:

$$h(t) = E_t^Q\left[e^{-\int_t^T c(s)ds} h(T)\right] = D(t,T) E_t^T\left[h(T)\right] \quad (8)$$

when the payment and collateral currencies are the same. This is consistent with the result of Piterbarg (2010). In addition, by setting $h(T) = 1$, it is easily seen by (5) that:

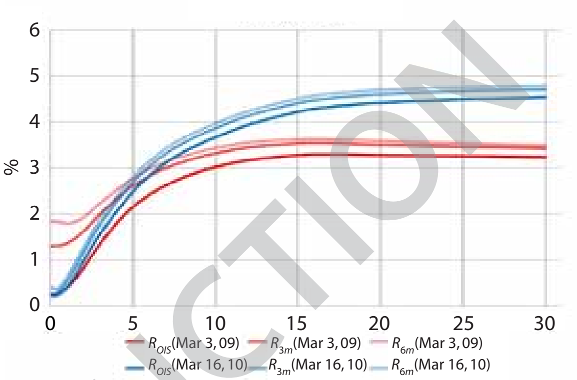
$$E_t^{T^i}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right]$$

is the ratio of two discount bonds, that is, a relative value of the discount bond collateralised in a different currency j in terms of the one collateralised in its payment currency i.

Curve construction in a single currency

Here, we will construct the relevant yield curves in a single currency market. For the details of the procedures, see Fujii, Shimada & Takahashi (2009a and 2010). We briefly summarise the set of formulas needed to strip the relevant discounting factors and forward Libors:

1 US dollar zero rate curves of the Fed-Fund rate, three-month and six-month Libors



$$OIS_N \sum_{n=1}^{N} \Delta_n D(0,T_n) = D(0,T_0) - D(0,T_N)$$

$$IRS_M \sum_{m=1}^{M} \Delta_m D(0,T_m) = \sum_{m=1}^{M} \delta_m D(0,T_m) E^{T_m}\left[L(T_{m-1},T_m;\tau)\right]$$

$$\sum_{n=1}^{N} \delta_n D(0,T_n)\left(E^{T_n}\left[L(T_{n-1},T_n;\tau_S)\right] + TS_N\right)$$
$$= \sum_{m=1}^{M} \delta_m D(0,T_m) E^{T_m}\left[L(T_{m-1},T_m;\tau_L)\right]$$

These are the consistency conditions to give the market quotes of various swaps.[4] We have denoted the market observed overnight index swap (OIS) rate, interest rate swap (IRS) rate and tenor swap (TS) spread respectively as OIS_N, IRS_M and TS_N, where the subscripts represent the lengths of swaps. $\{T_n\}_{n\geq 0}$ are the reset/payment times of each instrument. We distinguish the day-count fraction of fixed and floating legs by Δ and δ, which are not necessarily the same among different instruments. $L(T_{m-1}, T_m; \tau)$ is the Libor with tenor τ whose reset and payment times are T_{m-1} and T_m, respectively. In the third formula, we have distinguished the two different tenors by τ_S and τ_L ($> \tau_S$). If τ_S = 3m and τ_L = 6m, for example, then $N = 2M$ to match the length of two legs.

In figure 1, we have given examples of calibrated yield curves for the US dollar market on March 3, 2009 and March 16, 2010, where R_{OIS}, R_{3m} and R_{6m} denote the zero rates for OIS (Fed-Fund rate), three-month and six-month forward Libor, respectively. $R_{OIS}(\cdot)$ is defined as $R_{OIS}(T) = -\ln(D(0, T))/T$. For the forward Libor, the zero-rate curve $R_\tau(\cdot)$ is determined recursively through the relation:

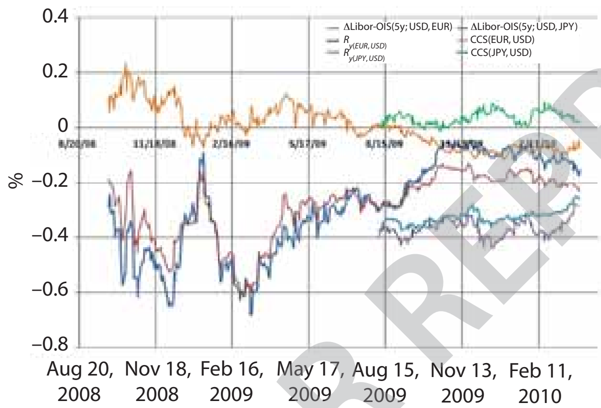
$$E^{T_m}\left[L(T_{m-1},T_m;\tau)\right] = \frac{1}{\delta_m}\left(\frac{e^{-R_\tau(T_{m-1})T_{m-1}}}{e^{-R_\tau(T_m)T_m}} - 1\right) \quad (9)$$

