Choice of Collateral Currency *

Masaaki Fujii†
joint work with Akihiko Takahashi*

Counterparty Risk Frontiers: May 4th, 2012 @Paris

*This research is supported by CARF (Center for Advanced Research in Finance) and the global COE program "The research and training center for new development in mathematics." All the contents expressed in this research are solely those of the authors and do not represent the views of any institutions. The authors are not responsible or liable in any manner for any losses and/or damages caused by the use of any contents in this research. M.Fujii is grateful for Y.Shimada, and former colleagues of Morgan Stanley, especially in IDEAS, IR option, and FX Hybrid desks in Tokyo for fruitful and stimulating discussions. The contents of the research do not represent any views or opinions of Morgan Stanley.

†Graduate School of Economics, The University of Tokyo

*Graduate School of Economics, The University of Tokyo

Outlines

Some facts on Collateralization

- **Collateralization**
  - **The most important credit risk mitigation tool.**
    - **CSA gives the details of collateral agreements.**
  - **Dramatic increase in recent years (ISDA Margin Survey)**
    - **30%(2003) → 70%(2011) in terms of trade volume for all OTC.**
    - **Coverage goes up to 79% (for all OTC) and 88% (for fixed income) among major financial institutions.**
    - **More than 80% of collateral is Cash.**
    - **About half of the cash collateral is USD.**
    - **Daily portfolio reconciliation is the market standard for large dealers.**
  - **More stringent collateral management for CCPs.**

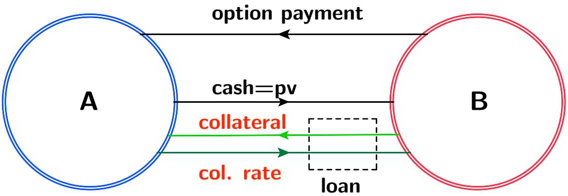
Impact of Collateralization

Impact of collateralization :

- **Reduction of Counter-party Exposure $\Rightarrow$ CVA/DVA.**
- **Change Clean Valuation Framework (main topic of my talk)**
  - **Assuming no meaningful counterparty risk.**
  - **Curve Construction and Term Structure Modeling**
  - **Funding Cost of Currency: Cross Currency Swap**
  - **"cheapest-to-deliver" option.**
  - **SCSA and USD Silo.**

A Simple Schematic Picture

- Collateralized (Secured) Contract (current picture)



  - No outright cash flow (collateral=PV)
  - No external funding is needed.
  - Funding is determined by the collateral rate.
  - Reference Rate : LIBOR $\neq$ Discounting Rate : OIS

Distinction among LIBORs and OIS

Historical behavior of IRS (1Y)-OIS (1Y) spreads (bps)

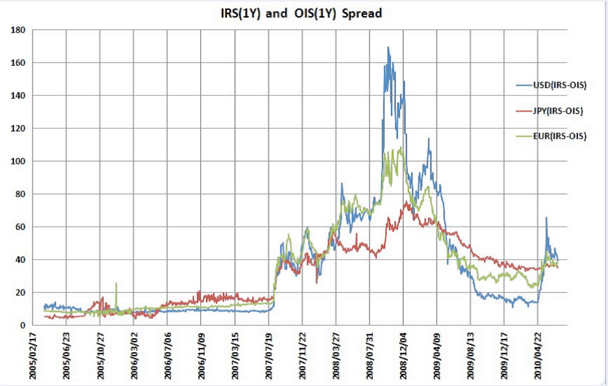


Figure: Source:Bloomberg

Distinction among LIBORs and OIS

Historical behavior of JPY TS spreads (bps)

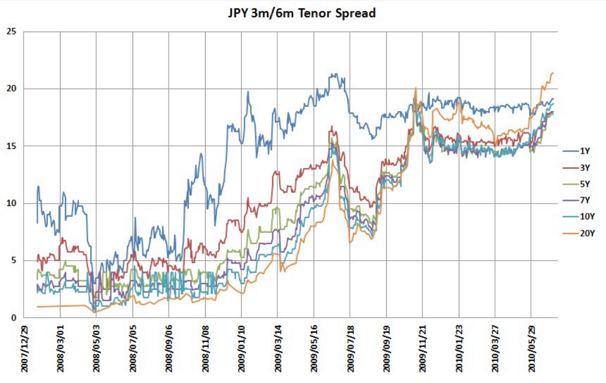


Figure: Source:Bloomberg

Distinction among LIBORs and OIS

Historical behavior of USD TS spreads (bps)

