Edu-Risk International

Financial Risk Management & Training

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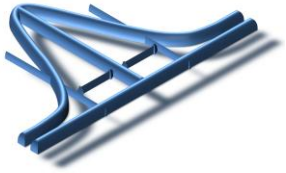

Constructing a Non-USD Swap Discounting Curve with USD Collateral

Introduction

During June 2010, LCH.Clearnet announced their intention to value swaps off the OIS curve instead of using the Swap Curve as was previously done. The reason for the move to the OIS curve was related to the collateral and margining arrangements of the majority of ISDA CSA agreements of trades cleared through LCH.Clearnet. A discussion on the rationale and effects of the change to OIS discounting is covered in another technical note produced by Edu-Risk International which can be found at the following link: Swap Discounting and the OIS Curve .

Edu-Risk has published a document which describes the construction of an OIS using a curve bootstrapping process and which takes into account rate behaviour around Monetary Policy Meeting dates. This technical note can be found at the following link: Constructing the OIS curve. In addition, Edu-Risk has also published a methodology where a Libor Curve can be generated from the constructed OIS curve and par swap rates. The document can be found at: Bootstrapping a Libor Curve off an OIS curve.

The methodology described to construct the OIS curve in the document above produces a discount curve which can be used successfully as a discount curve for swaps where the collateral placed on the swap is in the same currency as the swap itself. However, in many cases, collateral is not placed in the same currency as the swap.

This situation can arise where a swap consists of two distinct currencies (such as a cross currency basis swap) or when there is one ISDA CSA margining agreement which covers a large number of diverse trades with a counterparty. When this situation arises, it is no longer suitable (nor appropriate) to use an OIS curve derived from OIS prices in the same currency as the swap to discount swaps and the discount curve needs to reflect the relative cost of financing the swap's P&L (through the margining and collateral process) in a different currency.

Margining a swap in another currency raises a new set of pricing problems; in this paper we will describe how one can construct a correct swap discount curve so that the effect of the currency conversion implicit in the placement of collateral in another currency is taken into account.

This document describes a methodology that can be used to bootstrap or construct a swap discount curve in cases where collateral is placed in a currency other than that of the swap.

Methodology

The starting point for the computation is to generate the OIS curve in the currency in which the collateral is based using a method similar to that given in 'Constructing The OIS Curve'. This OIS curve becomes the only absolute curve in the whole process and all other discount or interest generation curves would be bootstrapped relative to this OIS curve.

Briefly, the construction process for the other currency (for the purposes of this paper, let us assume that the swap currency is EUR and the collateral currency is USD earning at the FED funds rate) can logically be thought as:

1. The swap is valued using the (as yet undetermined) EUR discount curve to produce a fair value of V_E Euros.
2. The V_E Euros will be swapped into USD at the current spot FX rate S. The USD value of the swap V_U is the amount of USD collateral posted.
3. A no arbitrage will result if the USD collateral is invested to each of the EUR swap cash flow dates and then converted at the forward FX rates into EUR exactly matching the cash flows of the swap.
4. The investment rate for the USD collateral will be the OIS rate (this is so because the collateral rate – the FED funds rate and the OIS rates are interchangeable with the use of OIS trades).
5. The forward exchange rate would be the forward rate quoted in the market. In many cases, the forward FX rate would not be observable with much liquidity beyond one or two years, so the forward FX rate would have to be inferred from the Cross Currency Basis Swap Market.
6. The correct EUR discount curve would consist of the set of discount factors which uniquely completes the investment 'round trip' and correctly present values the cash flows to the correct swap fair value.

The circumstances which leads to the circuitous argument presented above is not normally encountered in practice in the market and the construction of the implied EUR Swap discounting curve would require using interest rate swap and

cross currency basis swap rates and spreads. The following process is followed to achieve this:

Assume a set of 3-month USD swap par rates for interest rate swaps of maturity at time t, u_t, a set of 3-month Libor/Euribor Cross Currency Basis Swaps Spreads (to EUR) c_t and a set of 3-month EUR swap rates e_t (note that the normal EUR swap rates relate to annual fixed swaps and would require basis conversion). Each set would be interpolated to 3-month intervals.

Assume also we have constructed an USD OIS curve from bootstrapping USD OIS rates. The USD OIS curve would provide a set of USD discount factors $\{d_{UOIS}\}$. The discount factor time t is $d_{UOIS,t}$. This OIS curve would be used to discount any USD leg of a swap subject to USD margining and collateralisation under a CSA.

Also observable in the market are cross currency basis swaps which are quoted on a 3-month USD Libor/3-month Euribor basis.

To determine the EUR swap discount factors we complete the round-trip as described previously, but this time we use interest rate swaps and cross currency swaps instead of forward exchange rates and zero discount factors.

The cross currency basis swap spreads quoted in the market apply to floating-floating swaps, but without any loss of generality, these spreads would also apply to 3-month fixed swap legs, provided that the swap fixed rates are the par 3-month rates quoted vs. Libor in the market. If this were the case, then the PV of the EUR leg converted at spot should equate to the PV of the USD leg. Furthermore, the fixed EUR leg should include the cross currency basis spread as quoted on the Libor/Euribor swap.

