

The impossibility of DVA replication

Some have argued that the debit valuation adjustment – which measures the benefit to a bank from its own potential for default – is monetisable. They claim replication strategies involving the dealer buying its own bonds, or writing protection on its peers, can achieve this. Not so, says Antonio Castagna, who argues that these strategies ignore subtle effects on the firm's balance sheet

The debit valuation adjustment (DVA) that mark-to-market accounting rules insist should be included in derivatives' prices in order to capture a bank's own default risk is controversial because the dealer records paper profits – that could even be distributed as staff remuneration – when its creditworthiness deteriorates. Some argue that the DVA is not merely an accounting adjustment, but instead is monetisable through replication strategies. US bank Goldman Sachs, for example, has been quite open in pursuing a strategy of approximated replication, at least on some marked-to-market bonds, by selling credit protection on a basket of correlated names. Other US banks, forced to disclose their DVA by US generally accepted accounting principle rules, simply deduct it from their equity to avoid the public reputational risk.

But can banks replicate DVA? Some have argued that they can, by buying up outstanding bonds or writing protection on correlated names such as Goldman. Indeed, such a replication argument was used in Burgard & Kjaer (2010, 2011) to derive a partial differential equation (PDE) for the risky price by adapting the classical argument of Merton to take into account the possibility of default. If they are right, the DVA is a quantity that can be fairly deducted from the liabilities of a financial institution, since it can be hedged.

Generally speaking, their argument seems to ignore some subtle facts about such a strategy's effect on the bank's balance sheet. A dealer buying up its own bonds is not taking on a long position in them, but merely reducing or closing a short position. It cannot be net long in assets it issues itself.

Further, the circumstances under which a proxy hedge can replicate the DVA and give a funding benefit are very limited. As a result, the DVA should be considered as a cost to derivatives portfolios and as such should be deducted from equity, as laid down by the Basel Committee on Banking Supervision.¹

In this article, we review the existing replication arguments and explain why in practice they will not apply. We look in detail at the effect of DVA hedging on the balance sheet for a typical trade, and examine the (negative) consequences for the business. We conclude that it is in fact a cost, which in turn should be deducted from the bank's equity, as the committee recommends.

¹ www.bis.org/press/p120725b.htm

Dynamic replication of a defaultable claim

Dynamic replication relies on the ability to attain the same payout structure of a derivative via a trading strategy in primary securities such as stocks and bonds.

We wish to replicate the value to a counterparty C of a defaultable derivative \hat{V} on an underlying S , entered into with a bank B . The corresponding default-risk-free value is denoted by V . Beginning at time zero, we have to find a self-financing trading strategy whose value matches V up to its maturity. We recall Burgard & Kjaer (2010, 2011) for building a replicating portfolio in the case of an uncollateralised derivative.

The replicating portfolio is built out of positions in the stock S , a risk-free zero-coupon bond P , the bank's zero-coupon bonds P^B and those of the counterparty P^C . Dynamics for the three assets are given by:

$$\begin{aligned} dP_t &= r_t P_t dt \\ dP_t^B &= (r_t + s_t^B) P_t^B dt - dJ^B P_t^B \\ dP_t^C &= (r_t + s_t^C) P_t^C dt - dJ^C P_t^C \end{aligned}$$

where r_t is the deterministic time-dependent instantaneous risk-free interest rate and s^I is the yield spread, with $I \in \{B, C\}$, which in equilibrium should also be the instantaneous default intensity λ^I . The two intensities are assumed to be uncorrelated.

Burgard & Kjaer (2010, 2011) find that \hat{V}_t is replicated by a portfolio with the following weights on the stock, bank bonds and counterparty bonds respectively:

$$\begin{aligned} \delta_t &= \frac{\partial \hat{V}_t}{\partial S_t} \\ \alpha_t^B &= -\frac{\Delta \hat{V}_t^B}{P_t^B} = \frac{\hat{V}_t^B - (M^- + R^B M^+)}{P_t^B} \\ \alpha_t^C &= -\frac{\Delta \hat{V}_t^C}{P_t^C} = \frac{\hat{V}_t^C - (M^+ + R^C M^-)}{P_t^C} \end{aligned}$$

with the remainder reinvested into P to ensure the portfolio is self-financing. Here M is the mark-to-market value of the contract upon default of either counterparty and R^I , $I \in \{B, C\}$, the respective recovery rates.

