Danske Markets

From curves to surfaces

How plain vanilla grew complex

Den Danske Finansanalytikerforening
October 27, 2010

Ove Scavenius, ovs@danskebank.dk
Martin D. Linderstrøm, marlin@danskebank.dk

Agenda

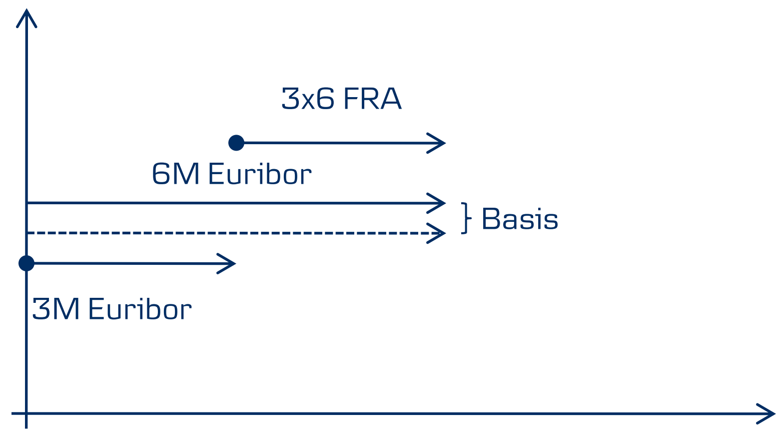
- Stylized facts
  - Money market basis
  - Cross currency basis
  - Turns
- Possible solutions
  - The rate surface setup
- Applications
  - Lessons learnt
  - Collateral consistent pricing
  - Forward starting swaps
  - Asset swaps

STYLIZED FACTS

Stylized facts – money market basis

- xIBOR fixings are supposed to reflect the price of unsecured term liquidity.
  - The deposit market is differentiating credit premia based on term length – so should then the xIBOR fixings.
- How to place money over 6M?
  - Can either do a single 6M deposit…
  - …or do two 3M deposits – which strategy is riskier?
- Term premia
  - Investors require higher compensation for taking on longer dated credit risk.
  - Funding managers are willing to pay this compensation in order to reduce their liquidity risk when re-financing.
- Consequence:
  - You cannot rely on the text-book replication argument between risk-free(!) zero coupon bond prices and (forward) xIBOR rates.

$$F(t,T,T+\delta)=\frac{1}{\delta}\left(\frac{P(t,T)}{P(t,T+\delta)}-1\right)$$



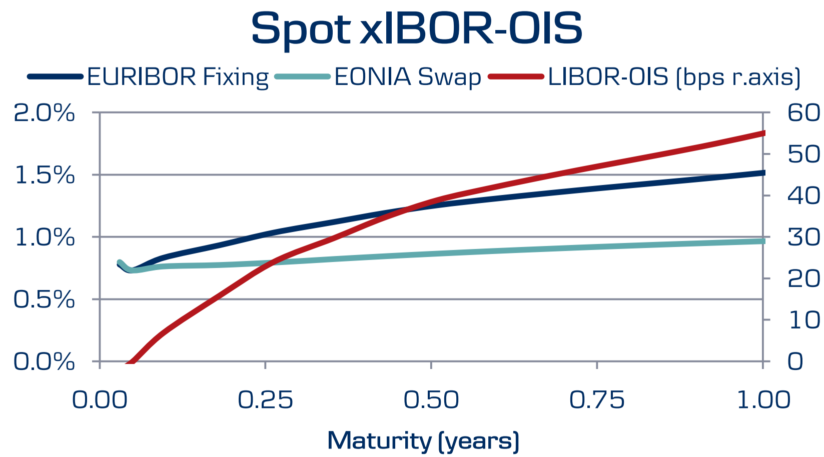
Example: 22 October 2010

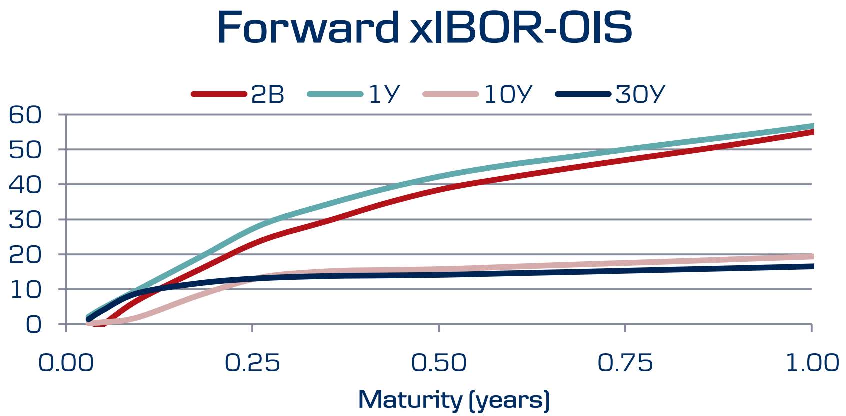
- 3M EURIBOR = 1.029%
- 3x6 FRA = 1.206%
- 6M EURIBOR = 1.250%

→3s6s break = 13.2 bps

Money market basis - markets

- xIBOR-OIS basis
  - We can measure the credit- and liquidity premia by comparing xIBOR rates to OIS rates.
  - This is a tradable spread – sometimes used as a proxy for money market distress.
  - Note that we can use this spread analysis in both spot-and forward terms.





Danske Markets

Money market basis – markets cont'd

03 CurncyDMUS

Danske Markets

USD Interest rate Derivatives Electronic Execution LIVE TRADEABLE

Semi - Bond vs 3m	IMM Swaps	Other
1) 6M - 21M	7) 1Y - 2Y	14) Check My Trades
2) 2Y - 30Y	OIS	15) Sign Up
Annual - Money vs 3m	8) 1M - 24M	16) Leave Feedback
3) 6M - 21M	9) FOMC Runs	
4) 2Y - 30Y		
Annual - Money vs 1m	FRA vs OIS	Contacts
5) 2M - 24M	*10) Whites n reds	Trading:
	Spot Basis	Kenneth Lycik +4545143277
	11) 3M vs 6M	Mikkel Regnarsson +4545147208
Swaps Spreads	Forward Basis	Interbank +4545147296
6) Treasury Spreads	*12) 1M vs 3M	
	*13) 3M vs 6M	

Visit DM <GO> for our complete IRS Offering *Call To Trade

01 CurncyDMEO

Danske Markets

EUR Short End Derivatives LIVE TRADEABLE

Swaps vs EONIA	Spot Basis	Other
1) 1W-12M Spot OIS	*11) OIS vs 1M	17) Check My Trades
2) 13M-3Y Spot OIS	*12) OIS vs 3M	18) Sign Up
3) 3M Fwd	13) 3M vs 6M	19) Leave Feedback
4) 6M Fwd		
5) ECB Runs		
Swaps vs 1m Euribor	Forward Basis	
6) 2M-3Y Spot	*14) OIS vs 3M	
	*15) 3M vs 6M	
Swaps vs 3m Euribor	Bond Future Cross	Contacts
7) 6M-3Y Spot	16) SCHATZ Future Cross	Meiko Dittrich/Trading
8) 1Y Fwd/IMM		Tel: +45 451 46844
Swaps vs 6m Euribor		Matthew Cross/Interbank
9) 9M-3Y Spot		Tel: +45 3334 1016
*10) 1Y Fwd/IMM		

Visit DMEU<GO> for our complete IRS Offering *Call To Trade

DMEU CurncyDMEU

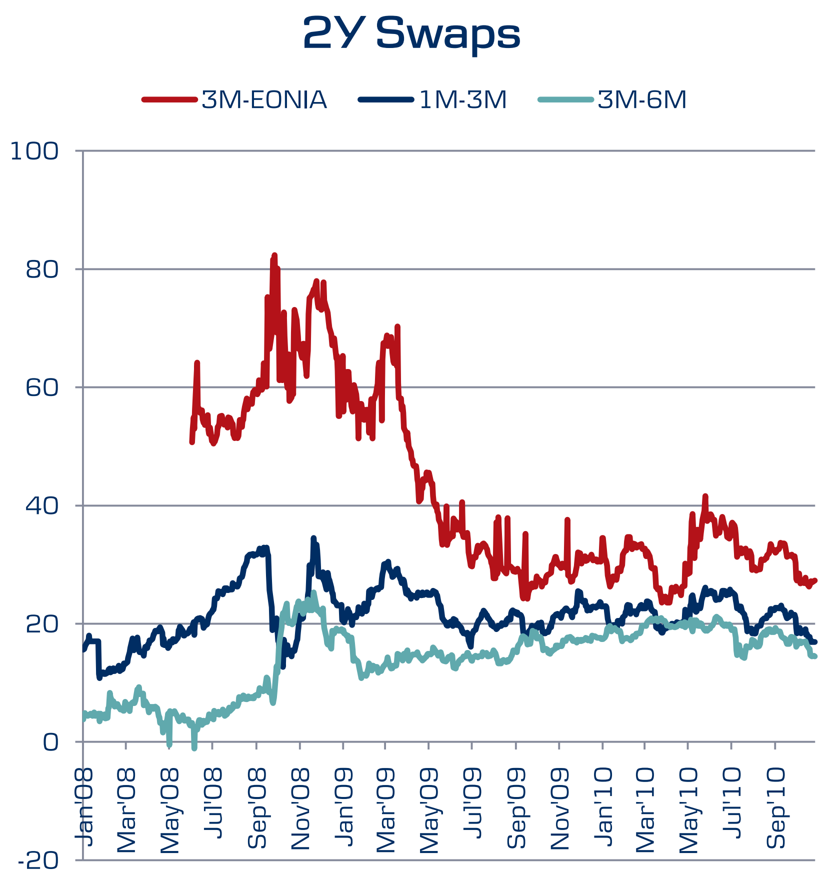
Danske Markets

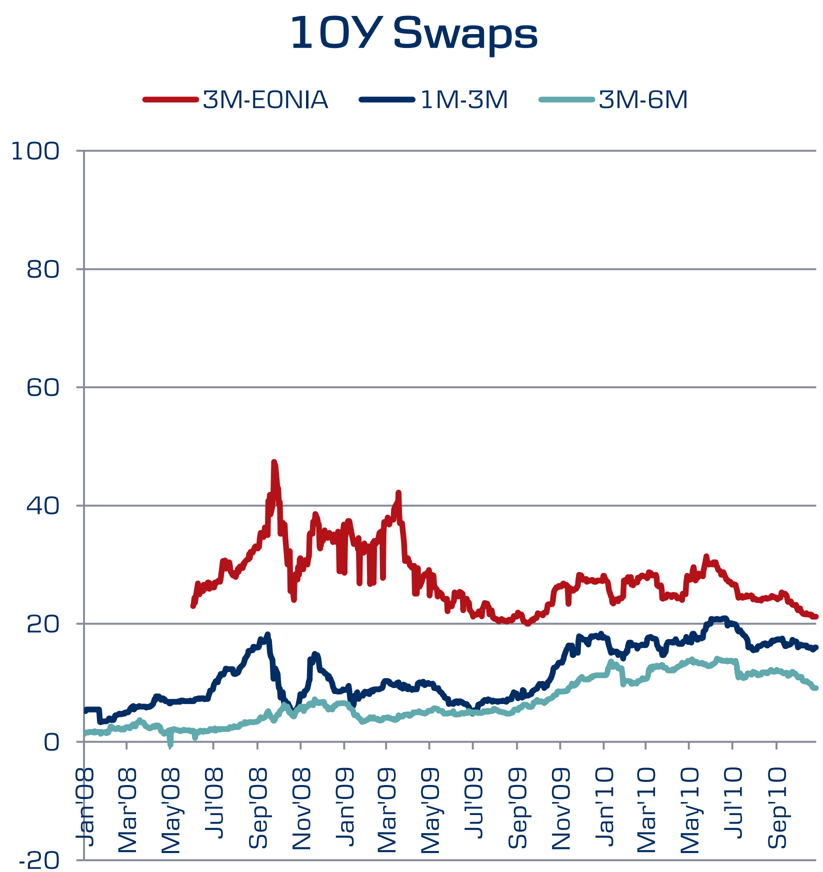
EUR Derivatives Live Tradeable

3 & 6M Euribor IRS	Euribor IRS Basis	Relative Value
1) Annual vs 6M EURIBOR	4) 3s6s Basis 1 Swap	8) IRS BONDFUTURE CROSS
2) Annual vs 3M EURIBOR	5) 3s6s Basis 2 Swap	9) Spot 6M IRS ASW
3) Forward Start IRS	6) 3s6s Fwd Basis 1 Swap	10) IRS Curve Spreads
	7) 3s6s Fwd Basis 2 Swap	11) IRS Curve Overview
		12) Butterflies
		Other
		13) DMEO EUR Short End
		14) Overview
		15) Research
		16) ASW Explanations
		17) Check My Trades
		18) Sign Up
		19) Leave Feedback

- xIBOR-xIBOR basis
  - We can trade floating legs of different tenors against each other.
  - Float-float basis swaps or as a basis swap package of two IRSs.