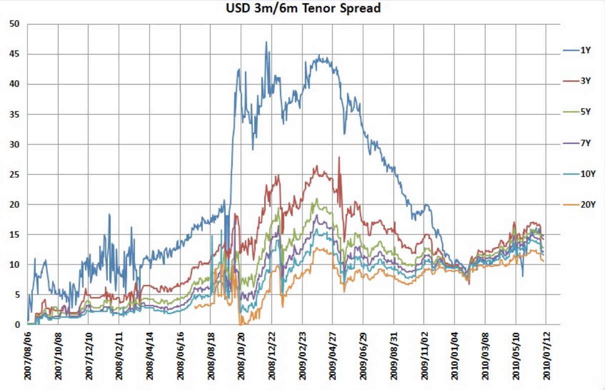


Figure: Source:Bloomberg

Distinction among LIBORs and OIS

Historical behavior of EUR TS spreads (bps)

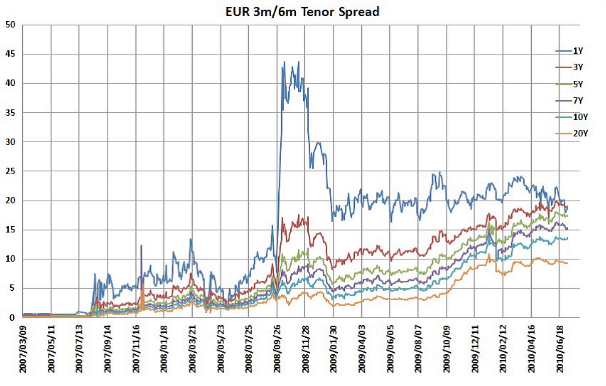


Figure: Source:Bloomberg

Funding Spread of Currency

The origin of multi-curve Setup (?)

- **Japan premium in late 1990s**
  - **Japanese financial firms had to pay extra premium to fund USD through Cross Currency Swap.**
- **At least, some of the firms started to calculate JPY related contracts with two curves, one for discounting and the other for reference rates around 1998 or so.**

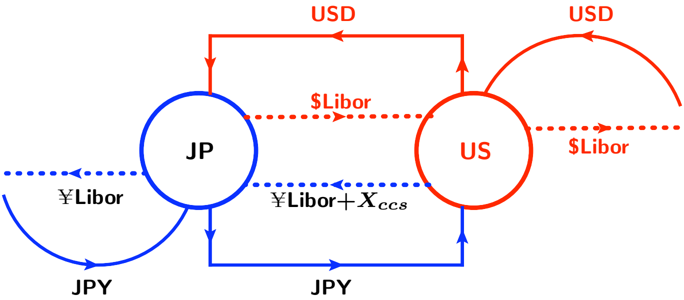
Currency Funding Spread

Funding Cost in the Domestic Market

$$\nparallel$$

Funding Cost through Cross Currency Swap

Cross Currency Swap



$X_{ccs} \neq 0$

$X_{ccs} < 0$ for JPY, for example

- USD funding cost for Japanese firm is higher than USD Libor.
- JPY funding cost for U.S. firm is lower than JPY Libor.

Cross Currency Swap

Historical behavior of USDJPY CCS spreads (bps)

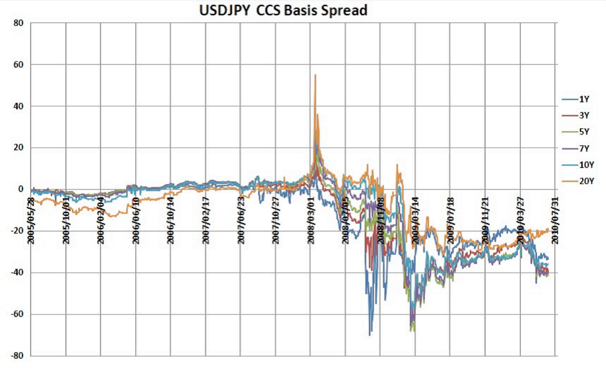


Figure: Source:Bloomberg

Cross Currency Swap

Historical behavior of EURUSD CCS spreads (bps)

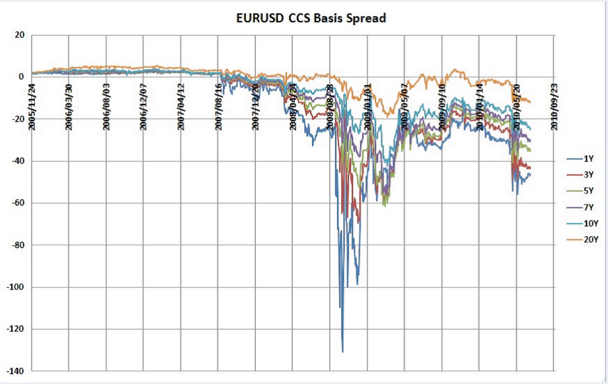


Figure: Source:Bloomberg

Cross Currency Swap

Recent history of 1Y CCS basis

Figure: Blue=JPY, Red=EUR, Green=GBP (Source:Bloomberg)