Equating the values of the fixed rate EUR and USD Legs we get the following equation, equation 1. From equation 1, the value of the USD leg (left hand side) equals the value of the euro leg (plus spread) :

$$\sum_{i=1}^{n}\left(u_i\tau_{i-1,i}d_{UOIS,i}\right)+d_{UOIS,n} \;=\; S\left(\sum_{i=1}^{n}\left((e_i+c_i)\tau_{i-1,i}d_{EOIS,i}\right)+d_{EOIS,n}\right) \qquad ..(1)$$

Equation 1 can be written in a matrix form to assist in the computation of the discount factors (in the example in equation 2 below, we have used a 4-period swap):

$$\begin{bmatrix} 1+u_1\tau_{0,1} & & & \\ u_2\tau_{0,1} & 1+u_2\tau_{1,2} & & \\ u_3\tau_{0,1} & u_3\tau_{1,2} & 1+u_3\tau_{2,3} & \\ u_4\tau_{0,1} & u_4\tau_{1,2} & u_4\tau_{2,3} & 1+u_4\tau_{3,4} \end{bmatrix}\begin{bmatrix} d_{UOIS,1} \\ d_{UOIS,2} \\ d_{UOIS,3} \\ d_{UOIS,4} \end{bmatrix} =$$

$$S\left(\begin{bmatrix} 1+(e_1+c_1)\tau_{0,1} & & & \\ (e_2+c_2)\tau_{0,1} & 1+(e_2+c_2)\tau_{1,2} & & \\ (e_3+c_3)\tau_{0,1} & (e_3+c_3)\tau_{1,2} & 1+(e_3+c_3)\tau_{2,3} & \\ (e_4+c_4)\tau_{0,1} & (e_4+c_4)\tau_{1,2} & (e_4+c_4)\tau_{2,3} & 1+(e_4+c_4)\tau_{3,4} \end{bmatrix}\begin{bmatrix} d_{EOIS,1} \\ d_{EOIS,2} \\ d_{EOIS,3} \\ d_{EOIS,4} \end{bmatrix}\right) \qquad ..2$$

Equation 2 can be simply solved for the discount factors d_{EOIS}, the EUR discount factors.

We can write equation 2 in matrix notation thus:

$$U\, d_U = S\, E\, d_E$$

The Euro discount factors d_E can then be determined by the following equation:

$$d_E = \frac{E^{-1}U\, d_U}{S}$$

Using these equations, the forward exchange rate at time t (Ft) in terms of USD/EUR can be defined as:

$$F_t = S\frac{d_E}{d_U}$$

The equation for the forward rate F_t can also be used instead of the equation above to determine the implied EUR discount rate at time t if both $d_{UOIS,t}$ and F_t are known.

Discussion Of The Meaning Of the EUR Discount Curve

It is not correct to view the EUR curve generated using the method described in the previous section as an OIS curve. The curve is strictly speaking a Swap Discount Curve under the conditions that collateral is placed in USD. The true EUR OIS curve would be expected to differ from this swap discount curve by approximately the value of the spread on the cross currency swap. The EUR OIS curve would be bootstrapped relative to the EUR swap discount curve where the EUR OIS curve would be used to generate floating OIS flows and all flows would be discounted using the EUR swap discount curve.

Other issues

The most important issue outstanding is the complication where one of the following issues may arise:

1. Collateral may be placed in one of a range of currencies, or non cash collateral is placed. In this case a certain degree of optionality arises as the collateral placer could select a 'cheapest to deliver' collateral currency.
2. Collateral is placed in a more than one currency, and one may have to attribute a mix of currencies to the collateral of a swap.
3. Portfolio effects where the collateral placed is offset by other swaps with the same counterpart.

The issues mentioned in the points above raise both significant computational issues as well as book management issues as far as the management of collateral is concerned. No doubt the market will evolve over time to address these issues. Further papers will be produced over time to deal with these issues.

Conclusion

The methodology presented in this document provides a framework for computing swap discount curves where the collateral currency is not the same as the swap currency. Constructing these discount curves curves is an essential requirement for the pricing of all collateralised interest rate derivative products in today's markets as collateral is often placed in currencies not equal to the swap currency. In a world where all collateral is placed in USD, the conclusion is that the USD OIS curve would be the only absolute swap curve bootstrapped in the market. All other curves (whether discount curves or interest generation curves) would be bootstrapped relative to this curve.

Edu-Risk International Courses

Edu-Risk International provides a large range of Financial Market and Risk Management in-house training and consultancy. Please consult our website to see the full range of courses on offer. All courses are customised to meet the specific objectives of the customer and to deal with specific issues facing the customer.

Of particular interest is our OIS Discounting course. A sample 1-day OIS Discounting course outline is:

Basis Curves & OIS Discounting

09h00 – 10h00

- Origin of basis risk arising from the financial crisis.
- Market changes observed since mid 2007.
- Challenges for curve construction

10h00 – 11h00

- Tenor Basis Risk – 1's, 3's & 6's spreads
- Cross currency basis risk.
- Basis risk products.
- Practical deal valuation in the presence of basis risk.
    - Swaps
    - Cross currency basis swaps.

11h00 – 12h00 & 13h00 – 15h30

- OIS Discounting Rationale
- History of OIS Discounting
- OIS Discounting & its links to CVA
- Curve Construction
    - OIS Curve
    - Libor Curves
    - Cross Currency Curves
- Deal Pricing with OIS Discounting
- Practical Implementation Approach.
- Review of the Market's approach to OIS Discounting.

15h30– 17h00

- Curve Construction Workshop

About Edu-Risk International

Edu-Risk International is an Irish based financial risk management consultancy and training company owned and managed by Justin Clarke. Justin has over 20 years experience in the banking industry, mainly in the area of Risk Management and related fields.

More information may be obtained at:

http://www.edurisk.ie