Money market basis – markets cont'd





Stylized facts - Cross currency basis

- The covered interest rate parity
  - The text books give us a no-arbitrage restriction between spot and forward exchange rates...
  - ...relative to risk-free interest rates.
  - Can be used to imply an interest rate:
    - Where can the average USD LIBOR bank fund itself in EUR relative to EURIBOR?
- Cross currency basis
  - Basically the difference between xIBOR and cash (liquidity) rates.

$$\underbrace{F_{\$/\text{€}}(0,T)P^{\$}(0,T)}_{\text{The time 0 cost in \$ of producing €1 at time T}} = \underbrace{S_{\$/\text{€}}P^{\text{€}}(0,T)}_{\text{The time 0 cost in \$ of producing €1 at time T}} \Leftrightarrow$$

$$F_{\$/\text{€}}(0,T)\frac{1}{1+\delta L^{\$}(0,T)} = S_{\$/\text{€}} * \frac{1}{1+\delta \tilde{L}^{\text{€}}(0,T)} \Leftrightarrow$$

$$\tilde{L}^{\text{€}}(0,T) = \frac{1}{\delta}\left(\frac{S_{\$/\text{€}}}{F_{\$/\text{€}}(0,T)}(1+\delta L^{\$}(0,T)) - 1\right)$$

This is not identical to 3M EURIBOR

Example: 22 October 2010

- 3M USD LIBOR = 0.2884%
- EUR/USD Spot = 1.4066
- 3M EUR/USD = -16.65

→3m FX implied EUR rate = 0.758%

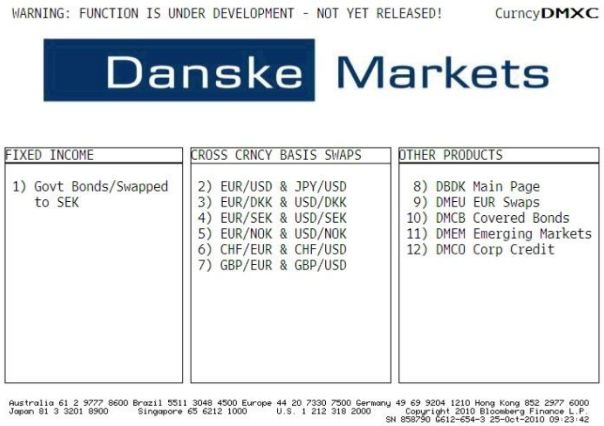
- 3M EURIBOR = 1.029%

→3M EUR/USD break = -27.1 bps

Danske Markets

Cross currency basis - markets

- Short-end
  - Trading activity in FX forward markets.
  - Most liquid until approx. 1Y
- Long-end basis
  - Cross currency swaps have initial and final exchange of notional against 3M xIBOR fixings…
  - …but can also be traded against other xIBOR tenors and even O/N.

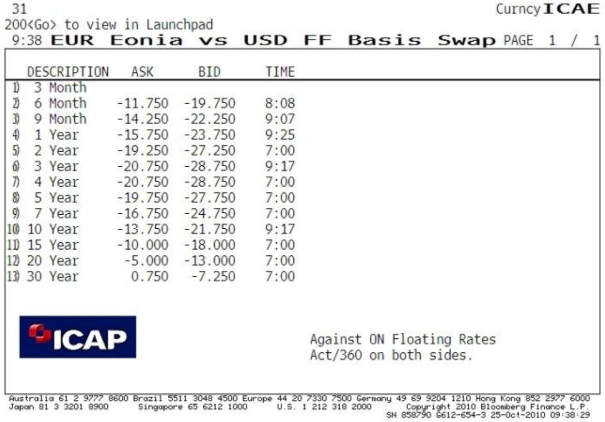


WARNING: FUNCTION IS UNDER DEVELOPMENT - NOT YET RELEASED! CurncyDMXC

Danske Markets

FIXED INCOME	CROSS CRNCY BASIS SWAPS	OTHER PRODUCTS
1) Govt Bonds/Swapped to SEK	2) EUR/USD & JPY/USD	8) DBDK Main Page
	3) EUR/DKK & USD/DKK	9) DMEU EUR Swaps
	4) EUR/SEK & USD/SEK	10) DMCB Covered Bonds
	5) EUR/NOK & USD/NOK	11) DMEM Emerging Markets
	6) CHF/EUR & CHF/USD	12) DMCO Corp Credit
	7) GBP/EUR & GBP/USD	

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
SN 858790 G612-654-3 25-Oct-2010 09:23:42



31 CurncyICAE
200<Go> to view in Launchpad
9:38 EUR Eonia vs USD FF Basis Swap PAGE 1 / 1

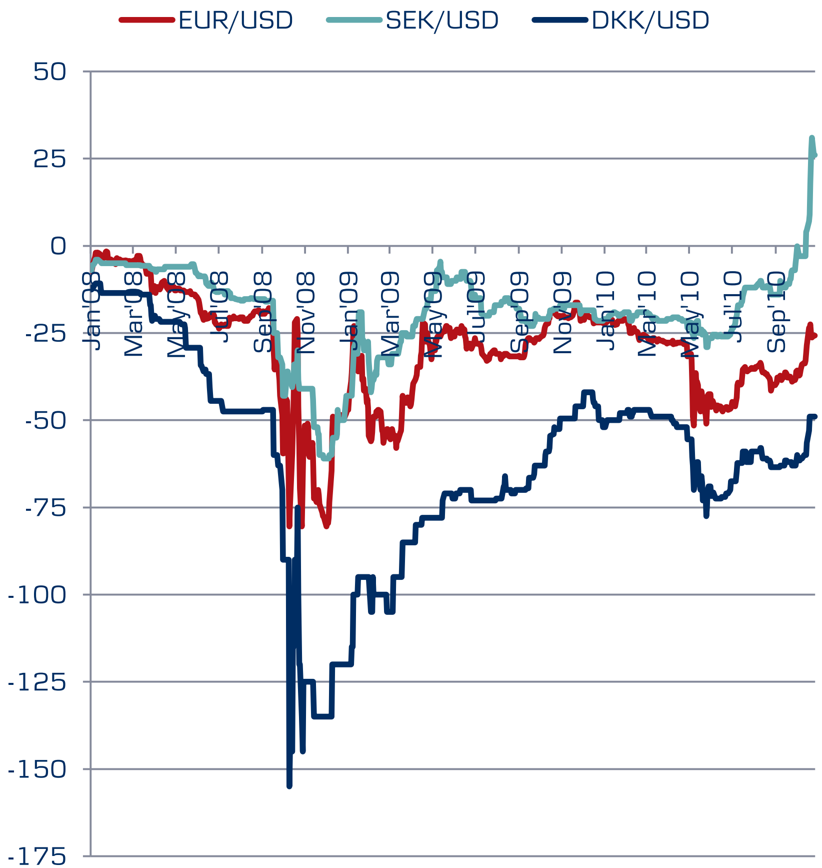
	DESCRIPTION	ASK	BID	TIME
1)	3 Month			
2)	6 Month	-11.750	-19.750	8:08
3)	9 Month	-14.250	-22.250	9:07
4)	1 Year	-15.750	-23.750	9:25
5)	2 Year	-19.250	-27.250	7:00
6)	3 Year	-20.750	-28.750	9:17
7)	4 Year	-20.750	-28.750	7:00
8)	5 Year	-19.750	-27.750	7:00
9)	7 Year	-16.750	-24.750	7:00
10)	10 Year	-13.750	-21.750	9:17
11)	15 Year	-10.000	-18.000	7:00
12)	20 Year	-5.000	-13.000	7:00
13)	30 Year	0.750	-7.250	7:00

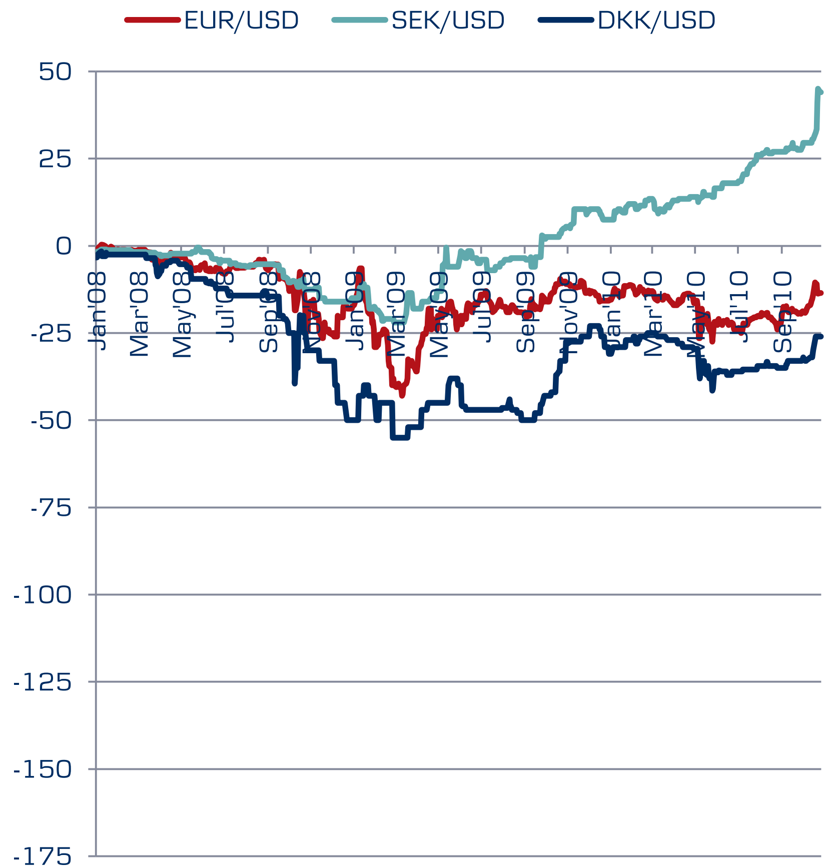
ICAP

Against ON Floating Rates
Act/360 on both sides.

