INTEREST RATES AFTER THE CREDIT CRUNCH

Markets and Models Evolution

IX RiskLab Meeting on Financial Risks
Madrid, 12 May 2011

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Disclaimer and acknowledgments

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Acknowledgments:
this work has benefitted of the contribution of many present and former colleagues, in particular M. Carlicchi, M. De Prato, M. Henrard, M. Joshi, C. Maffi, G. V. Mauri, F. Mercurio, N. Moreni, A. Pallavicini and M. Pucci, M. Trapletti.
A particular mention goes to M. Morini for the constant stimulating discussions and to F.M. Ametrano and to the QuantLib community for the open-source developments used.

Summary

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   - SABR model revisited
   - Multiple curve models: additive vs multiplicative basis
   - Revealing CSA discounting in plain vanilla quotes
   - Testing SABR calibration vs CSA discounting
4. **Switching to CSA discounting in practice**
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   - Methodological issues
   - Liquidity and collateral management issues
   - ALM issues
   - IT issues
   - Accounting issues
   - Risk Management issues
   - Management issues
5. **Conclusions**
6. **References**

1: Introduction and Motivations

The switch towards CSA discounting is not a minor technical issue for quant and IT addicted. It is, instead, a major change in the market triggered by the financial crisis started in August 2007 with deep and pervasive consequences at 360 degrees.

- The interbank market now quotes much higher and differentiated credit and liquidity premia. Even plain vanilla quotes have drastically changed. The new business of liquidity trading has emerged (unwindings, novations, etc.).
- The classical theoretical framework based on a single risk free curve and nice no-arbitrage relations must be abandoned in favour of a new, modern framework, reviewing from scratch the no-arbitrage models used for pricing and risk analysis and including a coherent pricing of liquidity and counterparty risk.
- The financial libraries and pricing systems implementation and usage must be carefully reviewed and re-engineered.
- The liquidity and collateral management must integrate coherently the cost of funding generated by derivatives and CSAs, thus inducing transfer of business among different areas inside banks.
- The ALM must take into account the basis risk in hedge accounting.
- The accounting, advisory and regulatory sides must evolve to take into account that the fair value of derivatives is CSA dependent.
- The management must lead the change and the corresponding frictions, taking business opportunities and controlling risks.

1: Introduction and Motivations
Basic interest rates

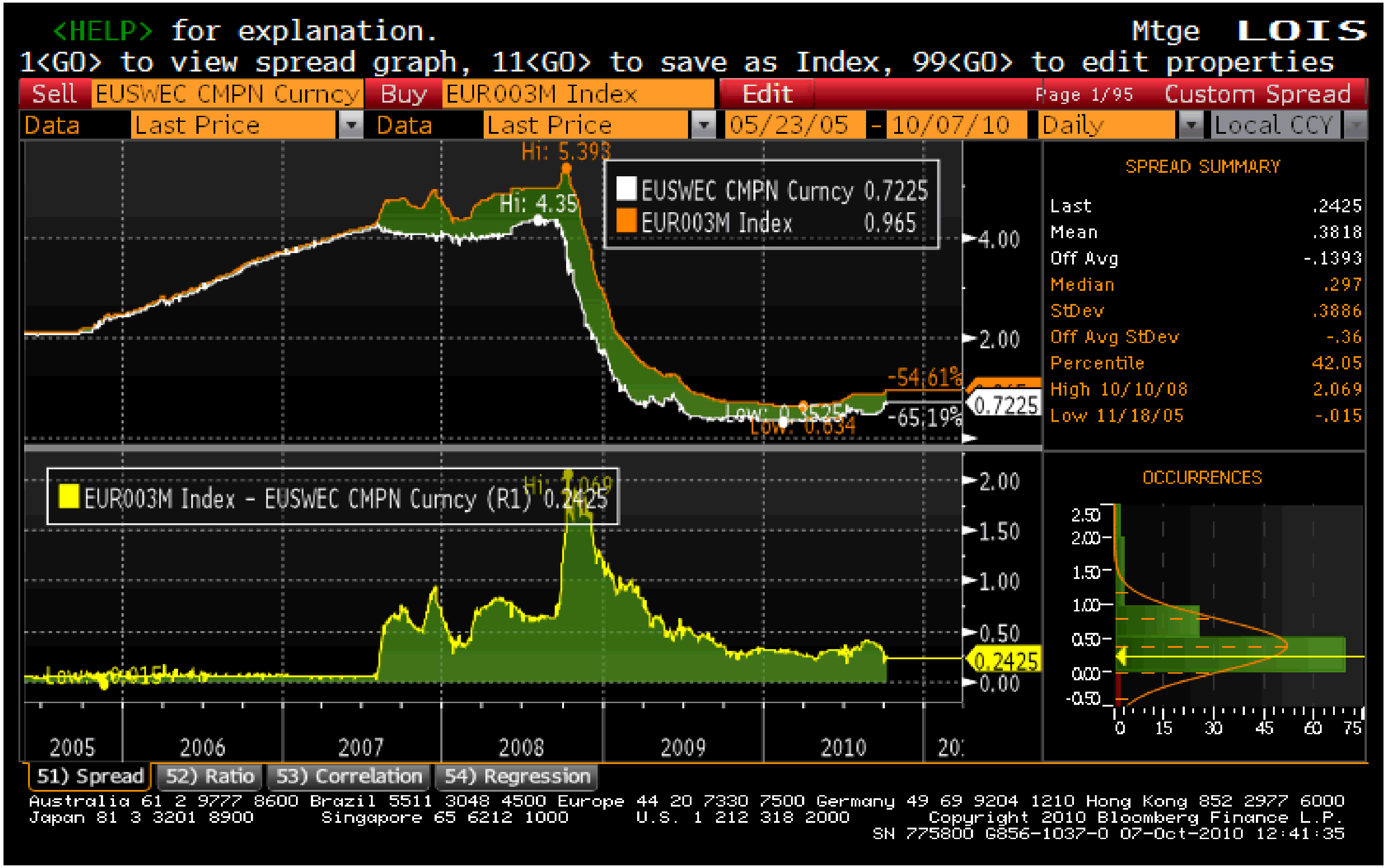
	Libor	Euribor	Eonia	Eurepo
Definition	**L**ondon **I**nter**B**ank **O**ffered **R**ate	**Eur**o **I**nter**B**ank **O**ffered **R**ate	**E**uro **O**ver**N**ight **I**ndex **A**verage	**Eur**o **Rep**urchase Ageement rate
Market	London Interbank	Euro Interbank	Euro Interbank	Euro Interbank
Side	Offer	Offer	Offer	Offer
Rate quotation specs	EURLibor = Euribor, Other currencies: minor differences (e.g. act/365, T+0, London calendar for GBPLibor).	TARGET calendar, settlement *T+2*, act/360, three decimal places, modified following, end of month, tenor variable.	TARGET calendar, settlement *T+1*, act/360, three decimal places, tenor 1d.	As Euribor
Maturities	1d-12m	1w, 2w, 3w,1m,…,12m	1d	T/N-12m
Publication time	12.30 CET	11:00 am CET	6:45-7:00 pm CET	As Euribor
Panel banks	8-16 banks (London based) per currency	42 banks from 15 EU countries + 4 international banks	Same as Euribor	34 EU banks plus some large international bank from non-EU countries
Calculation agent	Reuters	Reuters	European Central Bank	Reuters
Transactions based	No	No	Yes	No
Collateral	No (unsecured)	No (unsecured)	No (unsecured)	Yes (secured)
Counterparty risk	Yes	Yes	Low	Negligible
Liquidity risk	Yes	Yes	Low	Negligible
Tenor basis	Yes	Yes	No	No

2: The Market Across The Credit Crunch

Stylized facts

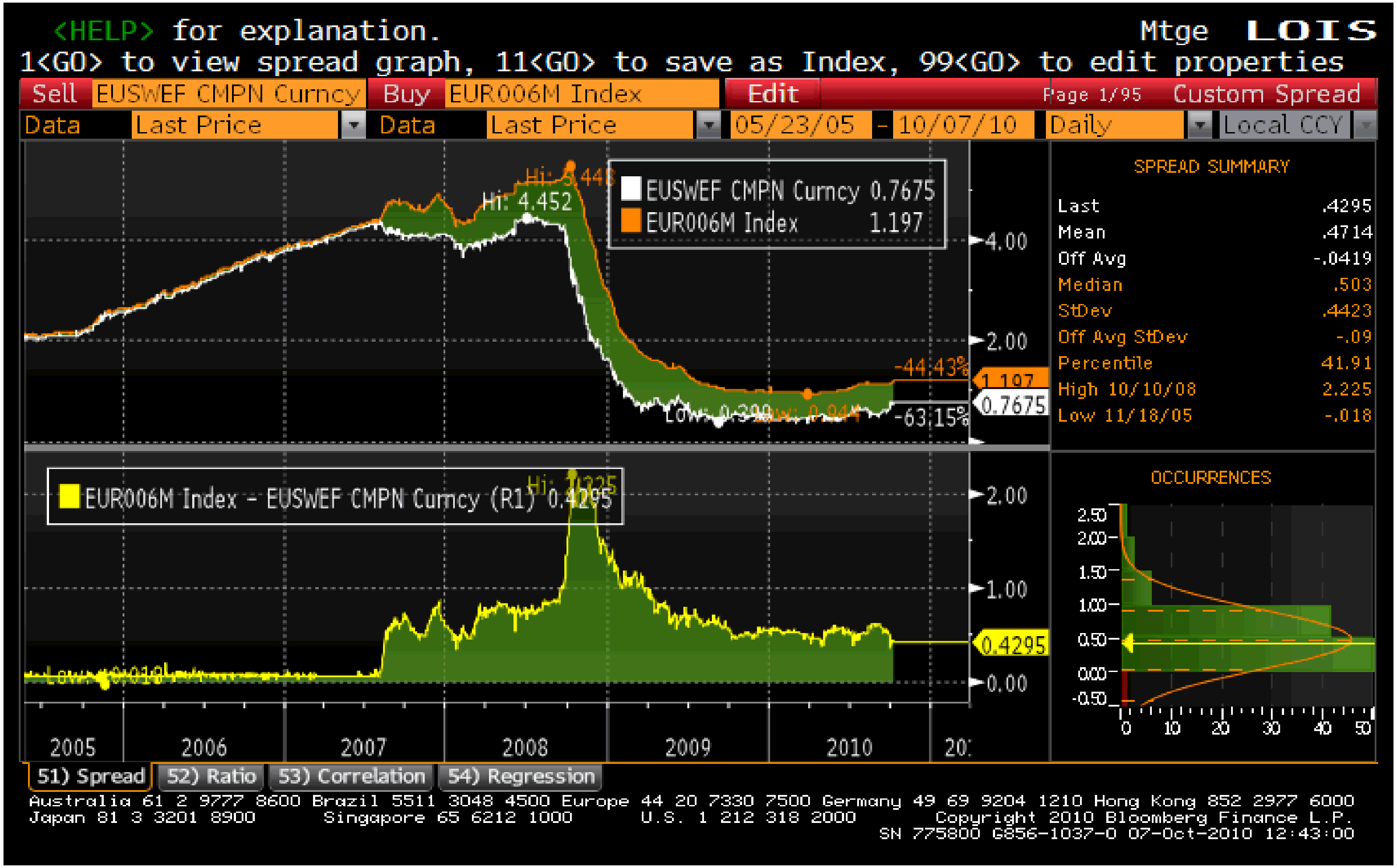
1. Divergence between deposit (Libor based) and OIS (Overnight based) rates.
2. Divergence between FRA rates and the corresponding forward rates implied by consecutive deposits.
3. Explosion of basis swap rates (based on Libor rates with different tenors).
4. Shift from unsecured towards secured market instruments.
5. Shift towards CSA discounting for collateralized cashflows: ICAP, Swapclear.

2: The Market Across The Credit Crunch
Deposit rates [1]



EUR 3M OIS rates vs 3M Depo (spot) rates
Quotations May. 2005 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch
Deposit rates [2]

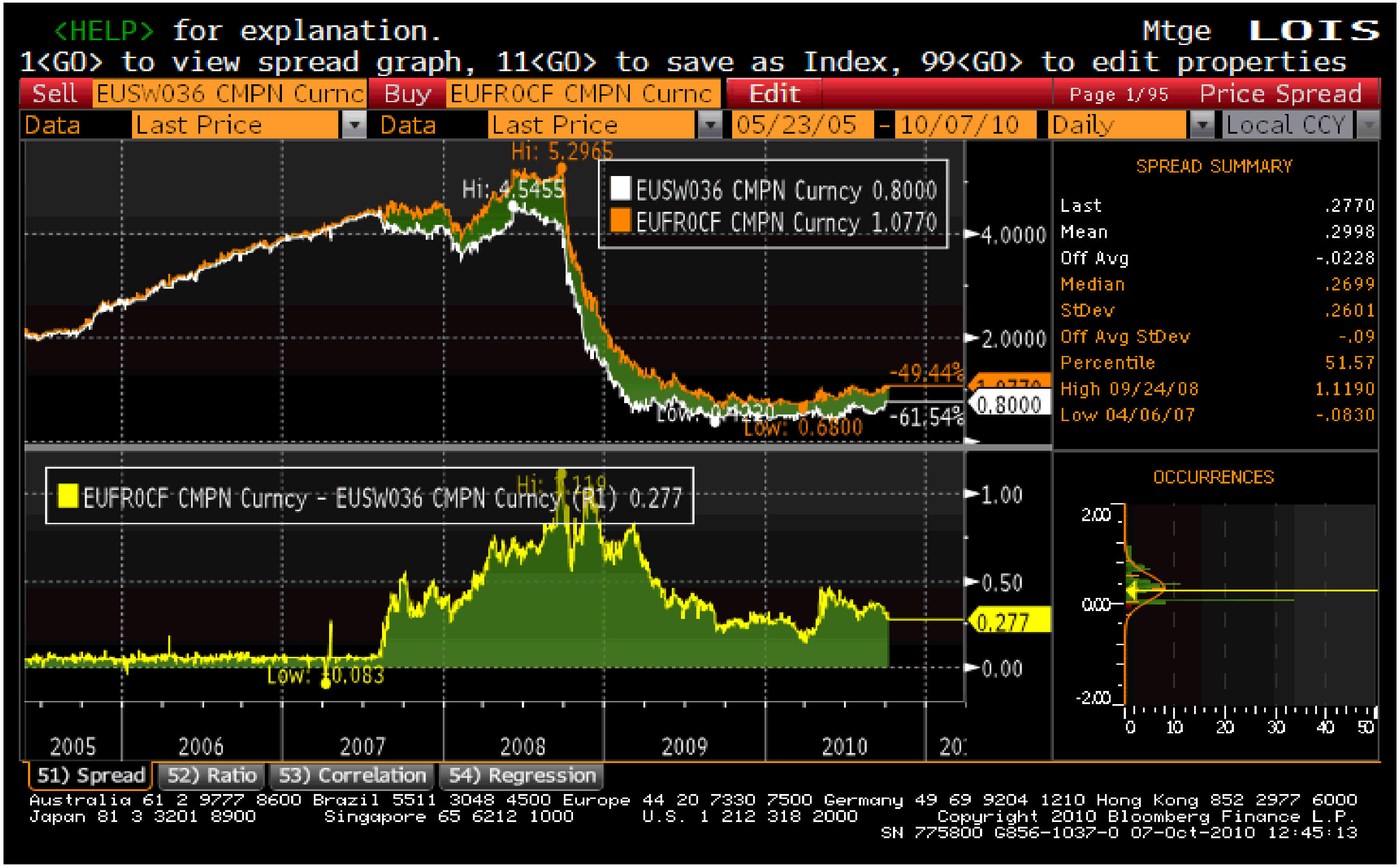


EUR 6M OIS rates vs 6M Depo (spot) rates

Quotations May. 2005 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch

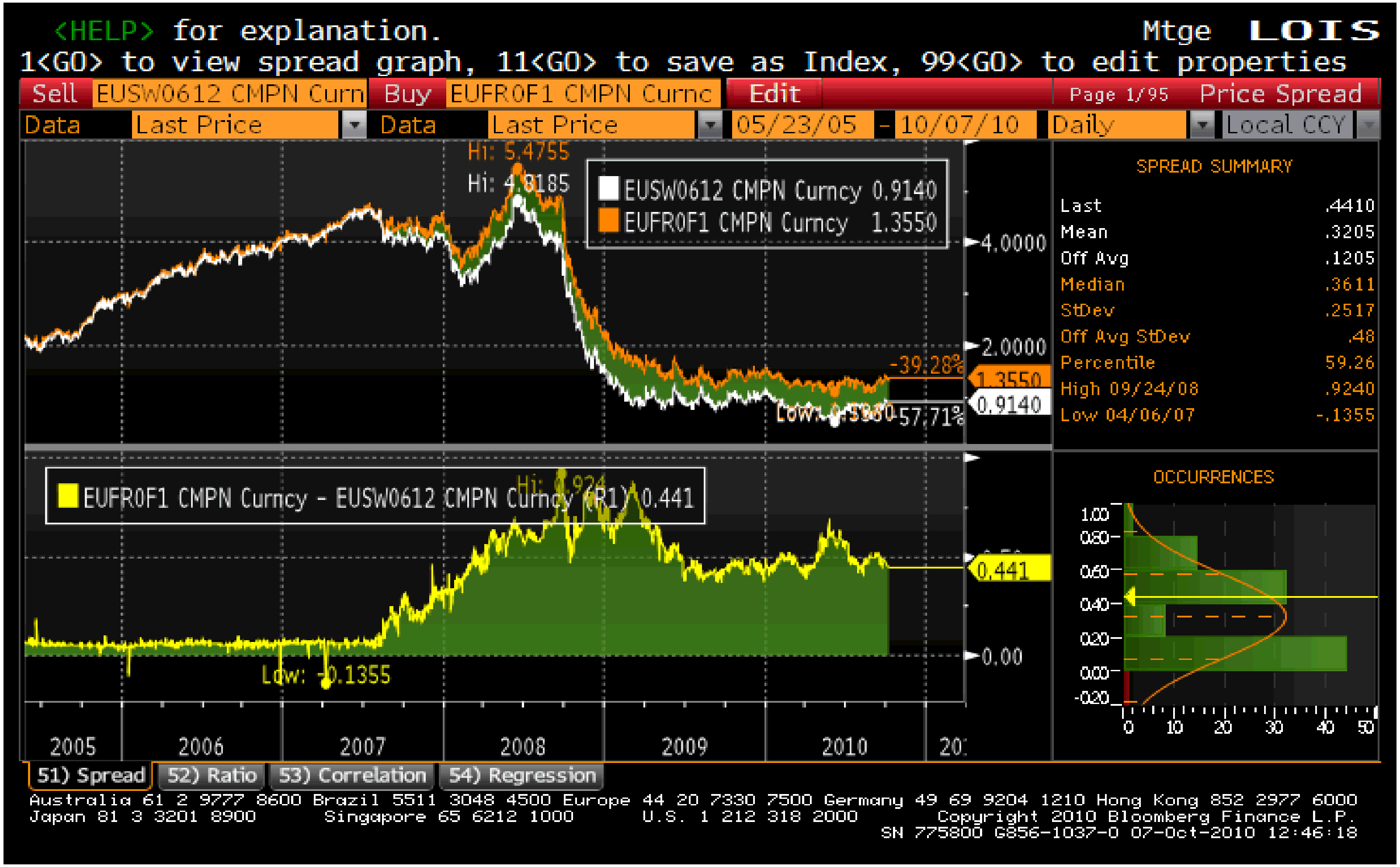
FRA rates [1]



