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Financial Risk Management & Training

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Swap Discounting & Pricing Using the OIS Curve

Introduction

Since August 2007 and the start of the financial crisis, swap pricing has undergone a significant revolution. Prior to that date, swap pricing was thought to be fully understood and pricing models were considered adequate. Market participants recognised that there were some simplifications that were made in the pricing of swaps such as accounting for counterparty risk, but these were minor in nature and had limited impact on pricing at the interbank level and were generally ignored. In the months subsequent to August 2007, tenor basis and cross currency basis was observed in swap pricing and accounted for by banks.

However, as the financial crisis and its effects persisted, market players started taking into account a wide range of other pricing anomalies arising primarily from credit and liquidity risk related issues in transactions. Different banks approached the issue in different ways, but since mid 2010, a common approach to swap pricing has emerged. The core issue that has now unified the approach to swap pricing has been the adoption of the OIS curve for swap discounting where swaps are under daily margining agreements with daily collateral calls.

Adoption of the OIS curve has compelled banks to reassess their approach to the following interrelated issues:

- Pricing swaps where collateral is placed in a different currency.
- Libor curve construction.
- Pricing of non-collateralised swaps.
- Specification of cross currency swap curves.

In this document, we will examine the justification for adopting the OIS curve and also touch on the issues enumerated above.

Market Dynamics Leading To OIS Discounting

Prior to 2007, the swap market priced and traded off a very simple set of yield curves which could be applied to swaps across a wide range of product features, counterparts and credit mitigation arrangements. The issues that have contributed to the change in approach are all inter-linked to some extent and are:

- Due to the problems experienced in a number of banks, the market became acutely aware of credit and liquidity risks associated with short-term interbank lending. High liquidity and credit premiums started being paid on interbank Libor funding.
- Libor rates of different tenors displayed different proportions of credit/liquidity spread which increased with term. As an example, this meant that rolling 1 month Libor rates for a longer term (e.g. for 6 months) resulted in a different spread being paid than on a single 6-month roll. The net result of this term-dependent credit spread was to create significant basis spreads between swaps with different floating leg reset tenors. The spread on a 6-month resetting swap was much greater than the spread for a 1-month resetting swap. Furthermore, spreads opened up between the various Libor rates and the equivalent term Treasury rate, the so-called TED spread.

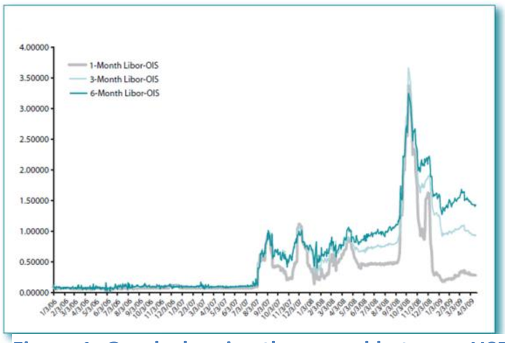

Figure 1: Graph showing the spread between USD Libor rates of different tenor and the OIS Rate which is a good illustration of tenor spreads.

- The market became more sensitive to the pricing of counterparty default in swap transactions. Prior to the crisis, the market was aware of the need to price-in the cost of counterparty default, but the calculation and application of such adjustments was inconsistently priced and applied across the market. The concept of Credit value Adjustments (CVA) is now widely accepted and applied across the markets. A consistent approach has developed and the wider acceptance of bilateral CVA adjustments has now meant banks have a method of comparing their assessments and approach to CVAs than existed in the past. In addition to a more consistent definition of CVA, banks have also taken into account the nature of credit mitigation arrangements (such as allowed for in ISDA Credit Support Annexes –

CSAs). The terms of the CSA modifies the effective counterparty credit risk exposure of a transaction or group of transactions and therefore would have a major impact on deal valuation.

- The market came to a realisation that all term inter-bank fixing rates (Libor) included some element of credit or liquidity premium. The only curve which is almost free from such effects was the OIS curve where fixings were based on central bank overnight collateralised accommodation rates and therefore did not suffer from interbank credit risk. The OIS curve is therefore recognised as being the only yield curve now which is practically free from basis effects due to credit or liquidity issues (provided the curve is built from the prices of OIS swaps subject to overnight margining).
- The bulk of inter-bank professional swap dealing is based on standard ISDA agreements which allow for netting of all deals traded under those ISDAs in the event of default. In addition a significant portion of the trades covered by an ISDA agreement are subject to regular collateral calls (daily in many cases) and inter counterparty settlement of outstanding MTMs. Collateral balances generally attract interest at the relevant overnight accommodation rate, such as the Fed Funds Rate, SONIA, EONIA or JIBAR rate. We will call this the rate the FED rate for convenience, irrespective of its currency.

