Choice of Collateral Currency *

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joint work with Akihiko Takahashi*

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Outlines

Some facts on Collateralization

- Collateralization
  - The most important credit risk mitigation tool.
    - CSA gives the details of collateral agreements.
  - Dramatic increase in recent years (ISDA Margin Survey)
    - 30%(2003) $\rightarrow$ 70%(2011) in terms of trade volume for all OTC.
    - Coverage goes up to 79% (for all OTC) and 88% (for fixed income) among major financial institutions.
    - More than 80% of collateral is Cash.
    - About half of the cash collateral is USD.
    - Daily portfolio reconciliation is the market standard for large dealers.
  - More stringent collateral management for CCPs.

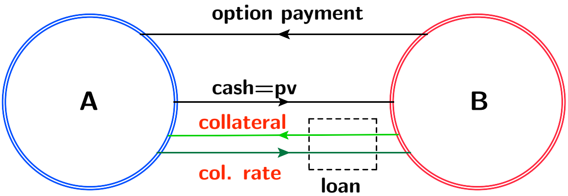
Impact of Collateralization

Impact of collateralization :

- **Reduction of Counter-party Exposure $\Rightarrow$ CVA/DVA.**
- **Change Clean Valuation Framework (main topic of my talk)**
  - **Assuming no meaningful counterparty risk.**
  - **Curve Construction and Term Structure Modeling**
  - **Funding Cost of Currency: Cross Currency Swap**
  - **"cheapest-to-deliver" option.**
  - **SCSA and USD Silo.**

A Simple Schematic Picture

- **Collateralized (Secured) Contract (current picture)**



  - **No outright cash flow (collateral=PV)**
  - **No external funding is needed.**
  - **Funding is determined by the collateral rate.**
  - **Reference Rate : LIBOR $\neq$ Discounting Rate : OIS**

Distinction among LIBORs and OIS

Historical behavior of IRS (1Y)-OIS (1Y) spreads (bps)

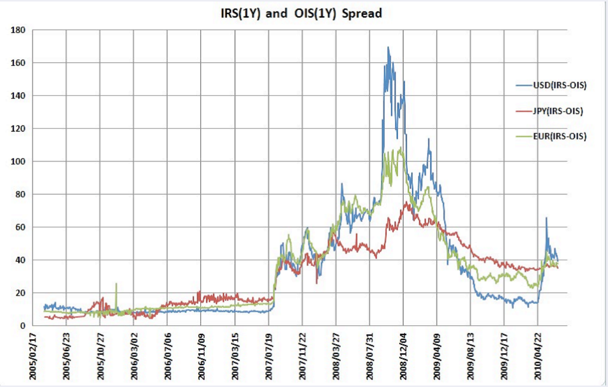


Figure: Source:Bloomberg

Distinction among LIBORs and OIS

Historical behavior of JPY TS spreads (bps)

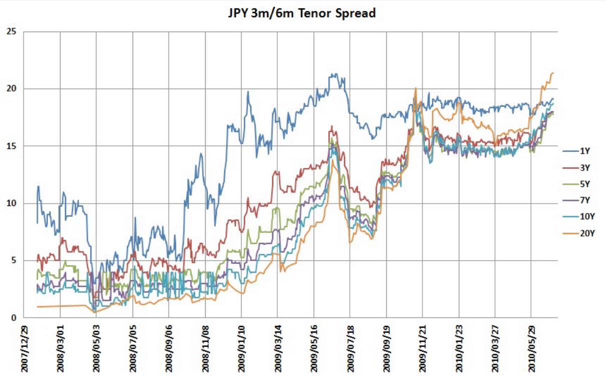


Figure: Source:Bloomberg

Distinction among LIBORs and OIS

Historical behavior of USD TS spreads (bps)

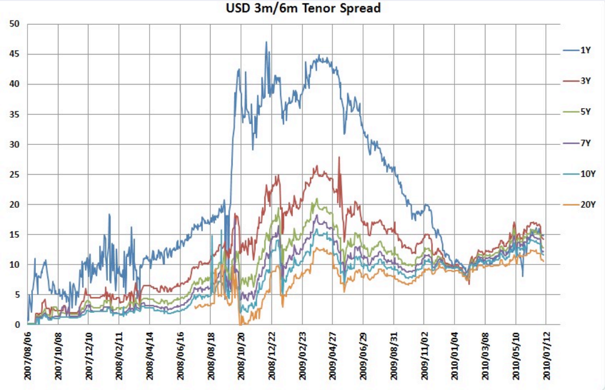


Figure: Source:Bloomberg

Distinction among LIBORs and OIS

Historical behavior of EUR TS spreads (bps)