In the actual calculation of $D(0, \cdot)$, we have used the Fed-Fund versus three-month Libor basis swap, where the two parties exchange three-month Libor and the compounded Fed-Fund rate

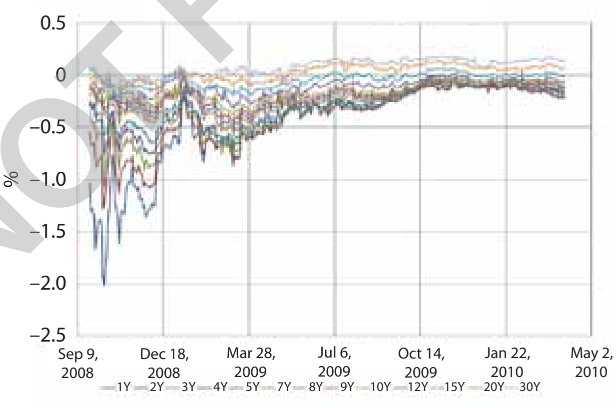
[2] Although we are dealing with continuous processes here, we obtain the same result as long as there is no simultaneous jump of underlying assets when the counterparty defaults
[3] Notice the difference from the usual forward measure where the numeraire is not collateralised
[4] If payments are compounded in TS, the formula becomes slightly more complicated. However, the effect from compounding is negligibly small and does not cause any meaningful change to the result

2 Difference between one-year IRS and OIS. Underlying floating rates are three-month Libor for US dollar and euro, and six-month Libor for yen

3 $R_{y(EUR,USD)}(5y)$, $R_{y(JPY,SD)}(5y)$, difference of Libor-OIS spreads and corresponding quotes of 5y MTMCCSs



4 Historical movement of calibrated $R_{y(EUR,USD)}$



with spread, which seems more liquid and has a larger number of quotes available than the usual OIS. In figure 2, one can see the historical behaviour of the spread between one-year IRS and OIS for the US dollar, the yen and the euro, where the underlying floating rates of the IRS are three-month Libor for the US dollar and the euro and six-month Libor for the yen.

In the above calculations, we have assumed that the conditions given in the previous section are satisfied, and also that all the instruments are collateralised by the cash of domestic currency or its payment currency. Cautious readers may worry about the possibility that the market quotes contain significant contributions from market participants who use a foreign currency as collateral. However, the induced changes in IRS/TS quotes are very small and impossible to distinguish from the bid/offer spreads in normal circumstances, because the correction appears both in the fixed and floating legs, which keeps the market quotes almost unchanged.[5]

Curve construction in multiple currencies

Calibration procedures. Here, we will discuss how to make the term structure consistent with the cross-currency swap (CCS) market. The current market is dominated by US dollar crosses, where three-month US dollar Libor is exchanged with three-month Libor of a different currency with additional basis spread. The most popular type of CCS is called the mark-to-market CCS (MTMCCS), in which the notional of the US dollar leg is reset at the start of every calculation period of Libor while the notional of the other leg is kept constant throughout the contract period.[6]

We consider a MTMCCS of the (i, j) currency pair, where the leg of currency i (intended to be US dollar) needs notional refreshments. We assume that the collateral is posted in currency i, which seems common in the market.

The value of the j-leg of a T_0-start T_N-maturing MTMCCS is calculated as:

$$PV_j = -D^{(j)}(0,T_0)E^{T_0^j}\left[e^{-\int_0^{T_0} y^{(j,i)}(s)ds}\right] + D^{(j)}(0,T_N)E^{T_n^j}\left[e^{-\int_0^{T_N} y^{(j,i)}(s)ds}\right] + \sum_{n=1}^{N}\delta_n^{(j)}D^{(j)}(0,T_n)E^{T_n^j}\left[e^{-\int_0^{T_n} y^{(j,i)}(s)ds}\left(L^{(j)}(T_{n-1},T_n;\tau)+B_N\right)\right] \quad (10)$$