Cross Currency Swap

Term Structure of CCS basis (4/19/2012)

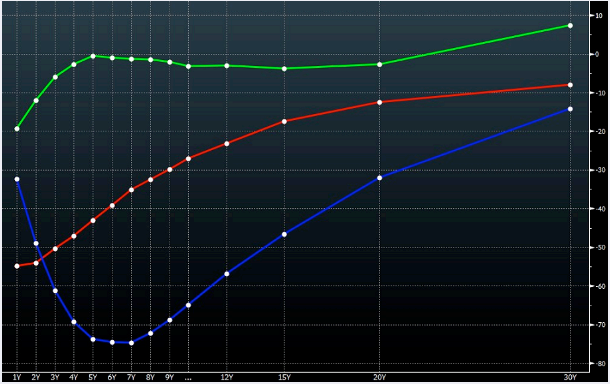


Figure: Blue=JPY, Red=EUR, Green=GBP (Source:Bloomberg)

Criteria for Valuation model for Clean Price

Criteria

- **Consistent discounting/forward curve construction**
- **Dynamic Term Structure Modeling**
  - **Price all types of IR swaps correctly under Collateralization:**
    - **OIS, IRS and TS (tenor swap)**
  - **Maintain consistency in multi-currency environment**
    - **CCS basis spreads need to be recovered $\leftrightarrow$ FX Forward.**
    - **Cost of cash collateral and its difference among major currencies should be taken into account.**

Inconsistency in pricing across different curves and currencies may provide a heavenly environment for rogue traders.

Assumptions for Collateralization

- **Assumptions**
  - **Continuous adjustment of collateral amount**
  - **Perfect collateralization by Cash**
  - **Zero minimum transfer amount**
  - **Focus on Clean Price.**

- **Comments**
  - **By making use of Repo market information, the same method can be applied to other collateral assets.**
  - **Longer term quotes are not typically available...**
  - **Liquidity swap may provide information in coming years (?)**

Pricing under the full Collateralization

Pricing Formula

PV of T-maturing payoff $h^{(i)}(T)$ in currency (i) collateralized in currency (j)

$$
\begin{aligned}
h^{(i)}(t) &= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}\left(e^{\int_t^T y^{(j)}(s)ds}\right)h^{(i)}(T)\right] \\
&= D^{(i)}(t,T)E_t^{T_{(i)}}\left[\left(e^{-\int_t^T y^{(i,j)}(s)ds}\right)h^{(i)}(T)\right]
\end{aligned}
$$

where,

$$
\begin{aligned}
y^{(j)}(s) &= r^{(j)}(s) - c^{(j)}(s) \quad , \quad y^{(i,j)}(s) = y^{(i)}(s) - y^{(j)}(s) \\
D^{(i)}(t,T) &= E_t^{Q_i}\left[e^{-\int_t^T c^{(i)}(s)ds}\right]
\end{aligned}
$$

- $h^{(i)}(T)$: option payoff at time T in currency i
- collateral is posted in currency j
- $c^{(j)}(s)$: instantaneous collateral rate of currency j at time s
- $r^{(j)}(s)$: instantaneous risk-free rate of currency j at time s
- $E^{T_{(i)}}[\cdot]$: expectation under the fwd measure associated with $D^{(i)}(\cdot,T)$

Pricing under the full Collateralization

Corollary: Single Currency Case

- **If payment and collateral currencies are the same (i), the option value is given by**

$$\begin{aligned} h^{(i)}(t) &= E_t^{Q_i}\left[e^{-\int_t^T c^{(i)}(s)ds} h^{(i)}(T)\right] \\ &= D^{(i)}(t,T) E_t^{T_{(i)}}\left[h^{(i)}(T)\right] . \end{aligned}$$

- **The discounting is determined by "collateral rate", which is consistent with the schematic picture seen before.**

Pricing under the full Collateralization

$f_x^{(i,j)}(t)$: Foreign exchange rate at time t representing the price of the unit amount of currency "j" in terms of currency "i".

- Collateral amount in currency j at time s is given by $\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}$, which is invested at the rate of $y^{(j)}(s)$:

$$h^{(i)}(t) = E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}h^{(i)}(T)\right]$$
$$+f_x^{(i,j)}(t)E_t^{Q_j}\left[\int_t^T e^{-\int_t^s r^{(j)}(u)du}y^{(j)}(s)\left(\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}\right)ds\right]$$
$$= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}h^{(i)}(T) + \int_t^T e^{-\int_t^s r^{(i)}(u)du}y^{(j)}(s)h^{(i)}(s)ds\right].$$

- This is a FBSDE, but with simple linear form.