Move to the OIS Curve

The last-mentioned point in the list above has had significant impact on the pricing of swaps and in June 2010, the London Clearing House (LCH.Clearnet) announced that[1]:

"LCH.Clearnet Ltd (LCH.Clearnet), which operates the world's leading interest rate swap (IRS) clearing service, SwapClear, is to begin using the overnight index swap (OIS) rate curves to discount its $218 trillion IRS portfolio. Previously, in line with market practice, the portfolio was discounted using LIBOR. After extensive consultation with market participants, LCH.Clearnet has decided to move to OIS to ensure the most accurate valuation of its portfolio for risk management purposes."

The rationale behind LCH's decision to move to OIS discounting for collateralised and daily margined trades was prompted mainly by the fact that most of the swaps clearing through their system were subject to the (now) standard CSA requiring daily collateral calls on the swap MTMs. The collateral placed would earn interest based on the prevailing O/N FED rate. The core reason for moving to the OIS curve for discounting is the fact that the collateral earns interest at the FED rate. To understand why the terms of interest accrual on the collateral translates into the selection of the discounting curve we can use the argument in the following section.

Rationale for OIS Discounting

Assume that a certain swap (S) has a MTM value of V(S), we can argue that V(S) is simply the present value of all the expected cash flows of the Swap. Assume the swap has n fixed flows (c_{fixed}) until maturity and m floating flows (c_{float})to maturity Using the notation d_i as the discount factor to time t_i, we have the following:

$$V(S) = \sum_{i=1}^{n} c_{fixed,i} d_i + \sum_{j=1}^{m} c_{float,j} d_j \qquad 1$$

(Note that the c_{fixed} and the c_{float} are normally of different sign as one would pay fixed/float and receive float/fixed. Also, the argument is presented for a fixed/float swap, but it could extend to any swap) .The collateral C placed by the counterparty would be equal to:

$$C = -\min(V(S), 0) \qquad 2$$

The above equation states that the collateral will only be placed if the swap counterpart is out of the money. If the swap is in the money, no collateral would be placed, but the other party would be out of the money and they would place the collateral. In all cases, collateral placed by either party will equal V(S).

The rate earned on collateral (equivalent to the FED rate) is an integral part of the OIS curve and lies on the OIS curve because this rate represents the floating rate index rate for OIS trades. Therefore if the collateral placed on a swap was invested at the OIS rate, the counterpart would be indifferent to earning the FED or OIS rate because of the ability to swap directly between the FED rate and the term OIS rate using an OIS.

Assume therefore that sufficient of the collateral was invested at the same set of cash flow dates as the swap flows using a series of OIS trades to ensure that the proceeds of the OIS trades exactly match the swap's expected cash flows. Assume that theoretically, this process was followed each time there was a collateral call.

In order for there to be no arbitrage possible, there should be no residual amount of collateral left over uninvested after each new set of OIS trades are executed. If there was some uninvested collateral, the swap counterparty could make an arbitrage return.

Alternatively, all the collateral could be invested at OIS rates to the swap cash flow dates and if the OIS and swap cash flows did not match exactly at the cash flow dates then an arbitrage free profit would result.

[1] LCH.Clearnet currently clears USD227 Trillion of derivatives.

For the non arbitrage condition to apply, the following must hold:

$$\sum_{i=1}^{n} c_{fixed,i} d_i + \sum_{j=1}^{m} c_{float,j} d_j = V(s) = C = \sum_{i=1}^{n} c_{fixed,i} d_{OIS,i} + \sum_{j=1}^{m} c_{float,j} d_{OIS,j} \quad (3)$$

Where $d_{OIS,i}$ is the discount factor read off the OIS curve to time t_i. Equation 3 simply states that the collateral posted (equal to V(s) as seen in equation 2) should equal the future expected cash flows of the swap discounted by the discount curve and also when discounted by the OIS curve. The only condition that generally satisfies equation 3 would be if $d_i = d_{OIS,i}$ for all i. Hence one can then say that the correct curve to use for swap discounting is the OIS curve.

Creating the OIS Curve

The OIS curve is generated directly off quoted OIS rates. OIS rates quoted in the market are simple interest rates for OIS trades of less than 1 year maturity and annual interest payments for trades of greater than 1 year maturity. OIS trades are quoted for regular intervals in the 0 to 1 year range and annually thereafter. Since the adoption of OIS discounting in July 2010, there has been a resurgence in the OIS markets. The OIS market has become increasingly more liquid, and quotes are made now for longer dated OIS trades.