We focus on the case in which $M = \hat{V}$ inclusive of default adjustments, but the analysis equally applies to the other close-out convention, in which $M = V$. Following Merton's replication argument for the derivation of pricing PDEs, it can be shown that $V = V + \hat{U}$, where V satisfies the traditional Black-Scholes-Merton equation, while the adjustment \hat{U} solves:

$$\begin{aligned} \mathcal{L}^{r-y} \hat{U}_t dt \\ = s_t^B (V + \hat{U})^- + (1 - R^B) \lambda_t^B (V + \hat{U})^+ + (1 - R^C) \lambda_t^C (V + \hat{U})^- \end{aligned}$$

where the operator:

$$\mathcal{L}^a = \frac{\partial \cdot}{\partial t} + a(S,t) \frac{\partial \cdot}{\partial S_t} + \frac{1}{2} \sigma^2(S,t) \frac{\partial^2 \cdot}{\partial S_t^2}$$

The solution is given by application of the Feynman-Kac theorem, as:

$$\begin{aligned} \hat{U}(S,t) = & -\left(1 - R^B\right) \int_t^T \lambda_s^B e^{-\int_s^t r_u du} \mathbf{E} \left[\left(V(S,s) + \hat{U}(S,s) \right)^+ \right] ds \\ & - \left(1 - R^C\right) \int_t^T \lambda_s^C e^{-\int_s^t r_u du} \mathbf{E} \left[\left(V(S,s) + \hat{U}(S,s) \right)^- \right] ds \\ & - \int_t^T s_s^B e^{-\int_s^t r_u du} \mathbf{E} \left[\left(V(S,s) + \hat{U}(S,s) \right)^- \right] ds \end{aligned}$$

From the bank's point of view, the expression on the right-hand side of the first line shows the DVA and the second shows the credit valuation adjustment (CVA) coming from its exposure to counterparty C 's default. From C 's point of view, these labels are of course reversed. The final term is a funding valuation adjustment representing the cost to B of funding the replication strategy at its funding spread s^B . We are interested in studying the effectiveness of the replication strategy of $U(S, t)$ from the bank's point of view.

Effectiveness of the replication strategy

The CVA can be replicated easily by selling an amount of bonds issued by counterparty C equal to $\alpha_t^C = (1 - R^C) V_t^C / P_t^C$. This can be achieved either by a repo agreement with a third party, or by buying credit protection via a credit default swap. The funding component is not problematic in principle either as issuing new bonds can account for any negative cashflow from the strategy.

However, DVA is trickier. It requires the bank to take a long position of its own bonds equal to $\alpha_t^B = (1 - R^B) V_t^B / P_t^B$. Unfortunately, unlike a short position, which can be achieved by issuance, this is not possible. Burgard & Kjaer (2010, 2011) suggest an apparently simple way for the bank to go long on its own bonds by buying back bonds issued in the past. This is not very difficult to implement – banks regularly issue debt and there are many bonds in the market to buy back. So, is buy-back a good strategy for the bank? The answer is a definite no.

There is a difference between buying a security and going long it. Buying bonds can only close or reduce the bank's existing structural short position in its debt. It can never go beyond this

point to a full long position. The replication strategy prescribes a long position in B 's bond, but when the bank buys back its own bonds, their gain process, that is, the quantity held times the price variations, simply sticks to amounts of profits or losses generated since their issuance. No other variation occurs. So replication of the DVA is not possible as there is no positive contribution from the gain process from the bank's bonds.

One could object to this assertion by saying it is true that the bank never really goes long its own bonds, but if one considers the replication strategy as a closed sub-system of the balance sheet, then the long position on the bond could actually be achieved. In reality, having a long position in bonds in the sub-system means a short position is opened somewhere else in the balance sheet, to counter-balance it at the aggregated level. So the net result is that the replication strategy leads to a loss – alternatively, a cost – for the bank.