Pricing under the full Collateralization

Economic Meanings of Spread y

- $y^{(i)} = r^{(i)} - c^{(i)}$
  - effective "dividend" yield from collateral of ccy (i)
  - cost of collateral from the view point of collateral payer
- $y^{(i,j)} = y^{(i)} - y^{(j)}$
  - Funding spread between currency (i) and (j).

- Full Collateralization $\Rightarrow$ Linear and Additive.
- Imperfect Collateralization $\Rightarrow$ Credit Risk, Funding Asymmetry...
  $\Rightarrow$ Non-linear FBSDE [1]
  - Zero-th order : Clean Price (full collateralization)
  - First order : Gateaux Derivative $\rightarrow$ CCA, CVA, DVA etc...
  - Higher orders: Non-linear FBSDE $\rightarrow$ Series of Linear FBSDEs

[1] depends on assumptions about collateral value, recovery scheme, etc..

Curve Construction in Single Currency

Collateralized Overnight Index Swap

- Assumptions:
  - payment and collateral currencies are the same
  - collateral rate is given by the overnight rate
- Condition for the length-N OIS rate:

$$\mathrm{OIS}_N^{(i)} \sum_{n=1}^{N} \Delta_n E^{Q_i}\left[e^{-\int_0^{T_n} c^{(i)}(s)ds}\right]$$
$$= \sum_{n=1}^{N} E^{Q_i}\left[e^{-\int_0^{T_n} c^{(i)}(s)ds}\left(e^{\int_{T_{n-1}}^{T_n} c^{(i)}(s)ds} - 1\right)\right]$$

or, equivalently,

$$\mathrm{OIS}_N^{(i)} \sum_{n=1}^{N} \Delta_n D^{(i)}(0,T_n) = D^{(i)}(0,T_0) - D^{(i)}(0,T_N)\ .$$

Then, the collateralized ZCB price can be bootstrapped as

$$D^{(i)}(0,T_N) = \frac{D^{(i)}(0,T_0) - \mathrm{OIS}_N^{(i)} \sum_{n=1}^{N-1} \Delta_n D^{(i)}(0,T_n)}{1 + \mathrm{OIS}_N^{(i)} \Delta_N}.$$

Curve Construction in Single Currency

- **Collateralized IRS**

$$\mathrm{IRS}_M^{(i)} \sum_{m=1}^{M} \Delta_m D^{(i)}(0,T_m) = \sum_{m=1}^{M} \delta_m D^{(i)}(0,T_m) E^{T_m^{(i)}}[L^{(i)}(T_{m-1},T_m;\tau)]$$

- **Collateralized Tenor (market basis) Swap**[2]

$$\sum_{n=1}^{N} \delta_n D^{(i)}(0,T_n) \left(E^{T_n^{(i)}} \left[L^{(i)}(T_{n-1},T_n;\tau_S) \right] + TS_N^{(i)} \right)$$

$$= \sum_{m=1}^{M} \delta_m D^{(i)}(0,T_m) E^{T_m^{(i)}} \left[L^{(i)}(T_{m-1},T_m;\tau_L) \right]$$

Market quotes of collateralized OIS, IRS, TS, and proper spline method allow us to determine

$$\{D^{(i)}(0,T)\}, \quad \{E^{T_m^{(i)}}\left[L^{(i)}(T_{m-1},T_m,\tau)\right]\}$$

for all the relevant T, T_m and tenor τ of Libor of currency (i).

[2] The short-tenor Leg may be compounded, and then exits additional small corrections.

Curve Construction: Multiple Currencies

Collateralized FX Forward: USD/JPY

- Suppose USD$= (i)$, JPY$= (j)$ and collateral currency is USD.
- Current time: t. Maturity: T
- At T, one unit of (i) is exchanged for K (fixed at t) unit of (j).
- FX forward is the break-even value of K.

$$K\mathbb{E}_t^{Q_j}\left[e^{-\int_t^T (c_s^{(j)}+y_s^{(j,i)})ds}\right] = f_x^{(j,i)}(t)\mathbb{E}_t^{Q_i}\left[e^{-\int_t^T c_s^{(i)}ds}\ 1\right]$$