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
SN 858790 G612-654-3 25-Oct-2010 09:38:29

Cross currency basis – markets cont's

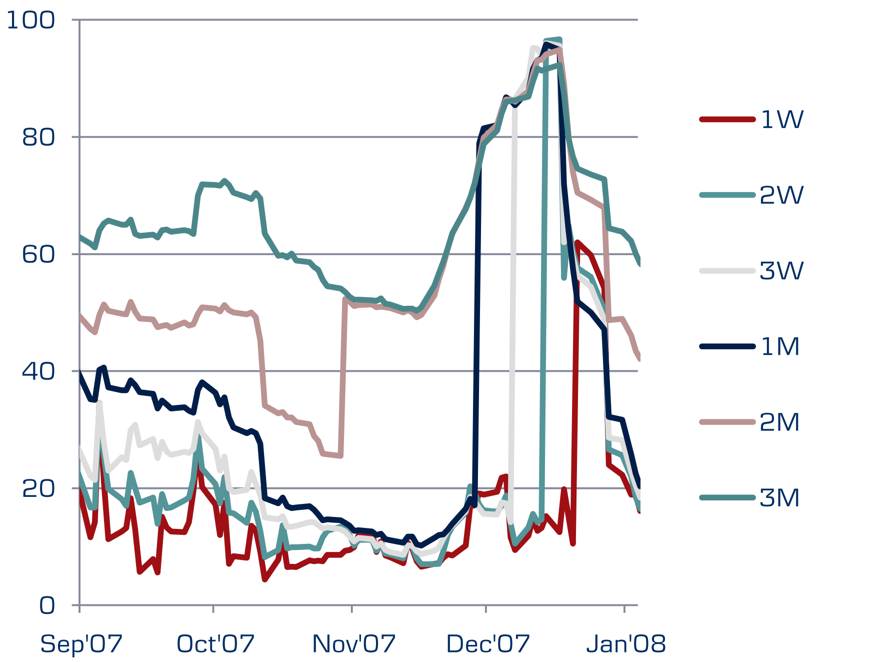




Turns

- Some fixing dates are special
  - Lending out money unsecured over quarter- and year-end is associated with an extra premium…
  - …so a 3M Deposit will generally be more expensive once it covers New Year's…
  - …and so will the xIBOR fixing.
- How much more expensive should the 31st of December be relative to other days?
  - The impact will obviously be bigger on shorter term
  - Size of impact varies from year to year.

EURIBOR-EONIA: 2007 turns



THE RATE SURFACE

The basic requirements

- Forward rates
  - Need to be able to project sensible forward xIBOR rates of all tenors on all future fixing days.
  - Should be consistent with money market basis – also for odd tenors such as 2W or 7M.
  - Basically, a 3D interpolation problem.
- Discount factors
  - One unique discount factor for all cash flows in the same currency.
  - Should be applied to all contracts (FX Forward, IRS, CCS, IRG, Swaptions, FXO, Inflation etc.) of the same credit quality.
  - No arbitrage between contracts.

A concrete example – Schatz spread futures cross

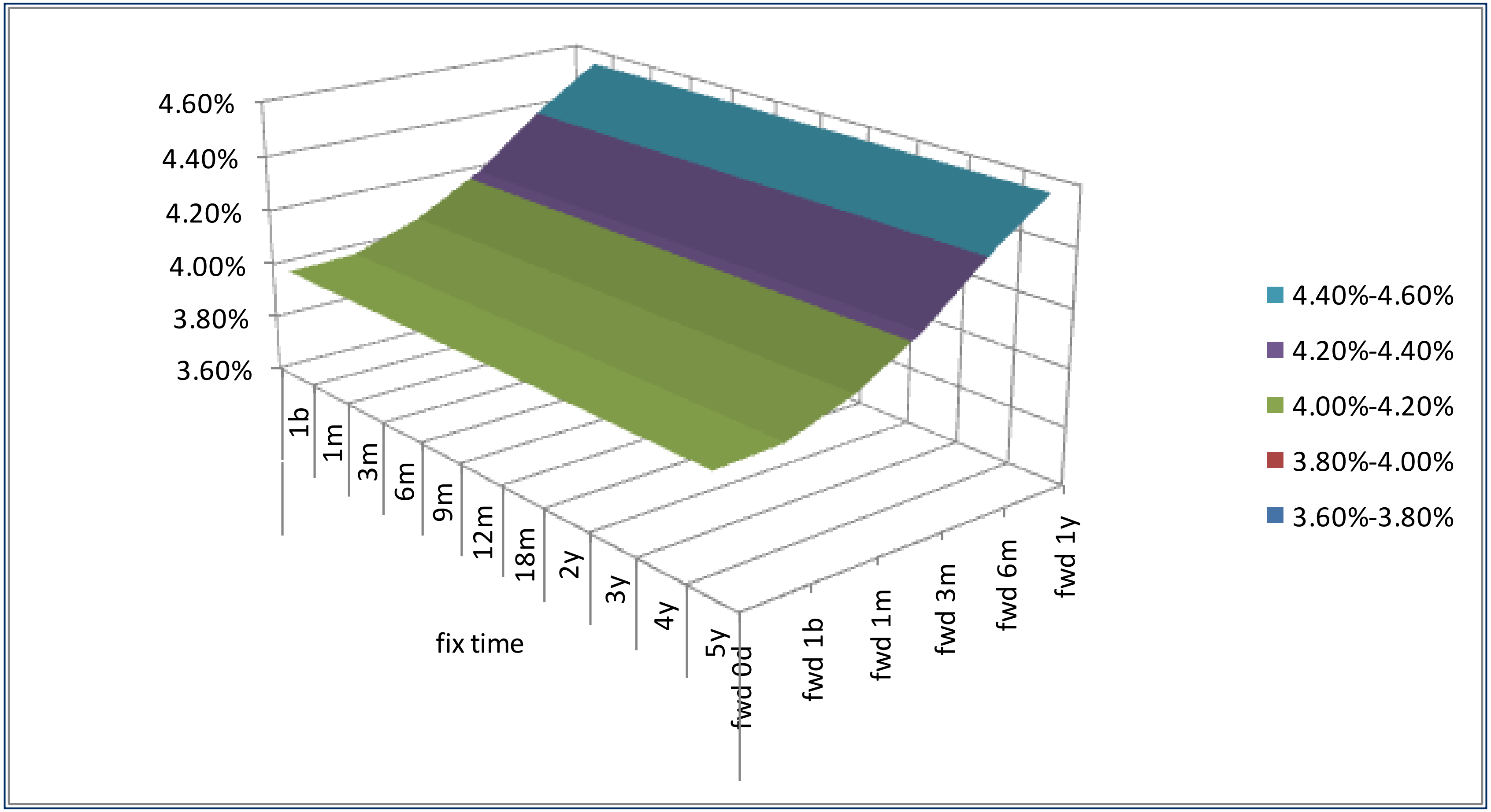
- Against 6M EURIBOR:
  - Forward starting out of 10Dec2010.
  - Maturing 14Sep2012.
  - 94D short stub.
- Challenge:
  - 6M market is not particularly liquid in the short end → Need to spread against 3M futures strip.
  - Want to quote 3M and EONIA indexed variant of the cross as well.

<u>Example: 22 October 2010</u>

- Par rate is 1.575%...
- ...with the stub rate interpolated at 1.156%.
- Had the stub been – incorrectly – calculated on the 6M EURIBOR curve, it had been ≈10 bps higher...
- ...implying a par-rate at 1.590%
- In a 1.3bp wide market.

→We cannot simply rely on using a single ”suitable” curve for each IRS.

The surface interpretation



The model and some formulas

- One curve bootstrapping, libor discounting
- Cross currency swaps and two curves
- Multi curves with tenor swaps
- Multi curves and OIS discounting

From the text-books: One curve bootstrapping, libor discounting

1. The short end of curve consists of fra/fut/dep and is trivial to calibrate, i.e. we just set discount factors using

$$D_j = \frac{D_{j-1}}{1 + F_{j-1} \cdot \delta_j}$$

2. For the long end we use swaps

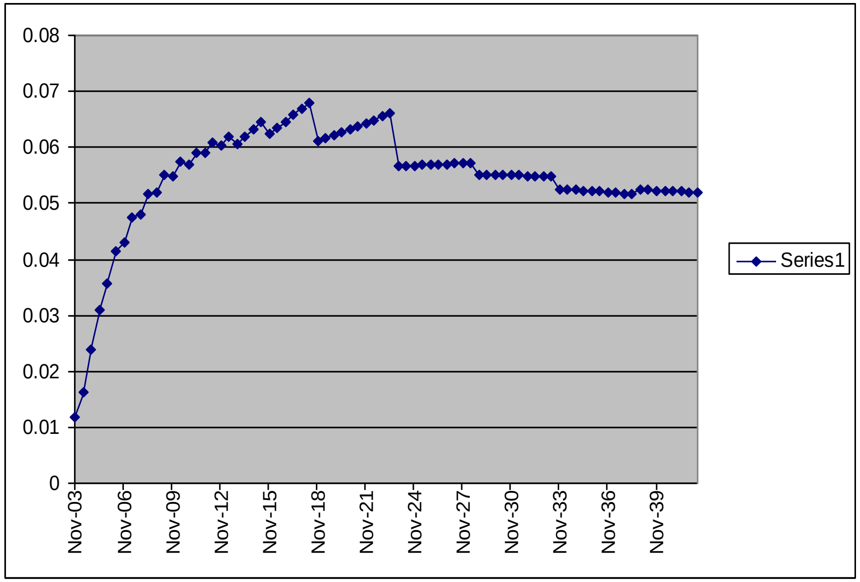
$$S_n \sum_{i=1}^{n} \delta_i \cdot D_i = \sum_{j=1}^{n} F_{j-1} \cdot \delta_j \cdot D_j$$

3. And using formula in 1 we get the formula

$$D_n = \frac{D_0 - S_n \sum_{i=1}^{n-1} \delta_i \cdot D_i}{1 + S_n \delta_i}$$

Old school – what many off-the-shelf systems still do

- Libor curves generated from many different instruments with different terms.
- For instance:
  - –ON
  - –TN
  - –1W
  - –2W
  - –…
  - –FUT MAR : 3m
  - –…
  - –SWAP1Y : 6m
  - –SWAP2Y : 6m
  - –…
- Linear interpolation (in zero coupon rates) gives this saw shape (in forwards)… special assumptions must be made regarding transition of curve segments etc etc.
- Some systems still use this methodology.
- Using e.g. cubic splines and simultaneous calibration will smooth the curve.



Cross currency swaps and two curves

- The system of equations now contains one more (omitting the deposits/fra/fut):

$$\sum_{i=1}^{n} F_{i-1}^{3m} + C \quad \cdot \delta_i \cdot D_i^D - D_0^D + D_n^D = \sum_{j=1}^{n} F_{j-1}^{3m} \cdot \delta_j \cdot D_j^{3m} - D_0^{3m} + D_m^{3m}$$

$$S_n \sum_{i=1}^{n} \delta_i \cdot D_i^D = \sum_{j=1}^{n} F_{j-1}^{3m} \cdot \delta_j \cdot D_j^D$$

- The right hand side of the equation for the CCS is the base part, e.g. USD using the 3M forward curve to discount.
- The value of such a leg with notional exchange is zero if the discount rate is that of libor (3m). So the base (domestic) curve can be constructed as above. Consequently the equation to solve for (foreign) discountfactors is:

$$\left(\sum_{i=1}^{n} (S_n \delta_i + C \delta_i) \cdot D_i^D \right) = D_0^D - D_n^D$$

- Then solve for the forward curve using the discount factors and the first equation. In practice this is done simultaniously using an optimiser. We want the discount curve as a spread and use non-linear interpolation.

Cross currency swaps and two curves

- Using this procedure, a consistent set of curve pairs can be obtained.
- The bank should choose one base currency to avoid internal arbitrage.
- The value of products that depends on discounting are strongly influenced when the basis spreads are large.
- Such products are for instance forward starting swaps.
- The risk of a swap desk becomes more complicated when we consider basis risk.

Multi curves with money market basis swaps

- The system of equations now contain one more (omitting the deposits/fra/fut):

$$\sum_{i=1}^{n} F_{i-1}^{3m} + C \cdot \delta_i \cdot D_i^D - D_0^D + D_n^D = \sum_{j=1}^{m} F_{j-1}^{3m} \cdot \delta_j \cdot D_j^{3m} - D_0^{3m} + D_m^{3m}$$

$$\sum_{i=1}^{n} (F_{i-1}^{3M} + B) \cdot \delta_i \cdot D_i^D = \sum_{j=1}^{m} F_{j-1}^{6M} \cdot \delta_j \cdot D_j^D$$

$$S_n \sum_{i=1}^{n} \delta_i \cdot D_i^D = \sum_{j=1}^{m} F_{j-1}^{6m} \cdot \delta_j \cdot D_j^D$$

- The base leg of the CCS equation is still zero. Note that the tenor of the swap equation is now 6m as is usually the convention, whereas the tenor of the CCY is 3m. We need an equation of the spread between 3m and 6m if the tenor spread is non-zero. Consequently the equation to solve for (foreign) discountfactors is:

$$\left(S_m \sum_{i=1}^{m} \delta_i \cdot D_i^D \right) + \left(\sum_{i=1}^{n} \delta_i \,(C - B) \cdot D_i^D \right) = D_0^D - D_n^D$$

- Then solve for the forward curve using the discount factors and the first equation. Again, in practice, this is done simultaneously using an optimiser.