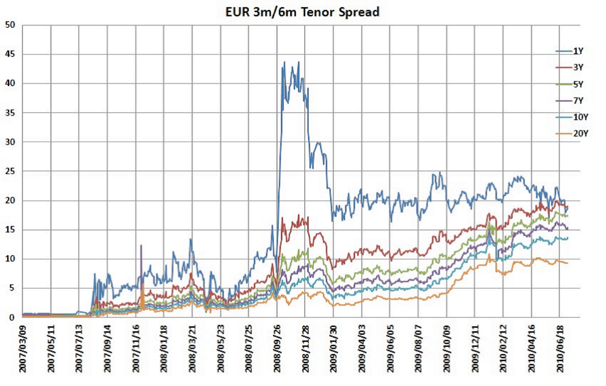


Figure: Source:Bloomberg

Funding Spread of Currency

The origin of multi-curve Setup (?)

- **Japan premium in late 1990s**
  - **Japanese financial firms had to pay extra premium to fund USD through Cross Currency Swap.**
- **At least, some of the firms started to calculate JPY related contracts with two curves, one for discounting and the other for reference rates around 1998 or so.**

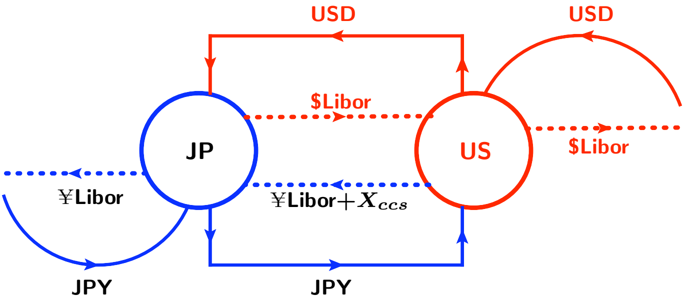
Currency Funding Spread

Funding Cost in the Domestic Market

$$\nparallel$$

Funding Cost through Cross Currency Swap

Cross Currency Swap



$X_{ccs} \neq 0$

$X_{ccs} < 0$ for JPY, for example

- USD funding cost for Japanese firm is higher than USD Libor.
- JPY funding cost for U.S. firm is lower than JPY Libor.

Cross Currency Swap

Historical behavior of USDJPY CCS spreads (bps)

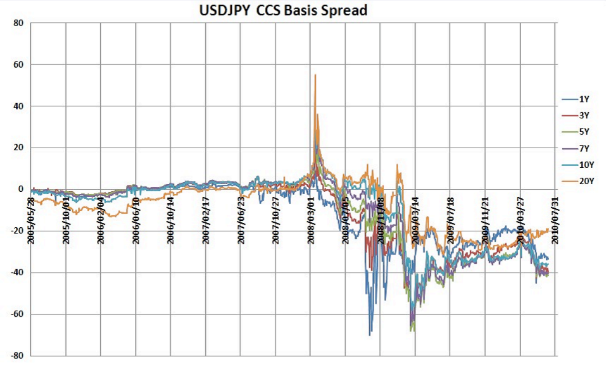


Figure: Source:Bloomberg

Cross Currency Swap

Historical behavior of EURUSD CCS spreads (bps)

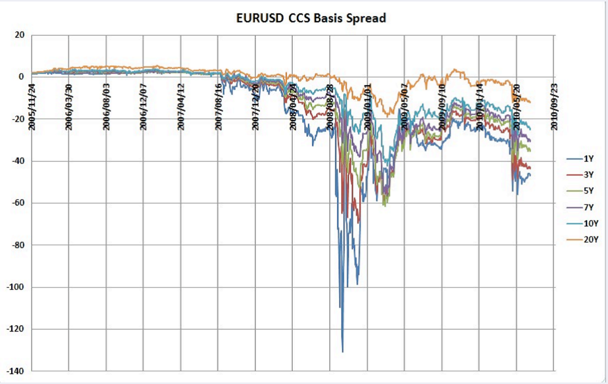


Figure: Source:Bloomberg

Cross Currency Swap

Recent history of 1Y CCS basis

Figure: Blue=JPY, Red=EUR, Green=GBP (Source:Bloomberg)

Cross Currency Swap

Term Structure of CCS basis (4/19/2012)