The more subtle objection to this is based on the funding benefit argument, that if the bank has cash to buy back its bonds issued in the past, then it gets a benefit in terms of lower funding costs it pays on that outstanding debt. The funding benefits are mentioned in Burgard & Kjaer (2011) and also in Morini & Prampolini (2011), although in vague terms. On first blush this argument is attractive, but it is not precisely a funding benefit, in our opinion. We loosely define a funding benefit as the deduction in the amount of paid interests one can obtain by reducing the total outstanding debt through buy-back.

Assume a very basic situation: the bank starts its activities at time zero, with an amount of capital E , deposited in an account D_1 . We observe the bank's activity at discrete time intervals of length T . At time zero, the bank also issues an amount K of zero-coupon bonds P^B with unit face value, expiring at $3T$. The amount of cash raised by the bank is $Ke^{-(r+s^B)3T}$. This is used to buy K zero-coupon bonds P^Y , issued by a third party Y , with unit face value and expiring at $3T$. The third party has a funding spread s^Y , so that the present value of the bond is $Ke^{-(r+s^Y)3T}$. If both bonds have the same funding spread, $s^B = s^Y$, then the money raised by the bank is enough to buy the bond of the issuer Y .

For the moment, we assume that the funding spread is due to some unspecified factors not linked to the default risk. We can then affirm that the bank is operating a very simple replication strategy for the asset A_1 by hedging it through assets with the opposite sign, that is, via the issuance of its own bonds. The marked-to-market balance sheet of the bank at time zero looks like:

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Time 0

Assets	Liabilities
$D_1 = E$	$L_1 = Ke^{-(r+s^B)3T}$
$A_1 = Ke^{-(r+s^Y)3T}$	E

Now, assume one period T elapses and the bank closes a derivatives contract. To avoid unnecessary complications (but with no loss of generality), we assume B sells to counterparty C an option on some underlying S whose value to C is $\hat{V} = V + \hat{U}$ (this will allow us to exclude from the analysis the CVA for the bank, which is zero for short options). Since the option has a negative value to the bank, the option is a liability; on the other hand, the premium paid by C increases the cash available to B and is kept in a deposit D_2 . The balance sheet will then be:

Time T

Assets	Liabilities
$D_1 = Ee^{rT}$	$L_1 = Ke^{-(r+s^B)2T}$
$D_2 = \hat{V}$	$L_2 = \hat{V}$
$A_1 = Ke^{-(r+s^Y)2T}$	E
	$II_1 = (Ee^{rT} - E)$

where we have also included the interest accrued on the assets and liabilities, producing a net profit, $Ee^{rT} - E$, between zero and T .

The bank starts the dynamic replication strategy immediately. For simplicity's sake, we focus only on the DVA part of the quantity $U(S, t)$ in (2) and neglect the risk-free part of the hedge. The bank has to buy back a quantity α^B of its own bonds. Without loss of generality, we can assume $\alpha^B = K$. The amount of available cash in D_2 is abated correspondingly so that the balance sheet reads as:

Time T

Assets	Liabilities
$D_1 = Ee^{rT}$	$L_1 = 0$
$D_2 = \hat{V} - Ke^{-(r+s^B)2T}$	$L_2 = \hat{V}$
$A_1 = Ke^{-(r+s^Y)2T}$	E
	$II_1 = (Ee^{rT} - E)$

Here, no bond appears among the bank's liabilities, and it even seems as if they have declined. But in reality liabilities did not decline – on the contrary, they increased, since the bond has been replaced by a short position in the option. In any case, the bond issued is no longer counterbalancing the asset A_1 . It looks like the bank has a long position in the asset that does not need to be financed by cash, whose availability for the bank increased, as is manifested by the new amount in the D_2 . This is what can be

thought of as a funding benefit, in the sense above, and it apparently makes it possible to have assets in the balance sheet while paying less funding costs.