Multi curves with money market basis swaps

- More tenors
  - Of course, any number of spread equations can be added, i.e. 1b, 1m and 12m
  - 1b or OIS is a bit special, we shall see that later
  - The risk picture is even more complicated now
  - One can start to understand why we call it a surface, in particular, when we think about how to obtain stubs
- Market restrictions
  - It is not trivial to obtain nice smooth surfaces for most currencies, not even he liquid ones as EUR and USD.
  - Some currencies like DKK are illiquid but can be constructed as spreads to liquid currencies e.g. EUR…
  - …this makes the risk quite evolved…
- Discounting revisited
  - Finally, the market has begun to shift towards OIS based discounting (more on this later)…
  - …so we need to allow for even more flexible surface calibration.

Multi curves and OIS discounting

- The system of equations for domestic rates now contain one more (omitting the deposits/fra/fut):

$$O_n \sum_{i=1}^{n} \delta_i^{fix} \cdot D_i^{OIS} = D_0^{OIS} - D_m^{OIS}$$

$$\sum_{i=1}^{n} (F_{i-1}^{3M} + B) \cdot \delta_i \cdot D_i^{OIS} = \sum_{j=1}^{m} F_{j-1}^{6M} \cdot \delta_j \cdot D_j^{OIS}$$

$$S_n \sum_{i=1}^{n} \delta_i \cdot D_i^{OIS} = \sum_{j=1}^{m} F_{j-1}^{6m} \cdot \delta_j \cdot D_j^{OIS}$$

- In this case, we first need to solve the above system, before we can solve for the foreign system.
- The reason is that the base leg of the CCS is no longer zero!
  - Many off-the-shelf systems disregard this.
- So first solve the OIS part, and then solve the rest as before.
- Now let us look at the foreign system:

Multi curves and OIS discounting

- Basically, the system looks as before,

$$S_n \sum_{i=1}^{n} \delta_i \cdot D_i^D = \sum_{j=1}^{m} F_{j-1}^{6M} \cdot \delta_j \cdot D_j^D$$

$$\sum_{i=1}^{n} F_{i-1}^{3M} + C \cdot \delta_i \cdot D_i^D - D_0^D + D_n^D = \sum_{j=1}^{m} F_{j-1}^{3M} \cdot \delta_j \cdot D_j^{OIS} - D_0^{OIS} + D_m^{OIS}$$

$$\sum_{i=1}^{n} (F_{i-1}^{3M} + B) \cdot \delta_i \cdot D_i^D = \sum_{j=1}^{m} F_{j-1}^{6M} \cdot \delta_j \cdot D_j^D$$

- Again, the base leg of the CCS is no longer zero!
- Solve for discount factors.
- And then solve for the forward curve using the discount factors and the first equation. Again, in practice, this is done simultaneously using an optimiser.

$$\left(S_m \sum_{i=1}^{m} \delta_i \cdot D_i^D \right) + \left(\sum_{i=1}^{n} \delta_i \, (C - B) \cdot D_i^D \right) - V_{base} = D_0^D - D_n^D$$

Multi curves and OIS discounting

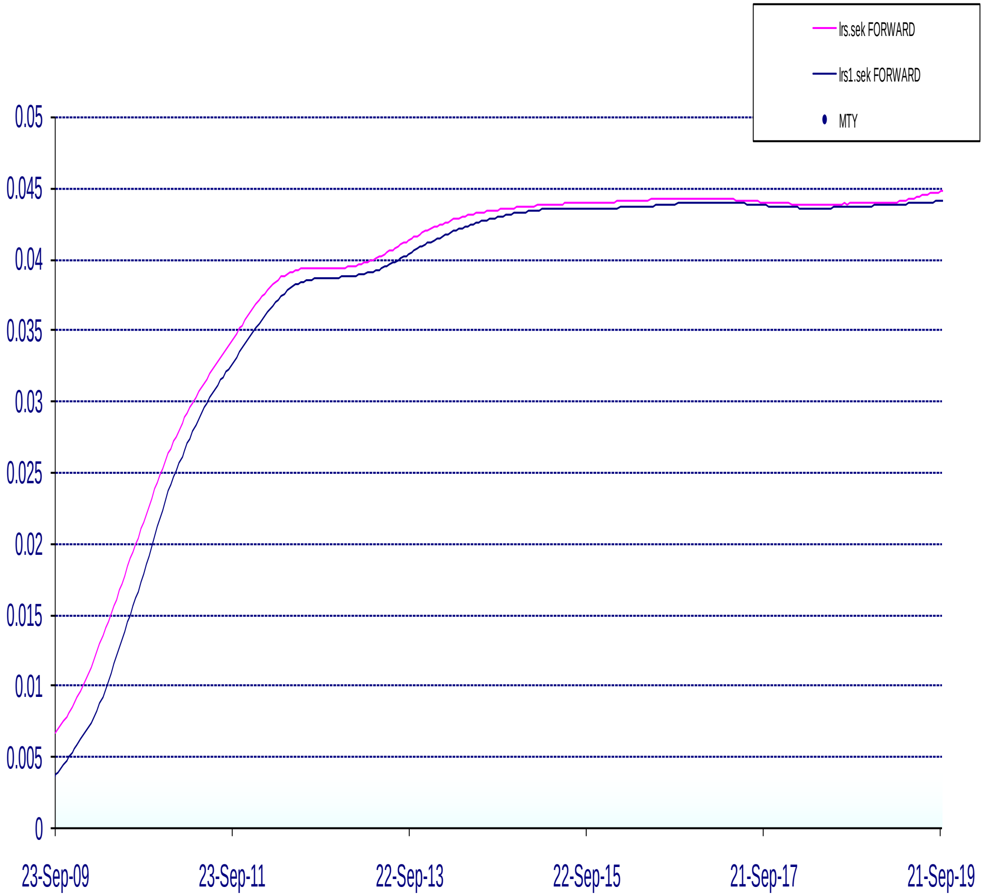
- OIS discounting, CCSs and money market basis swaps make the construction quite non-trivial.
- Even more so the risk…
- But correct discounting absolutely crucial in order to value terminations and forward starting swaps correctly.

The model in practice

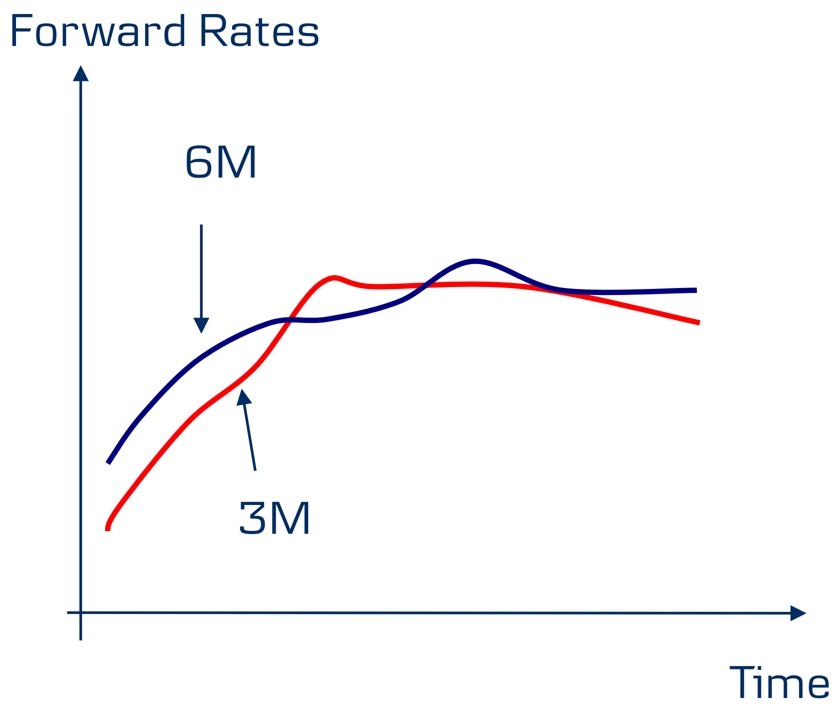
1. Introduction
2. Spreading
3. Setup
4. Instruments
5. Calibration
6. Direct fixing
7. Direct fixing and zero coupon simultaneously
8. Ois
9. Turns
10. Stubs
11. How it is used in the bank
12. examples

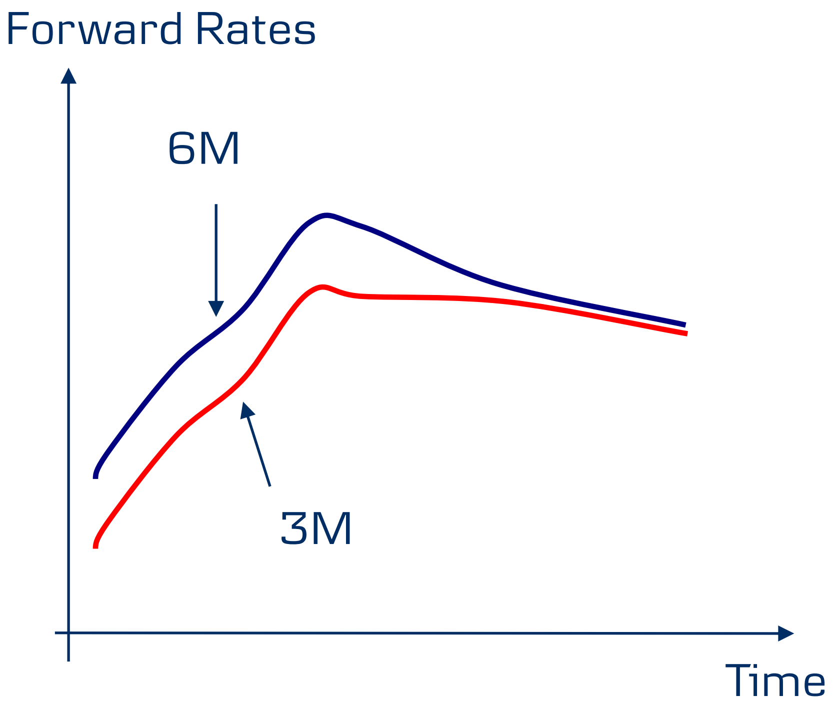
Introduction

- Term specific curves needed, i.e. different curves for OIS (overnight indexed swap), 1m, 3m 6m, 12m.
- We need a discount curve as well.
- For instance 3m:
  - Deposit 2b 3m
  - FUT MAR : 3m
  - ...
  - SWAP1Y : 3m
  - SWAP2Y : 3m
  - ...
- Can cause problems for zero coupon based curves as extrapolation is needed in start of curve: Wiggles or other irregularities.
- Also difficult to get the relative position of curves right.