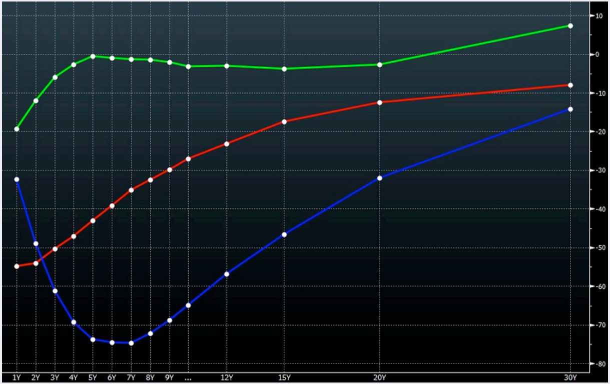


Figure: Blue=JPY, Red=EUR, Green=GBP (Source:Bloomberg)

Criteria for Valuation model for Clean Price

Criteria

- **Consistent discounting/forward curve construction**
- **Dynamic Term Structure Modeling**
  - **Price all types of IR swaps correctly under Collateralization:**
    - **OIS, IRS and TS (tenor swap)**
  - **Maintain consistency in multi-currency environment**
    - **CCS basis spreads need to be recovered $\leftrightarrow$ FX Forward.**
    - **Cost of cash collateral and its difference among major currencies should be taken into account.**

Inconsistency in pricing across different curves and currencies may provide a heavenly environment for rogue traders.

Assumptions for Collateralization

- **Assumptions**
  - Continuous adjustment of collateral amount
  - Perfect collateralization by Cash
  - Zero minimum transfer amount
  - Focus on Clean Price.

- **Comments**
  - By making use of Repo market information, the same method can be applied to other collateral assets.
  - Longer term quotes are not typically available...
  - Liquidity swap may provide information in coming years (?)

Pricing under the full Collateralization

Pricing Formula

PV of T-maturing payoff $h^{(i)}(T)$ in currency (i) collateralized in currency (j)

$$\begin{aligned} h^{(i)}(t) &= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}\left(e^{\int_t^T y^{(j)}(s)ds}\right)h^{(i)}(T)\right] \\ &= D^{(i)}(t,T)E_t^{T_{(i)}}\left[\left(e^{-\int_t^T y^{(i,j)}(s)ds}\right)h^{(i)}(T)\right] \end{aligned}$$

where,

$$\begin{aligned} y^{(j)}(s) &= r^{(j)}(s)-c^{(j)}(s) \quad , \quad y^{(i,j)}(s)=y^{(i)}(s)-y^{(j)}(s) \\ D^{(i)}(t,T) &= E_t^{Q_i}\left[e^{-\int_t^T c^{(i)}(s)ds}\right] \end{aligned}$$

- $h^{(i)}(T)$: option payoff at time T in currency i
- collateral is posted in currency j
- $c^{(j)}(s)$: instantaneous collateral rate of currency j at time s
- $r^{(j)}(s)$: instantaneous risk-free rate of currency j at time s
- $E^{T_{(i)}}[\cdot]$: expectation under the fwd measure associated with $D^{(i)}(\cdot,T)$

Pricing under the full Collateralization

Corollary: Single Currency Case

- **If payment and collateral currencies are the same (i), the option value is given by**

$$
\begin{aligned}
h^{(i)}(t) &= E_t^{Q_i}\left[e^{-\int_t^T c^{(i)}(s)ds} h^{(i)}(T)\right] \\
&= D^{(i)}(t,T) E_t^{T_{(i)}}\left[h^{(i)}(T)\right] .
\end{aligned}
$$

- **The discounting is determined by "collateral rate", which is consistent with the schematic picture seen before.**

Pricing under the full Collateralization

$f_x^{(i,j)}(t)$: Foreign exchange rate at time t representing the price of the unit amount of currency "j" in terms of currency "i".

- Collateral amount in currency j at time s is given by $\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}$, which is invested at the rate of $y^{(j)}(s)$:

$$h^{(i)}(t) = E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}h^{(i)}(T)\right]$$

$$+f_x^{(i,j)}(t)E_t^{Q_j}\left[\int_t^T e^{-\int_t^s r^{(j)}(u)du}y^{(j)}(s)\left(\frac{h^{(i)}(s)}{f_x^{(i,j)}(s)}\right)ds\right]$$

$$= E_t^{Q_i}\left[e^{-\int_t^T r^{(i)}(s)ds}h^{(i)}(T) + \int_t^T e^{-\int_t^s r^{(i)}(u)du}y^{(j)}(s)h^{(i)}(s)ds\right].$$

- This is a FBSDE, but with simple linear form.