EUR 3x6 Euribor FRA vs 3x6 OIS forward rates

Quotations May. 2005 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch
FRA rates [2]



EUR 6x12 Euribor FRA vs 6x12 OIS forward rates
Quotations May. 2005 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch
Basis Swap rates [1]

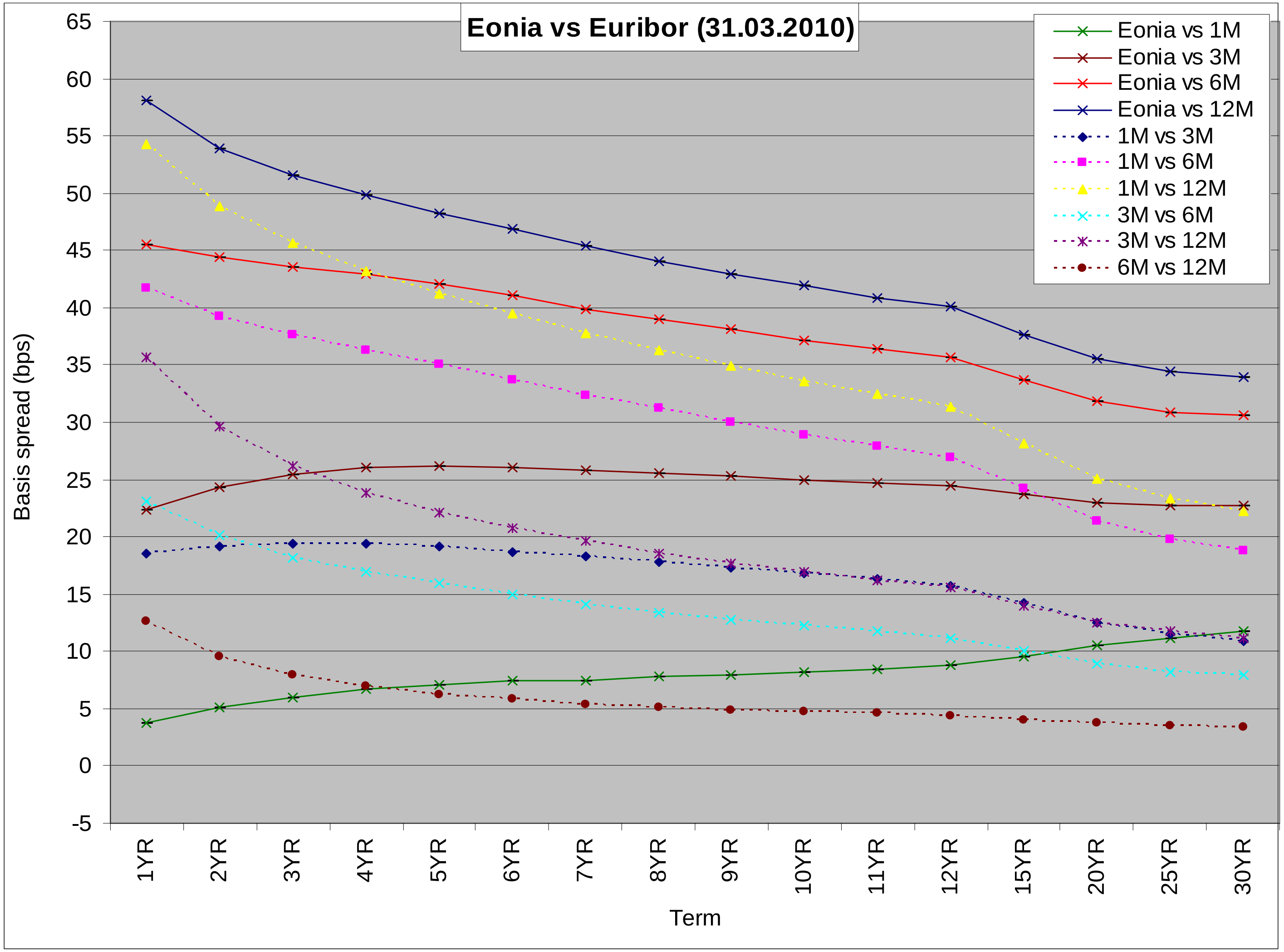
EUR Basis Swap Euribor3M vs Euribor6M, 5Y
Quotations Oct. 2005 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch
Basis Swap rates [2]

EUR Basis Swap Eonia vs Euribor3M, 5Y

Quotations Jun. 2008 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch
Basis Swap rates [3]



EUR Basis Swaps term structure

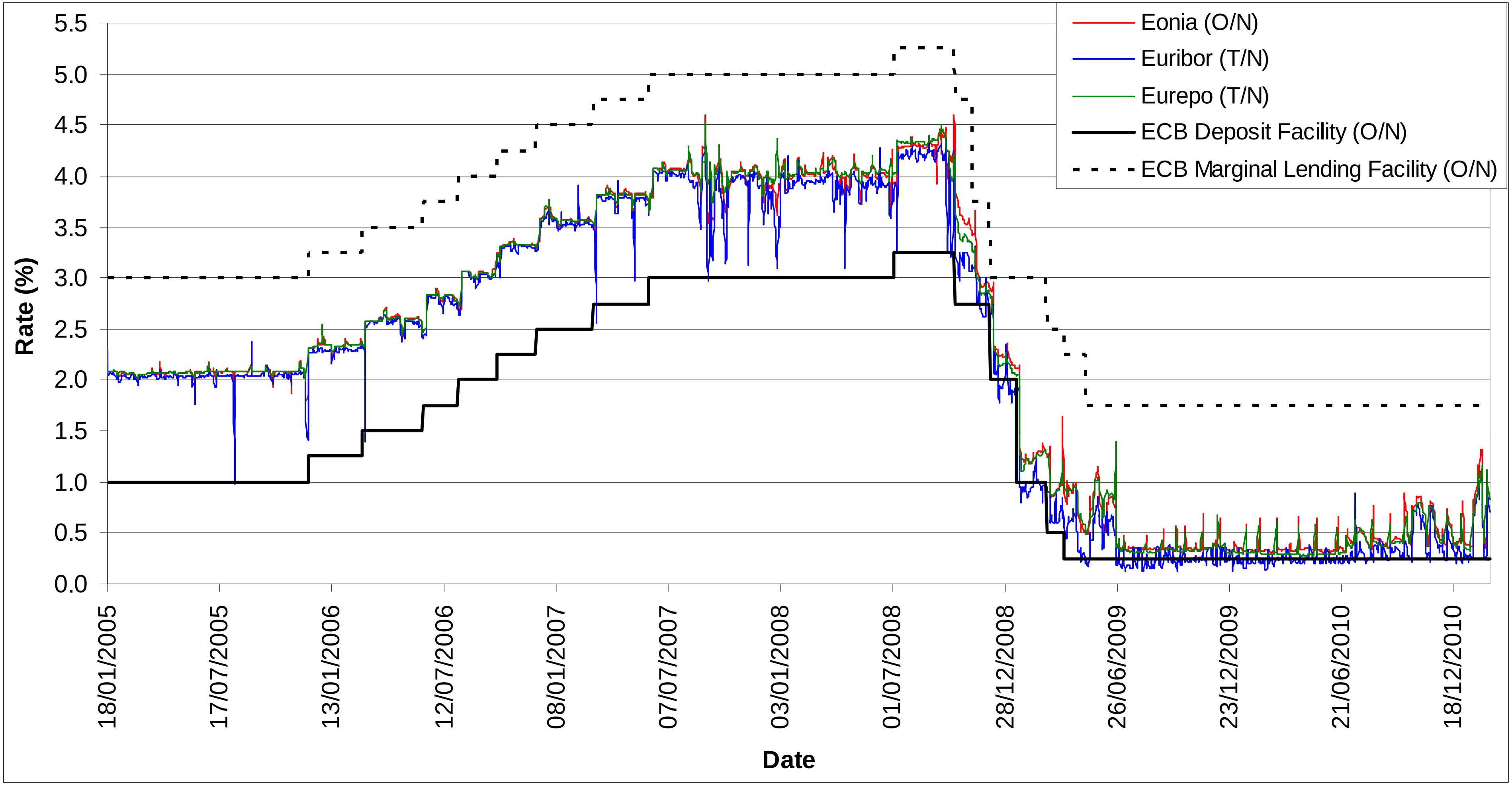
Quotations as of 31 Mar. 2010 (source: Reuters, ICAP)

2: The Market Across The Credit Crunch
Basis Swaps vs CDSs vs Bonds

Bond prices vs CDS spreads vs Euribor6M-Eonia6M basis

Quotations Jun. 2008 – Oct. 2010 (source: Bloomberg)

2: The Market Across The Credit Crunch
Interest rate corridor

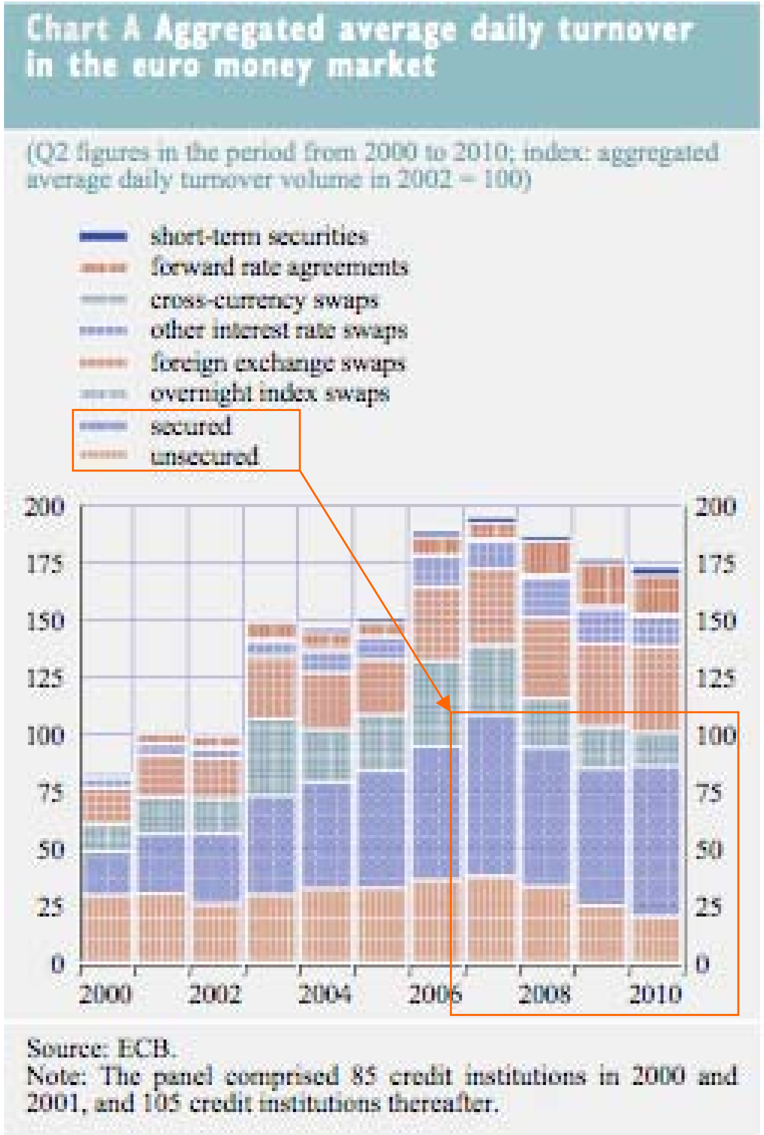


EUR interest rate corridor

Quotations Jan. 2005 – Dec. 2010

(sources: European Central Bank press releases and Bloomberg)

2: The Market Across The Credit Crunch
Unsecured vs secured transactions



EUR money market average daily turnover

The crisis has:

- inverted the overall trend, from increasing to decreasing turnover;
- shifted transactions from the unsecured money market (Eonia, bottom sections, -18% in Q2-2010 vs Q2-2009) to the secured money market (Eurepo, second bottom sections, +8% in Q2-2010 vs Q2-2009).

(source: European Central Bank Financial Stability Review, Dec. 2010, p. 65)

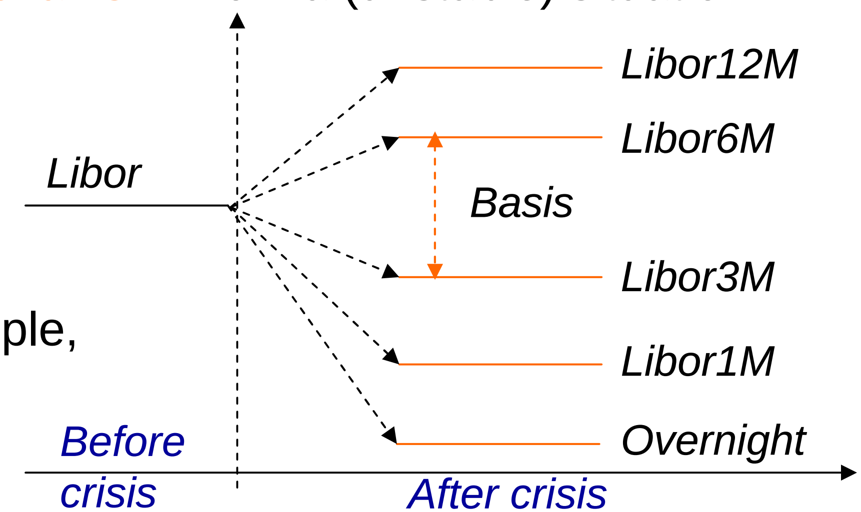
2: The Market Across The Credit Crunch
Libor questioned during the crisis

- The Bank for International Settlements reported that "*available data do not support the hypothesis that contributor banks manipulated their quotes to profit from positions based on fixings*" (see J. Gyntelberg, P. Wooldridge, "*Interbank rate fixings during the recent turmoil*", BIS Quarterly Review, Mar. 2008).
- Risk Magazine reported rumors that "*Libor rates are still not reflective of the true levels at which banks can borrow*" (see P. Madigan, "*Libor under attack*", Risk, Jun. 2008)
- Peng et al. from Citigroup (one of the largest Libor contributors) argue that "*...any Bank posting an high Libor level runs the risk of being perceived as needing funding*" (see Peng et al. "*Is Libor Broken*?", Fixed Income Strategies, Citigroup, 2008).
- The Wall Street Journal reported that some banks "*have been reporting significantly lower borrowing costs for the Libor, than what another market measure suggests they should be*" (see C. Mollenkamp, M. Whitehouse, The Wall Street Journal, 29 May 2008).
- The British Banker's Association commented that Libor continues to be reliable, and that other proxies are not necessarily more sound than Libor at times of financial crisis.
- The International Monetary Fund reported that "*it appears that U.S. dollar LIBOR remains an accurate measure of a typical creditworthy bank's marginal cost of unsecured U.S. dollar term funding*" (see Global Financial Stability Report, Oct. 2008, ch. 2).

2: The Market Across The Credit Crunch
Symmetry breaking and market segmentation

- Apparently similar interest rate instruments with different underlying rate tenors are characterised, in practice, by different liquidity and credit risk premia, reflecting the different views and interests of the market players.

- Thinking in terms of more fundamental variables, e.g. a short rate, the credit crunch has acted as a sort of symmetry breaking mechanism: from a (unstable) situation in which an unique short rate process was able to model and explain the whole term structure of interest rates of all tenors, towards a sort of market segmentation into sub-areas corresponding to instruments with different underlying rate tenors, characterised, in principle, by distinct dynamics, e.g. different short rate processes (the Zeeman effect in finance…).



- Notice that market segmentation was already present (and well understood) before the credit crunch (see e.g. B. Tuckman, P. Porfirio, "*Interest Rate Parity, Money Market Basis Swaps, and Cross-Currency Basis Swaps*", Lehman Brothers, Jun. 2003) but not effective due to negligible basis spreads.

2: The Market Across The Credit Crunch
Libor and counterparty/liquidity risk

1. Suppose an investor interested to enter into a 6M deposit on Libor rate. There are (at least) two different strategies:
   - choose Bank A, enter today into a 6M deposit, and get your money plus interest back in 6 months if Bank A has not defaulted;
   - choose Bank A, enter today into a 3M deposit, get your money plus interest back in 3 months if Bank A has not defaulted, then rechoose a second Bank B (the same or another), enter into a second 3M deposit and get your money plus interest back in 3 months if Bank B has not defaulted.