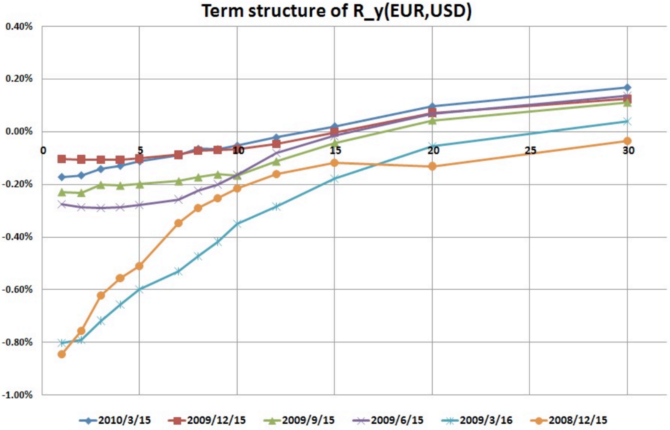
$$f_x^{(j,i)}(t,T;(i)) = f_x^{(j,i)}(t)\frac{D^{(i)}(t,T)}{D^{(j)}(t,T)}\exp\left(\int_t^T y^{(j,i)}(t,u)du\right)$$

where

$$y^{(j,i)}(t,T) = -\frac{\partial}{\partial T}\ln\left(\mathbb{E}_t^{T(j)}\left[e^{-\int_t^T y_s^{(j,i)}ds}\right]\right)$$

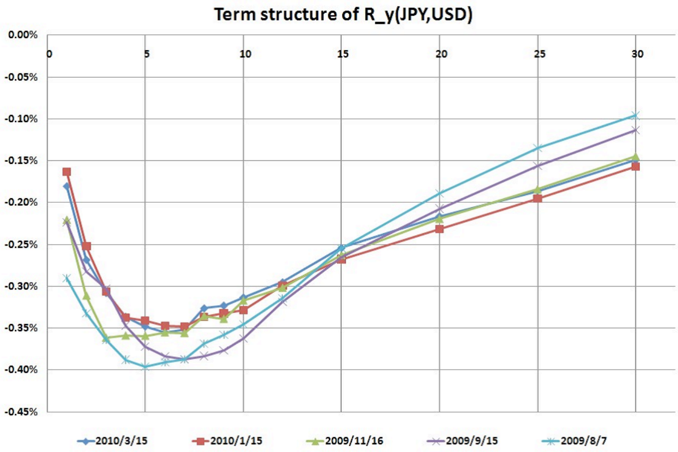
- FX Forward $\Rightarrow$ Forward curve of funding spread.
- CCS for longer maturities.

Term Structure of Funding Spread (EUR↔USD)

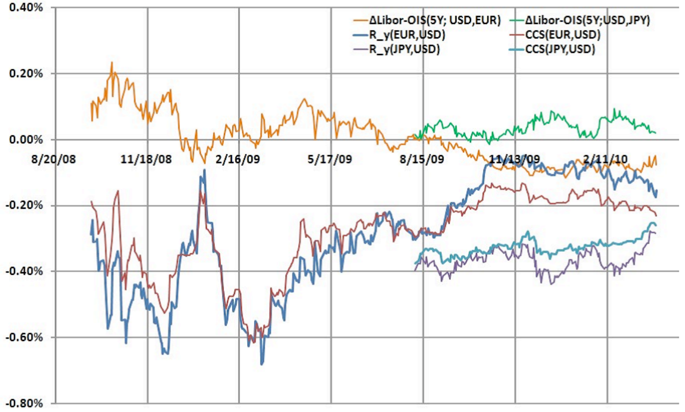


$$R_{y(i,j)}(T) = -\frac{1}{T}\ln\left(E^{T(i)}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right]\right) = \frac{1}{T}\int_0^T y^{(i,j)}(0,s)ds$$

Term Structure of Funding Spread (JPY↔USD)



CCS Basis and Funding Spread



CCS Basis Spread $\leftrightarrow$ Funding Spread $y^{(i,j)}$

LIBOR-OIS (and hence credit risk) seems to have only minor effects...

Constant Notional CCS and MtM-CCS

Against USD

USD-LIBOR ⇔ X-LIBOR + basis spread

- **Constant Notional CCS (CNCCS)**
  - **Notional of both legs are kept constant.**
- **Mark-to-Market CCS (MtMCCS)**
  - **Notional of currency X is kept constant.**
  - **Notional of USD is readjusted to $f_x^{(USD,X)} \times N_X$ at every start of LIBOR accrual period.**

Prices of two contracts were quite close and the difference was not paid enough attention. Their quotes were not clearly distinguished on broker screens...[a]

[a] I am not sure the very recent situation.