We would like to check if this apparent saving is effective in the replication strategy. Actually we will see it could only be effective if a certain set of circumstances is true. Indeed, after one more period has elapsed, the balance sheet reads as:

Time 2T

Assets	Liabilities
$D_1 = Ee^{r2T}$	$L_1 = 0$
$D_2 = \left(\hat{V} - Ke^{-(r+s^B)2T} \right) e^{rT}$	$L_2 = \hat{V}$
$A_1 = Ke^{-(r+s^Y)T}$	E
	$II_1 = (Ee^{r2T} - E)$
	$II_2 = \left(\hat{V} - Ke^{-(r+s^B)2T} \right) (e^{rT} - 1)$
	$P \& L = K \left[e^{-(r+s^Y)T} - e^{-(r+s^B)2T} \right]$

The asset has increased in value to $Ke^{-(r+s^Y)T}$, accrued interest on the deposit account D_1 is $II_1 = E(E^{r2T} - 1)$, while on the account D_2 it is $II_2 = (\hat{V} - Ke^{-(r+s^B)2T})(e^{rT} - 1)$. Assuming the risk-free part of the derivative is fully and properly hedged so that we can cancel any contribution of V , this yields a profit and loss of $P\&L = K[e^{-(r+s^Y)T} - e^{-(r+s^B)2T}]$. This profit would not be generated if the bonds issued by the bank were not bought back – the issued bond would generate a perfectly counterbalancing loss $K[e^{-(r+s^B)T} - e^{-(r+s^B)2T}]$ since $s^B = s^Y$ and the total effect in the balance sheet would be zero. On the other hand, and for the same reason of equal funding spreads, the profit appearing in this case is the same as the profit that the bank could earn if it had a true long position in its own bonds.

The gain process is working and the replication strategy for the DVA is operating as expected. So, is the funding benefit argument correct? Let us analyse the hidden assumptions under which the replication strategy is working.

First, the profit earned after the bank's bonds are bought back is equal to the profit that the bank would have earned if it were able to actually buy its own bonds only because we have assumed an equal spread for both the bank B and the issuer Y . This is the reason we can be sure that the profit generated by the bond P^Y is exactly equal to that generated by the bond P^B .

This profit and loss will not be replicated if the spreads do not match exactly. We can relax the assumption of constant spread by introducing, for both bank and issuer, a more realistic time-dependent spread $s^I, I \in \{B, Y\}$, but if the spreads are a deterministic function of time, they both have to be the same one. Alternatively, if they are stochastic processes, we have to ensure that they follow the same paths.

Second, we have to explicitly consider the possibility of default for the bank and the issuer Y – the existence of a non-zero spread indicates that this probability is not zero. In an environment with no recovery upon default, no liquidity premium or intermediation costs, it is well known that $s^I = \lambda^I$, the instantaneous default

intensity for issuer I . Assume now that the condition for the identity of time functions for deterministic spreads, or of the perfect correlation for stochastic spreads, is fulfilled. Then the equality $\alpha_t^B dP_t^B = \alpha_t^Y dP_t^Y$ needed to match the profit and loss is guaranteed only if neither bank B nor issuer Y defaults in the interval $[0, T]$. Either default, though, affects the effectiveness of the replication strategy differently.

If the bank goes bankrupt before the issuer Y , then the replication strategy would still work, although it is very likely to be stopped along with the rest of the bank's activities as creditors are paid. So in this case, up to the bank's default time, the replication strategy works, but afterwards it does not matter to the replication whether Y defaults.

If Y 's default occurs first, then the strategy is completely spoiled and the replication is not achieved. Another condition we must add is that issuer Y 's default has to occur after the bank B 's default, or at least that they happen together. Again in this case the replication is attained up to the last instant needed by the bank and hence does not produce negative consequences for the strategy.

So, in summary, the conditions under which the 'funding benefit' argument is valid, and the DVA is effectively replicated, are:

- The funding spreads over the risk-free rate of the asset and of the bank's bond must coincide at all times.
- The default times of the bank B and the issuer Y must be the same.

These conditions are trivially fulfilled when the issuer Y is the bank B , but in this case it is impossible for the bank to go long its own bonds. As satisfying these conditions is highly unlikely in practice, and this is the only way both cashflows can be identical under all circumstances, effective replication is equally unlikely.

DVA replication and franchise business

Value created by a bank's systems, people and customer base is known as its franchise business. In effect, this is created by being able to buy assets that yield more than their risks are worth.