In practice, OIS curve bootstrapping is normally split into two regimes. The first is the short end of the curve (possibly out to 2 months) and the other is for longer periods.

In the short-dated regime, the curve can be constructed using flat expectations of FED rates up to the next (or subsequent) Monetary Policy Committee (MPC) meeting, where FED rates are set. The assumption is that the FED rate will remain (nearly) constant until the next MPC. In addition, if there is general market certainty about future MPC views on rates, this expectation can persist beyond the next MPC.

In practice, daily discount factors are compounded to produce OIS rates in this area of the curve. Short-dated OIS rates are quoted in the market in this part of the curve, but care must be taken to interpolate these rates due to the step-function characteristics of FED rates around the times of MPC meetings. In some markets OISs are quoted to MPC meeting dates, and these can be used to back out daily discount factors between each MPC date, and these discount factors can be used to bootstrap the short portion of the curve.

Due to the greater uncertainty of MPC actions over longer horizons, longer dated OIS rates do not suffer from the same 'step-function' behaviour of shorter OIS curves. Here a traditional bootstrapping and interpolation approach can be followed to construct the curve in this area.

Bootstrapping Libor Curves in an OIS World

In the past, the normal method of bootstrapping a swap curve relied on the fact that the interest generation curve (hereinafter called the Libor Curve) and the discount curve were identical. With identical discounting and Libor curves, a single set of swap rates would be sufficient to bootstrap the curve where the discount curve and Libor curve are bootstrapped in tandem

With the adoption of OIS discounting methodology, a different method of bootstrapping becomes necessary. In effect, the unknown Libor curve must be bootstrapped relative to the known discount curve. The approach taken in the bootstrapping process follows the same approach as the traditional bootstrapping methodology except that the discount curve is known in advance.

In essence the Libor curve is constructed where the objective is to ensure that the swap prices to par when the flows generated off the Libor curve are discounted using the OIS curve.

Appendix A demonstrates one possible method of bootstrapping a Libor curve off an OIS curve.

Libor Curves

Libor curves need to be bootstrapped for each tenor encountered in the swap world. In practice, a set of 1-month, 3-month and 6-month Libor curves are produced for each currency. All of these curves would be bootstrapped relative to the OIS curve. The concept of bootstrapping an interest generating curve relative to a already known discount curve (or vice versa) would become a standard process in an OIS discounting world.

Traditionally, 3-Month Short term interest rate futures such as Eurodollar futures are used in the 3-month to 2-year portion of the curve, however, these futures cannot be used in the construction of the 1-month or 6-month curves. For these curves, 1's-3's basis swaps are used for the 1-month curve and 6-month FRAs can be used for the 6-month curve.

In order to bootstrap a Libor curve using OIS discounting, all previous issues that existed when the Libor curve was used for discounting and interest generation still exist. These issues include:

- Selection of curve instruments.
- Usage of convexity adjustments to Future prices.
- Generation of smooth forward curves.
- Interpolation of the curve during construction.

Generation of an OIS Curve When Collateral is Posted in a Different Currency

In many cases, a common CSA agreement may cover a diverse set of trades making up a counterparty's portfolio. Most often than not, the CSA would assume that collateral is placed in a single currency, irrespective of the currency of the underlying transactions covered by the CSA. Many CSAs allow the counterparty to place collateral in a range of currencies, and the counterpart could decide what currency to place. A common example would be where USD collateral is placed on a world-wide portfolio of trade of many different currencies. In the discussion below, we assume that we are obliged to place USD collateral on all the trades covered under a CSA.

In this case, it would not be sufficient to use the home currency OIS curve to discount trades in a non-collateral currency. The reason for the home currency OIS curve being inappropriate is because on could not satisfy the the non-arbitrage argument presented earlier relating to the reinvestment of the collateral to the cash flow dates of the trade.

In order to fulfil the non-arbitrage argument, one would follow a process where firstly the USD collateral is invested out to the cash flow date using the USD OIS rate. Secondly, the forward USD cash flows resulting from step 1 would be converted to the home currency at the forward exchange rate (the cross currency basis swap rate).

The new effective discounting curve for a trade where collateral calls are in a different currency would therefore be an equivalent forward implied currency discount curve as described above.