Constant Notional CCS and MtM-CCS

USD-JPY CCS (Spot-start, T_N-maturing)
USD: currency-(i), JPY: currency-(j)

$$X_{ccs}^{\mathrm{MtM}} - X_{ccs}^{\mathrm{CN}}$$
$$= \frac{\sum_{n=1}^{N} \delta_n D^{(i)}(0,T_n)\mathbb{E}^{T_n(i)}\left[\left(\frac{f_x^{(j,i)}(0)}{f_x^{(j,i)}(T_{n-1})} - 1\right) B^{(i)}(T_{n-1},T_n)\right]}{\sum_{n=1}^{N} \delta_n D^{(j)}(0,T_n;i)}$$

where

$$D^{(j)}(t,T_n;i) = E^{Q_j}\left[e^{-\int_0^{T_n}(c_s^{(j)}+y_s^{(j,i)})ds}\right]$$
$$B^{(i)}(T_{n-1},T_n) = L^{(i)}(T_{n-1},T_n) - \frac{1}{\delta_n}\left(\frac{1}{D^{(i)}(T_{n-1},T_n)} - 1\right)$$

HJM-framework under the collateralization

SDEs for HJM-framework

$$dc^{(i)}(t,s) = \sigma_c^{(i)}(t,s) \cdot \left(\int_t^s \sigma_c^{(i)}(t,u)du \right) dt + \sigma_c^{(i)}(t,s) \cdot dW_t^{Q_i}$$

$$dy^{(i,k)}(t,s) = \sigma_y^{(i,k)}(t,s) \cdot \left(\int_t^s \sigma_y^{(i,k)}(t,u)du \right) dt + \sigma_y^{(i,k)}(t,s) \cdot dW_t^{Q_i}$$

$$\frac{dB^{(i)}(t,T;\tau)}{B^{(i)}(t,T;\tau)} = \sigma_B^{(i)}(t,T;\tau) \cdot \left(\int_t^T \sigma_c^{(i)}(t,s)ds \right) dt + \sigma_B^{(i)}(t,T;\tau) \cdot dW_t^{Q_i}$$

$$\frac{df_x^{(i,j)}(t)}{f_x^{(i,j)}(t)} = \left(c^{(i)}(t) - c^{(j)}(t) + y^{(i,j)}(t) \right) dt + \sigma_X^{(i,j)}(t) \cdot dW_t^{Q_i}$$

where

$$B^{(i)}(t,T_k;\tau) = E_t^{T_{k(i)}} \left[L^{(i)}(T_{k-1},T_k;\tau) \right] - \frac{1}{\delta_k^{(i)}} \left(\frac{D^{(i)}(t,T_{k-1})}{D^{(i)}(t,T_k)} - 1 \right)$$

is forward LIBOR-OIS spread.

Choice of Collateral Currency

Role of $y^{(i,j)}$

- Payment currency i with Collateral currency j

$$D^{(i)}(t,T) \Rightarrow E_t^{T(i)}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right] D^{(i)}(t,T)$$

  after neglecting small corrections from possible non-zero correlations.

- To choose "strong" currency, such as USD, is expensive (for the collateral payer).

Choice of Collateral Currency

Role of $y^{(i,j)}$

Optimal behavior of collateral payer can significantly change the derivative value.

- **Payment currency: (i), Eligible Collateral Set: $\mathcal{C}$.**

$$D^{(i)}(t,T) \Rightarrow E_t^{T(i)}\left[e^{-\int_t^T \max_{j\in\mathcal{C}}\{y^{(i,j)}(s)\}ds}\right] D^{(i)}(t,T)$$

- **Payment currency: (i), Eligible Collateral Set: (i, USD)**

$$D^{(i)}(t,T) \Rightarrow E_t^{T(i)}\left[e^{-\int_t^T \max\{y^{(i,USD)}(s),0\}ds}\right] D^{(i)}(t,T)$$

- **Volatility of $y^{(i,j)}$ is an important determinant.**

Choice of Collateral Currency

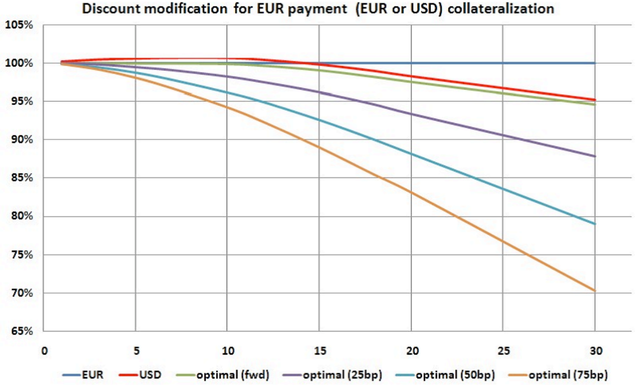


Figure: Modification of EUR discounting factors based on HW model for $\boldsymbol{y}^{(EUR,USD)}$ as of 2010/3/16. The mean-reversion parameter is $\mathbf{1.5\%}$, and the volatility is given at each label.