where the basis spread B_N is available as a market quote. In Fujii, Shimada & Takahashi (2009b), it is assumed that all the $\{y^{(k)}(\cdot)\}$ and hence $\{y^{(i,j)}(\cdot)\}$ are deterministic functions of time to make the curve construction simpler. Here, we slightly relax the assumption allowing randomness of $\{y^{(i,j)}(\cdot)\}$. As long as we assume that $\{y^{(i,j)}(\cdot)\}$ is independent of the dynamics of Libors and collateral rates, the procedures of bootstrapping given in Fujii, Shimada & Takahashi (2009b) can be applied in the same way.[7] Under this assumption, we obtain:

$$PV_j = -D^{(j)}(0,T_0)e^{-\int_0^{T_0} y^{(j,i)}(0,s)ds} + D^{(j)}(0,T_N)e^{-\int_0^{T_N} y^{(j,i)}(0,s)ds} + \sum_{n=1}^{N}\delta_n^{(j)}D^{(j)}(0,T_n)e^{-\int_0^{T_n} y^{(j,i)}(0,s)ds}\left(E^{T_n^j}\left[L^{(j)}(T_{n-1},T_n;\tau)\right]+B_N\right) \quad (11)$$

Here, we have defined, $y^{(j,i)}(t, s)$, the forward rate of $y^{(j,i)}(s)$ at time t as[8]:

$$e^{-\int_t^T y^{(j,i)}(t,s)ds} = E_t^{Q^j}\left[e^{-\int_t^T y^{(j,i)}(s)ds}\right] \quad (12)$$

[5] *As for cross-currency swaps, the change can be a few basis points, which can be comparable with the market bid/offer spreads*
[6] *For the details of MTMCCS and a different type of CCS, see Fujii, Shimada & Takahashi (2009b, 2010)*
[7] *In practice, it would not be a problem even if there is a non-zero correlation as long as it does not meaningfully change the model implied quotes compared with the market bid/offer spreads*
[8] *Since we are assuming the independence from the collateral rate, the measure change within the same currency gives no difference*

Note that non-zero correlations among $\{y^{(i,k)}\}_{i,k}$ themselves do not pose any difficulty on curve construction.

On the other hand, the present value of the i-leg in terms of currency j is given by:

$$PV_i = -\sum_{n=1}^{N} E^{Q^i}\left[e^{-\int_0^{T_{n-1}} c^{(i)}(s)ds} f_x^{(i,j)}(T_{n-1})\right] / f_x^{(i,j)}(0)$$
$$+\sum_{n=1}^{N} E^{Q^i}\left[e^{-\int_0^{T_n} c^{(i)}(s)ds} f_x^{(i,j)}(T_{n-1})\right.$$
$$\left.\left(1+\delta_n^{(i)} L^{(i)}(T_{n-1},T_n;\tau)\right)\right] / f_x^{(i,j)}(0) \quad (13)$$
$$= \sum_{n=1}^{N} \delta_n^{(i)} D^{(i)}(0,T_n) E^{T_n^i}\left[\frac{f_x^{(i,j)}(T_{n-1})}{f_x^{(i,j)}(0)} B^{(i)}(T_{n-1},T_n;\tau)\right]$$

where:

$$B^{(i)}(t,T_k;\tau) = E_t^{T_k^i}\left[L^{(i)}(T_{k-1},T_k;\tau)\right] - \frac{1}{\delta_k^{(i)}}\left(\frac{D^{(i)}(t,T_{k-1})}{D^{(i)}(t,T_k)} - 1\right) \quad (14)$$

which represents a Libor-OIS spread. Since we found no persistent correlation between the forex and Libor-OIS spread in historical data, we have treated them as independent variables. Even if a non-zero correlation exists in a certain period, the expected correction seems numerically unimportant relative to the typical size of bid/offer spreads for MTMCCSs (about a few basis points at the time of writing). Since the three-month timing adjustment of forex is negligible, an approximate value of the i-leg is obtained as:

$$PV_i \simeq \sum_{n=1}^{N} \delta_n^{(i)} D^{(i)}(0,T_n) \frac{D^{(j)}(0,T_{n-1})}{D^{(i)}(0,T_{n-1})} e^{-\int_0^{T_{n-1}} y^{(j,i)}(0,s)ds} B^{(i)}(0,T_n;\tau) \quad (15)$$

where we have used the following result of the forex forward collateralised with currency i:

$$f_x^{(i,j)}(t,T) = f_x^{(i,j)}(t)\frac{D^{(j)}(t,T)}{D^{(i)}(t,T)} e^{-\int_t^T y^{(j,i)}(t,s)ds} \quad (16)$$

Using equations (11) and (15), the term structure of $\{y^{(j,i)}(0,\cdot)\}$ can be extracted from the equality $PV_i = PV_j$, a consistency condition for the observed market spread.