2. Suppose a Bank with excess liquidity (cash) to lend today at Libor rate for 6 month. There are (at least) two different strategies:
   - the Bank checks its liquidity today, it loans the excess liquidity today for 6M and gets cash plus interest back in 6M if the borrower has not defaulted;
   - the Bank checks its liquidity today, it loans the excess liquidity today for 3M and gets cash plus interest back in 3M if the borrower has not defaulted, then it rechecks its liquidity, loans the excess liquidity for the next 3M and gets cash plus interest back in 6M if the borrower has not defaulted;

2: The Market Across The Credit Crunch
Counterparty risk and collateral [1]

Collateral mechanics		
	Regulated markets	Over the counter markets
Collateralisation:	All trades are collateralised	Not all trades are collateralised, it depends on the agreements between the counterparties
Financial instruments:	highly standardised	highly customised
Clearing House:	There is a Clearing House that acts as counterparty for any trade and establish settlement and margination rules	There is no Clearing House, direct interaction between the counterparties, ad hoc contracts are used
Settlement and margination execution:	Daily settlement and margination, collateral in cash of main currencies or highly rated bonds (govies)	Most used contracts are: ✓ISDA Master Agreement ✓Credit Support Annex (CSA)
Collateral interest:	Overnight rate	Depend on the agreements

2: The Market Across The Credit Crunch

Counterparty risk and collateral [2]

Collateral Pros

- Counterparty risk reduction
- Credit management optimization
- Capital ratios reduction (Basilea II)
- Increased business opportunities
- Funding at overnight rate
- Periodic check of credit exposure and portfolio NPV

Collateral Cons

- Funding volatility sensitivity
- Possible liquidity squeeze
- Structural and running costs
- Operational risks: settlement and MTM mismatch
- Legal risk
- Pricing impact

2: The Market Across The Credit Crunch
Counterparty risk and collateral [3]

CSA diffusion
(ISDA Margin Survey, 2010)

- Survey on ISDA members, 89 respondents, 14 of the largest OTC derivatives dealers, 53% in Europe, Middle East or Africa, 29% in the Americas, 18% Asia.
- 80% of the collateral value exchanged is cash collateral.
- The number of collateral agreements is in constant increase (+14% in 2009 vs 2008).
- 92% of CSA (of ISDA members) is an ISDA CSA.
- 83% of CSA are bilateral CSA (75% in 2008).
- 78% of OTC derivatives transactions of large dealers are subject to CSA (see next slide).
- 56% of CSA of large dealers are subject to daily margination (see next slide).

2: The Market Across The Credit Crunch
Counterparty risk and collateral [4]

CSA diffusion (cont'd)
(ISDA Margin Survey, 2010)

Table 3.2 Percent of trades subject to collateral agreements, by size of program

	Percent of trades						
	All OTC derivatives	Fixed Income derivatives	Credit derivatives	FX derivatives	Equity derivatives	Precious & base metals derivatives	Energy and other commodity derivatives
All Respondents	70	79	93	57	71	60	64
Large dealers	78	84	97	63	68	69	62
Medium and Small	68	77	91	54	72	52	65

Table 4.1 Frequency of portfolio reconciliation:percentange of trades reconciled at stated intervals

Percent of trades	**Daily**	**Weekly**	**Monthly**	**Ad hoc/ Dispute driven**
Total Sample	29	10	15	47
Fed 14	56	5	3	37

2: The Market Across The Credit Crunch

From Libor to OIS discounting: LCH.Clearnet

June 17th 2010 LCH press release:

"*LCH.Clearnet Ltd […] which operates […] SwapClear, is to begin using the overnight index swap (OIS) rate curves to discount its $218 trillion IRS portfolio. Previously, in line with market practice, the portfolio was discounted using LIBOR. However, an increasing proportion of trades are now priced using OIS discounting. After extensive consultation with market participants, LCH.Clearnet has decided to move to OIS to ensure the most accurate valuation of its portfolio for risk management purposes. LCH.Clearnet already uses OIS rates to price the rate of return on cash collateral. From 29 June 2010, USD, Euro and GBP trades in SwapClear will be revalued using OIS*. […]"

2: The Market Across The Credit Crunch
From Libor to OIS discounting: Swaptions quotations [1]

August 11th 2010: ICAP communicates that "*We are planning to us the following methodology for pricing and for the publication of our information. We plan to gross up our spot premium to fwd premiums and intend to run two pages concurrently. This process will be driven by the spot premia until the 1st of Sept and vice-versa from then on. We will be using a eonia curve to discount from the moment we publish both numbers, thus immediately affecting our bp vol page VCAP6 (the vols will be lower). All pages will remain the same with the addition of VCAP2A for fwd premia. It will be made clear that VCAP2 is for indicative purposes only, discounted using our eonia curve and therefore CSA dependant.*"

September 15°, 2010: ICAP explains that "*Until very recently all prices quoted in the Euro IR Swaption market were quoted as spot premium. About three months ago, Deutsche started to show fwd premium prices in an attempt to convince the market to move to fwd premium. This was unsuccessful as the market showed no interest in changing methodology.*
Around 6 weeks ago, Deutsche began showing prices in the market specifying that these were only for names with whom they did not have a $$ CSA. Those counterparties responded in kind. The result was a two-tier market with very little prospect of any long-term liquidity. The obvious solution was to move to forward premium. A discussion was started by Nick Moore from Merrill with most of the main dealers who all agreed to migrate to forward premium from Sept 1st.
We have moved all of our pricing to this methodology and currently still give the market an indication of where we expect the spot premium price to be, using our discount curve. Clearly this price is entirely CSA dependant.
At the moment 95% of our prices and trades are forward premium."

2: The Market Across The Credit Crunch

From Libor to OIS discounting: Swaptions quotations [2]

Quote: VCAP2A

15:12 30SEP10 ICAP UK69580 VCAP2A

EUR ATM Swaption Straddles - Fwd Premium Mids

Please call +44 (0)20 7532 3050 for further details

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
1M Opt	10.5	25.5	41.0	57.5	76.5	96.5	117	138	161	185	270	346	421	495
2M Opt	15.5	37.5	60.0	86.0	115	141	169	198	228	261	379	486	592	701
3M Opt	20.0	49.5	78.5	110	142	176	209	243	278	315	459	584	712	842
6M Opt	32.0	76.5	116	160	206	250	294	341	386	431	622	799	964	1135
9M Opt	44.5	99.0	150	203	257	310	363	417	471	526	756	959	1155	1366
1Y Opt	55.0	118	178	237	299	363	423	483	545	605	859	1083	1311	1539
18M Opt	76.0	151	225	300	374	447	519	590	664	739	1035	1303	1564	1817
2Y Opt	94.5	183	268	351	437	520	601	681	762	842	1179	1479	1767	2042
3Y Opt	116.5	225	327	426	528	625	722	814	906	1002	1376	1726	2064	2388
4Y Opt	131.5	254	369	480	588	695	798	900	999	1108	1507	1878	2244	2585
5Y Opt	141.5	273	396	514	630	744	856	967	1076	1189	1618	2001	2374	2754
7Y Opt	152.0	295	431	564	691	819	942	1059	1177	1302	1757	2168	2567	2974
10Y Opt	170.5	329	487	638	785	927	1065	1195	1327	1463	1960	2419	2848	3255
15Y Opt	193.0	374	552	725	890	1055	1217	1373	1532	1688	2256	2810	3309	3780
20Y Opt	209.0	410	608	804	994	1181	1363	1539	1714	1866	2523	3110	3645	4115
25Y Opt	218.0	424	633	843	1047	1244	1430	1607	1791	1961	2622	3209	3755	4241
30Y Opt	220.5	425	629	833	1035	1227	1414	1590	1771	1939	2613	3226	3803	4369

screen closed

Reuters page **VCAP2A** showing market At-The-Money Swaption Straddle premia with settlement at option expiry (forward premia).

Quote: VCAP2

15:12 30SEP10 ICAP UK69580 VCAP2

EUR ATM Swaption Straddles - Implied Spot Premium Mids

Please call +44 (0)20 7532 3050 for further details

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
1M Opt	10.5	25.5	41.0	57.5	76.5	96.5	117	138	161	185	270	346	420	494
2M Opt	15.5	37.5	60.0	85.5	115	141	168	198	227	261	379	486	592	700
3M Opt	20.0	49.5	78.5	110	142	176	209	243	278	315	458	583	711	841
6M Opt	32.0	76.5	116	159	205	250	293	340	385	430	620	796	961	1131
9M Opt	44.5	98.5	149	202	255	308	361	414	468	523	751	953	1148	1358
1Y Opt	54.5	117	177	235	296	360	419	479	540	599	852	1074	1300	1526
18M Opt	75.0	149	222	296	369	441	512	582	655	729	1020	1286	1542	1792
2Y Opt	92.5	179	262	344	428	510	589	667	747	825	1156	1449	1731	2002
3Y Opt	112.5	217	315	411	509	604	697	785	874	967	1328	1665	1992	2305
4Y Opt	124.5	240	349	455	556	658	756	852	946	1049	1427	1778	2125	2447
5Y Opt	131.0	253	367	476	583	689	792	895	996	1100	1497	1852	2197	2549
7Y Opt	133.5	259	377	494	606	717	825	928	1031	1141	1539	1900	2249	2605
10Y Opt	136.0	263	388	508	626	739	849	953	1058	1166	1563	1928	2270	2594
15Y Opt	130.0	251	371	487	598	709	817	922	1029	1134	1515	1888	2223	2539
20Y Opt	119.5	235	348	460	569	676	780	881	981	1068	1444	1780	2087	2356
25Y Opt	110.0	214	320	426	528	628	722	811	904	990	1324	1621	1896	2142
30Y Opt	101.5	196	289	384	477	565	651	732	816	893	1203	1485	1751	2012

screen closed

Reuters page **VCAP2** showing market At-The-Money Swaption Straddle premia with settlement at spot date (spot premia).

These prices are obtained from the corresponding forward premia in page VCAP2A using an Eonia discounting curve.

2: The Market Across The Credit Crunch

From Libor to OIS discounting: Swaptions quotations [3]

Quote: VCAP1A

VCAP1A

15:12 30SEP10 ICAP UK69580 VCAP1A

EUR ATM Swaption Straddles - Implied Volatilities

Please call +44 (0)20 7532 3050 for further details

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
1M Opt	40.6	38.4	37.9	36.8	36.3	35.8	35.5	35.4	35.8	36.4	34.4	34.4	36.1	38.6
2M Opt	40.4	38.6	37.6	37.1	37.1	35.8	34.9	34.7	34.6	35.1	33.2	33.3	35.0	37.7
3M Opt	41.1	40.8	39.5	38.3	37.1	36.1	35.1	34.6	34.4	34.5	32.8	32.7	34.4	37.0
6M Opt	44.1	42.7	39.7	38.0	36.7	35.3	34.1	33.6	33.1	32.8	31.2	31.5	33.0	35.3
9M Opt	46.9	43.0	40.2	37.8	36.0	34.5	33.4	32.7	32.3	32.1	30.6	30.6	32.1	34.5
1Y Opt	47.5	42.5	39.6	36.8	35.1	34.0	32.9	32.2	31.7	31.3	29.8	29.8	31.4	33.6
18M Opt	48.7	41.0	37.9	35.6	33.8	32.5	31.6	30.9	30.5	30.3	28.8	29.0	30.5	32.3
2Y Opt	47.6	39.8	36.4	33.8	32.4	31.3	30.5	29.8	29.4	29.1	28.0	28.3	29.7	31.4
3Y Opt	40.5	34.6	31.9	30.0	29.2	28.4	27.9	27.4	27.0	27.0	26.0	26.6	28.2	29.8
4Y Opt	34.0	29.9	28.1	27.0	26.2	25.7	25.3	25.0	24.8	24.9	24.2	24.9	26.5	27.9
5Y Opt	28.9	26.3	25.2	24.4	23.9	23.6	23.3	23.2	23.2	23.4	23.0	23.7	25.1	26.7
7Y Opt	23.2	22.0	21.5	21.1	20.8	20.8	20.8	20.8	20.9	21.2	21.1	22.0	23.3	24.7
10Y Opt	19.6	18.9	19.0	19.1	19.2	19.3	19.4	19.5	19.7	20.0	20.4	21.6	22.6	23.5
15Y Opt	18.8	18.4	18.6	18.8	19.0	19.4	19.8	20.3	20.8	21.4	22.2	23.3	23.9	24.2
20Y Opt	20.6	20.9	21.6	22.4	23.1	23.9	24.5	25.2	25.8	26.0	26.0	25.8	25.2	24.7
25Y Opt	27.2	27.0	28.1	29.0	29.4	29.6	29.4	29.2	29.3	29.6	27.2	25.3	24.1	23.7
30Y Opt	29.2	26.9	26.7	26.9	28.0	27.1	27.2	27.0	27.1	26.8	23.9	22.3	22.0	22.3

screen closed

Reuters page **VCAP1A** showing market volatilities implied in spot premia from page VCAP2, obtained by Black's formula inversion using an Euribor discounting curve.

Quote: VCAP1

VCAP1

15:12 30SEP10 ICAP UK69580 VCAP1

EUR ATM Swaption Straddles - Implied Volatilities (Euribor disc)

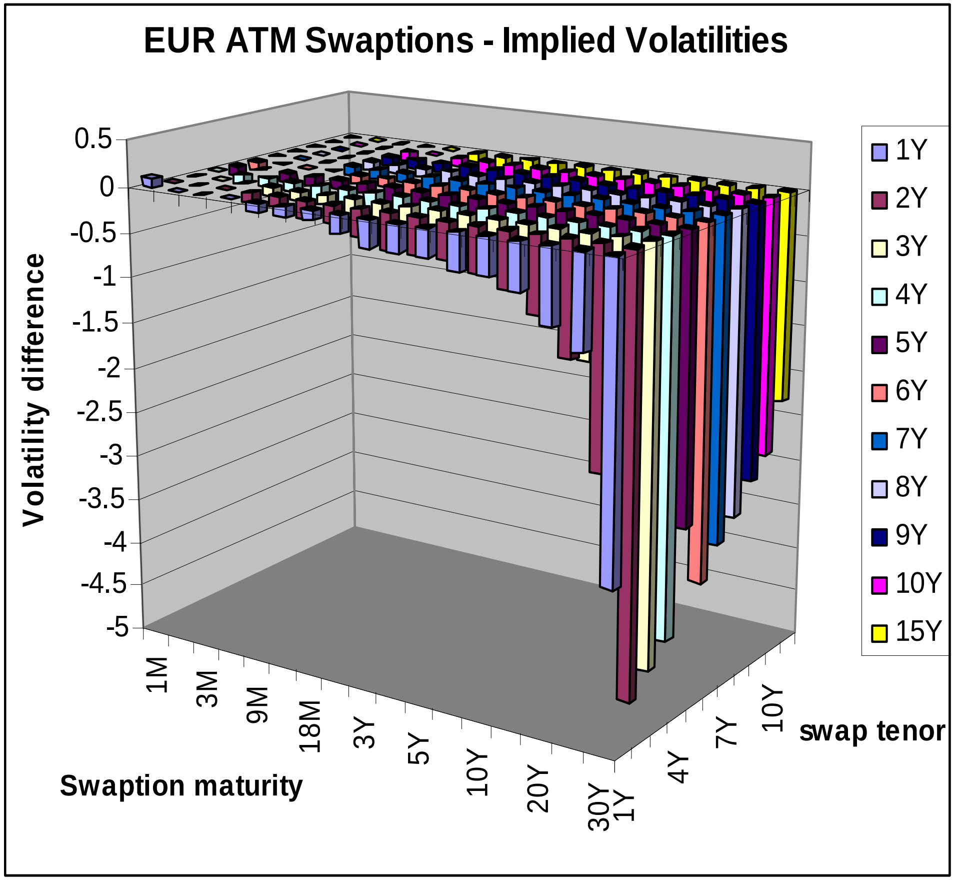
Please call +44 (0)20 7532 3050 for further details

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
1M Opt	40.5	38.4	37.9	36.8	36.4	35.9	35.5	35.4	35.8	36.4	34.4	34.4	36.1	38.6
2M Opt	40.4	38.6	37.6	37.2	37.1	35.8	34.9	34.7	34.6	35.1	33.2	33.3	35.0	37.7
3M Opt	41.1	40.8	39.5	38.4	37.2	36.1	35.1	34.6	34.4	34.5	32.8	32.7	34.5	37.1
6M Opt	44.1	42.8	39.8	38.1	36.8	35.3	34.2	33.7	33.2	32.9	31.2	31.6	33.0	35.4
9M Opt	47.0	43.1	40.3	38.0	36.1	34.6	33.5	32.8	32.4	32.1	30.6	30.7	32.1	34.6
1Y Opt	47.6	42.7	39.7	37.0	35.2	34.1	33.0	32.3	31.8	31.4	29.9	29.9	31.5	33.7
18M Opt	48.8	41.2	38.1	35.8	34.0	32.7	31.8	31.1	30.7	30.5	29.0	29.2	30.6	32.5
2Y Opt	47.8	40.1	36.7	34.1	32.6	31.5	30.7	30.0	29.6	29.3	28.2	28.5	29.9	31.6
3Y Opt	40.8	35.0	32.2	30.4	29.5	28.7	28.2	27.6	27.3	27.3	26.3	26.9	28.5	30.2
4Y Opt	34.3	30.3	28.5	27.4	26.6	26.1	25.6	25.3	25.1	25.3	24.5	25.3	26.9	28.4
5Y Opt	29.2	26.7	25.6	24.8	24.3	24.0	23.7	23.6	23.6	23.8	23.4	24.1	25.6	27.2
7Y Opt	23.6	22.5	22.0	21.6	21.3	21.2	21.3	21.3	21.4	21.7	21.6	22.6	23.9	25.4
10Y Opt	20.0	19.5	19.6	19.7	19.8	19.9	20.0	20.1	20.3	20.6	21.1	22.3	23.4	24.3
15Y Opt	19.3	19.2	19.4	19.6	19.8	20.2	20.7	21.2	21.7	22.3	23.3	24.4	25.1	25.4
20Y Opt	21.4	22.1	22.9	23.7	24.5	25.3	26.1	26.8	27.4	27.7	27.8	27.5	27.0	26.3
25Y Opt	28.2	29.3	30.3	31.4	32.1	32.6	32.6	32.5	32.6	32.3	29.6	27.5	26.1	25.5
30Y Opt	32.5	31.5	31.1	31.1	31.1	30.9	30.7	30.3	30.1	29.6	26.2	24.4	23.9	24.4

screen closed

Reuters page **VCAP1** showing market volatilities implied in spot premia from page VCAP2, obtained by Black's formula inversion using an Eonia discounting curve.

2: The Market Across The Credit Crunch
From Libor to OIS discounting: Swaptions quotations [4]



Implicit market volatility differences from previous pages (VCAP1A - VCAP1, Eonia - Euribor).