In recent times, the relative cost of placing collateral in different currencies may vary greatly. A good example of this was in the year subsequent to the financial crisis in September 2008, the effective funding rate in EUR was much less than the funding rate of USD due to the actions of the ECB compared to the FED and also the demand for USD assets resulting from a flight to quality in times of crisis. In practice, many market participants do not price in the effect of multi-currency collateral placing, but there is a wide recognition of the effects of the selection of currency on swap pricing. Many participants are keeping a watching brief on this issue and will modify swap pricing in the future to account for arbitrage opportunity arising from the selection of collateral currency.

An important feature of many CSA agreements is that they allow for collateral to be placed in a selection of different currencies at the collateral placer's discretion. With the significant basis spreads between funding in different currencies, the party placing the collateral can select the 'cheapest to deliver' currency and place that. This feature has some significant impact on deal pricing because it introduces an optionality element in the pricing of collateral and the use of an appropriate discount curve.

Discounting Of Cross Currency Swaps[2]

A question exercising many market participants is the method for determining the discount curve for cross currency swaps. The floating leg for a cross currency swap would generally have one leg at zero spread to Libor (generally the USD leg) and the leg of the other currency would be at a spread to its Libor. There are a number of ways in which such a curve can be constructed. The two major methods possible with currently quoted products would be:

- Maintain the OIS curve for discounting both interest rate and cross currency swaps. A natural consequence of this approach is that Cross Currency Libor rates bootstrapped using this method would be different to bootstrapped Swap Libor rates This would not be the preferred method of discounting the cross currency swaps because the Libor interest flows would not net off between a swap and cross currency swap.
- The other method is to use the Swap Libor curves for a cross currency swap as well. This will ensure that identical interest flows are generated for Interest Rate and Cross Currency Swaps. Using this Libor curve means that one has to reverse bootstrap the discount curve off the Libor Curve (plus spread)

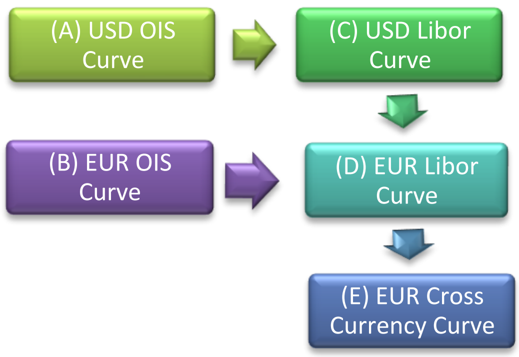

Figure 2: Cross Currency Curve Bootstrapping Sequence

[2] The term Cross Currency Swap is used to refer to Cross Currency Basis Swaps as they are also often called.

Figure 2 shows the sequence for bootstrapping the cross currency curve for an example USD-EUR cross currency swap. The starting point for the sequence is the USD and EUR OIS curves, A and B. Using Interest Rate swap rates, the USD & EUR Libor curves (C & D) are bootstrapped. The USD Leg of the Cross Currency Swap would discount using the USD OIS curve. It will be seen that the USD leg will not price to Par because the USD OIS curve would not equal the USD Libor Curve; consequently, because of the bootstrapping process the other currency leg will not price to Par as well. This has a large impact on the management of currency deltas in the pricing of the cross currency swap.

It is expected that cross currency swaps based on floating OIS rates will begin to be quoted in the market and will provide the pricing benchmarks for other cross currency swaps with Libor tenors. OIS based cross currency swaps will also provide the benchmarks for correct calculation of currency deltas. How this will work (we assume) will be as follows, refer to Figure 3 where a USD/EUR example is given:

- The USD discounting curve will be the USD OIS curve (A). The USD interest flows on the OIS cross currency swap will be calculated with the same OIS curve (B). This will result in full Par pricing of USD legs.
- For the non USD leg, the interest is generated off a standard OIS curve plus the cross currency spread (C), the spread is quoted in the market. The discounting curve (D) is reverse-bootstrapped off the interest generation curve (C) whist ensuring that the leg prices to Par.
- For other Libor cross currency swaps, the same Libor curves used for interest rate swaps should be used and the discount curves (A &D) used.

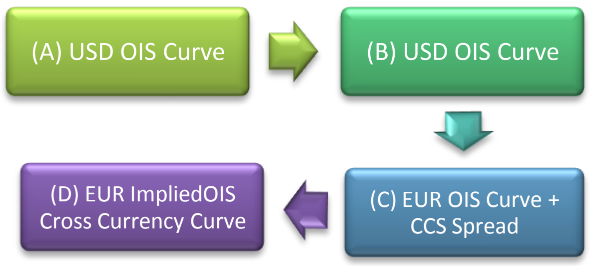

Figure 3: Proposed OIS-based discounting methodology for Cross currency swaps.