Pricing under the full Collateralization

Economic Meanings of Spread y

- $y^{(i)} = r^{(i)} - c^{(i)}$
  - effective "dividend" yield from collateral of ccy (i)
  - cost of collateral from the view point of collateral payer
- $y^{(i,j)} = y^{(i)} - y^{(j)}$
  - Funding spread between currency (i) and (j).

- Full Collateralization $\Rightarrow$ Linear and Additive.
- Imperfect Collateralization $\Rightarrow$ Credit Risk, Funding Asymmetry...
  $\Rightarrow$ Non-linear FBSDE [1]
  - Zero-th order : Clean Price (full collateralization)
  - First order : Gateaux Derivative $\rightarrow$ CCA, CVA, DVA etc...
  - Higher orders: Non-linear FBSDE $\rightarrow$ Series of Linear FBSDEs

[1] depends on assumptions about collateral value, recovery scheme, etc..

Curve Construction in Single Currency

Collateralized Overnight Index Swap

- **Assumptions:**
  - payment and collateral currencies are the same
  - collateral rate is given by the overnight rate
- **Condition for the length-N OIS rate:**

$$\mathrm{OIS}_N^{(i)} \sum_{n=1}^{N} \Delta_n E^{Q_i}\left[e^{-\int_0^{T_n} c^{(i)}(s)ds}\right]$$

$$= \sum_{n=1}^{N} E^{Q_i}\left[e^{-\int_0^{T_n} c^{(i)}(s)ds}\left(e^{\int_{T_{n-1}}^{T_n} c^{(i)}(s)ds} - 1\right)\right]$$

or, equivalently,

$$\mathrm{OIS}_N^{(i)} \sum_{n=1}^{N} \Delta_n D^{(i)}(0,T_n) = D^{(i)}(0,T_0) - D^{(i)}(0,T_N)\ .$$

Then, the collateralized ZCB price can be bootstrapped as

$$D^{(i)}(0,T_N) = \frac{D^{(i)}(0,T_0) - \mathrm{OIS}_N^{(i)} \sum_{n=1}^{N-1} \Delta_n D^{(i)}(0,T_n)}{1 + \mathrm{OIS}_N^{(i)} \Delta_N}.$$

Curve Construction in Single Currency

- **Collateralized IRS**

$$\mathrm{IRS}_M^{(i)} \sum_{m=1}^{M} \Delta_m D^{(i)}(0,T_m) = \sum_{m=1}^{M} \delta_m D^{(i)}(0,T_m) E^{T_m^{(i)}}[L^{(i)}(T_{m-1},T_m;\tau)]$$

- **Collateralized Tenor (market basis) Swap**[2]

$$\sum_{n=1}^{N} \delta_n D^{(i)}(0,T_n) \left(E^{T_n^{(i)}} \left[L^{(i)}(T_{n-1},T_n;\tau_S) \right] + TS_N^{(i)} \right)$$

$$= \sum_{m=1}^{M} \delta_m D^{(i)}(0,T_m) E^{T_m^{(i)}} \left[L^{(i)}(T_{m-1},T_m;\tau_L) \right]$$

Market quotes of collateralized OIS, IRS, TS, and proper spline method allow us to determine

$$\{D^{(i)}(0,T)\}, \quad \{E^{T_m^{(i)}} \left[L^{(i)}(T_{m-1},T_m,\tau) \right] \}$$

for all the relevant T, T_m and tenor τ of Libor of currency (i).

[2] The short-tenor Leg may be compounded, and then exits additional small corrections.

Curve Construction: Multiple Currencies

Collateralized FX Forward: USD/JPY

- Suppose USD= (i), JPY= (j) and collateral currency is USD.
- Current time: t. Maturity: T
- At T, one unit of (i) is exchanged for K (fixed at t) unit of (j).
- FX forward is the break-even value of K.

$$K\mathbb{E}_t^{Q_j}\left[e^{-\int_t^T (c_s^{(j)}+y_s^{(j,i)})ds}\right] = f_x^{(j,i)}(t)\mathbb{E}_t^{Q_i}\left[e^{-\int_t^T c_s^{(i)}ds}\ 1\right]$$

