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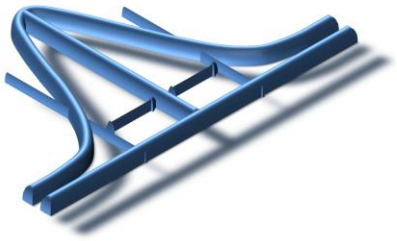

Bootstrapping A Libor Curve Off An OIS Curve

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Method

Assume that we have a generalised discount curve {D} used to present value the flows of interest rate derivatives. In practice this curve would be derived off OIS rates. The discount curve would be able to produce discount factors d_i at each t_i.

The problem is to bootstrap a basis curve that can be used to generate the flows and hence price any series of products which trade at a basis to the OIS curve. An example of such a curve would be a 3-month Libor curve. For the purposes of this discussion, we will assume that we wish to bootstrap a swap basis curve. Let us call this curve {D'} and the discount factors read off this curve d'_i

To calculate {D'} we use the following methodology:

Assume we have a series of swap prices {S} where we have equally spaced swap rates which are consistent with the swap coupon dates. Assume the swap rate for the swap maturing at time t_i is s_i. A price P'_i for each fixed leg of each swap is obtained from the following equation (using the OIS discount curve D and treating the fixed leg of the swap as a bond):

$$\begin{bmatrix} s_1+100 & & & \\ s_2 & s_2+100 & & \\ s_3 & s_3 & s_3+100 & \\ s_4 & s_4 & s_4 & s_4+100 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \\ P'_4 \end{bmatrix}$$

The property of a swap is that the price of the fixed leg must equal the price of the floating leg. Using the notation a_i, where a_i is the accumulation factor (generalised relationship of a to a discount factor d is: a =1/d-1 and also $100a_i$ is the interest on the floating leg in period i) for each of the forward periods in the Libor interest generation curve applicable to the swap, we have the following equation:

$$\begin{bmatrix} a_1 & & & \\ a_1 & a_2 & & \\ a_1 & a_2 & a_3 & \\ a_1 & a_2 & a_3 & a_4 \end{bmatrix} \begin{bmatrix} 100d_1 \\ 100d_2 \\ 100d_3 \\ 100d_4 \end{bmatrix} = \begin{bmatrix} P_1' - 100d_1 \\ P_2' - 100d_2 \\ P_3' - 100d_3 \\ P_4' - 100d_4 \end{bmatrix}$$ or alternatively (for easier computation):

$$\begin{bmatrix} 100d_1 & & & \\ 100d_1 & 100d_2 & & \\ 100d_1 & 100d_2 & 100d_3 & \\ 100d_1 & 100d_2 & 100d_3 & 100d_4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} P_1' - 100d_1 \\ P_2' - 100d_2 \\ P_3' - 100d_3 \\ P_4' - 100d_4 \end{bmatrix}$$

In the equation above, one sees that the present value of the floating flows implied off the Libor curve would equate to the value of the fixed leg P', minus the present value of the principal at maturity on the float leg. If one had to solve the equation above for the a_is one can then calculate a d'_n for the Libor basis curve {D'} each time period t_n thus:

$$d'_n = \prod_{i=1}^{n} \frac{1}{a_i + 1}$$

A numerical example is provided on the next page.

Calculated Example

Input Data

03/10/2010

Term	Date	OIS Rate (%) (Annual) Fixed	Discount Factor		Swap Rate (s_i)
ON	04/10/2010	0.50	0.9999863		
1Y	05/10/2011	0.60	0.9940032	**d1**	0.75
2Y	05/10/2012	0.75	0.9851066	**d2**	0.9
3Y	07/10/2013	1.00	0.9704579	**d3**	1.15
4Y	06/10/2014	1.35	0.9476352	**d4**	1.5
5Y	05/10/2015	1.55	0.9258603	**d5**	1.75

Fixed Leg - Calc of Fixed Leg PVs (s_i)					P'
100.75	0	0	0	0	100.1458
0.9	100.9	0	0	0	100.2919
1.15	1.15	101.15	0	0	100.4378
1.5	1.5	1.5	101.5	0	100.6093
1.75	1.75	1.75	1.75	101.75	101.0264

$$\begin{bmatrix} s_1+100 & & & & \\ s_2 & s_2+100 & & & \\ s_3 & s_3 & s_3+100 & & \\ s_4 & s_4 & s_4 & s_4+100 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} P_1' \\ P_2' \\ P_3' \\ P_4' \end{bmatrix}$$

Calc Of Float Leg Flows ($100d_i$)					P'-100d
99.40032	0	0	0	0	0.745502
99.40032	98.510658	0	0	0	1.781199
99.40032	98.510658	97.045786	0	0	3.392003
99.40032	98.510658	97.045786	94.763516	0	5.845804
99.40032	98.510658	97.045786	94.763516	92.58603	8.44036

$$\begin{bmatrix} 100d_1 & & & & \\ 100d_1 & 100d_2 & & & \\ 100d_1 & 100d_2 & 100d_3 & & \\ 100d_1 & 100d_2 & 100d_3 & 100d_4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} P_1'-100d_1 \\ P_2'-100d_2 \\ P_3'-100d_3 \\ P_4'-100d_4 \end{bmatrix}$$

Calc Of Results

	a_i		a_i+1	d'_i	
a1	0.007500		1.007500	0.992556	d'1
a2	0.010514		1.010514	0.989596	d'2
a3	0.016598		1.016598	0.983673	d'3
a4	0.025894		1.025894	0.97476	d'4
a5	0.028023		1.028023	0.972741	d'5

Check

Fixed Leg Flows					Float Leg Flows					Net Flows PV'd				
100.75	0.90	1.15	1.50	1.75	- 100.75	- 0.75	- 0.75	- 0.75	- 0.75	-	0.14910	0.39760	0.74550	0.99400
-	100.90	1.15	1.50	1.75	-	- 101.05	- 1.05	- 1.05	- 1.05	-	- 0.14910	0.09718	0.44196	0.68824
-	-	101.15	1.50	1.75	-	-	- 101.66	- 1.66	- 1.66	-	-	- 0.49478	- 0.15512	0.08750
-	-	-	101.50	1.75	-	-	-	- 102.59	- 2.59	-	-	-	- 1.03235	- 0.79544
-	-	-	-	101.75	-	-	-	-	- 102.80	-	-	-	-	- 0.97430
								Sum		0.000	0.000	0.000	0.000	0.000