Under the above approximation, the (i, j)-MTMCCS par spread is expressed as:

$$B_N = \left[\left\{\sum_{n=1}^{N} \delta_n^{(i)} D_{T_n}^{(i)} \left(\frac{D_{T_{n-1}}^{(j)}}{D_{T_{n-1}}^{(i)}}\right) e^{-\int_0^{T_{n-1}} y^{(j,i)}(0,s)ds} B_{T_n}^{(i)}\right.\right.$$
$$\left. -\sum_{n=1}^{N} \delta_n^{(j)} D_{T_n}^{(j)} e^{-\int_0^{T_n} y^{(j,i)}(0,s)ds} B_{T_n}^{(j)}\right\} \quad (17)$$
$$\left. -\sum_{n=1}^{N} D_{T_{n-1}}^{(j)} e^{-\int_0^{T_{n-1}} y^{(j,i)}(0,s)ds}\left(e^{-\int_{T_{n-1}}^{T_n} y^{(j,i)}(0,s)ds} - 1\right)\right]$$
$$/ \sum_{n=1}^{N} \delta_n^{(j)} D_{T_n}^{(j)} e^{-\int_0^{T_n} y^{(j,i)}(0,s)ds}$$

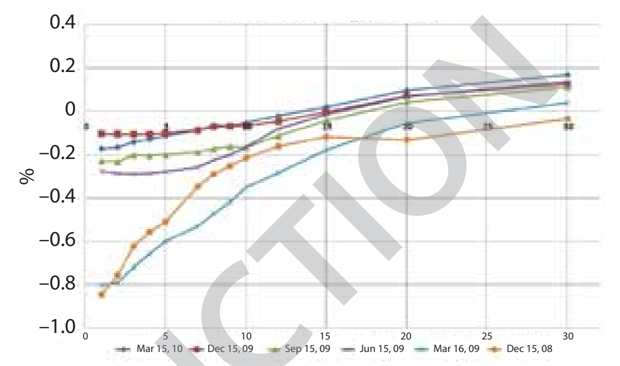
where we have shortened the notations as $D^{(k)}(0, T) = D_T^{(k)}$ and $B^{(k)}(0, T; \tau) = B_T^{(k)}$.

■ **Historical behaviour.** Now, let us check the historical behaviour of $R_{y(EUR,USD)}$ and $R_{y(JPY,USD)}$ given in figures 3–6.[9] Here, the spread R_y is defined as:

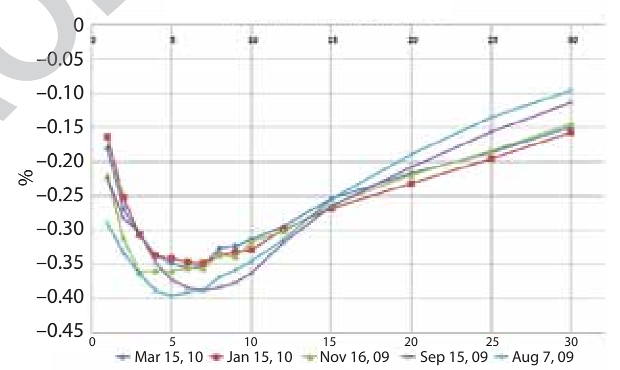
$$R_{y(j,i)}(T) = -\frac{\ln\left(E^{Q^j}\left[e^{-\int_0^T y^{(j,i)}(s)ds}\right]\right)}{T} = \frac{1}{T}\int_0^T y^{(j,i)}(0,s)ds \quad (18)$$

In figure 3, we show the historical behaviour of the basis spreads of five-year MTMCCSs, corresponding $R_{y(X,USD)}(5y)$, and the difference of $R_{3m}(5y) - R_{OIS}(5y)$ between the two currency pairs denoted by $\Delta Libor\text{-}OIS(5y; USD, X)$.[10] Here, X stands for either the euro or the yen. As expected from equation (17), $R_{y(X,USD)}(5y) + \Delta Libor\text{-}OIS(5y; USD, X)$ agrees well with the five-year MTMCCS spread, with a typical error being smaller than a few basis points. From the figure, we observe that a significant portion of the movement of CCS spreads stems from the change of $y^{(i,j)}$, rather than the difference of the Libor-OIS spread between two currencies. In fact, the level (difference)-correlation between R_y and the CCS spread is quite high, about 93% (69%) for the euro and about 70% (48%) for the yen for the historical series used in the figure. On the other hand, the same quantities between $\Delta Libor\text{-}OIS$ and the CCS spread are given by –56% (3%) for the