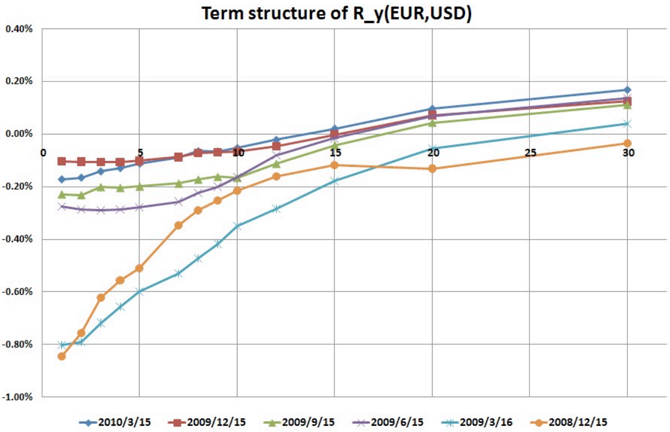
$$f_x^{(j,i)}(t,T;(i)) = f_x^{(j,i)}(t)\frac{D^{(i)}(t,T)}{D^{(j)}(t,T)}\exp\left(\int_t^T y^{(j,i)}(t,u)du\right)$$

where

$$y^{(j,i)}(t,T) = -\frac{\partial}{\partial T}\ln\left(\mathbb{E}_t^{T(j)}\left[e^{-\int_t^T y_s^{(j,i)}ds}\right]\right)$$

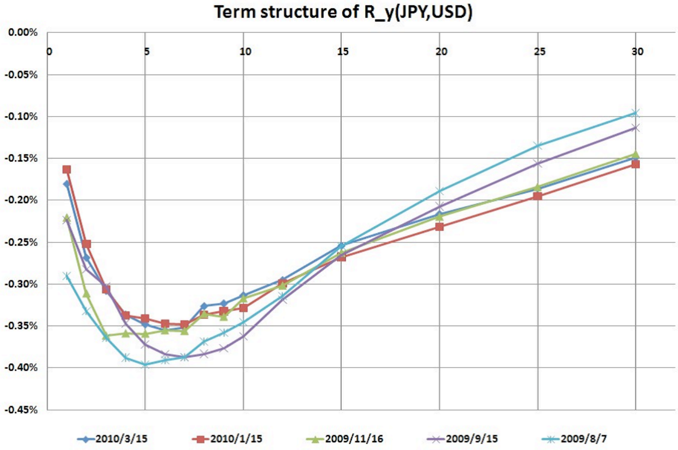
- FX Forward ⇒ Forward curve of funding spread.
- CCS for longer maturities.

Term Structure of Funding Spread (EUR↔USD)

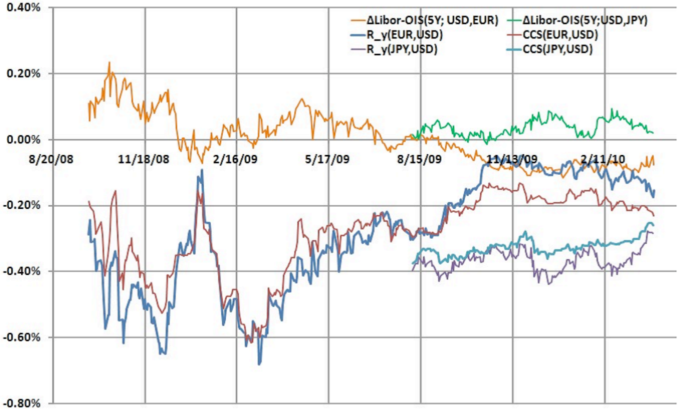


$$R_{y(i,j)}(T) = -\frac{1}{T}\ln\left(E^{T(i)}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right]\right) = \frac{1}{T}\int_0^T y^{(i,j)}(0,s)ds$$

Term Structure of Funding Spread (JPY↔USD)



CCS Basis and Funding Spread



CCS Basis Spread $\leftrightarrow$ Funding Spread $y^{(i,j)}$

LIBOR-OIS (and hence credit risk) seems to have only minor effects...

Constant Notional CCS and MtM-CCS

Against USD

$$\text{USD-LIBOR} \Leftrightarrow \text{X-LIBOR} + \text{basis spread}$$

- Constant Notional CCS (CNCCS)
  - Notional of both legs are kept constant.
- Mark-to-Market CCS (MtMCCS)
  - Notional of currency X is kept constant.
  - Notional of USD is readjusted to $f_x^{(USD,X)} \times N_X$ at every start of LIBOR accrual period.

Prices of two contracts were quite close and the difference was not paid enough attention. Their quotes were not clearly distinguished on broker screens...[a]

[a] I am not sure the very recent situation.