Interest Rate Derivative Pricing in the Absence of a Collateralised CSA

The discussion presented above deals with situations where the trades being priced are subject to a typical CSA with daily collateral calls and the collateral accrues at the relevant FED funds rate, this will be the case with the bulk of transactions between large market players. However, in the case where transactions are done that do not fall under such CSA agreements a different approach to deal pricing is required. The pricing of such deals are based on the computation of a credit value adjustment (CVA) which affects the deal pricing considerably. It is not the intention of this paper to deal with the pricing of such deals, but a future paper will be produced to introduce the concepts and methodologies involved.

Conclusion

It was almost unthinkable prior to 2007 that such a fundamental reassessment of interest rate derivative pricing was going to take place in the near future. The changes have been so fundamental that many banks have been unsure of exactly how to price certain derivative products which in the past were considered standard. The changes seen in the market however have perversely not highlighted new issues inherent in the market, but have rather exposed basis risks which were always present in the products, but which had been previously been ignored or deemed to be insignificant.

The field of interest rate derivative pricing is changing rapidly and as the market becomes more comfortable with the pricing of the credit and liquidity risks in these products, we fully expect that further enhancements and refinements will be made to the way in which the products are priced.

Edu-Risk International

Edu-Risk International is an Irish based financial risk management consultancy and training company owned and managed by Justin Clarke. Justin has 20 years experience in the banking industry, mainly in the area of Risk Management and related fields.

More information may be obtained at: www.edurisk.ie

Appendix: Method to Bootstrap the Libor Curve

Assume that we have a generalised discount curve {D} used to present value the flows of interest rate derivatives. In practice this curve would be derived off OIS rates. The discount curve would be able to produce discount factors d_i at each t_i.

The problem is to bootstrap a basis curve that can be used to generate the flows and hence price any series of products which trade at a basis to the OIS curve. An example of such a curve would be a 3-month Libor curve. For the purposes of this discussion, we will assume that we wish to bootstrap a swap basis curve. Let us call this curve {D'} and the discount factors read off this curve d'_i

To calculate {D'} we use the following methodology:

Assume we have a series of swap prices {S} where we have equally spaced swap rates which are consistent with the swap coupon dates. Assume the swap rate for the swap maturing at time t_i is s_i. A price P'_i for each fixed leg of each swap is obtained from the following equation (using the OIS discount curve {D} and treating the fixed leg of the swap as a bond):

$$\begin{bmatrix} s_1+100 & & & \\ s_2 & s_2+100 & & \\ s_3 & s_3 & s_3+100 & \\ s_4 & s_4 & s_4 & s_4+100 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \\ P'_4 \end{bmatrix}$$

The property of a swap is that the price of the fixed leg must equal the price of the floating leg. Using the notation a_i, where a_i is the accumulation factor (generalised relationship of a to a discount factor d is: a =1/d-1 and also $100a_i$ is the interest on the floating leg in period i) for each of the forward periods in the Libor interest generation curve applicable to the swap, we have the following equation:

$$\begin{bmatrix} a_1 & & & \\ a_1 & a_2 & & \\ a_1 & a_2 & a_3 & \\ a_1 & a_2 & a_3 & a_4 \end{bmatrix} \begin{bmatrix} 100d_1 \\ 100d_2 \\ 100d_3 \\ 100d_4 \end{bmatrix} = \begin{bmatrix} P'_1-100d_1 \\ P'_2-100d_2 \\ P'_3-100d_3 \\ P'_4-100d_4 \end{bmatrix}$$

or alternatively (for easier computation):

$$\begin{bmatrix} 100d_1 & & & \\ 100d_1 & 100d_2 & & \\ 100d_1 & 100d_2 & 100d_3 & \\ 100d_1 & 100d_2 & 100d_3 & 100d_4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} P'_1-100d_1 \\ P'_2-100d_2 \\ P'_3-100d_3 \\ P'_4-100d_4 \end{bmatrix}$$

In the equation above, one sees that the present value of the floating flows implied off the Libor curve would equate to the value of the fixed leg P', minus the present value of the principal at maturity on the float leg. If one had to solve the equation above for the a_is one can then calculate a d'_n for the Libor basis curve {D'} each time period t_n thus:

$$d'_n = \prod_{i=1}^{n} \frac{1}{a_i+1}$$