5 Examples of $R_{y(EUR,USD)}$ term structure

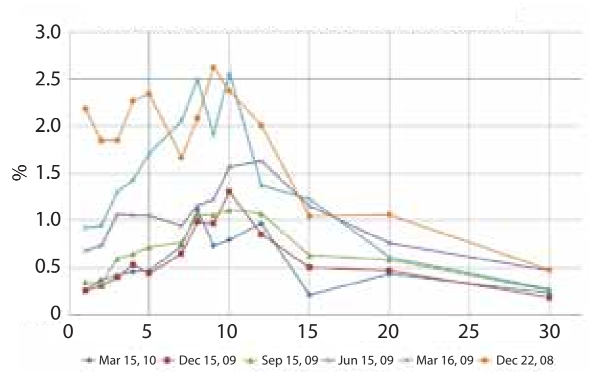


6 Examples of $R_{y(JPY,USD)}$ term structure

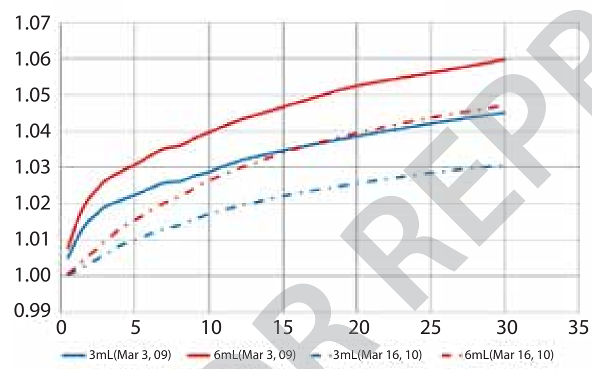


[9] Due to the lack of OIS data for the yen market, we have only limited data for the yen/US dollar pair. We have used a cubic monotone spline for calibration although the figures are given in linear plots for ease
[10] It can be interpreted as the difference of the Libor-OIS spread between the US dollar and X

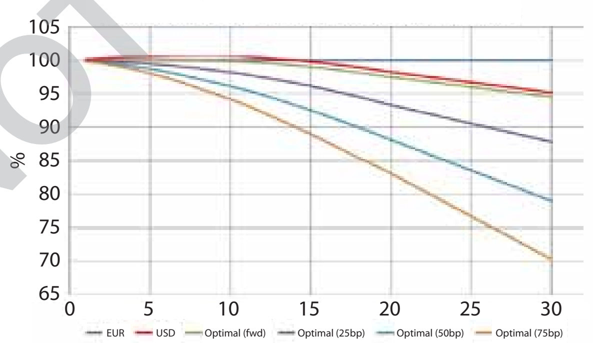
7 Three-month-roll historical volatility of $y_{(EUR,USD)}$ instantaneous forward, annualised in absolute terms



8 Present value of US dollar Libor stream with final principal (= 1) payment



9 Modification of euro discounting factors based on the Hull-White model for $y^{(EUR,USD)}$ as of March 16, 2010



euro and 9% (4%) for the yen.

The three-month-roll historical volatilities of $y^{(EUR,USD)}$ instantaneous forwards, which are annualised in absolute terms, are shown in figure 7. In a calm market, they tend to be 50bp or so, but they were more than 1 percentage point just after the market crisis, which reflects a significant widening of the CCS basis spread to seek US dollar cash in an illiquid market. Apart from the CCS basis spread, y does not seem to have persistent correlations with other market variables such as OIS, IRS and forex forwards.

Implications for derivatives pricing and summary

We now consider the implications of collateralisation for derivatives pricing. It is straightforward to see when payment and collateral currencies are the same. As in equation (8), the discounting rate is now determined by the collateral (or overnight) rate rather than Libors. Hence, in the presence of the current level of the Libor-OIS spread of 10 – 20bp, the conventional Libor discounting method results in significant underestimation of the value of future payments, which can even be a few percentage points for long maturities. Considering the mechanism of collateralisation, financial firms need to hedge the move of OISs in addition to Libors. In particular, the risk of floating-rate payments needs to be checked carefully, since the overnight rate can move in the opposite direction to Libor, as was observed in this financial crisis. In figure 8, the present values of Libor floating legs with final principal payment equal to one:

$$PV = \sum_{n=1}^{N} \delta_n D(0,T_n) E^{T_n}\left[L(T_{n-1},T_n;\tau)\right] + D(0,T_N) \qquad (19)$$

are given for various maturities. If traditional Libor discounting is used, the stream of Libor payments has the constant present value 1, which is obviously wrong from our results. This point is very important from a risk management point of view, since financial firms may overlook the quite significant interest rate risk exposure when they use traditional interest rate models in their system.