The Eonia Black's implied volatilities (VCAP1A) are smaller than the corresponding Euribor volatilities (VCAP1) because, lowering the discounting rate from Euribor to Eonia, a larger discount factor is obtained, thus leading, at constant spot premium, a smaller implied volatility.

The larger differences correspond to the longer option maturities (30Y) with the shorter underlying swap tenors (1Y).

Source: Reuters, 30 Sep. 2010.

From Libor to OIS discounting: IRD quotations [1]

Main Broker standard for interest rate derivatives quotations		
Instrument	**Classical quotations**	**Modern quotations**
Swap	forwarding = Euribor xM discounting = Euribor xM	forwarding = Euribor xM discounting = Eonia
Basis Swap	forwarding1 = Euribor xM1 forwarding2 = Euribor xM2 discounting = Min(EuriborxM1,EuriborxM2)	forwarding1 = EuriborxM1 forwarding2 = EuriborxM2 discounting = Eonia
CMS	As Basis Swaps	As Basis Swaps
CMS S.O.	As CMSs	As CMSs
CCS	As Basis Swaps	As Basis Swaps
Caps/Floors/ Swaptions	forwarding = Euribor xM discounting = Euribor xM	forwarding = Euribor xM discounting = Eonia Forward premium Eonia options ?

Source: personal reverse engineering of market quotes.

2: The Market Across The Credit Crunch
PwC Survey (Nov. 2010)

Survey conducted by PwC London Product Control Discussion Group in November 2010 Including 14 out of the largest US and European investment and retail banks.

Main results:

1. Trades under CSA: see tables

Fair value adjustment of books and records for trades under CSA			
Planned before	**IR plain vans**	**IR exotics**	**Other**
2010 end of year	6	5	
2011-Q1	5		1
2011-Q2	2	9	
2011-S2	1		

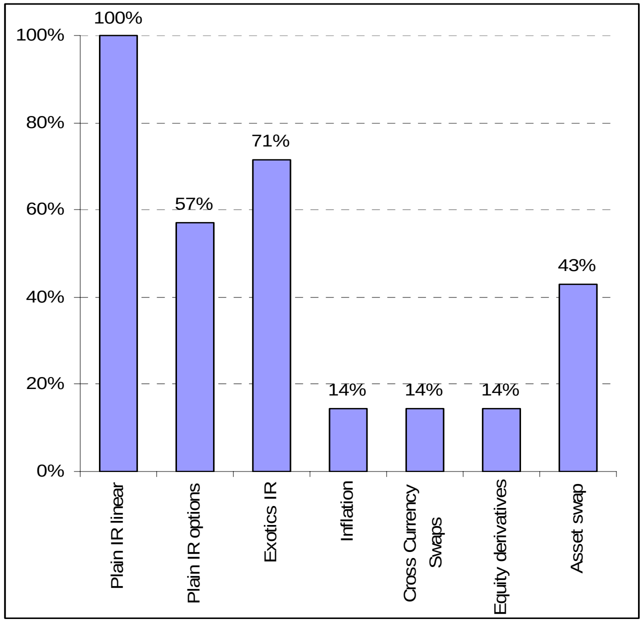
Adoption of OIS discounting for trading instruments under CSA	
Interest rate plain vanillas	14
Interest Rate exotics	12
Inflation	?
Equity	4
Credit	4
Fx	3
Commodity	1

2. Trades without CSA: "*Libor based valuations of uncollateralised derivatives as at 31 Dec. 2010 are considered appropriate*".
3. Collateral: "*the majority of banks currently call margin and issue customer valuations statements on a Libor basis*".

Source: PwC + personal synthesis.

2: The Market Across The Credit Crunch
Ernst & Young Survey (Dec. 2010)

- Overview on the European banks:
  qualitative survey, no standardized questionnaire, big players only:
  - around 50% is ready to switch within 2010.
  - the other 50% is working to switch in 2011.
- Survey on Italian banks:
  quantitative survey, standardized questionnaire + direct interview, 2 big players + 5 national banks in FTSE MIB index (89% of banking sector capitalisation):
  1. Present pricing framework:
     - 71% (5/7): multiple-tenor forward Libor curves + classical discount curve
     - 29% (2/7): multiple-tenor Libor curves
  2. Market experience of OIS discounting:
     - 71% (5/7): clear evidence
     - 29% (2/7): evidence limited to some ctps
     - Plain vanilla IR linears: 100% (7/7)
     - Plain vanilla IR options: 71% (4/7)
     - Exotics IR: 57% (5/7)
     - Inflation: 14% (1/7)
     - Cross Currency Swap: 14% (1/7)
     - Equity derivatives: 14% (1/7)
     - Asset swap: 43% (3/7)



2: The Market Across The Credit Crunch
Ernst & Young Survey (Dec. 2010)

3. Market evidence of CSA-dependent discounting :
    - 29% (2/7): no evidence
    - 43% (3/7): evidence limited to some counterparties
    - 29% (2/7): clear evidence
4. Opinion about CSA-discounting (multiple choice):
    - Business opportunity: 43% (3/7)
    - Problems for pricing, hedge accounting, fair value accountancy, IT: 100% (7/7)
5. Presence of internal working group on CSA discounting (multiple choice):
    - Risk Management: 57% (4/7)
    - Front Office: 57% (4/7)
    - IT: 43% (3/7)
    - Administration: 14% (1/7)
    - Preliminary analysis: 29% (2/7)
    - No working group: 14% (1/7)
    - Other: 14% (1/7)

Source: "OIS discounting", survey Ernst & Young, Dec. 2010 + personal synthesis.

3: Classical vs Modern Market Practice & Modeling

Summary & motivation

- Classical vs modern pricing of interest rate derivatives
- SABR model revisited
- Multiple curve models: additive vs multiplicative basis
- Revealing CSA discounting in plain vanilla quotes
- Testing SABR calibration vs CSA discounting

In order to understand the modern interest rate market after the credit crunch, we must set up a theoretical framework able to explain the observed market data.

As a first step, we must go back to basics and restart from scratch the interest rate theory, with the aim to refresh the foundations and to (re)discover hidden assumptions, in particular concerning the single versus the multiple-curve approach.

At the end we will be ready to (re)set the foundations and to construct the modern interest rate pricing framework on a solid theoretical basis.

3: Classical vs Modern Market Practice & Modeling
Basic assumptions and notation [1]

The observed market segmentation can be included into the interest rate modelling framework using two different approaches.

1. Modeling the joint evolution of a default-free rate plus counterparty's default times: this is not easy because it implies to model the default of the interbank sector, not of a precise counterparty.

2. Modeling the joint evolution of multiple distinct rates: this implies taking the approach of multiple-curves constructions to its logical consequences, and to introduce a generalised interest rate model where such distinct curves are modeled jointly.

We will follow the second route, as described in the recent financial literature (see bibliography). In particular we will borrow mainly from recent papers:

- F. Mercurio (2009-2010)
- Kijima et al. (2008)
- Fujii et al. (2009-2010)

3: Classical vs Modern Market Practice & Modeling

Basic assumptions and notation [2]

We remind and extend here the basic definitions and notations described before

1. Time grids:

$$\begin{aligned}
&\mathbf{S} = \{S_0, ..., S_n\}, \text{ fixed leg schedule,} \\
&\mathbf{T} = \{T_0, ..., T_m\}, \text{ floating leg schedule,} \\
&S_0 = T_0, S_n = T_m,
\end{aligned}$$

   where the floating leg fixing and cash flow frequency δ is both regular and compatible with the tenor of the floating leg rate (e.g. if the rate is Euribor6M, then the floating leg frequency is $2\ y^{-1}$ and the times T_i are six-month spaced).

2. Spot floating rates and Zero Coupon Bonds:
   we denote with with $L_x(T_{i-1},T_i)$ the spot floating rate, where x indexes the different rate tenors. For example in the EUR market we have, typically: $x = \{d, 1M, 3M, 6M, 12M\}$. The simbol L, reminiscent of "Libor", is still used for continuity.
   We can define a Zero Coupon Bond associated to the spot Libor rate as

$$P_x(T_{i-1};T_i) := \frac{1}{1 + L_x(T_{i-1},T_i)\tau_x(T_{i-1},T_i)}.$$

3: Classical vs Modern Market Practice & Modeling
Basic assumptions and notation [3]

3. Risk Free Bank Account, Zero Coupon Bond and Forward Rate:we assume the existence of basic risk free instruments

$$B_d(t) = \exp \int_0^t r_d(u)\, du,$$

$$D_d(t,T) = \frac{B_d(t)}{B_d(T)} = \exp\left[-\int_t^T r_d(u)\, du\right],$$

$$P_d(t;T) = \mathbb{E}_t^{Q_{B_d}}[D_d(t,T)],$$

$$F_d(t;T_{i-1},T_i) = \frac{1}{\tau_d(T_{i-1},T_i)}\left[\frac{P_d(t;T_{i-1})}{P_d(t;T_i)} - 1\right].$$

4. Forward rates:
   Forward rates $F_x(t;T_{i-1},T_i)$ are not defined, see the following.

3: Classical vs Modern Market Practice & Modeling
Pricing vanilla FRAs

Standard FRA: the payoff at time T_i of the standard FRA tied to risky Libor $L_x(T_{i-1},T_i)$ is

$$\mathbf{FRA}_{\mathrm{Std}}(T_i;\mathbf{T},K,\omega) = N\omega\left[L_x(T_{i-1},T_i) - K\right]\tau_x(T_{i-1},T_i),$$

The price at time $t<T_{i-1}$ is given by

$$\mathbf{FRA}_{\mathrm{Std}}(t;\mathbf{T},K,\omega) = P_d(t;T_i)\mathbb{E}_t^{Q_d^{T_i}}\left[\mathbf{FRA}_{\mathrm{Std}}(T_i;\mathbf{T},K,\omega)\right]$$
$$= N\omega P_d(t;T_i)\left\{\mathbb{E}_t^{Q_d^{T_i}}\left[L_x(T_{i-1},T_i)\right] - K\right\}\tau_x(T_{i-1},T_i),$$

and we can define the generalised FRA rate as

$$\tilde{R}_{x,\mathrm{Std}}^{\mathrm{FRA}}(t;\mathbf{T}) := \tilde{F}_{x,i}(t) := \mathbb{E}_t^{Q_d^{T_i}}\left[L_x(T_{i-1},T_i)\right]$$
$$\neq R_{\mathrm{Std}}^{\mathrm{FRA}}(t;T_{i-1},T_i)$$

such that the standard FRA price can be written as

$$\mathbf{FRA}_{\mathrm{Std}}(t;\mathbf{T},K,\omega) = N\omega P_d(t;T_i)\left[\tilde{F}_{x,i}(t) - K\right]\tau_x(T_{i-1},T_i).$$

3: Classical vs Modern Market Practice & Modeling
The generalised FRA rate

Properties of the generalised FRA rate:

1. It coincides with the Libor rate at fixing date T_{i-1}

$$\tilde{F}_{x,i}(T_{i-1}) = L_x(T_{i-1}, T_i).$$

2. It is a martingale under the T_i- forward discounting measure $Q_d^{T_i}$ associated to the numeraire $P_d(t;T_i)$:

$$\boxed{\tilde{F}_{x,i}(t) = \mathbb{E}_t^{Q_d^{T_i}}\left[L_x(T_{i-1}, T_i)\right] = \mathbb{E}_t^{Q_d^{T_i}}\left[\tilde{F}_{x,i}(T_{i-1})\right],}$$

3. In the single curve limit it coincides with the classical single-curve value

$$\begin{aligned}\tilde{F}_{x,i}(t) &:= \mathbb{E}_t^{Q_d^{T_i}}\left[L_x(T_{i-1}, T_i)\right] \rightarrow \mathbb{E}_t^{Q^{T_i}}\left[L(T_{i-1}, T_i)\right] \\ &= \mathbb{E}_t^{Q^{T_i}}\left[F(T_{i-1}; T_{i-1}, T_i)\right] = F(t; T_{i-1}, T_i) := F_i(t),\end{aligned}$$

   thanks to the (single-curve) martingality property of the forward rate $F(t;T_{i-1},T_i)$ under the forward measure Q^{T_i}.

4. Usually, FRAs are quoted in terms of the FRA rate, thus it is "what you read on the screen". Notice that we do not even need to talk about "forward rates" anymore: the FRA rate is the basic building block of the new theoretical interest rate framework.

Pricing vanilla FRAs

	FRA pricing formulas
Classical (single-curve)	$\mathbf{FRA}_{\mathrm{Std}}(t;T_{i-1},T_i,K,\omega) = N\omega P(t;T_i)\left[F_i(t)-K\right]\tau(T_{i-1},T_i),$ $R^{\mathrm{FRA}}_{\mathrm{Std}}(t;\mathbf{T}) = F_i(t) = \mathbb{E}_t^{Q^{T_i}}\left[L(T_{i-1},T_i)\right],$ $\mathbf{FRA}_{\mathrm{Mkt}}(t;T_{i-1},T_i,K,\omega) = \mathbf{FRA}_{\mathrm{Std}}(t;T_{i-1},T_i,K,\omega),$ $R^{\mathrm{FRA}}_{\mathrm{Mkt}}(t,\mathbf{T}) = R^{\mathrm{FRA}}_{\mathrm{Std}}(t,\mathbf{T}).$
Modern (multiple-curve)	$\mathbf{FRA}_{\mathrm{Std}}(t;T_{i-1},T_i,K,\omega) = N\omega P_d(t;T_i)\left[\tilde{F}_{x,i}(t)-K\right]\tau_x(T_{i-1},T_i),$ $\tilde{R}^{\mathrm{FRA}}_{x,\mathrm{Std}}(t;\mathbf{T}) = \tilde{F}_{x,i}(t) := \mathbb{E}_t^{Q_d^{T_i}}\left[L_x(T_{i-1},T_i)\right],$ $\mathbf{FRA}_{\mathrm{Mkt}}(t;\mathbf{T},K,\omega) = N\omega P_d(t;T_{i-1})\left[1-\frac{1+\tau_x(T_{i-1},T_i)K}{1+\tau_x(T_{i-1},T_i)\tilde{F}_{x,i}(t)}e^{C_x^{\mathrm{FRA}}(t;T_{i-1})}\right],$ $\tilde{R}^{\mathrm{FRA}}_{x,\mathrm{Mkt}}(t;\mathbf{T}) = \frac{1}{\tau_x(T_{i-1},T_i)}\left\{\left[1+\tau_x(T_{i-1},T_i)\tilde{F}_{x,i}(t)\right]e^{C_x^{\mathrm{FRA}}(t;T_{i-1})}-1\right\}.$

See Mercurio (2010) for an explicit calculation of the market FRA convexity adjustment.

3: Classical vs Modern Market Practice & Modeling

Pricing Futures

	Futures pricing formulas
Classical (single-curve)	$\mathbf{Futures}(t;\mathbf{T}) = N\left[1 - R^{\mathrm{Fut}}(t;\mathbf{T})\right],$ $R^{\mathrm{Fut}}(t;\mathbf{T}) := \mathbb{E}_t^{Q_B}\left[L(T_{i-1},T_i)\right] = F_i(t) + C^{\mathrm{Fut}}(t,T_{i-1}).$
Modern (multiple-curve)	$\mathbf{Futures}(t;\mathbf{T}) = N\left[1 - R_x^{\mathrm{Fut}}(t;\mathbf{T})\right],$ $R_x^{\mathrm{Fut}}(t;\mathbf{T}) := \mathbb{E}_t^{Q_{B_d}}\left[L_x(T_{i-1},T_i)\right] = \tilde{F}_{x,i}(t) + C_x^{\mathrm{Fut}}(t,T_{i-1}).$

See Mercurio (2010) for an explicit calculation of the Futures convexity adjustment.

3: Classical vs Modern Market Practice & Modeling

Pricing vanilla Swaps

	Swap pricing formulas
Classical (single-curve)	$\mathbf{Swap}(t;\mathbf{T},\mathbf{S},K,\omega) = N\omega\left[R^{\text{Swap}}(t;\mathbf{T},\mathbf{S}) - K\right]A_d(t,\mathbf{S}),$ $R^{\text{Swap}}(t;\mathbf{T},\mathbf{S}) = \frac{\sum_{j=1}^{m} P(t,T_j)F(t;T_{j-1},T_j)\tau_L(T_{j-1},T_j)}{A(t,\mathbf{S})}$ $\simeq \frac{P(t,T_0)-P(t,T_m)}{A(t,\mathbf{S})}.$
Modern (multiple-curve)	$\mathbf{Swap}(t;\mathbf{T},\mathbf{S},K,\omega) = N\omega\left[\tilde{R}_x^{\text{Swap}}(t;\mathbf{T},\mathbf{S}) - K\right]A_d(t,\mathbf{S}),$ $\tilde{R}_x^{\text{Swap}}(t;\mathbf{T},\mathbf{S}) = \frac{\sum_{j=1}^{m} P_d(t,T_j)\tilde{F}_{x,j}(t)\tau_x(T_{j-1},T_j)}{A_d(t,\mathbf{S})}.$

Notice that the legs of a spot-starting swap do not need to be worth par (when a fictitious exchange of notionals is introduced at maturity).
However, this is not a problem, because the only requirement for quoted spot-starting swaps is that their initial total value must be equal to zero.