Inefficient Collateral Management

What happens if an investor can select the cheapest collateral but the counterparty cannot?

- **Asymmetric CSA.**
- **Symmetric CSA but the counterparty has only limited access to the cheapest collateral.**
- **The counterparty is much less sophisticated in collateral management.**

$$\Downarrow$$

Maybe disastrous for the counterparty....

Collateral Management

Suppose the situation

- **The investor "1" can select collateral currency from the set $\mathcal{C}$**
- **The counterparty "2" can only use the currency (i).**

Pricing Formula

$$V_t = E_t^Q\left[\int_{]t,T]} \exp\left(-\int_t^s (r_u - \mu(u, V_u))du\right) dD_s\right]$$

$$\mu(u, V_u) = y_u^1 1_{\{V_u<0\}} + y_u^2 1_{\{V_u\geq 0\}}$$

$$y_u^1 = \min_{j\in\mathcal{C}}(r^{(j)} - c^{(j)})$$

$$y_u^2 = r^{(i)} - c^{(i)}$$

- **V is the contract value from the view point of the investor.**
- **D denotes a cumulative cash-flow of the contract.**

Collateral Management

One sees

$$r_u - \mu(u, V_u) = c_u^{(i)} + \max_{j \in \mathcal{C}} y_u^{(i,j)} 1_{\{V_u < 0\}}$$

First order approximation:

$$V_t \simeq \overline{V}_t + \mathbb{E}_t \left[\int_t^T e^{-\int_t^s c_u^{(i)} du} [-\overline{V}_s]^+ \max_{j \in \mathcal{C}} y_s^{(j,i)} ds \right]$$

$$\overline{V}_t = \mathbb{E}_t \left[\int_{]t,T]} e^{-\int_t^s c_u^{(i)} du} dD_s \right]$$

- **Similar to CVA formula for an uncollateralized contract (from the counterparty point of view.)**
- **If the counterparty does not recognize the optionality, it may lose significant value.**

Collateral Management: Numerical Example

For demonstration, consider a simplistic system:

- **USD: currency (i)**
- **JPY: currency (j)**
- **Home currency: (j)**

$$dc_t^{(j)} = \left(\theta^{(j)}(t) - \kappa^{(j)} c_t^{(j)}\right)dt + \sigma_c^{(j)} \cdot dW_t$$

$$dc_t^{(i)} = \left(\theta^{(i)}(t) - \sigma_c^{(i)} \cdot \sigma_x^{(j,i)} - \kappa^{(i)} c_t^{(i)}\right)dt + \sigma_c^{(i)} \cdot dW_t$$

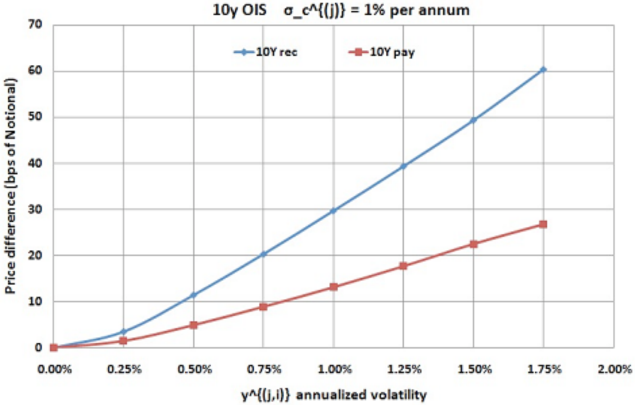
$$dy_t^{(j,i)} = \left(\theta^{(j,i)}(t) - \kappa^{(j,i)} y_t^{(j,i)}\right)dt + \sigma_y^{(j,i)} \cdot dW_t$$

$$d\ln f_x^{(j,i)}(t) = \left(c_t^{(j)} - c_t^{(i)} + y_t^{(j,i)} - \frac{1}{2}||\sigma_x^{(j,i)}||^2\right)dt + \sigma_x^{(j,i)} \cdot dW_t$$

Collateral Management: Numerical Example

JPY OIS (10y):

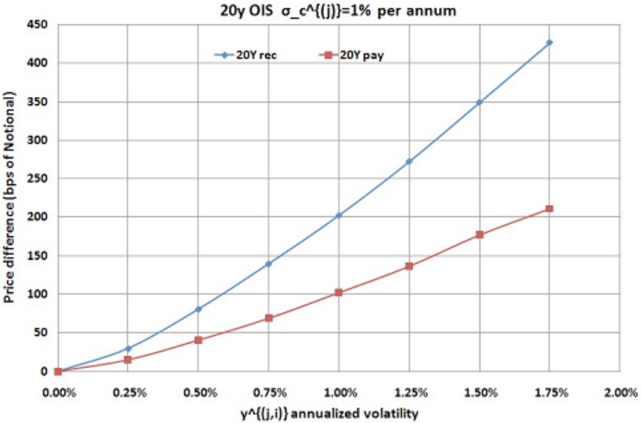
- **investor: party-1 can select USD, JPY as collateral.**
- **counterparty: party-2 can only use JPY as collateral.**