If a trade with payment currency j is collateralised by foreign currency i, an additional modification to the discounting factor appears (see the theorem above with $h(T) = 1$)[11]:

$$e^{-\int_t^T y^{(j,i)}(t,s)ds} = E_t^{Q^j}\left[e^{-\int_t^T y^{(j,i)}(s)ds}\right] \qquad (20)$$

From figures 5 and 6, one can see that posting US dollars as collateral tends to be expensive from the view point of collateral payers, which is particularly the case when everyone seeking US dollar cash in an illiquid market. For example, from figure 6, one can see that the value of a yen payment in 10 years' time is more expensive by around 3% when it is collateralised by the US dollar instead of the yen. The effects should be more profound for emerging currencies, where the implied CCS basis spread can easily be −100bp or more.

We now discuss the embedded CTD option in a collateral agreement. In some cases, financial firms make contracts with the CSA, allowing several currencies as eligible collateral. Suppose that the payer of collateral has a right to replace a collateral currency whenever he wants. In this case, the collateral payer should choose the cheapest collateral currency to post, which leads to the modification of the discounting factor of currency j as:

$$E_t^{Q^j}\left[e^{-\int_t^T \max_{i \in C}\left\{y^{(j,i)}(s)\right\}ds}\right] \qquad (21)$$

where C is the set of eligible currencies. Note that collateral payers want to make $-PV > 0$ as small as possible. Although there is a tendency towards CSAs allowing only one collateral currency to reduce the operational burden, it does not seem uncommon to accept the domestic currency and the US dollar as eligible collat-

[11] *Here, we are assuming independence of y from reference assets*

eral, for example. In this case, the above factor turns out to be:

$$E_t^{Q^j}\left[e^{-\int_t^T \max\left\{y^{(j,USD)}(s),0\right\}ds}\right] \quad (22)$$

In figure 9, we have plotted the modification factor given in equation (22), for $j = EUR$ as of March 16, 2010. We have used the Hull-White model for the dynamics of $y^{(EUR,USD)}(\cdot)$ with a mean-reversion parameter of 1.5% a year and the set of volatilities, σ = 0, 25, 50 and 75bp[12], respectively. As can be seen from the historical volatilities shown in figure 7, σ can be much higher in a volatile environment. The curve labelled US dollar (euro) denotes the modification of the discount factor when only the US dollar (euro) is eligible collateral for the ease of comparison. One can easily see that there is a significant impact when the collateral currency is chosen optimally. For example, from figure 9, one can see that if the parties choose the collateral currency from the euro and the US dollar optimally, it increases the effective discounting rate by roughly 50bp annually even when the annualised volatility of the spread $y^{(EUR,USD)}$ is 50bp. We have qualitatively the same results for the (*JPY*, *USD*) pair, although they are omitted due to space limitations.[13] Although we expect various obstacles to implementing the optimal strategy in practice, the development of a common electronic platform for collateral management as well as brisk start-ups for central clearing houses will make the optimal collateral strategy an important issue in coming years.

Finally, let us emphasise a potential danger with using the traditional Libor-discounting model, which still seems quite common among financial firms. First, it can overlook large delta exposure to Libor-OIS and MTMCCS (or closely related y) spreads. Note that, even if a desk is only dealing with single-currency products, it inevitably has exposure to CCS spreads through modifications of discounting factors if it accepts foreign currencies as collateral. Furthermore, if the firm adopts a CSA allowing free replacement of collateral currency, there may be non-negligible exposure on CCS volatility (with large negative gamma) through the embedded CTD options. Although we have cut the details of HJM framework under collateralisation due to space limitations, the full list of relevant stochastic differential equations could be provided upon request. We emphasise that every building block of the framework is observable in the market, that is, the collateral rate $c^{(i)}$, the Libor-OIS spread $B^{(i)}$, the $y^{(i,j)}$ spread and $f_x^{(i,j)}$ for each currency and pairs, where the unobserved risk-free rate is embedded in c and y. See Fujii, Shimada & Takahashi (2009b) for related discussions. ■

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[12] *These are annualised volatilities in absolute terms*
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