Pricing Overnight Indexed Swaps

	OIS pricing formulas
Classical (single-curve)	$\mathbf{OIS}(t;\mathbf{T},K,\omega) = N\omega\left[\sum_{i=1}^{n} P(t;T_i)R^{\mathrm{on}}(t;\mathbf{T}_i)\tau_R(T_{i-1},T_i) - KA(t;\mathbf{T})\right],$ $R^{\mathrm{OIS}}(t;\mathbf{T}) = \frac{\sum_{i=1}^{n} P(t;T_i)R^{\mathrm{on}}(t;\mathbf{T}_i)\tau_R(T_{i-1},T_i)}{A(t;\mathbf{T})}.$
Modern (multiple-curve)	$\mathbf{OIS}(t;\mathbf{T},K,\omega) = N\omega\left[\sum_{i=1}^{n} P_d(t;T_i)R^{\mathrm{on}}(t;\mathbf{T}_i)\tau_R(T_{i-1},T_i) - KA_d(t;\mathbf{T})\right],$ $R_d^{\mathrm{OIS}}(t;\mathbf{T}) = \frac{\sum_{i=1}^{n} P_d(t;T_i)R^{\mathrm{on}}(t;\mathbf{T}_i)\tau_R(T_{i-1},T_i)}{A_d(t;\mathbf{T})}.$

3: Classical vs Modern Market Practice & Modeling

Pricing vanilla Basis Swap

	Basis Swap pricing formulas
Classical (single-curve)	$\Delta(t;\boldsymbol{T}_x,\boldsymbol{T}_y,\boldsymbol{S},\omega) = \Delta(t;\boldsymbol{T}_x,\boldsymbol{T}_y) = 0$!!
Modern (multiple-curve)	$\Delta(t;\boldsymbol{T}_x,\boldsymbol{T}_y,\boldsymbol{S},\omega) = \dfrac{\mathbf{Swap}_{\text{float}}(t;\boldsymbol{T}_x) - \mathbf{Swap}_{\text{float}}(t;\boldsymbol{T}_y)}{N\omega A_d(t;\boldsymbol{S})},$ $\Delta(t;\boldsymbol{T}_x,\boldsymbol{T}_y) = \dfrac{\mathbf{Swap}_{\text{Float}}(t;\boldsymbol{T}_x,\omega) - \mathbf{Swap}_{\text{Float}}(t;\boldsymbol{T}_y,\omega)}{N\omega A_d(t;\boldsymbol{T}_y)}.$

3: Classical vs Modern Market Practice & Modeling
Pricing vanilla Floating Rate Bonds

	Floating rate Bond pricing formulas
Classical (single-curve)	$\mathbf{Bond}(t;\mathbf{T}) \simeq N P_d(t, T_0)$
Modern (multiple-curve)	$\mathbf{Bond}(t;\mathbf{T}) = N \sum_{j=1}^{m} P_d(t, T_j) \tilde{F}_{x,j}(t) \tau_x(T_{j-1}, T_j) + N P_d(t, T_m)$

Notice that the spot-starting swap do not need to be worth par.

3: Classical vs Modern Market Practice & Modeling
Pricing vanilla Caps/Floors/Swaptions

	Caplet/floorlet pricing formulas
Classical (single-curve)	$NP(t, T_i)\text{Black}\left[F_i(t), K, v(t; T_{i-1}), \omega\right]\tau(T_{i-1}, T_i)$
Modern (multiple-curve)	$NP_d(t, T_i)\text{Black}\left[\tilde{F}_{x,i}(t), K, \tilde{v}_x(t; T_{i-1}), \omega\right]\tau_x(T_{i-1}, T_i)$

	Swaption pricing formulas
Classical (single-curve)	$NA(t, \mathbf{S})\text{Black}\left[R^{\text{Swap}}(t, \mathbf{T}, \mathbf{S}), K, v(t, \mathbf{T}, \mathbf{S}), \omega\right]$
Modern (multiple-curve)	$NA_d(t, \mathbf{S})\text{Black}\left[\tilde{R}_x^{\text{Swap}}(t, \mathbf{T}, \mathbf{S}), K, \tilde{v}_x(t, \mathbf{T}, \mathbf{S}), \omega\right]$

3: Classical vs Modern Market Practice & Modeling
SABR model revisited

The classical SABR model derivation by Hagan et al. (2002) makes neither explicit nor hidden assumptions regarding the nature of the yield curves underlying the FRA rates. Hence, the extension of the classical model to the modern framework is trivial, just requiring the replacement of the classical forward rate with the modern FRA rate and of the T_i - forward Libor measure associated with the classical single curve numeraire $P(t;T_i)$ with the modern T_i–forward measure associated with the discounting numeraire $P_d(t;T_i)$.

	Classical vs modern SABR
Classical SABR	$\frac{dF_i(t)}{F_i^\beta(t)} = V(t)dW_i^{Q^{T_i}}(t),$ $\frac{dV(t)}{V(t)} = \nu dZ^{Q^{T_i}}(t),\ V(t_0) = \alpha,$ $dW_i^{Q^{T_i}}(t)dZ^{Q^{T_i}}(t) = \rho dt,\ \forall\, i = 1, ..., n.$
Modern SABR	$\frac{d\tilde{F}_{x,i}(t)}{\tilde{F}_{x,i}^\beta(t)} = V_x(t)dW_{x,i}^{Q_d^{T_i}}(t),$ $\frac{dV_x(t)}{V_x(t)} = \nu dZ_x^{Q_d^{T_i}}(t),\ V_x(t_0) = \alpha,$ $dW_{x,i}^{Q_d^{T_i}}(t)dZ_x^{Q_d^{T_i}}(t) = \rho dt,\ \forall\, i = 1, ..., n.$

3: Classical vs Modern Market Practice & Modeling
Modern multiple curves market practice [1]

In case of vanilla linear derivatives the modern procedure is as follows:

1. build a single discounting curve $\mathcal{C}_d$ using the preferred bootstrapping procedure;
2. build multiple distinct forwarding curves $\mathcal{C}_{f1}, \ldots, \mathcal{C}_{fn}$ using distinct selections of vanilla interest rate instruments, each homogeneous in the underlying rate tenor (typically 1M, 3M, 6M, 12M);
3. compute the FRA/Swap rates with tenor *f* on the corresponding forwarding curve $\mathcal{C}_f$ and calculate the corresponding cash flows;
4. compute the corresponding discount factors using the discounting curve $\mathcal{C}_d$ and work out prices by summing the discounted cashflows;
5. compute the delta sensitivity and hedge the resulting delta risk using the suggested amounts (hedge ratios) of the corresponding set of vanillas.

3: Classical vs Modern Market Practice & Modeling
Modern multiple curves market practice [2]

In case of volatility derivatives the procedure above must be extended as follows:

1. build multiple distinct volatility surfaces $\Sigma_{f1} \dots \Sigma_{fn}$ using distinct selections of vanilla interest rate options, each homogeneous in the underlying rate tenor, typically 1M, 3M, 6M, 12M for Euribor rate and swap rate volatilities;

2. compute_the FRA/Swap rates and volatilities with tenor *f* on the corresponding curves $\mathcal{C}_f$ and volatility surfaces Σ_{f1} , and calculate the corresponding cashflows;

3. compute the corresponding discount factors using the discounting curve $\mathcal{C}_d$ and work out prices by summing the discounted cashflows;

4. compute the delta and vega sensitivities and hedge the resulting delta and vega risk using the suggested amounts (hedge ratios) of the corresponding set of vanillas.

3: Classical vs Modern Market Practice & Modeling
Modern multiple curves market practice [3]

In case of non-vanilla derivatives the procedure above must be extended as follows:

- Choose the fundamental variables:
  - Multiple short rates $\Rightarrow$ multi-curve short rate models.
  - Multiple instantaneous forward rates $\Rightarrow$ multi-curve HJM models.
  - Multiple discrete FRA rates $\Rightarrow$ multi-curve Black's model, SABR, Libor Market Model.
  - Multiple forward Swap rates $\Rightarrow$ multi-curve Black's model, SABR, Swap Market Model.
- Assume a dynamics for the time evolution of the fundamental variables.
- Derive (arbitrage free) pricing formulas for plain vanilla instruments.
- Calibrate the model parameters to the market quotes of a chosen set of plain vanilla instruments (calibration instruments).
- Price other derivatives using the calibrated model.
- Derive sensitivities and hedge ratios with respect to a choosen subset of calibration instruments (hedging instruments).

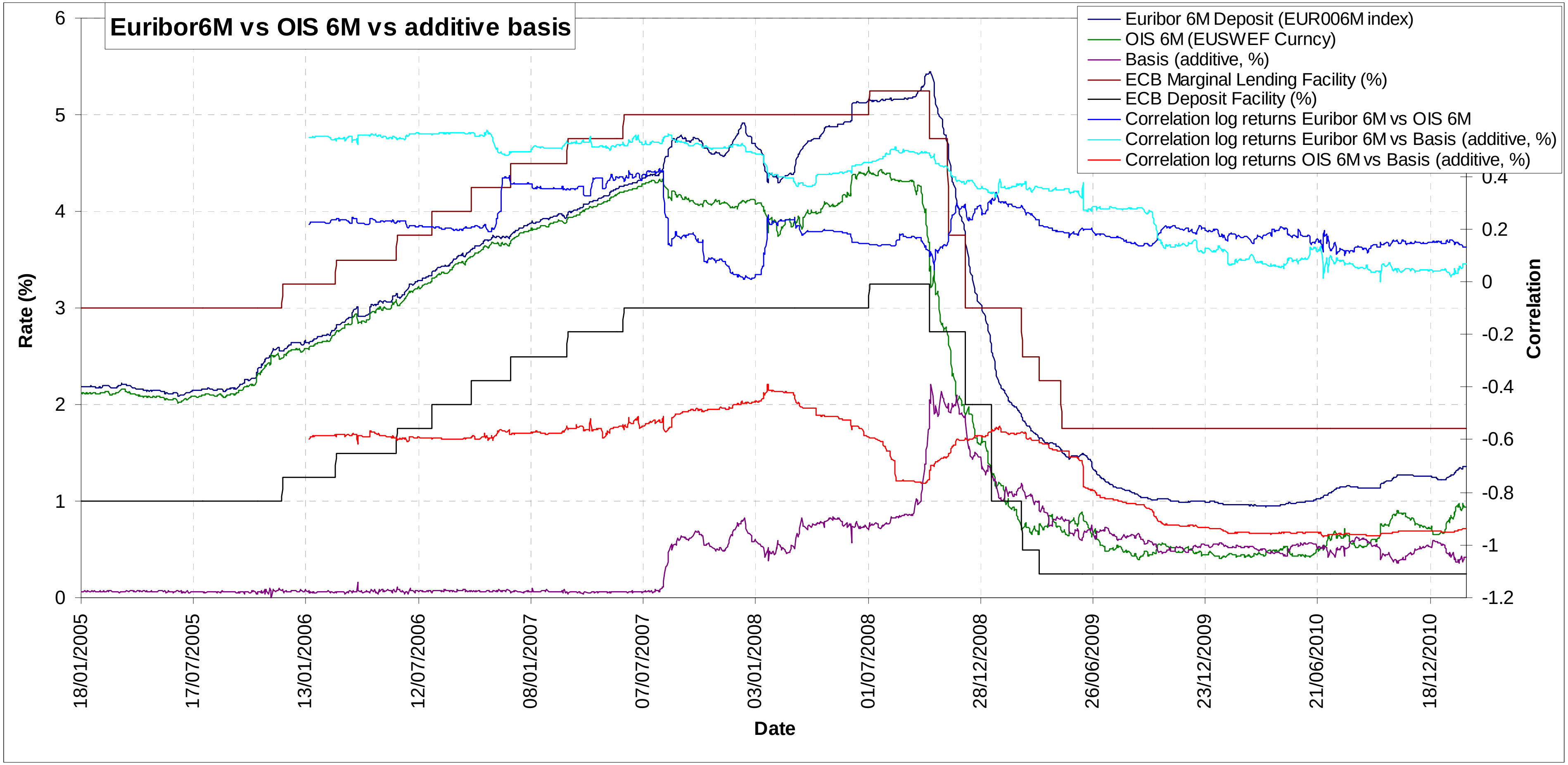
3: Classical vs Modern Market Practice & Modeling
Multiple curve models

But research is just at the beginning:

- Libor Market Models: see e.g. F. Mercurio, "*A LIBOR Market Model with Stochastic Basis*", Risk Magazine, Dec. 2010 and refs. therein.
- HJM models: see e.g N. Moreni, A. Pallavicini, "*Parsimonious HJM Modelling for Multiple Yield-Curve Dynamics*", Oct. 2010, SSRN working paper, *http://ssrn.com/abstract=1699300*
- Short rate models: see e.g. C. Kenyon, "*Post Shock Short-Rate Pricing*", Risk Magazine, Oct. 2010.

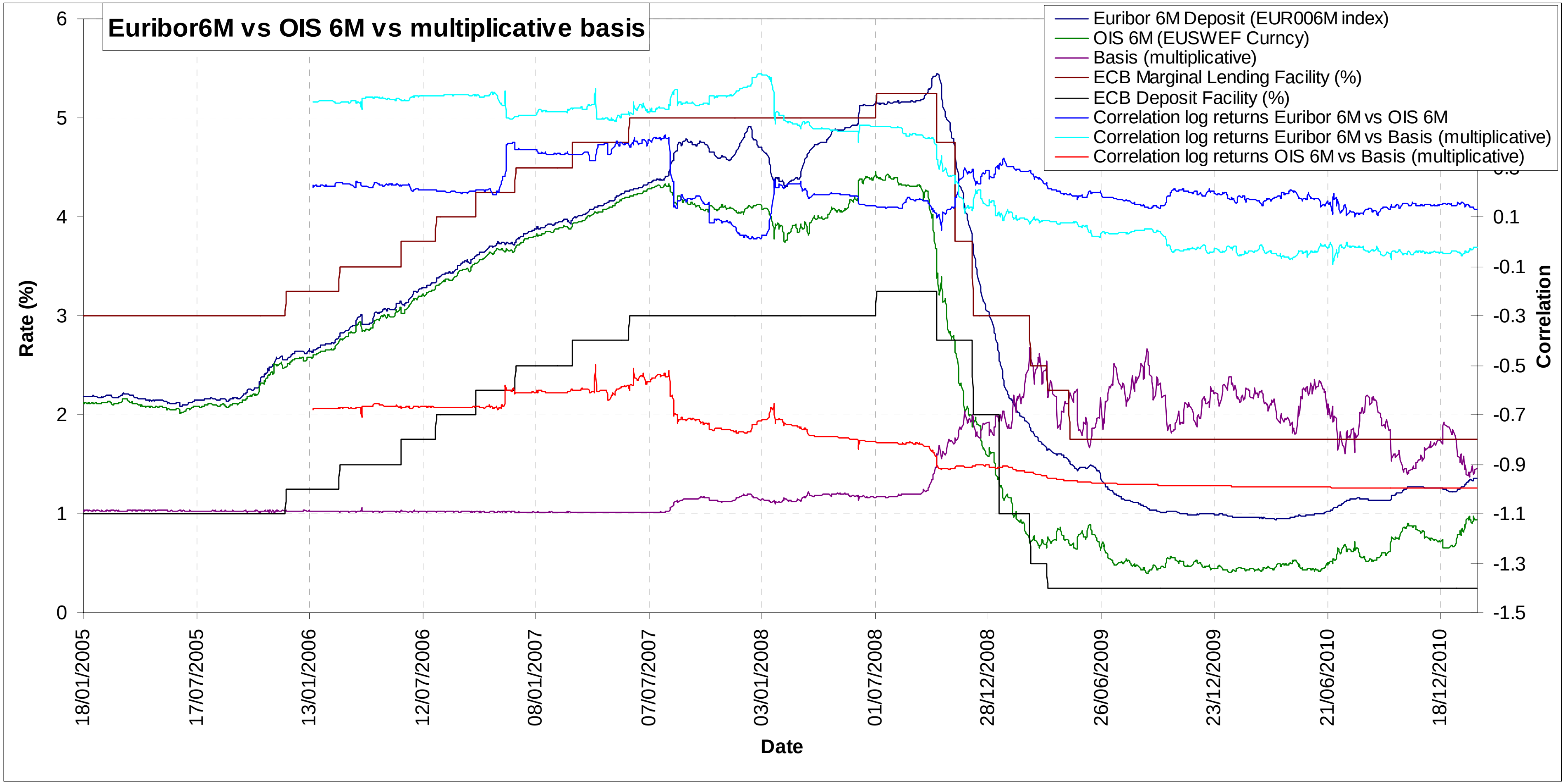
3: Classical vs Modern Market Practice & Modeling

Multiple curve models: additive basis



3: Classical vs Modern Market Practice & Modeling

Multiple curve models: multiplicative basis



3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Swaps [1]

The Forward Start Interest Rate Swaps (FSIRS) contracts quoted on the Euro market are characterized by a floating leg on Euribor 6M with 6-month frequency vs a fixed leg with annual frequency, a forward start date and maturity dates ranging from 1 to 25 years.

FSIRS are more sensible of spot start swaps to the choice of the pricing methodology.

For each pricing methodology:

- Single-Curve (Libor),
- Multiple-Curve No-CSA (Libor discounting and forwarding by tenor),
- Multiple Curve CSA (OIS discounting, Libor forwarding by tenor),

and two valuation dates:

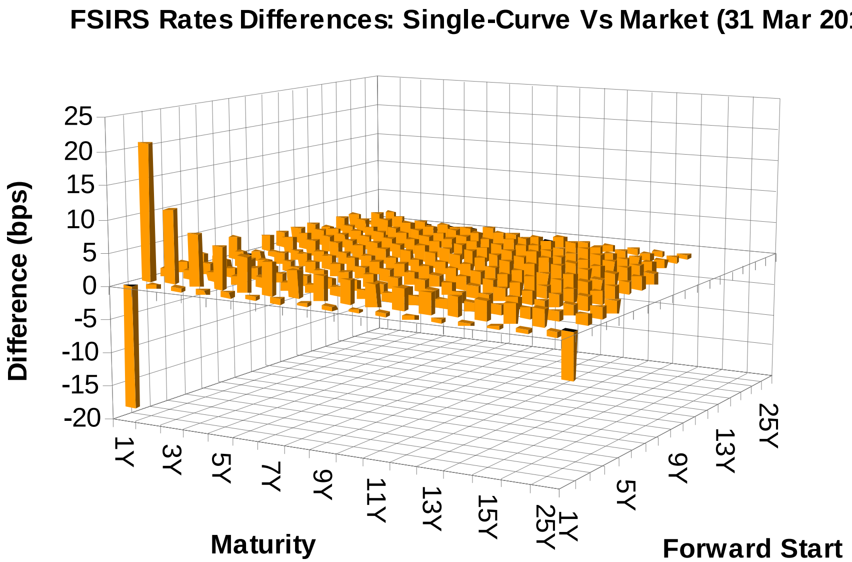
- 31st March 2010,
- 31st August 2010,

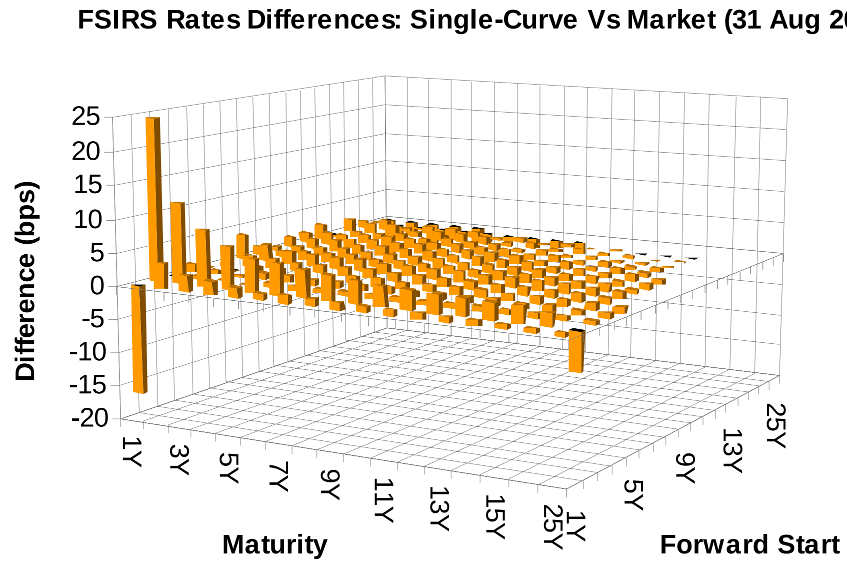
we computed the theoretical equilibrium FSIRS rates and we compared them with the market quotes.