Collateral Management: Numerical Example

JPY OIS (20y):

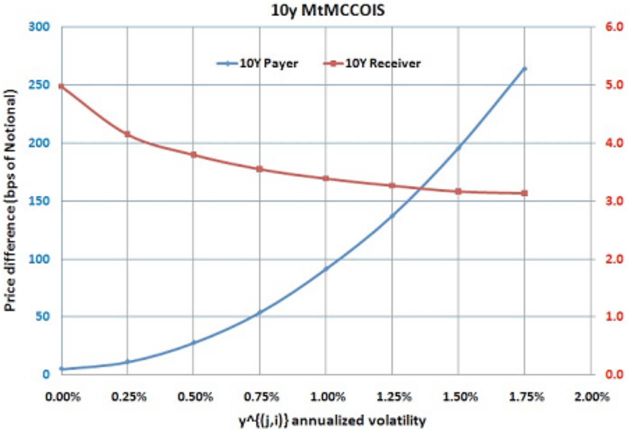
- **investor: party-1 can select USD, JPY as collateral.**
- **counterparty: party-2 can only use JPY as collateral.**



Collateral Management: Numerical Example

USD/JPY Cross Currency OIS (10y):

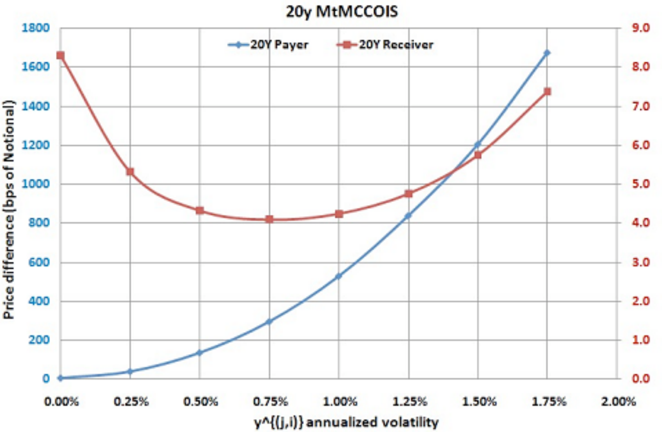
- investor: party-1 can select USD, JPY as collateral.
- counterparty: party-2 can only use USD as collateral.



Collateral Management: Numerical Example

USD/JPY Cross Currency OIS (20y):

- investor: party-1 can select USD, JPY as collateral.
- counterparty: party-2 can only use USD as collateral.



Collateral Management

Numerical example suggests...

- **It is best to avoid to make a flexible agreement on eligible collaterals if there is no capability of choosing the cheapest collateral.**
- **Winning positions with flexible CSA may be suffering from significant negative gammas.**

Collateral Management

Some implications for netting

Assume some regularity conditions and perfect collateralization. Suppose that, for each party i, its collateral funding cost y^i does not explicitly depend on the value process of contract. Let V^a, V^b, and V^{ab} be, respectively, the value processes (from view point of the party 1) of contracts with cumulative dividend processes D^a, D^b, and $D^a + D^b$ (ie., netted portfolio). Then, we have,

$$\begin{cases} V^{ab} \geq V^a + V^b & \text{if} \quad y^1 \geq y^2 \\ V^{ab} \leq V^a + V^b & \text{if} \quad y^1 \leq y^2 \end{cases}$$

Standard CSA

Embedded optionality in CSA

- **no price transparency**
- **difficult to do unwinding and novation of trades**
- **difficult to hedge**

$\Downarrow$

Standard Credit Support Annex (SCSA) [3]

- **17 Currency Silos**
- **emerging currencies, (most of ?) multi-currency trades $\Rightarrow$ USD silo**

[3] Not sure for the contents of final decisions.

Remarks on USD Silo

Curve Construction under the USD Silo

- **No domestic OIS market.**
- **Separation of $c^{(i)}$ and $y^{(i,USD)}$ is impossible...**
- **However, what we need are only the discounting and reference rates under USD collateralization.**

$\Downarrow$

Simultaneous calibration of USD-collateralized domestic IRS and USD-collateralized CCS provides relevant curves.

Thank You!