Constant Notional CCS and MtM-CCS

USD-JPY CCS (Spot-start, T_N-maturing)
USD: currency-(i), JPY: currency-(j)

$$X_{ccs}^{\mathrm{MtM}} - X_{ccs}^{\mathrm{CN}}$$
$$= \frac{\sum_{n=1}^{N} \delta_n D^{(i)}(0,T_n) \mathbb{E}^{T_n(i)} \left[\left(\frac{f_x^{(j,i)}(0)}{f_x^{(j,i)}(T_{n-1})} - 1 \right) B^{(i)}(T_{n-1},T_n) \right]}{\sum_{n=1}^{N} \delta_n D^{(j)}(0,T_n;i)}$$

where

$$D^{(j)}(t,T_n;i) = E^{Q_j}\left[e^{-\int_0^{T_n}(c_s^{(j)}+y_s^{(j,i)})ds} \right]$$

$$B^{(i)}(T_{n-1},T_n) = L^{(i)}(T_{n-1},T_n) - \frac{1}{\delta_n}\left(\frac{1}{D^{(i)}(T_{n-1},T_n)} - 1 \right)$$

HJM-framework under the collateralization

SDEs for HJM-framework

$$dc^{(i)}(t,s) = \sigma_c^{(i)}(t,s)\cdot\left(\int_t^s \sigma_c^{(i)}(t,u)du\right)dt + \sigma_c^{(i)}(t,s)\cdot dW_t^{Q_i}$$

$$dy^{(i,k)}(t,s) = \sigma_y^{(i,k)}(t,s)\cdot\left(\int_t^s \sigma_y^{(i,k)}(t,u)du\right)dt + \sigma_y^{(i,k)}(t,s)\cdot dW_t^{Q_i}$$

$$\frac{dB^{(i)}(t,T;\tau)}{B^{(i)}(t,T;\tau)} = \sigma_B^{(i)}(t,T;\tau)\cdot\left(\int_t^T \sigma_c^{(i)}(t,s)ds\right)dt + \sigma_B^{(i)}(t,T;\tau)\cdot dW_t^{Q_i}$$

$$\frac{df_x^{(i,j)}(t)}{f_x^{(i,j)}(t)} = \left(c^{(i)}(t) - c^{(j)}(t) + y^{(i,j)}(t)\right)dt + \sigma_X^{(i,j)}(t)\cdot dW_t^{Q_i}$$

where

$$B^{(i)}(t,T_k;\tau) = E_t^{T_k(i)}\left[L^{(i)}(T_{k-1},T_k;\tau)\right] - \frac{1}{\delta_k^{(i)}}\left(\frac{D^{(i)}(t,T_{k-1})}{D^{(i)}(t,T_k)} - 1\right)$$

is forward LIBOR-OIS spread.

Choice of Collateral Currency

Role of $y^{(i,j)}$

- **Payment currency i with Collateral currency j**

$$D^{(i)}(t,T) \Rightarrow E_t^{T(i)}\left[e^{-\int_t^T y^{(i,j)}(s)ds}\right] D^{(i)}(t,T)$$

after neglecting small corrections from possible non-zero correlations.

- **To choose "strong" currency, such as USD, is expensive (for the collateral payer).**

Choice of Collateral Currency

Role of $y^{(i,j)}$

Optimal behavior of collateral payer can significantly change the derivative value.

- **Payment currency:** (i), **Eligible Collateral Set:** $\mathcal{C}$.

$$D^{(i)}(t,T) \Rightarrow E_t^{T^{(i)}}\left[e^{-\int_t^T \max_{j\in\mathcal{C}}\{y^{(i,j)}(s)\}ds}\right] D^{(i)}(t,T)$$

- **Payment currency:** (i), **Eligible Collateral Set:** (i, USD)

$$D^{(i)}(t,T) \Rightarrow E_t^{T^{(i)}}\left[e^{-\int_t^T \max\{y^{(i,USD)}(s),0\}ds}\right] D^{(i)}(t,T)$$

- **Volatility of $y^{(i,j)}$ is an important determinant.**

Choice of Collateral Currency

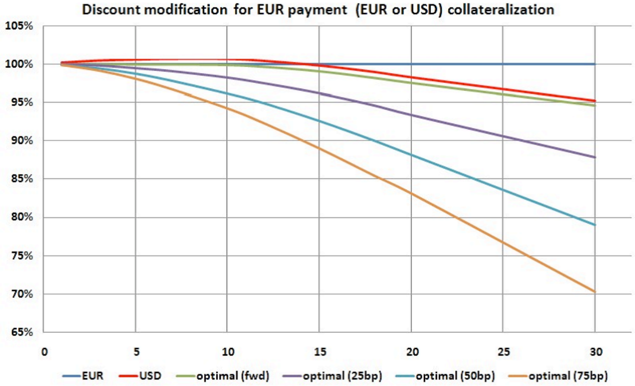


Figure: Modification of EUR discounting factors based on HW model for $y^{(EUR,USD)}$ as of 2010/3/16. The mean-reversion parameter is 1.5%, and the volatility is given at each label.