See M. Bianchetti, M. Carlicchi, "Interest Rates After The Credit Crunch: Markets and Models Evolution", http://ssrn.com/abstract=1783070.

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Swaps [2]



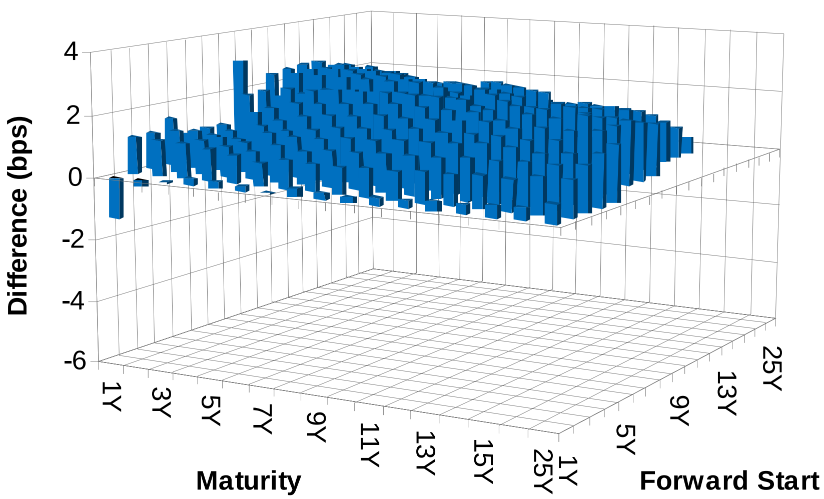


Forward Start IRS: differences between theoretical and market rates (bps).
Single-Curve methodology.
Valuation dates: 31st March 2010 (left side) and 31st August 2010 (right side).
Source: Reuters.

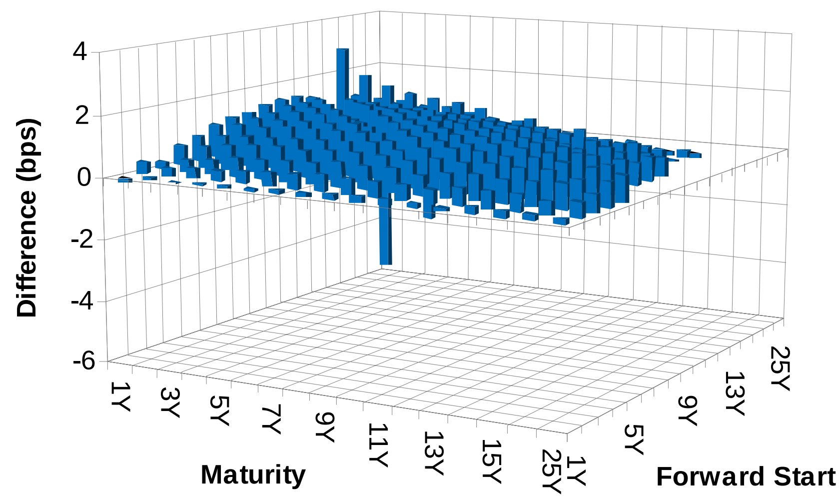
3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Swaps [3]

FSIRS Rates Differences: Multiple-Curve No-CSA Vs Market (31 Mar 2010)



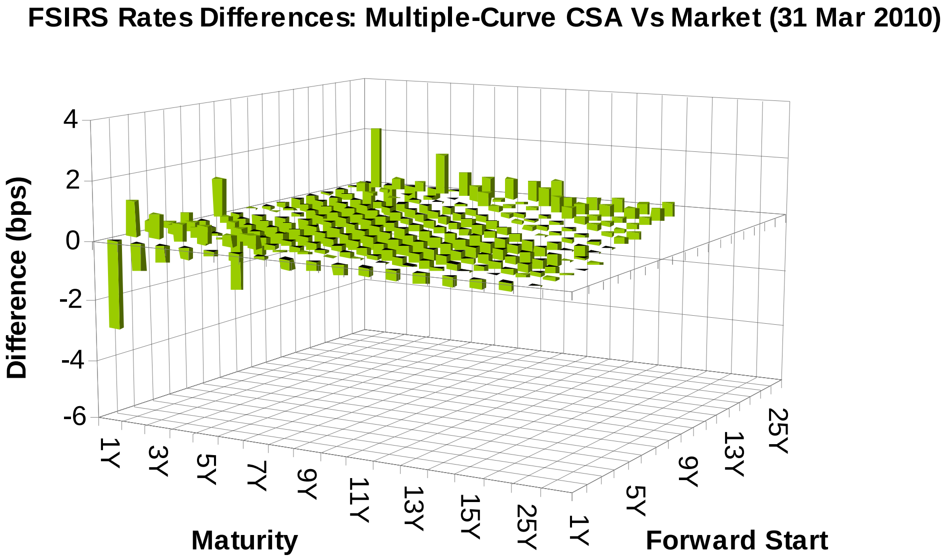
FSIRS Rates Differences: Multiple-Curve No-CSA Vs Market (31 Aug 2010)

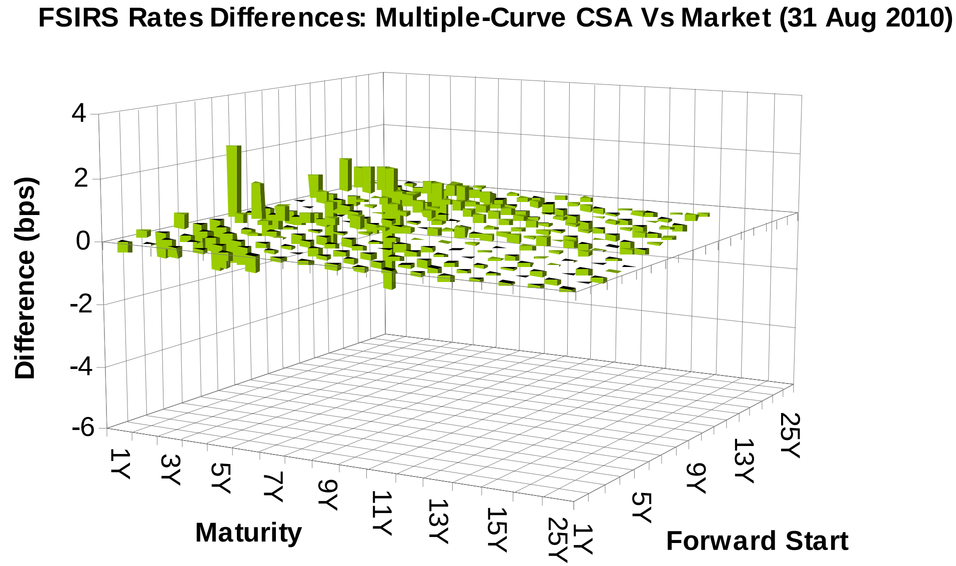


Forward Start IRS: differences between theoretical and market rates (bps).
Multiple-Curve (no-CSA) methodology.
Valuation dates: 31st March 2010 (left side) and 31st August 2010 (right side).
Source: Reuters.

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Swaps [4]





Forward Start IRS: differences between theoretical and market rates (bps).
Multiple-Curve CSA methodology.
Valuation dates: 31st March 2010 (left side) and 31st August 2010 (right side).
Source: Reuters.

3: Classical vs Modern Market Practice & Modeling
Revealing CSA discounting: Swaps [5]

Forward Start Interest Rate Swaps Differences								
	31st March 2010				31st August 2010			
	Range		Standard deviation		Range		Standard deviation	
Single-Curve	[-18.4;+20.8]	[-3.2;+2.7]	2.84	1.89	[-16.3;+24.4]	[-3.9;+1.9]	2.58	1.15
Multiple-Curve No-CSA	[-2.9;+3.1]	[-2.9;+2.6]	1.77	1.86	[-5.7;+2.9]	[-3.7;+1.7]	1.11	1.09
Multiple-Curve CSA	[-2.9;+2.3]	[-1.0;+1.5]	0.53	0.37	[-4.1;+2.4]	[-1.4;+1.0]	0.47	0.26

Forward Start IRS: differences (in basis points).
For each pricing methodology and valuation date, we show the range of minimum and maximum discrepancies and the standard deviation, both considering all FSIRS (columns on the left) and excluding the two 1Y-2Y stripes (columns on the right).

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Swaps [6]

The spikes observed in the figures for start dates below 3Y may be explained in terms of differences in the short term yield curve construction, where there is a significant degree of freedom in choosing the bootstrapping instruments (Deposits, FRAs and Futures). Smaller spikes are also present for short tenor FSIRS with maturity below 3Y because these swaps depend on a few forwards and discounts and, thus, are more sensitive to minor differences in the yield curves. Hence we show results both including and excluding the two “stripes” below 2 years start/maturity date.

Overall we observe that, on both dates,

- the first methodology has the worst performance, producing, on average, overestimated FSIRS rates.
- The second methodology introduces small improvements, at least below 3 years. This is expected, because the two curves used are very similar after 3 years, both using standard Euro Swaps on Euribor 6M.
- The third methodology is by far the best in reproducing the market data. The remaining differences around 1 basis points may be explained with minor differences with respect to the market yield curves.

We conclude that the market of Interest Rate Swaps has abandoned, at least since March 2010, the classical Single-Curve pricing methodology, typical of the pre-credit crunch interest rate world, and has adopted the modern Multiple-Curve CSA approach, thus incorporating into market prices the credit and liquidity effects.

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Caps/Floors [1]

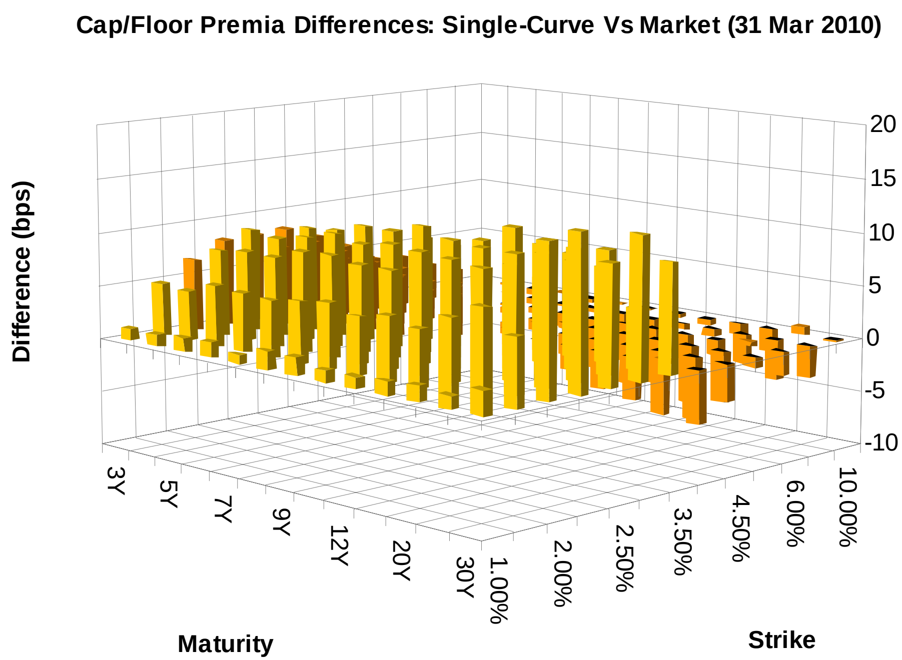
The European Cap/Floor options quoted on the Euro are characterized by floating payments with 6M frequency indexed to Euribor 6M, spot start date, maturity dates ranging from 3 to 30 years, and strikes ranging from 1% to 10%. The first caplet/floorlet, already known at spot date, is not included in the cap/floor premium. The market quotes Floor premia for strikes below the at-the-money (ATM) and Cap premia for strikes above ATM.

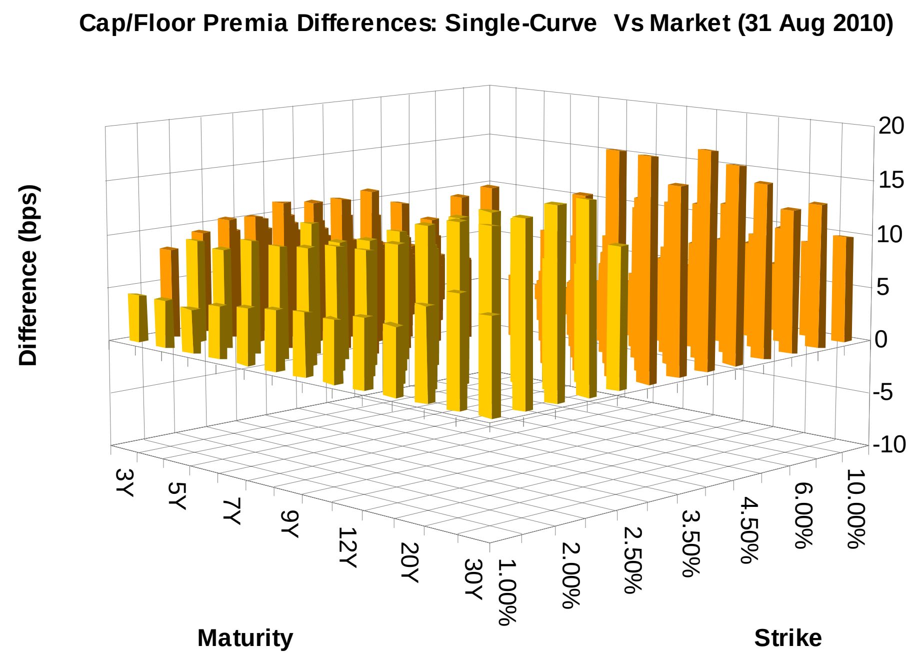
For each methodology and each valuation date (31st March and 31st August 2010) we computed the theoretical Caps/Floors premia and we compared them with the market premia.

See M. Bianchetti, M. Carlicchi, "Interest Rates After The Credit Crunch: Markets and Models Evolution", http://ssrn.com/abstract=1783070.

3: Classical vs Modern Market Practice & Modeling

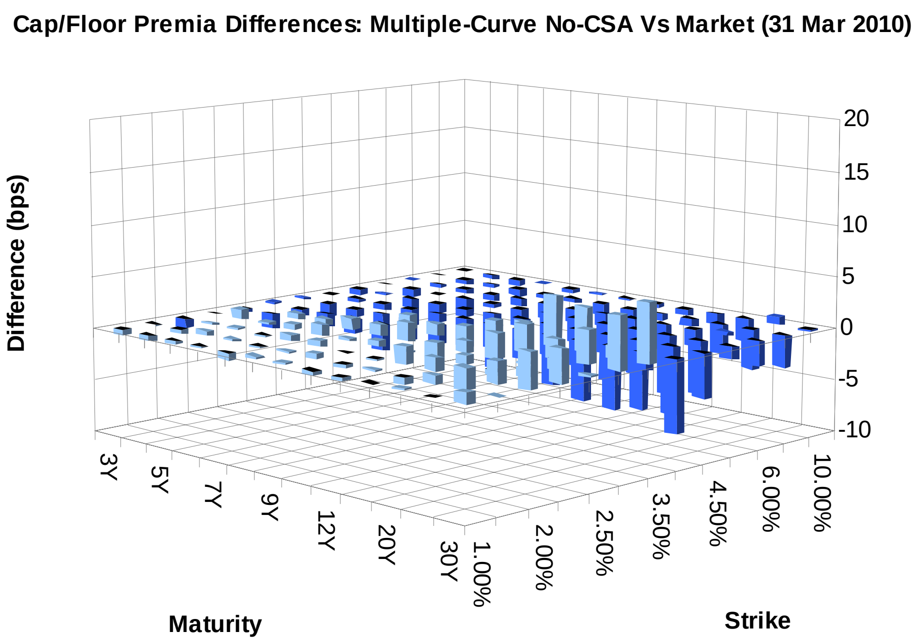
Revealing CSA discounting: Caps/Floors [2]

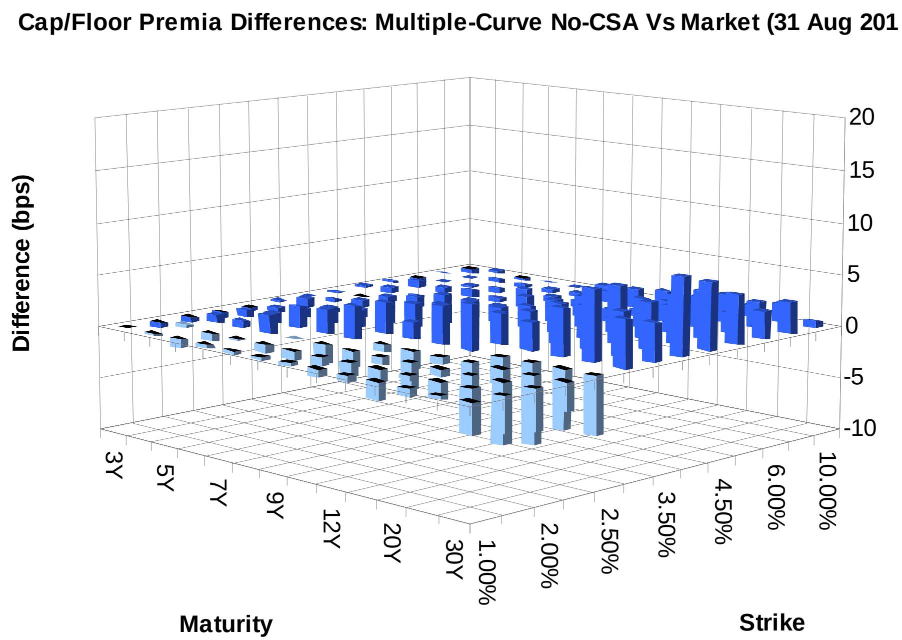




Cap/floor options premia differences (light colours: Floors, dark colours: Caps). Single-Curve methodology. Valuation dates: 31st March 2010 (left side) and 31st August 2010 (right side). (source: Reuters).