Spreading





Seperate curve generation:

- Indirect basis risk
- Very difficult to build

Spreading to other curve:

- Direct basis risk
- Better basis PnL
- Easy to build

Setup

- Discount curve:
    - Base ccy via OIS or 3M or ?
    - Other ccys using cross currency swap (CCS) and/or FX Forwards
- Term curves:
    - 1M, 3M, 6M,and12M
    - OIS curve (1B)
- A specification for obtaining other terms, e.g. 2M or 70d
- Interpolation/extrapolation: Flat, linear, hermite, tension splines, cubic splines, akima etc

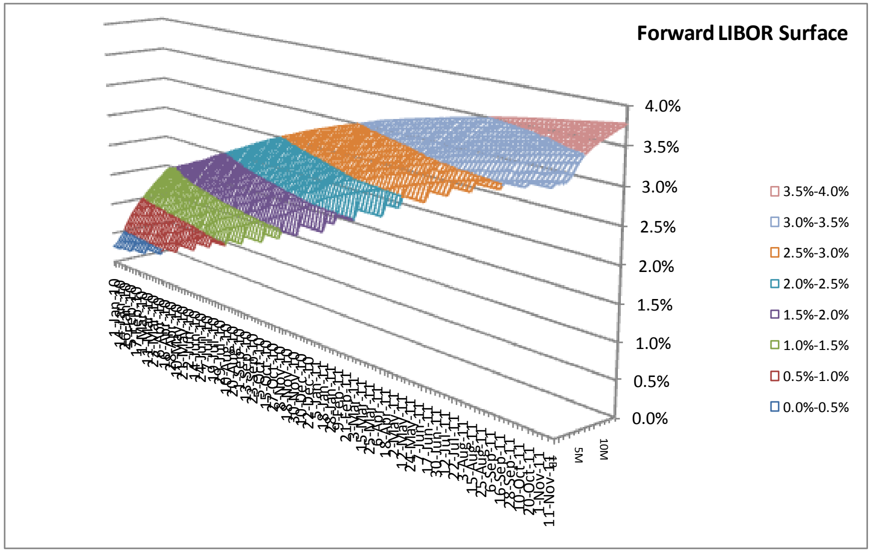
Instruments

- Forward/fixing and discount curves:
  - FRA/FUT/DEPOSIT
  - SWAP
- Forward/fixing and discount curves, spread instruments:
  - BASIS SWAP
  - FORWARD BASIS SPREAD
  - SPREAD SWAP
- Only discount curve :
  - FX forward and CCS with possible non-par base leg and special mark to market feature.
- For spot starting swaps, tenor swaps and CCSs, the first fixing is fixed during the day and should be possible to set from outside.

Danske Markets

Super Surface

- Input
  - This surface is normally generated from OIS, 1M, 3M, 6M and 12M quotes
- Calibration
  - In general, it must be generated simultaneously using an optimiser.
  - However, calibration times can be optimised by not calibrating all together, independent slices can be calibrated one at a time.
  - The calibration time for each model can be reduced to less than 1 second by extensive use of caching.
- Dependencies
  - Note also that some models need other models when calibrating, i.e. if USD is base ccy and we discount with OIS or we use FX forwards in the short end…
  - …can even let the model depend on the bond futures that drive most markets.



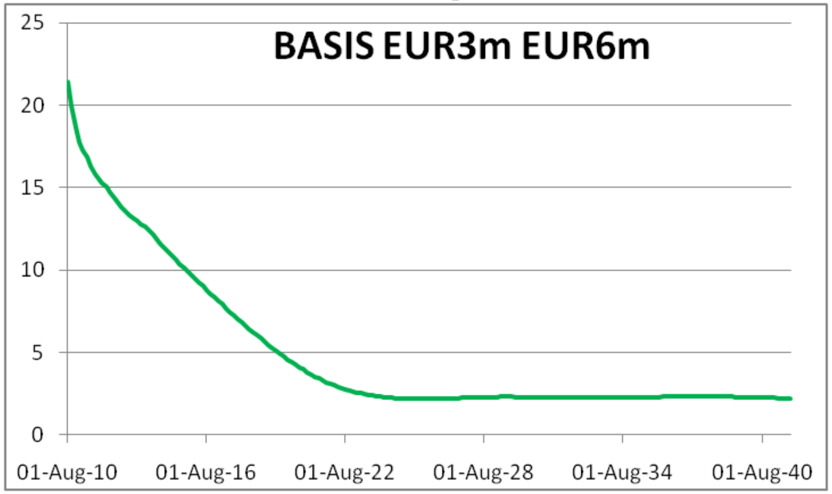
Direct fixing?

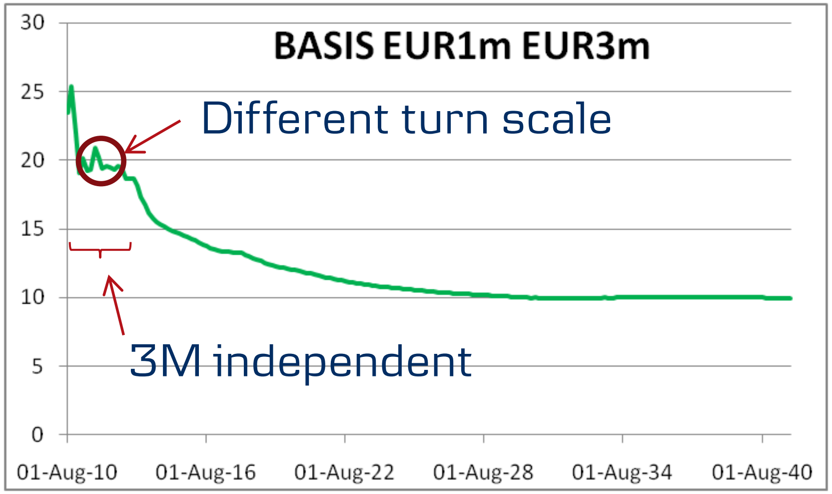
- Zero coupon representation:
  - For the forward curve, a discount factor or zero coupon rate is a rather virtual object.
  - It does not make sense to calculate 3 month forward rate on a 12 month curve.
- We really just need a fixing curve
  - So it makes actually sense to generate an underlying direct forward curve instead of zero coupon curve .
  - The curve usually looks nicer in short end, however, the long end can get unstable.
  - Moreover, the short end of 3m seems to be linear in forward rates, rather difficult to reconstruct in zero coupon rates

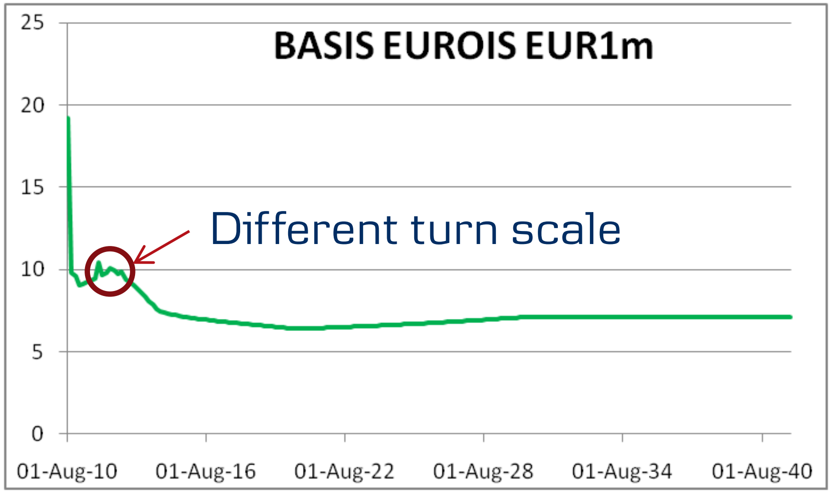
Direct fixing and zero coupon at the same time

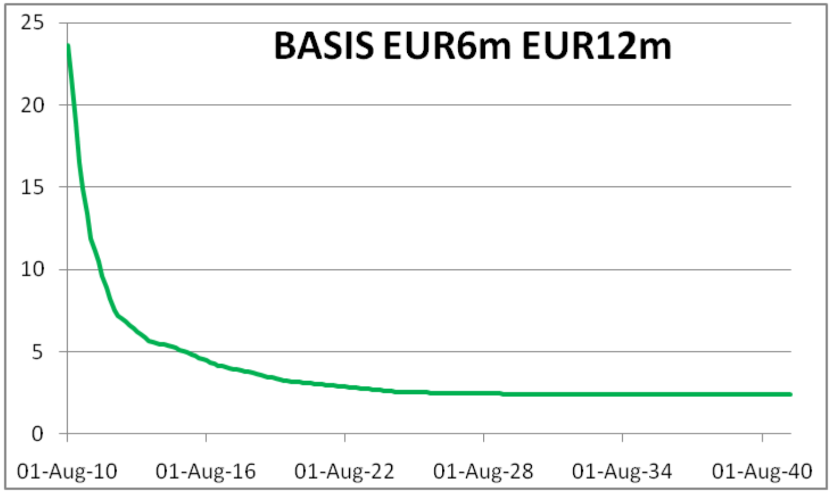
- Market observation
  - Curves for many currencies are linear in the short end, say up till 3 years after which they become more smooth
  - So need to be able to mix interpolation, direct fixing and zero coupon methods in the same curve or slice of surface
- Transition solution
  - One could make a smooth style mixing of direct forward versus zero coupon style by allowing curve to be both, as function of a time dependent parameter.
  - This method probably requires adjustable interpolation, like tension splines
- Complete seperation
  - Another way, which is implemented now is a brute force way of having two curves, e.g. one independent for short end and one dependent for long end. This is implemented recently, with success
  - Moreover, fwd surfaces usually look nice, but traders also want basis to look nice, and direct fixing and spreading help in that respect.