Inefficient Collateral Management

What happens if an investor can select the cheapest collateral but the counterparty cannot?

- **Asymmetric CSA.**
- **Symmetric CSA but the counterparty has only limited access to the cheapest collateral.**
- **The counterparty is much less sophisticated in collateral management.**

$\Downarrow$

Maybe disastrous for the counterparty....

Collateral Management

Suppose the situation

- **The investor "1" can select collateral currency from the set $\mathcal{C}$**
- **The counterparty "2" can only use the currency (i).**

Pricing Formula

$$V_t = E_t^Q\left[\int_{]t,T]} \exp\left(-\int_t^s (r_u - \mu(u,V_u))du\right) dD_s\right]$$

$$\mu(u,V_u) = y_u^1 1_{\{V_u<0\}} + y_u^2 1_{\{V_u \geq 0\}}$$

$$y_u^1 = \min_{j\in\mathcal{C}}(r^{(j)} - c^{(j)})$$

$$y_u^2 = r^{(i)} - c^{(i)}$$

- **V is the contract value from the view point of the investor.**
- **D denotes a cumulative cash-flow of the contract.**

Collateral Management

One sees

$$r_u - \mu(u, V_u) = c_u^{(i)} + \max_{j \in \mathcal{C}} y_u^{(i,j)} 1_{\{V_u < 0\}}$$

First order approximation:

$$V_t \simeq \overline{V}_t + \mathbb{E}_t \left[\int_t^T e^{-\int_t^s c_u^{(i)} du} [-\overline{V}_s]^+ \max_{j \in \mathcal{C}} y_s^{(j,i)} ds \right]$$

$$\overline{V}_t = \mathbb{E}_t \left[\int_{]t,T]} e^{-\int_t^s c_u^{(i)} du} dD_s \right]$$

- **Similar to CVA formula for an uncollateralized contract (from the counterparty point of view.)**
- **If the counterparty does not recognize the optionality, it may lose significant value.**

Collateral Management: Numerical Example

For demonstration, consider a simplistic system:

- **USD: currency (i)**
- **JPY: currency (j)**
- **Home currency: (j)**

$$dc_t^{(j)} = \left(\theta^{(j)}(t) - \kappa^{(j)} c_t^{(j)}\right) dt + \sigma_c^{(j)} \cdot dW_t$$

$$dc_t^{(i)} = \left(\theta^{(i)}(t) - \sigma_c^{(i)} \cdot \sigma_x^{(j,i)} - \kappa^{(i)} c_t^{(i)}\right) dt + \sigma_c^{(i)} \cdot dW_t$$

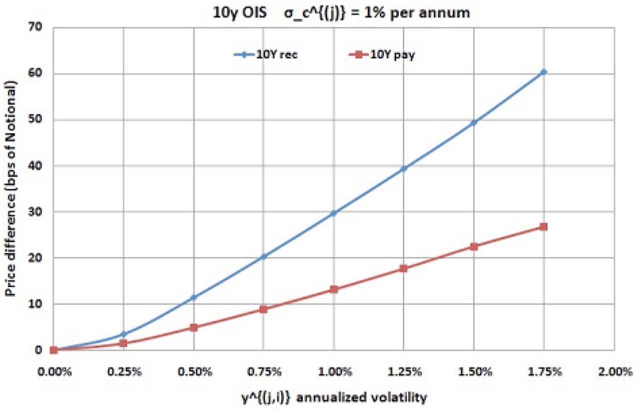
$$dy_t^{(j,i)} = \left(\theta^{(j,i)}(t) - \kappa^{(j,i)} y_t^{(j,i)}\right) dt + \sigma_y^{(j,i)} \cdot dW_t$$

$$d \ln f_x^{(j,i)}(t) = \left(c_t^{(j)} - c_t^{(i)} + y_t^{(j,i)} - \frac{1}{2} ||\sigma_x^{(j,i)}||^2\right) dt + \sigma_x^{(j,i)} \cdot dW_t$$

Collateral Management: Numerical Example

JPY OIS (10y):

- **investor: party-1 can select USD, JPY as collateral.**
- **counterparty: party-2 can only use JPY as collateral.**