3: Classical vs Modern Market Practice & Modeling
Revealing CSA discounting: Caps/Floors [3]

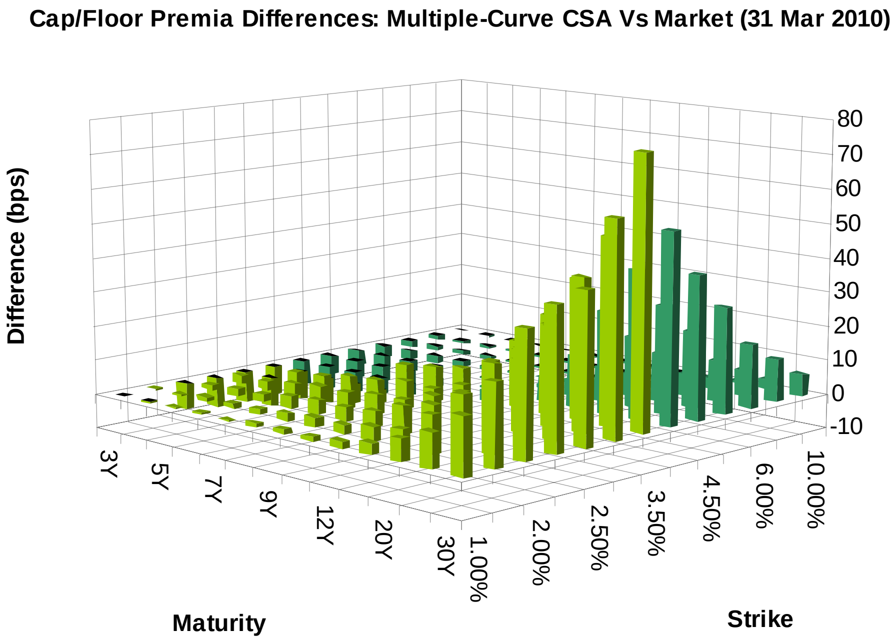


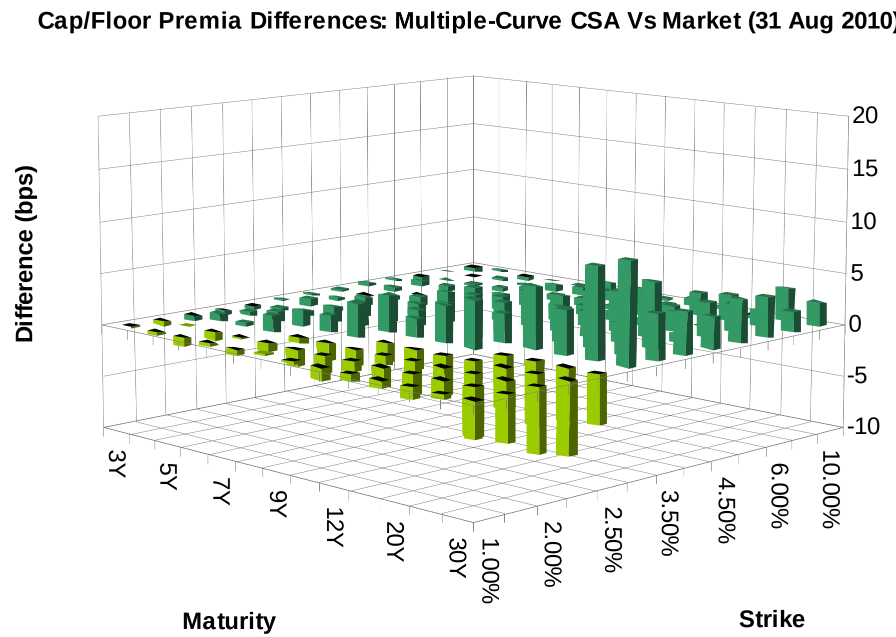


Cap/floor options premia differences (light colours: Floors, dark colours: Caps).
Multiple-Curve No-CSA methodology.
Valuation dates: 31st March 2010 (left side) and 31st August 2010 (right side).
(source: Reuters).

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Caps/Floors [4]





Cap/floor options premia differences (light colours: Floors, dark colours: Caps).
Multiple-Curve methodology.
Valuation dates: 31st March 2010 (left side) and 31st August 2010 (right side).
(source: Reuters).

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Caps/Floors [5]

Cap/Floor Premia Differences				
	31st March 2010		31st August 2010	
	Range	Standard deviation	Range	Standard deviation
Single-Curve	[-5.8;+14.1]	6.3	[+0.2;+20.0]	9.7
Multiple-Curve No-CSA	[-7.0;+5.8]	2.1	[-6.3;+7.4]	2.3
Multiple-Curve CSA	[-8.9;+77.7]	15.8	[-6.8;+9.6]	2.4

Cap/floor options premia differences (in basis points).
For each pricing methodology and valuation date we show the range of minimum and maximum discrepancies and the standard deviation.

3: Classical vs Modern Market Practice & Modeling

Revealing CSA discounting: Caps/Floors [6]

Overall, we notice again that, on both dates:

- The Single-Curve methodology has the worst performance.
- The Multiple-Curve No-CSA methodology has a good performance on both dates, with an absolute average difference of 1.4/1.6 bps over a total of 169 options and a standard deviation of 2.06/2.28 bps.
- The Multiple-Curve CSA methodology shows a bad performance on the first date (standard deviation 15.82 bps) and a performance as good as that of the Multiple-Curve CSA methodology on the second date, with absolute average difference of 1.7 bps and standard deviation of 2.43 bps.

We conclude that the results discussed above are coherent with our findings for Forward Start IRS:

- First of all, the market, at least since March 2010, has abandoned the classical Single-Curve pricing methodology, typical of the pre-credit crunch interest rate world, and has adopted the modern Multiple-Curve approach.
- Second, the transition to the CSA-discounting methodology for options has happened just in August 2010. In this case, contrary to FSIRS, both the two modern Multiple-Curve methodologies (if correctly applied) lead to good repricing of the market premia, because the change in the yield curves (switching from Euribor discounting to Eonia discounting) are compensated by the corresponding changes in the Black implied volatilities (at constant market premia).

3: Classical vs Modern Market Practice & Modeling

Testing SABR vs CSA discounting: Caps/Floors [1]

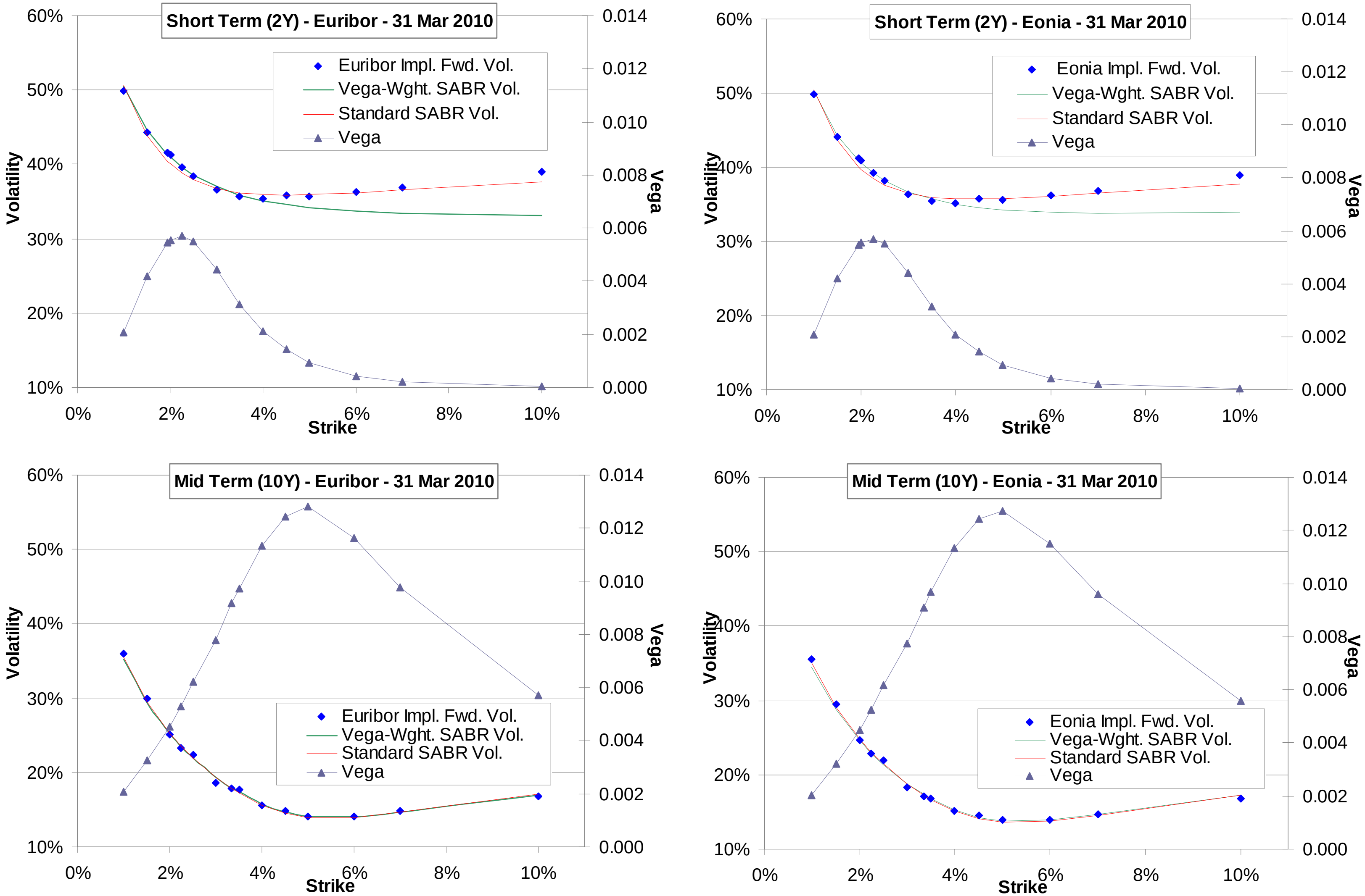
We stripped the two forward volatility surfaces implied in the Cap/Floor premia published by Reuters on the 31st March and on 31st August 2010, using the two Multiple-Curve methodologies (Multiple-Curve No-CSA, Multiple-Curve CSA).

For the two dates (31st March and on 31st August 2010) and the two pricing methodologies associated to the two corresponding forward volatility surfaces (Euribor, Eonia), we performed two minimizations using two distinct error functions: non vega-weighted and vega weighted.

See M. Bianchetti, M. Carlicchi, "Interest Rates After The Credit Crunch: Markets and Models Evolution", http://ssrn.com/abstract=1783070.

3: Classical vs Modern Market Practice & Modeling

Testing SABR vs CSA discounting: Caps/Floors [2]

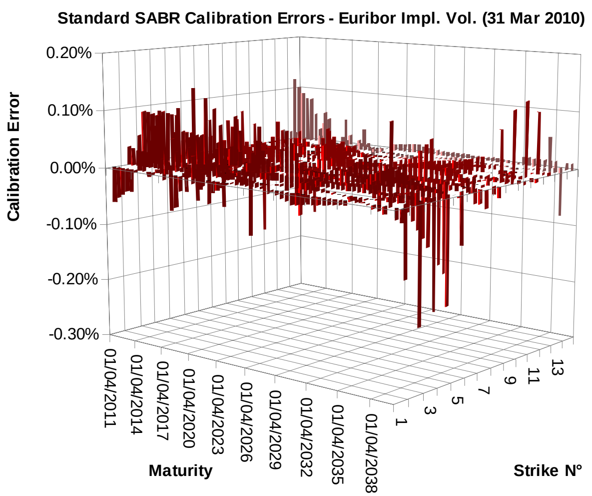


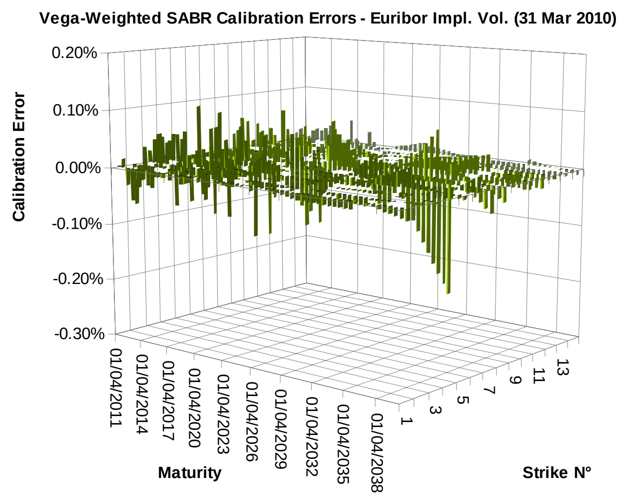
SABR model calibration results.

Valuation date: 31st March 2010 (source: Reuters).

3: Classical vs Modern Market Practice & Modeling

Testing SABR vs CSA discounting: Caps/Floors [3]

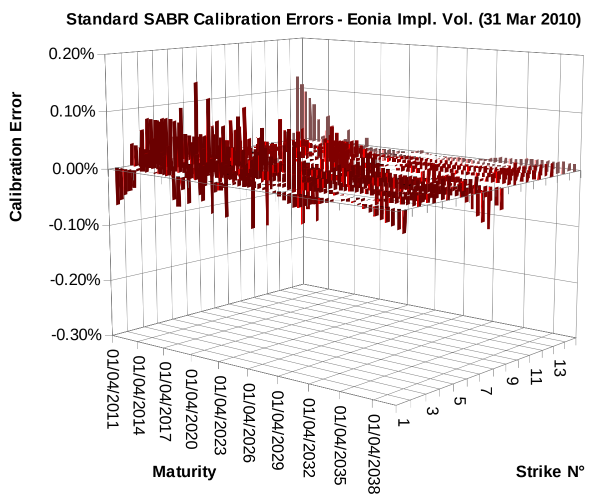


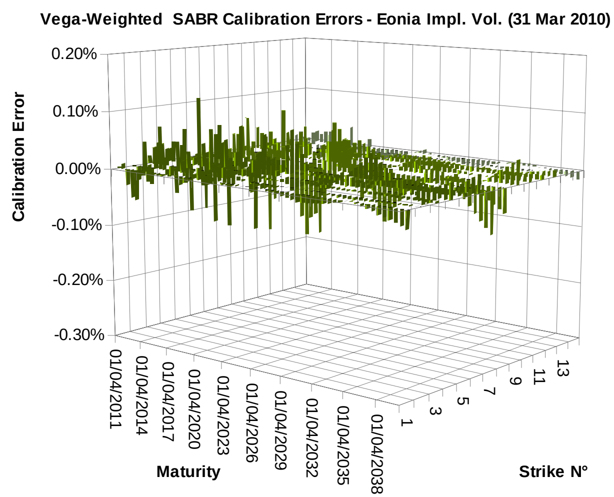


SABR calibration errors.

Upper/lower panels: SABR calibration on the Euribor/Eonia implied volatility surface.

Left/right panels: standard/vega-weighted SABR calibration. Valuation date: 31st March 2010 (source: Reuters).





3: Classical vs Modern Market Practice & Modeling

Testing SABR vs CSA discounting: Caps/Floors [4]

SABR Calibration Errors								
	31st March 2010				31st August 2010			
	Implied Volatility Euribor		Implied Volatility Eonia		Implied Volatility Euribor		Implied Volatility Eonia	
	Range	Standard Deviation	Range	Standard Deviation	Range	Standard Deviation	Range	Standard Deviation
Standard Calibration	[-0.2%;+0.1%]	0.0003	[-0.1%;+0.1%]	0.0003	[-0.3%;+0,2%]	0.0004	[-0.3%;+0.2%]	0.0004
Vega-Weighted Calibration	[-0.2%;+0.1%]	0.0003	[-0.1%;+0.1]	0.0002	[-0.1%;+0.1]	0.0004	[-0.1%;+0.1%]	0.0004

SABR model calibration errors over all the market volatility smile.
For each calibration procedure (standard and vega-weighted) and for each valuation date (31st March and 31st August 2010), we report the range of minimum and maximum calibration errors and the standard deviation of the errors (equally-weighted for standard calibration and vega-weighted for vega-weighted calibration).

3: Classical vs Modern Market Practice & Modeling

Testing SABR vs CSA discounting: Caps/Floors [5]

- Overall, the SABR model performs very well at both dates with both pricing methodologies. In particular, we notice that in the short term (2-year) the standard SABR calibration (red line) seems, at first sight, closer to the market volatility (blue dots) and to better replicate the trend in the OTM regions. However, a closer look reveals that there are significant differences in the ATM area, where even small calibration errors can produce sensible price variations. Instead, the vega-weighted SABR calibration (green line) gives a better fit of the market volatility smile in the ATM region, in correspondence of the maximum vega sensitivity, and allows larger differences in the OTM regions where the vega sensitivity is close to zero. Thus the vega-weighted calibration permits a more efficient fit in the volatility surface regions that are more critical for option pricing. The effects is less visible for long terms because of the higher vega sensitivity in the OTM regions.
- Both the standard and the vega-weighted approaches lead to similar results in terms of range of minimum and maximum errors and standard deviation. In particular, the standard deviation measures of the errors over the 30-year term structure are almost the same: this is due to the fact that only in the short term (up to 4 years) the two calibration differ and using a vega-weighted minimization can ensure a more better fitting of the market data.
- We conclude that the SABR model is quite robust under generalisation to the modern pricing framework and can be applied to properly fit the new dynamics of the market volatility smile and to price off-the-market options coherently with the new market evidences.