EUR surface – spreads



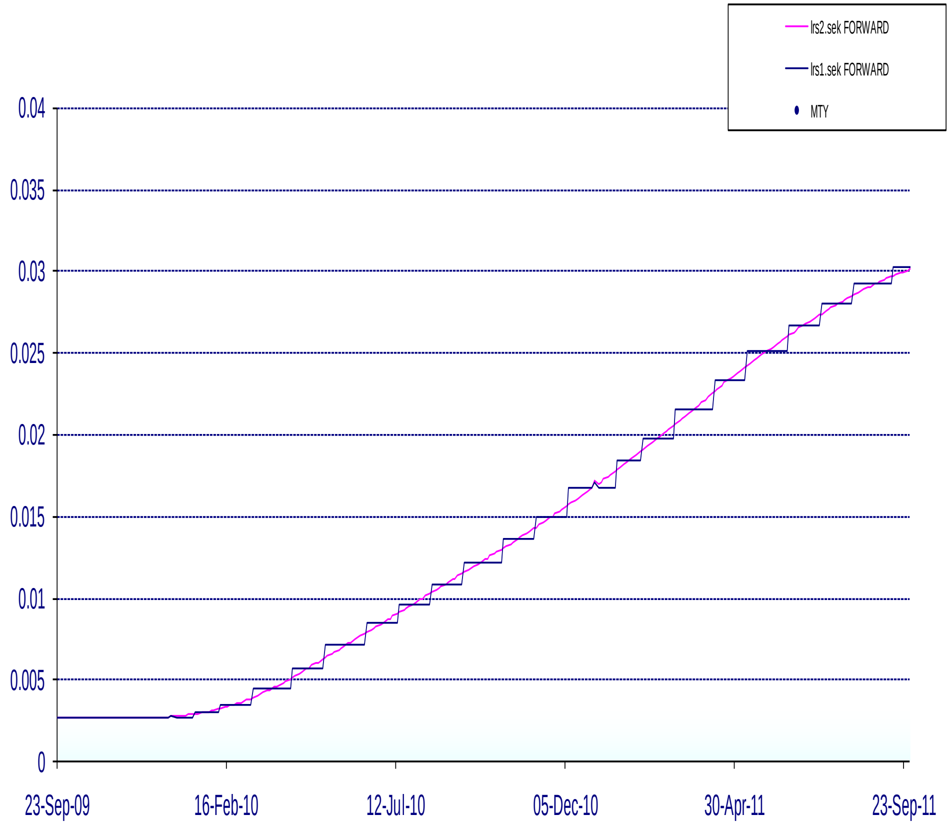




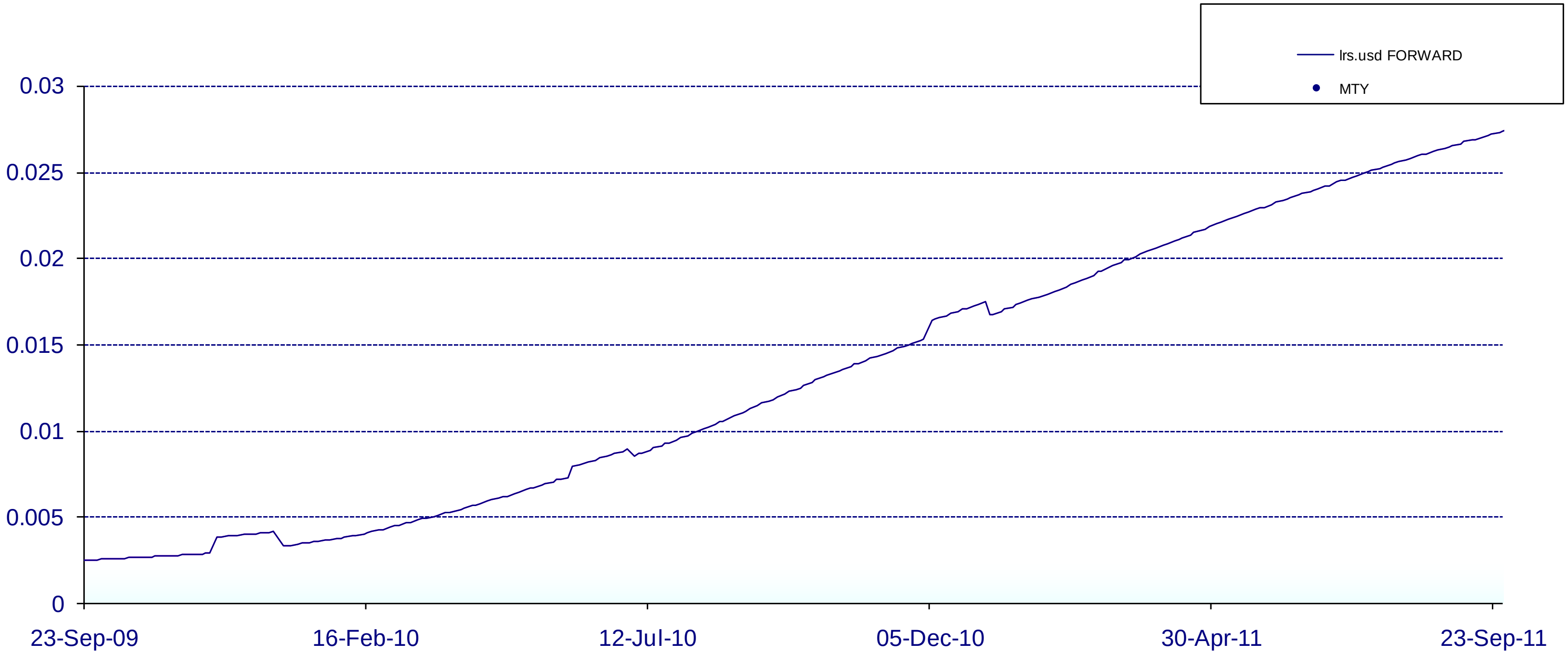


OIS

- Is usually generated via 3m IMM dated FRA-OIS spreads, deposits and swaps
- The 1d rate is constant between central bank meeting days.
- This reflects itself in a staircase shaped curve, at least in the short end.
- Is generated by ”overlaying” a step curve with knots at meeting dates.
- For zero coupon based OIS, reciprocal interpolation must be used.
- Step dates are inputs.



Turns



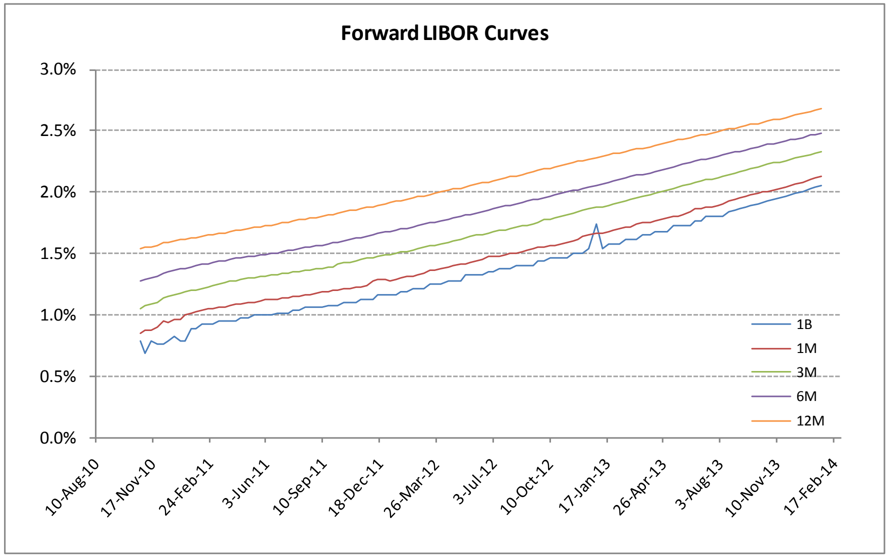
- Is seen as ”boxes” on the forward curve,
- (Date, Value) is input, where value is term specific
- Challenge is to ensure smooth underlying money market basis curves.

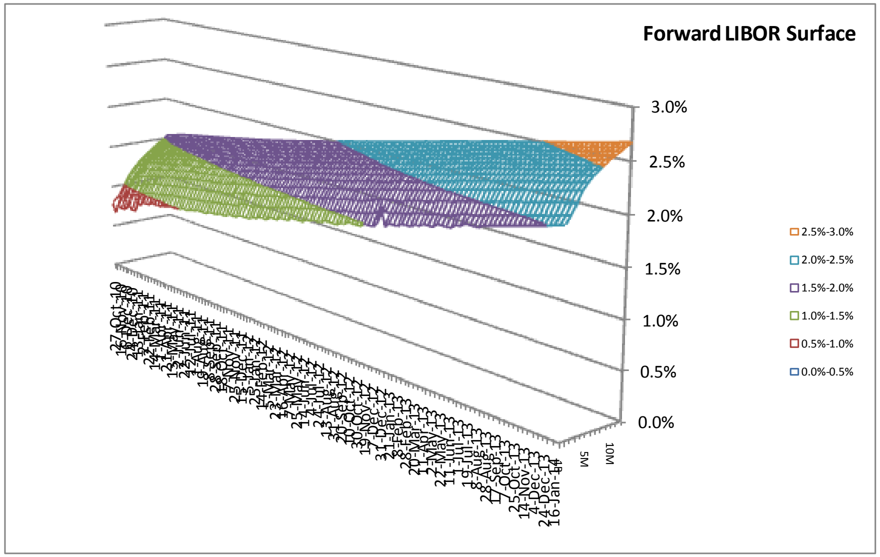
Stubs

- The surface is usually spanned by the terms: 1B, 1M,3M,6M and 12M
  - However, in principle, the fixing entities (EBF, BBA etc.) also have fixings for other terms: 1W, 2W, 3W, 1M, 2M, 3M, 4M, 5M, 6M, 7M, 8M, 9M, 10M, 11M and 12M.
  - The stub floating rates are interpolated by using a spline through the surface spanning fixings(i.e. (1b) ,1m,3m,6m and 12m). But without adding their respective turns.
- The turn for each market fixing is added after interpolation.
- Everything in between is linearly interpolated with turns.

Danske Markets

EUR

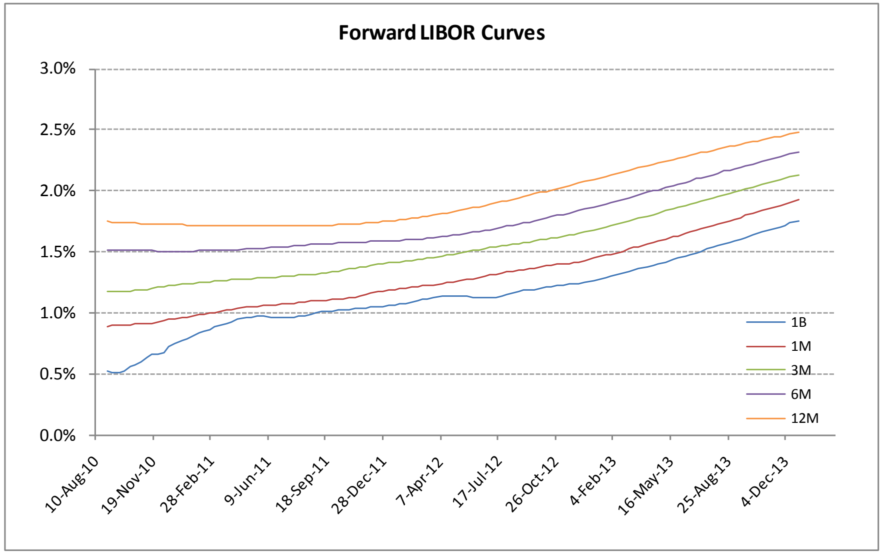


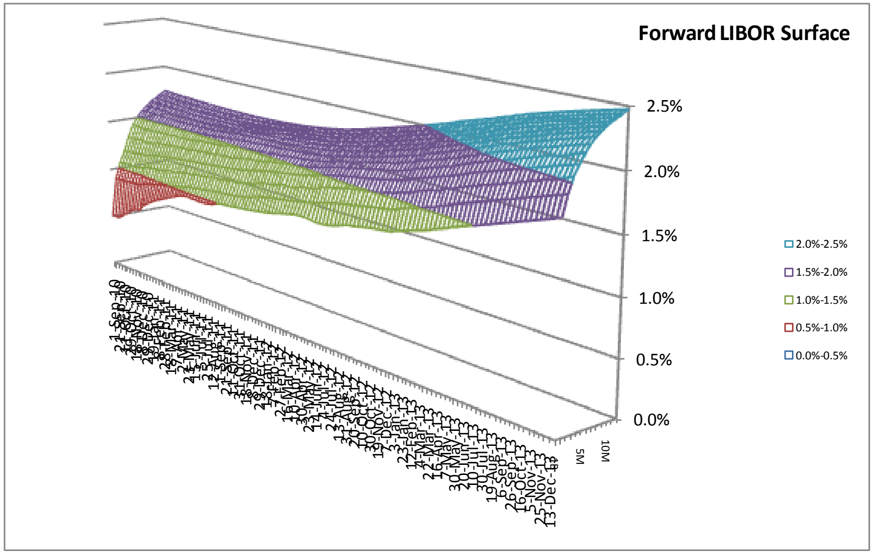


- The most liquid surface – many market quotes to check against…
- …but also the smallest margin of error.

Danske Markets

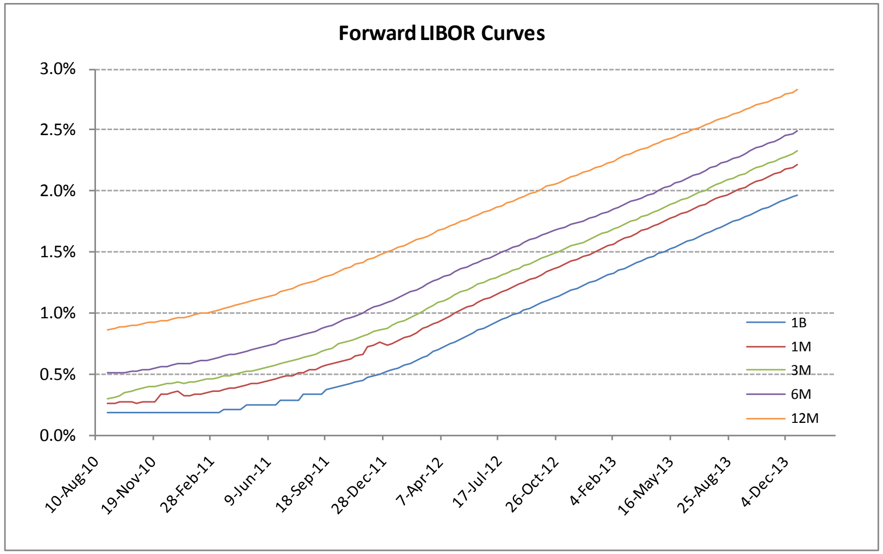
DKK

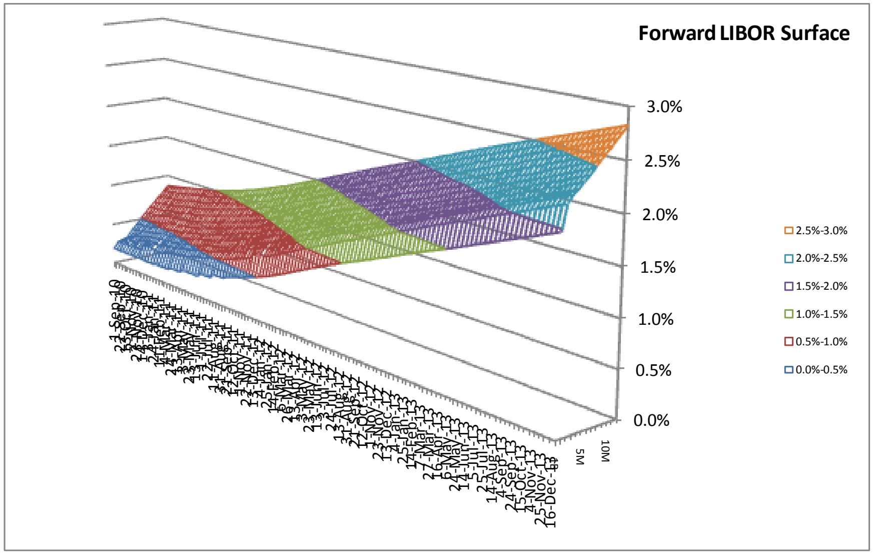




- Trades as a spread to EUR and can be calibrated as such…
- …the DKK model can ”inherit” shapes from EUR in terms of both outright and money market basis spreads.