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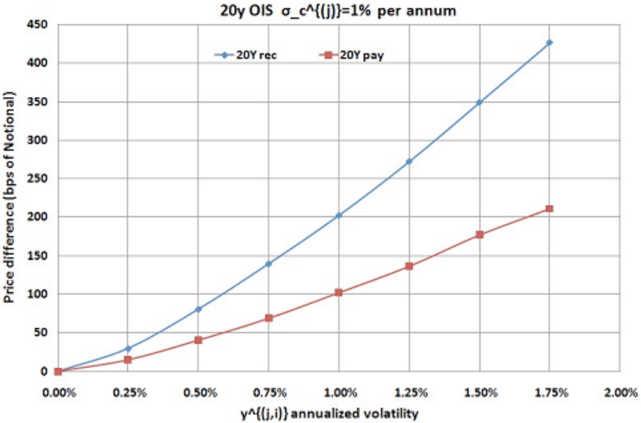
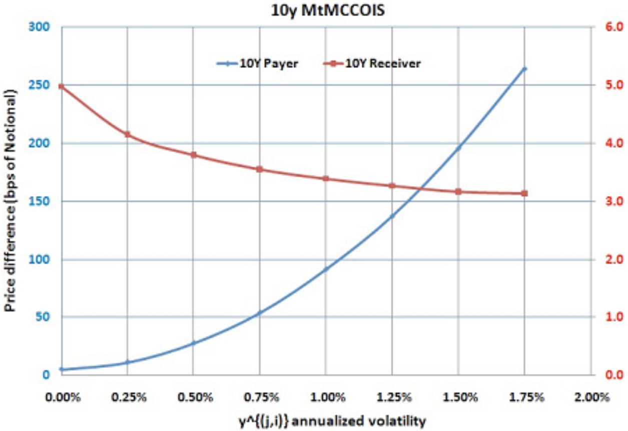


Collateral Management: Numerical Example

USD/JPY Cross Currency OIS (10y):

- investor: party-1 can select USD, JPY as collateral.
- counterparty: party-2 can only use USD as collateral.



Collateral Management: Numerical Example

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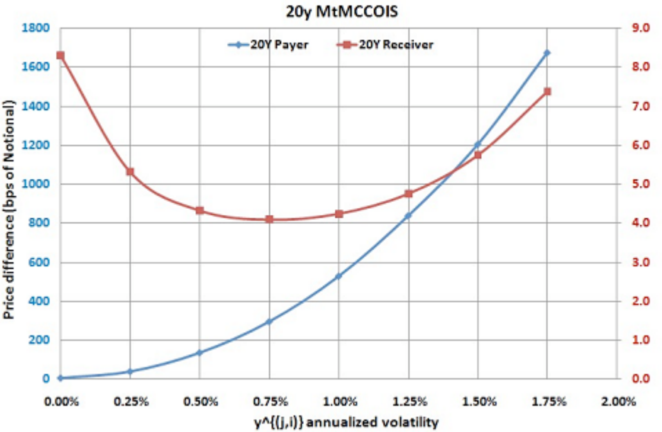

Collateral Management

Numerical example suggests...

- **It is best to avoid to make a flexible agreement on eligible collaterals if there is no capability of choosing the cheapest collateral.**
- **Winning positions with flexible CSA may be suffering from significant negative gammas.**

Collateral Management

Some implications for netting

Assume some regularity conditions and perfect collateralization. Suppose that, for each party i, its collateral funding cost y^i does not explicitly depend on the value process of contract. Let V^a, V^b, and V^{ab} be, respectively, the value processes (from view point of the party 1) of contracts with cumulative dividend processes D^a, D^b, and $D^a + D^b$ (ie., netted portfolio). Then, we have,

$$\begin{cases} V^{ab} \geq V^a + V^b & \text{if} \quad y^1 \geq y^2 \\ V^{ab} \leq V^a + V^b & \text{if} \quad y^1 \leq y^2 \end{cases}$$

Standard CSA

Embedded optionality in CSA

- **no price transparency**
- **difficult to do unwinding and novation of trades**
- **difficult to hedge**

$\Downarrow$

Standard Credit Support Annex (SCSA) [3]

- **17 Currency Silos**
- **emerging currencies, (most of ?) multi-currency trades $\Rightarrow$ USD silo**

[3] Not sure for the contents of final decisions.

Remarks on USD Silo

Curve Construction under the USD Silo

- **No domestic OIS market.**
- **Separation of $c^{(i)}$ and $y^{(i,USD)}$ is impossible...**
- **However, what we need are only the discounting and reference rates under USD collateralization.**

$\Downarrow$

Simultaneous calibration of USD-collateralized domestic IRS and USD-collateralized CCS provides relevant curves.

Thank You!