4: Switching To CSA Discounting In Practice:
Main Issues

- Market issues
- Methodological issues
- Liquidity and collateral management issues
- ALM issues
- IT issues
- Accounting issues
- Risk Management issues
- Management issues

4: Switching To CSA Discounting In Practice:
Market Issues [1]

- Slow market transition:
  some market quotations have already shifted towards OIS discounting (e.g. interest rate swaps and options), others have not, or are unclear.

- Controversial or lacking market evidences of CSA-dependent fair values, especially for non-interest rate derivatives and for non-CSA counterparties.

4: Switching To CSA Discounting In Practice: *Methodological Issues*

The theoretical framework is still incomplete:

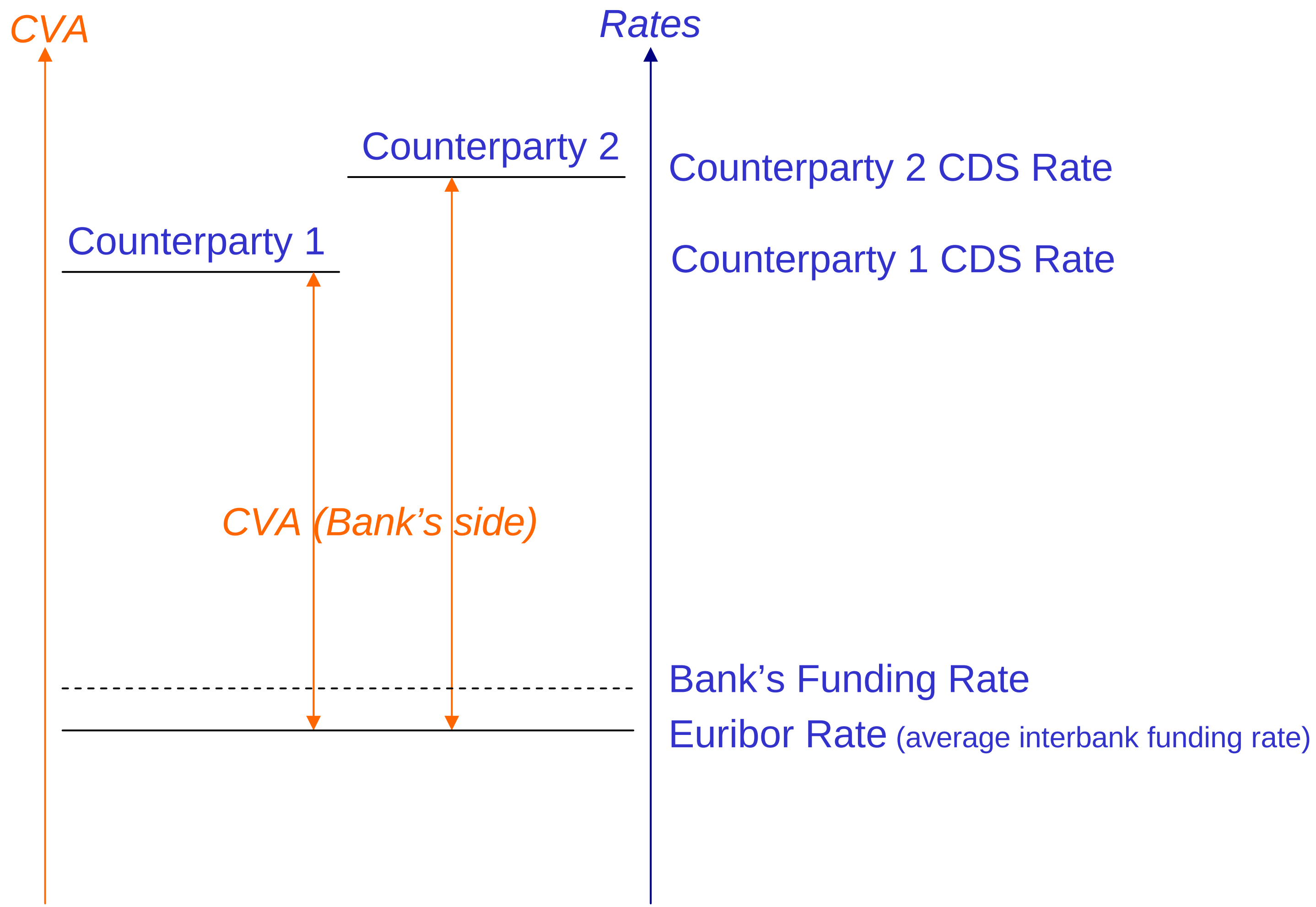
- Pricing trades under CSA:
  - some market quotations have already shifted towards OIS discounting (e.g. interest rate swaps and options), others have not, or are unclear.
  - Multiple curve models still under construction.
  - CSA chaos: pricing the option to chose the most convenient currency for collateral margination.

- Pricing trades without CSA:
  - Funding curve construction including all sources of funding (money market, repo, collateral, bond issuance, retail bank accounts, etc.)
  - Coherent pricing including Credit Value Adjustment (CVA), Debt Value Adjustment (DVA), Funding Value Adjustment (FVA).

4: Switching To CSA Discounting In Practice:
Methodological issues: trades under CSA

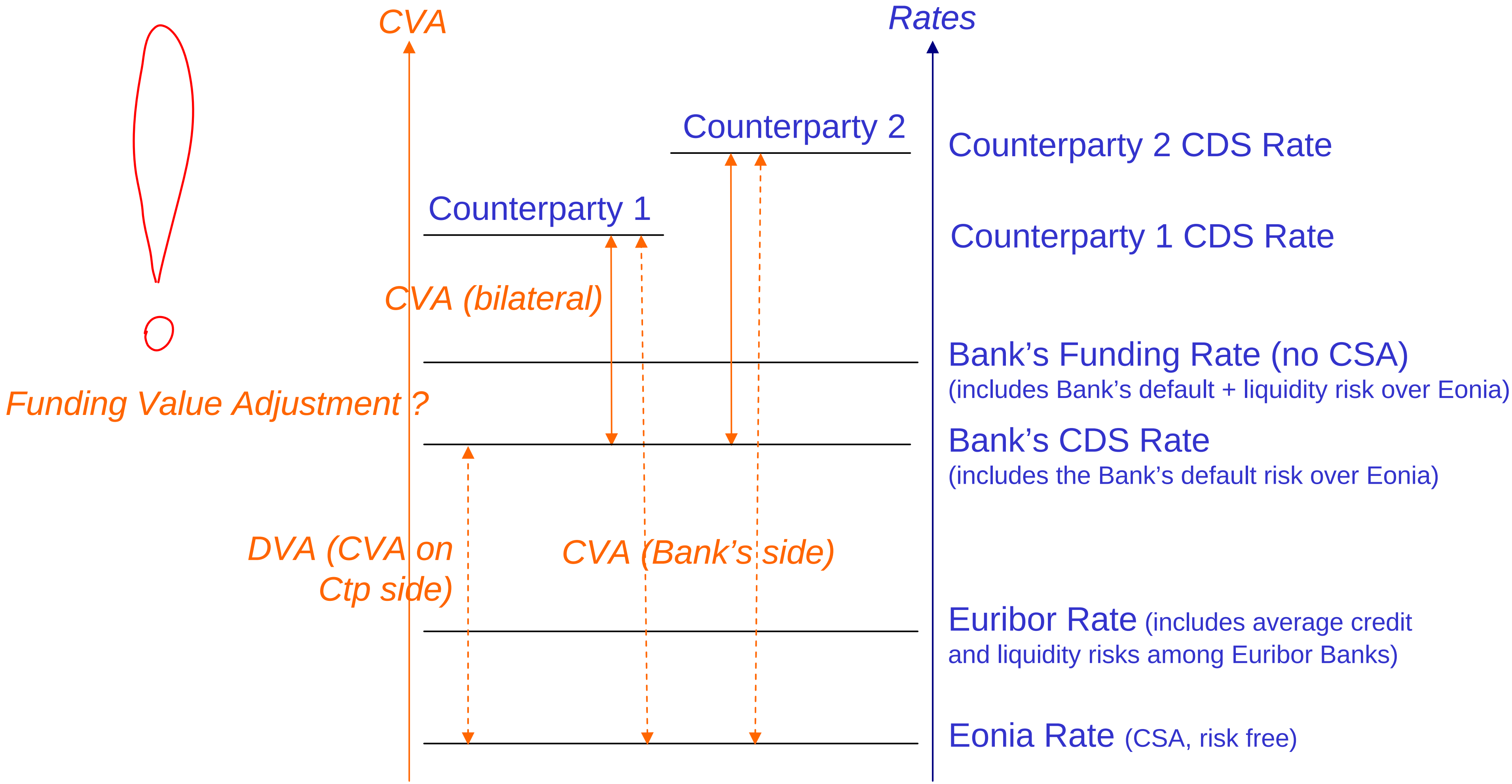
How the market is changing	
Triggers	Main dealers, main brokers, SwapClear.
Currencies	EUR (Eonia), USD (Fed Fund rate) GBP (Sonia) at the beginning.
IR Swaps	Spot starting: NPV = 0, constant swap rate quotation. Forward starting: NPV = 0, changes the swap rate quotation.
IR Options	Shift to forward premium quotation => changes the spot premium, the Black's implied volatility, the smile (changes the ATM), the SABR calibrations.
CMS	NPV = 0, constant CMS spreads, changes beta SABR.
CMS Spread Options	Constant premium, change the (bilognormal) implied correlation.
Inflation Swaps	NPV = 0, constant ZC rate, changes YoY rate.
Inflation Options	Constant spot premiums, changes the Black's implied volatility.
Equity Options	Constant premiums, changes the implied dividends and/or Repo Rates, changes the Black's implied volatility.
CDS	NPV = 0, constant CDS spread, changes the default probability.
Bonds	Constant prices, changes the credit spread absorbing the liquidity/credit risk inside Libor vs Eonia.

4: Switching To CSA Discounting In Practice:
Methodological issues: CVA/DVA/FVA puzzle [1]



Classical pre-credit crunch discounting at Euribor for interbank counterparties + CVA for non-interbank counterparties.

4: Switching To CSA Discounting In Practice:
Methodological issues: CVA/DVA/FVA puzzle [2]



Modern post-credit crunch discounting at Eonia for CSA counterparties + CVA + DVA for non-CSA counterparties. Pay attention to the double counting of DVA.
See M. Morini & A. Prampolini, Risk Magazine Mar. 2011.

4: Switching To CSA Discounting In Practice:
Methodological issues: CVA/DVA/FVA puzzle [3]

Credit Value Adjustment (CVA), Debt Value Adjustment (DVA) and Liquidity Value Adjustment (LVA) are the main issues in the modern interest rate market
A consistent pricing framework is still under development.

See e.g.

- V. V. Piterbarg, "*Funding beyond discounting: collateral agreements and derivatives pricing*", Risk, Feb. 2010, *http://www.risk.net/digital_assets/735/piterbarg.pdf*.
- M. Fujii, A. Takahashi, "*Asymmetric and Imperfect Collateralization, Derivative Pricing, and CVA*", Dec. 2010, SSRN working paper, *http://ssrn.com/abstract=1731763*.
- D. Brigo, A. Capponi, A. Pallavicini, V. Papatheodorou, "*Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment Including Re-Hypotecation and Netting*", Jan. 2011, SSRN working paper, *http://ssrn.com/abstract=1744101*.
- M. Morini, A. Prampolini, "*Risky Funding with countarparty and liquidity charges*", Risk, Mar. 2011, SSRN working paper, 30 Aug. 2010, *http://ssrn.com/abstract=1669930*.
- C. Burgard, M. Kjaer, *"In the Balance"*, 14 Mar. 2011, SSRN working paper *http://ssrn.com/abstract=1785262*.
- D. Lu, J. Frank, "*Credit Value Adjustment and Funding Value Adjustment All Together*", 5 Apr. 2011), SSRN working paper *http://ssrn.com/abstract=1803823*.

4: Switching To CSA Discounting In Practice: *Liquidity/Collateral issues*

- Controversial collateral evidences: someone asks for overnight, someone for Libor, depending on the situation, time, side.
- The liquidity and collateral management must integrate coherently the cost of funding generated by derivatives and CSAs, thus inducing transfer of business among different areas inside banks.
- Special cases: internal deals with other legal entities within the same Bank Group, deals with Vehicles, etc…
- CSA revision: CSA should be renegotiated to keep uniform conditions (haircuts, daily margination, currency, etc.)
- CSA chaos: collateral currency arbitrages.

4: Switching To CSA Discounting In Practice: *ALM issues*

- Hedge accounting methodology (ad hoc pricing procedures in particular) must be revisited to take into account the basis risk between the bond leg and the swap legs.

- Hedges that display large NPV jumps after switch to OIS discounting must be renegotiated or sterilised.

4: Switching To CSA Discounting In Practice: *ICT issues*

- Financial libraries must be carefully reviewed and re-engineered to be multiple curve compliant.
- Multiple curve bootstrapping must be properly implemented and configured.
- Booking of trades and systems configuration must be reviewed to be CSA-compliant, allowing proper re-assignment of CSA-dependent yield curves.
- System integration and alignment must be carefully checked to avoid the classical "two systems two prices" problem.
- Price and risk calculation must be fully reviewed to avoid hidden assumptions regarding discounting and forwarding (automatic default yield curve usage withoout explicit assignment).

4: Switching To CSA Discounting In Practice: *Accounting issues*

Sections of IAS 39 and FRS 26 relevant for fair value determination:

1. Fair value is defined as "*the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm's length transaction*".
2. In determining the valuation of OTC derivative "*a valuation technique (a) incorporates all factors that market participants would consider in setting a price (b) is consistent with accepted economic methodologies for pricing financial instruments*" (AG76).
3. "*The objective of determining fair value for a financial instrument that is traded in an active market is to arrive at the price at which a transaction would occur at the end of the reporting period in that instrument [...] in the most advantageous active market to which the entity has immediate access*" (AG 71).

Thus there is a judgemental area where the estimation of fair value is based on market (multilateral) consensus. We must consider the market as a whole.

The accounting, advisory and regulatory sides must evolve to take into account that the fair value of derivatives is CSA dependent.

4: Switching To CSA Discounting In Practice:
Risk Management issues

- P&L impacts estimation is very difficult thanks to the reasons above.

4: Switching To CSA Discounting In Practice: *Management issues*

The management is called to lead the change, and the corresponding frictions, taking business opportunities and controlling risks.

Main management decisions may regard:

- Cost of funding for non CSA counterparties (funding curve)
- Market coherent pricing and hedging of trading books (basis risk)
- Collateral management reorganization and CSAs review/update
- Business transfer inside the bank (holding vs subsidiaries)
- Upgrade of internal IT systems: (pricing, booking, reporting, etc.)
- P&L impacts and fair value accounting (auditors and regulators)

Notice that quant people play a cross-critical role…

5: Conclusions

1. We have reviewed the changes in the market across the credit crunch

2. We have set up the foundations of a modern theoretical framework for plain vanilla derivatives. In particular we have revisited:
   - the pricing of vanilla linear instruments: FRAs, Futures, Swaps, Basis Swaps
   - the pricing of vanilla caps/floors/swaptions with the Black's model
   - the SABR model for smile-consistent pricing of vanilla options.

3. We have addressed various important issues relevant to the practical switch towards CSA discounting.

6: Main References:
Main references on interest rates

Textbooks:

- D. Brigo, F. Mercurio, "*Interest Rate Models - Theory and Practice*", 2nd ed., Springer, 2006.
- L. B. G. Andersen, V. V. Piterbarg, "*Interest Rate Modeling",* Atlantic Financial Press, 2010.

Websites:

- Euribor, Eonia, Eurepo official website: *http://www.euribor.org*
- Libor official website: *http://www.bbalibor.com*

6: Main References:
Recent references on interest rate markets evolution [1]

I. Euribor ACI – The Financial Markets Association, "€onia Swap Index", Nov. 2009, http://www.euribor.org

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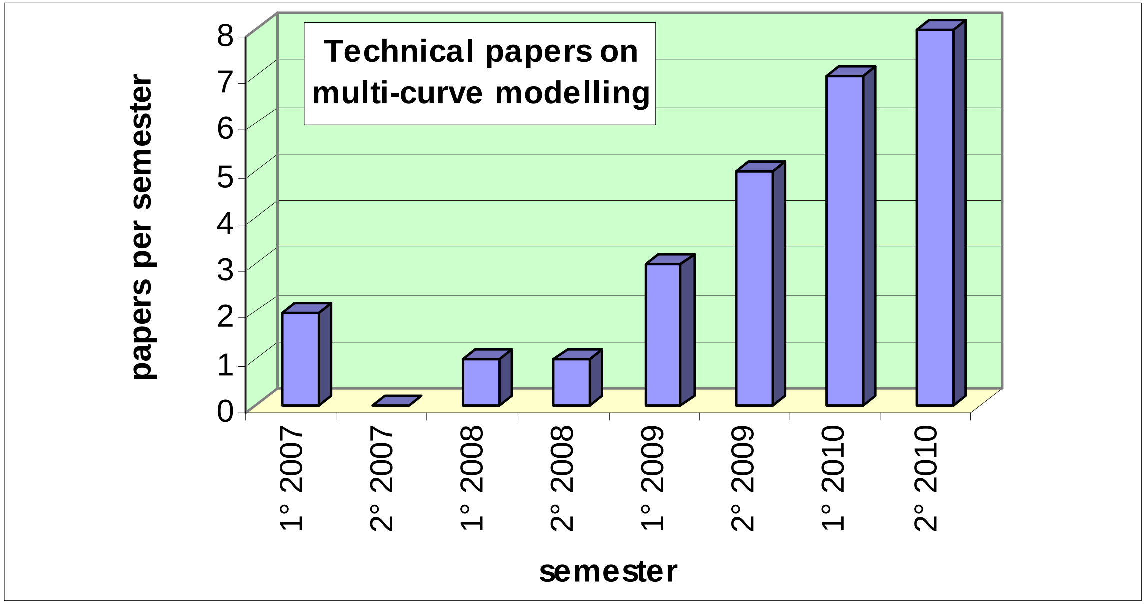

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