USD



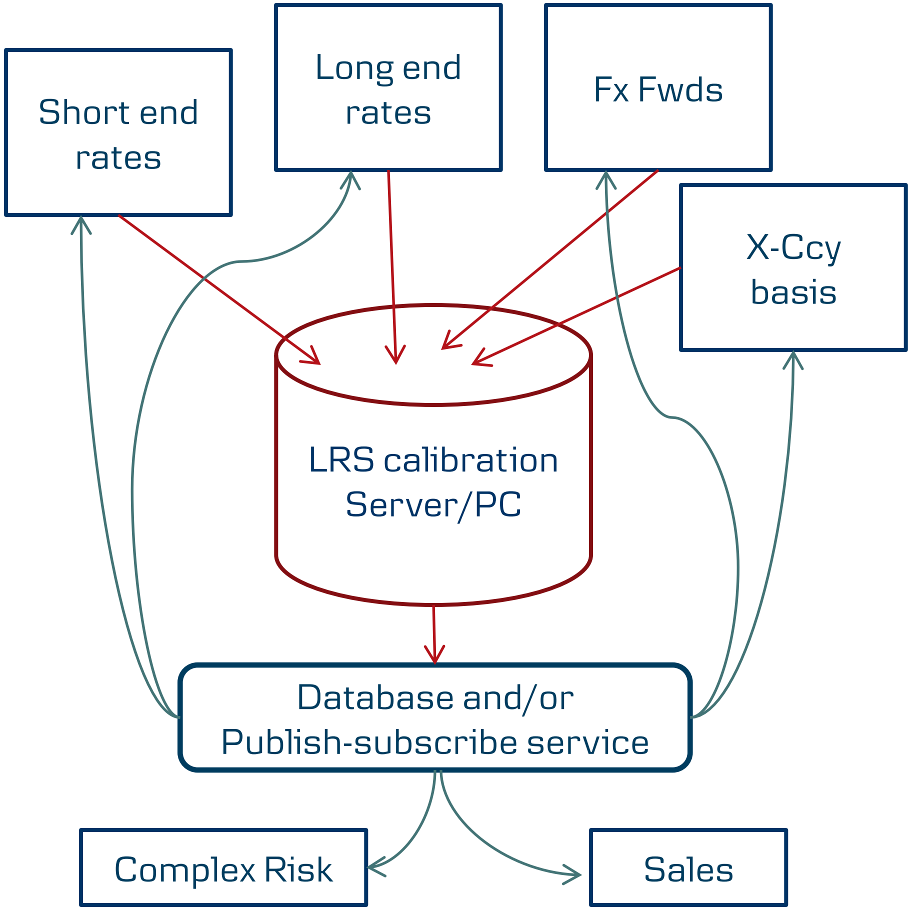


- Is ”special” in the sense that most CCS swaps are quoted against USD…
- …so a natural input to other models.
- Note that the DKK model can be spreaded to EUR but can still CCS quotes against USD.

APPLICATIONS

Leasons learned: One Linear Rate Surface for everyone

- Collecting calibration inputs from the different trading units and building one LRS model per currency
  - Central calibration giving consistent pricing across the desks
  - Presents a cultural "revolution"...
  - ...and presents many practical problems when running across 5 (6) geographically seperated trading floors.
  - Requires much attention to fall-back schemes and back-up data feeds.
  - Need for real-time data transmission with multiple subscribers.



Leasons learned – how much complexity do we need?

- 1st shot – abuse existing single curve system.
  - Continue to use a single zero curve for forwarding and discounting – let the curve depend on the instrument you are pricing.
  - This means pricing a 4% fixed leg different when against 3s or 6s.
  - Makes CCS pricing impossible.
- 2nd shot – seperate forward and discounting curves.
  - This can be done in many of-the-shelf systems.
  - By treading carefully, you can potentially price most swaps except float-float MM basis swaps and stub handling.
  - But the standard systems typically fail, when it comes to calibrating CCS consistent curves where the USD-leg is not at par or if you want spread instruments in calibration.
- 3rd shot – full surface implementation.
  - Needed for sharp pricing in a multi product trading environment.

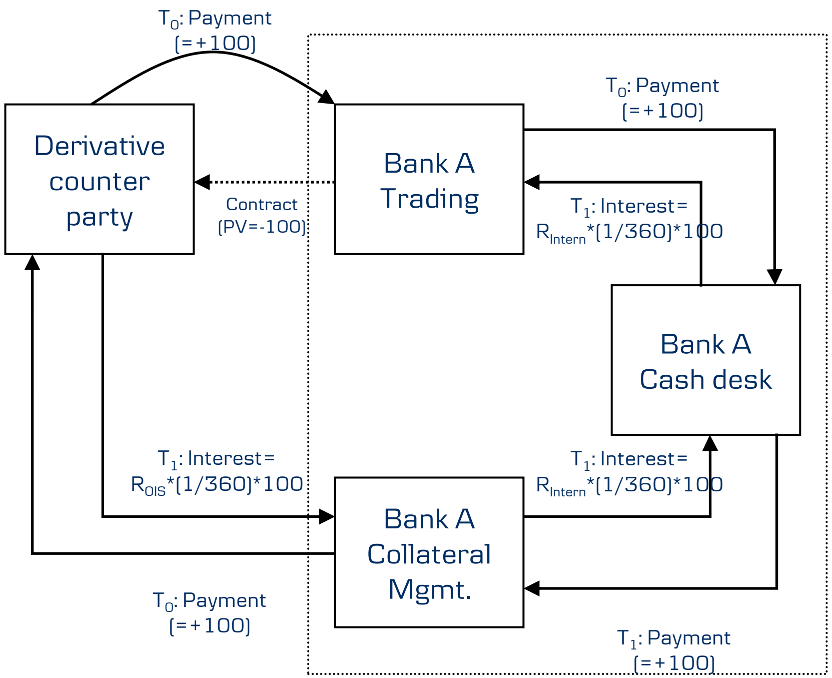
Collateral consistent pricing

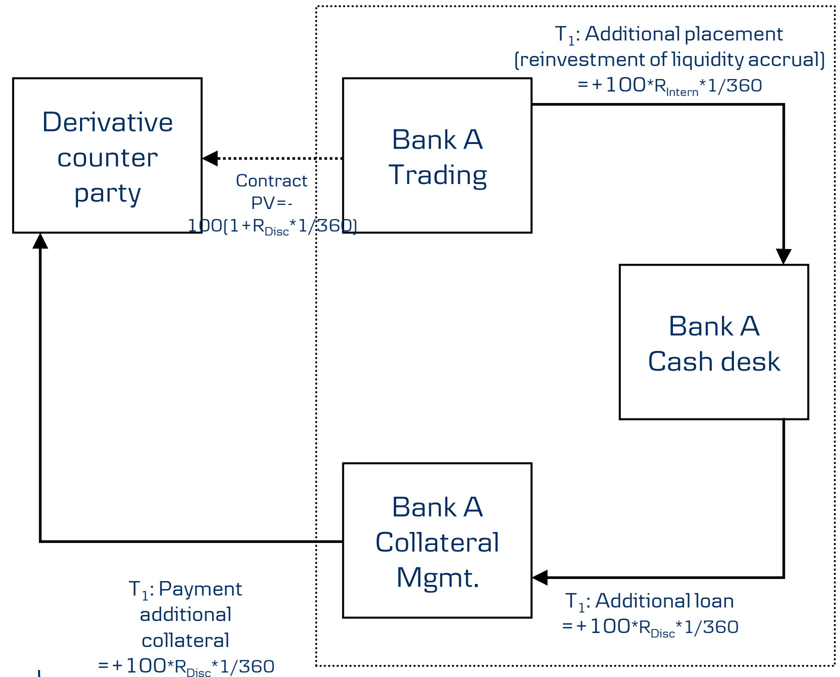
- The surface allows for an arbitrary choice of discount curve.
  - We only need to choose an anchor discount curve in a single currency
  - CCS and FX Forward markets can then be used to infer the discount curves in every other currency.
- The important choices:
  - What is your anchor currency (EUR or USD)?
  - What is your anchor credit premium (O/N, 3M xIBOR or funding curve)?
- We need to make this choice consistent with our collateral.
  - Typically we either have none or have a Collateral Support Annex (CSA) in place under the ISDA Master Agreement.

Collateral

- Credit Support Annex (CSA).
  - What does it do?
  - Paragraph 11/13 – Elections and variables → Tells you implicitly where you can place/borrow funds in the eligible currencies…
  - …or rather, where you are required to place/borrow.
- What are the market standard terms?
  - There is no such thing…
  - …but many Euro zone banks have begun to use Euro cash as collateral (for operational reasons) and this typically earns EONIA → This is then you collateral rate.
- OIS discounting
  - Is thus derived from the underlying CSA – but there can be large variations to the EONIA benchmark.

The possible setup in a capital markets operation





- Cash desk is passing through the liquidity – no haircuts or disagreement on valuation.
- Loop can be ”closed” if $r_{Intern}=r_{OIS}$

- For the setup to be arbitrage free, the trader needs to be discounted at the rate his cash position earns.
- He could in principle hedge his cash exposure via an EONIA swap.

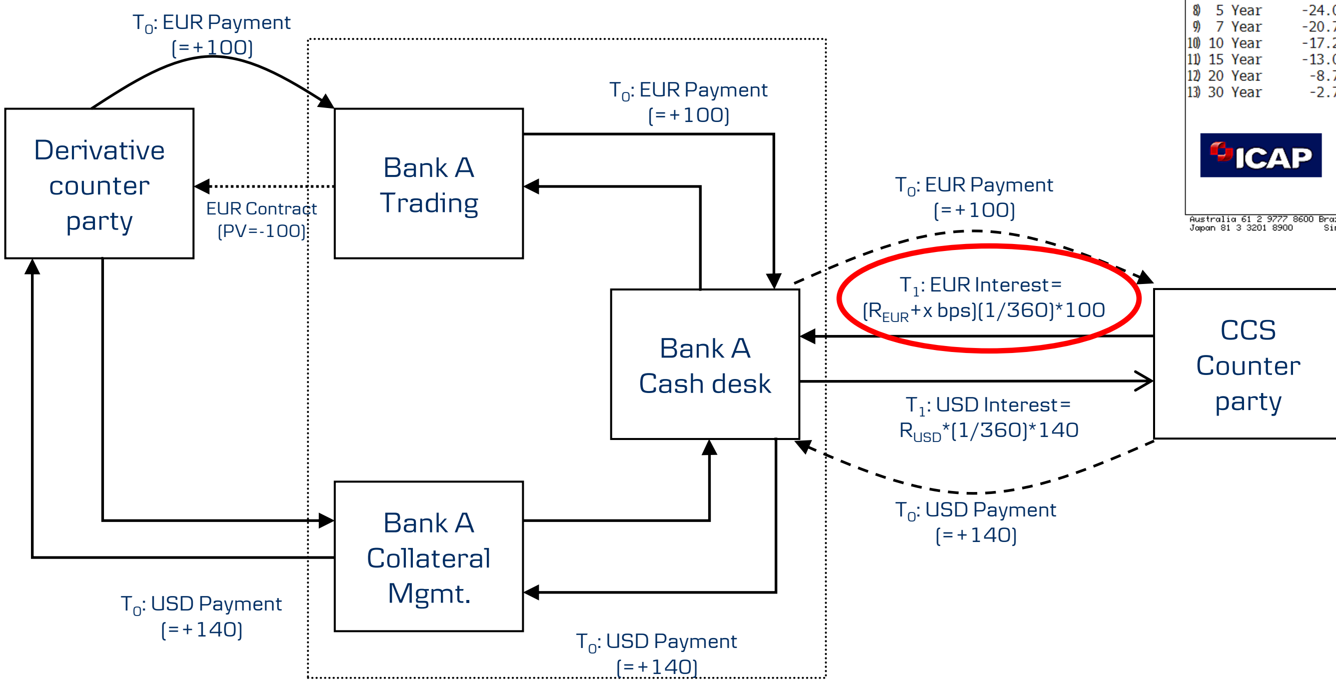
31 Curncy ICAE
200<Go> to view in Launchpad
8:43 EUR Eonia vs USD FF Basis Swap PAGE 1 / 1

	DESCRIPTION	ASK	BID	TIME
1)	3 Month			
2)	6 Month	-22.250	-30.250	8:23
3)	9 Month	-23.750	-31.750	8:04
4)	1 Year	-26.250	-34.250	8:00
5)	2 Year	-28.000	-36.000	7:00
6)	3 Year	-28.000	-36.000	8:14
7)	4 Year	-25.750	-33.750	7:00
8)	5 Year	-24.000	-32.000	7:00
9)	7 Year	-20.750	-28.750	7:00
10)	10 Year	-17.250	-25.250	7:00
11)	15 Year	-13.000	-21.000	7:00
12)	20 Year	-8.750	-16.750	7:00
13)	30 Year	-2.750	-10.750	7:00

ICAP

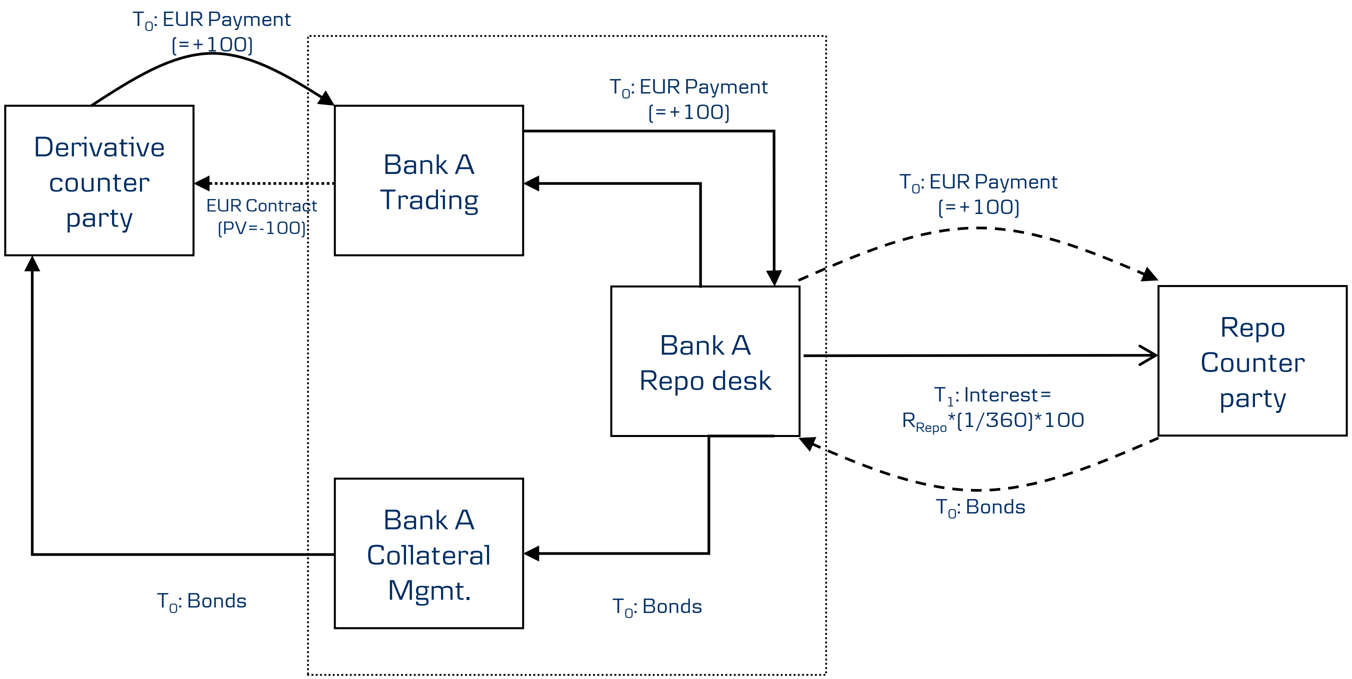
Against ON Floating Rates
Act/360 on both sides.

Introducing cross currency basis



- By introducing a cross currency mismatch, we can no longer simply discount at the OIS rate in the contract currency – we need to look at the CSA currency instead.
- Using FF and EONIA at the same time (for USD, respectively, EUR trades), also implies that you cannot match CCS spreads from the market.
- Importantly, multi currency CSAs have a cheapest-to-deliver element

Introducing non-cash collateral



Bond	EUR CCS Repo vs. EONIA*
US Govt	-15
DE Govt	-10
FR Govt	-5
NL Govt	-3
FI Govt	-3
UK Govt	0
DK Govt	0
SE Govt	-25
NO Govt	0
DK MBS	+5-10
SE MBS	-15

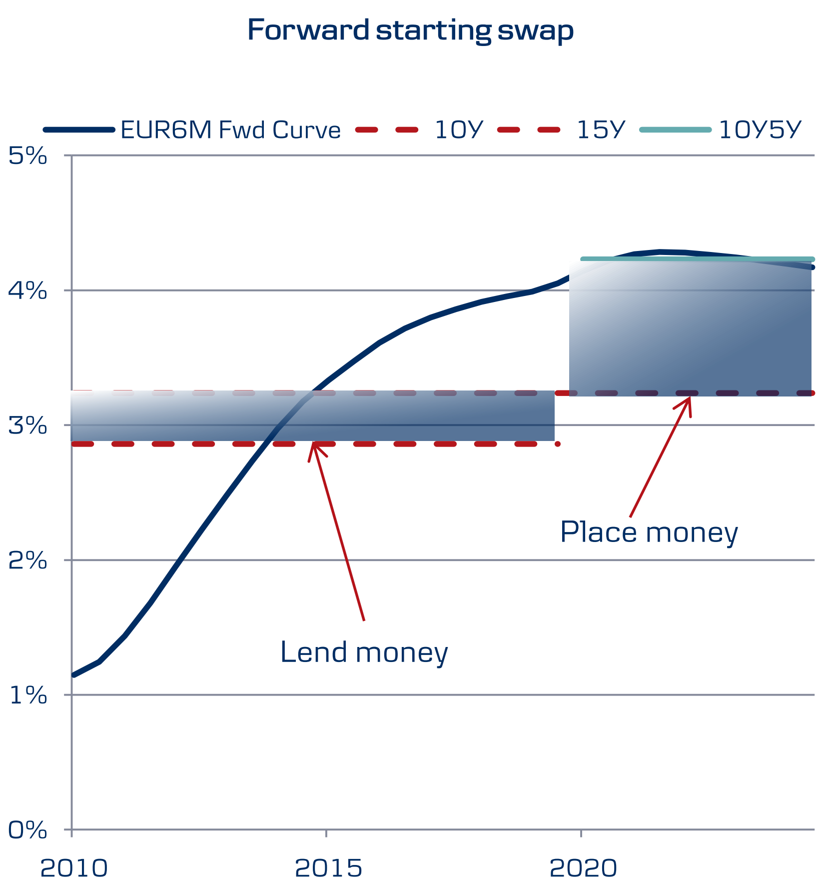
*Indicative open ended repo rates.

- When looking at non-cash collateral, we can use the repo market to find our discounting rate.
- Again, the cross currency element will play a role…
- …and the cheapest-to-deliver element is again important.

The importance of discounting

- Discounting matters for:
  - Terminations and novations (which is now more a tri- rather than bilateral transaction).
  - Forward starting swaps (on a non-flat yield curve, we have a liquidity impact even for par swaps).
  - Par-par asset swaps.
  - Options (premium today vs. expected payoff in the future).
- How many discount curves can we have?
  - One for each counter party in each currency…
  - …but we need to agree on what constitutes a par swap.
  - With which counter parts can you actually trade the par-swap?
  - If you insist on different discount curves you must either accept a multitude of forward surfaces or accept different par levels for different counter parties.

Forward starting swaps



Difference between EONIA and CCS-adjusted 3M EURIBOR (bps)

Fwd\Swap Tenor	1Y	5Y	10Y	15Y	30Y
Spot	0.0	0.0	0.0	0.0	0.0
1Y	0.0	0.0	0.1	0.1	0.2
5Y	0.2	0.4	0.5	0.6	0.7
10Y	0.7	0.7	0.8	0.8	0.4
15Y	0.7	0.7	0.4	0.0	-0.6
30Y	-3.4	-3.4	-3.3	-2.9	-2.3

Danske Markets

Asset swaps

Par-par ASW spreads against 3M EURIBOR	DBR 4 1/4 07/04/18	DBR 3 3/4 01/04/19	DBR 2 1/4 09/04/20
Clean Price	114.215	110.745	98.235
3M USD LIBOR Discounting	-0.176%	-0.165%	-0.211%
EONIA Discounting	-0.173%	-0.163%	-0.213%
Fed Funds Discounting	-0.155%	-0.150%	-0.215%

- In par-par asset swaps the upfront payment is offset over the life of the trade...
- ...this introduces a sensitivity towards the choice of discount curve.
- This effect is bigger the further away from par the bond is trading.

The risk methodology – a brief overview

- Pricing is the easy part…
  - Risk is closely related to calibration.
  - We want the sensitivities of a position with respect to model calibration instruments (market) as well as to possible model parameters…
  - …but sometimes traders want to view risk in other instruments.
  - Risk should be calculated via a jacobian that describes the dependency between model parameters and calibration instruments.
  - In the linear case this could be between knot points on zero coupon curves and values of calibration instruments.
- Dependencies to other models
  - Models are arranged into tree structures: During calibration, dynamic models call models for European option prices and those models call Linear models for forwards and discount factors.
  - Linear Models->…->Linear Models->European Models->Dynamic models

Conclusions and references

- Stylized facts
- Possible solutions
  - Curve construction today is quite non-trivial.
  - The corresponding risk can be challeging to understand.
- Applications
  - Differing levels of complexity.

- References:

  - A note on construction of multible swap curves with and without callateral, M Fujii, Y Shimada, A Takahashi, http://ssrn.com/abstract=1440633
  - Funding beyond discounting: collateral agreements and derivatives pricing, RISK, 2:97-210, 2010
  - Igor Smirnov, talk at RISK 2010

Disclaimer

This presentation has been prepared by Danske Markets for information purposes only and should be viewed solely in conjunction with the oral presentation provided by Danske Markets. It is not an offer or solicitation of any offer to purchase or sell any financial instrument. Whilst reasonable care has been taken to ensure that its contents are not untrue or misleading, no representation is made as to its accuracy or completeness and no liability is accepted for any loss arising from reliance on it. Danske Bank, its affiliates or staff, may perform services for, solicit business from, hold long or short positions in, or otherwise be interested in the investments (including derivatives), of any issuer mentioned herein. Danske Markets´ research analysts are not permitted to invest in securities under coverage in their research sector.

This presentation is not intended for retail customers in the UK or any person in the US. Danske Markets is a division of Danske Bank A/S, which is regulated by the FSA for the conduct of designated investment business in the UK and is a member of the London Stock Exchange.