In practice, this is of course very difficult, as the efficiency of financial markets mean that such arbitrage is difficult to find. For example, let us go back to the case we analysed in the previous section. If we assume that at time zero the spread over the risk-free rate yielded by the bond P^Y is only due to the default risk and that the recovery is zero, so that $s^Y = \lambda^Y$, then if the market prices the risks correctly, the expected return over a small period dt is:

$$\mathbf{E}\left[dP_t^Y\right] = \mathbf{E}\left[\left(r_t + s_t^Y\right)P_t^Y dt - dJ^Y P_t^Y\right] = r_t P_t^Y dt$$

In this case, the bank's franchise is not really increasing, even if the spread over the risk-free rate is positive. On the contrary, the bank is actually losing money on average, since the funding spread on its liabilities has to be paid anyway, and they instantaneously accrue interest at a rate $r_t + s_t^B$ with certainty, and the asset $A_1 = P^Y$ yields only the risk-free rate r_t .

A more feasible way to create the franchise is to charge a margin over the fair rate that remunerates risks and costs, and that provides for a profit, when the bank lends money to clients that have a weaker bargaining power, especially retail ones that do not have easy direct access to the capital markets. For example, let us assume that the bond P^Y is issued by a very particular obligor who is default risk-free and cannot have access to the capital market but can borrow money only from bank B . In this case, the bank may apply the spread m over the risk-free rate so

that the expected return on the asset $A_1 = P^Y$ is $r + m$. So, if $m = s^B + m' > s^B$ then the bank is increasing its franchise since it is able to generate profits in the future on a sound basis, after covering its funding costs.

This is also true if the bank is lending money to defaultable obligors such that it is able to charge a spread $s^Y = \lambda^Y + s^B + m'$ that remunerates the costs and the risks, that is, the bank's funding spread s^B and the default risk λ^Y , with a positive margin m' .

When we considered the two conditions under which the DVA can be effectively replicated, we mentioned that the default times of the bank and of the obligor Y must be perfectly correlated. This condition is tantamount, from the bank's perspective, to assuming the possibility of buying an asset that is default-risk-free and yet yields more than the risk-free rate. In fact, when B does not default, neither does the obligor. So when pricing the asset issued by Y and evaluating it against the costs and the risks borne by the bank, the obligor's default need not be considered.

Under these conditions, only if the spread $s^Y > s^B$ is B creating franchise, notwithstanding the fact that the asset is defaultable. If $s^Y = s^B$, then the bank is just covering the funding costs without any profit margin. This is hardly a realistic situation, but if it does happen then it is precisely the bank's power to apply this rate over the risk-free rate that is used to replicate the DVA. Again as above, at the balance-sheet level the replication of the DVA would end up as a cost to be covered by a margin above the risk-free rate on other contracts.

Since the perfect coincidence between the funding spreads of Y and B and the perfect correlation between their default times are not very likely to be matched in reality, the spread s^Y is really the remuneration for the default risk of the obligor Y and cannot be used for replicating DVA. But, if the bank is able to apply a margin over the rate needed to remunerate the default risk, so as to compensate the funding costs, so that the total spread is $s^Y = \lambda^Y + m$, this can be effective in replicating the DVA. However, the bank should be able to update it frequently so as to track the variations of its own funding spread, $m = s^B$, and at any time for the bank the following should hold true:

$$\alpha_t^B dP_t^B = \alpha_t^B \mathbf{E}\left[dP_t^Y\right] = \left(r_t + s_t^B\right)P_t^B dt = \left(r_t + m_t\right)P_t^Y dt$$

In other words, the assets cannot be fixed-rate bonds, and the spreads have to be reviewed to reflect not only the obligor's default risk but also the bank's default risk. Also in this case, the bank is using its ability to finance some investments yielding enough to cover the losses represented by the DVA. So the DVA is formally hedged, but the cost has been indirectly charged to other business areas. Eventually, considering the total level of funding available, the bank will always bear the same total funding cost.

In summary, if when closing a derivatives contract the bank receives some cash, this can be used to buy back a quantity of the bank's own bonds. In this case, the balance sheet shrinks, because an asset (the cash received) is used to reduce liabilities (bank's bonds). Given the reduced amount of liabilities, there is a smaller absolute cost to pay. If the bank was able, bonds would be issued before the closing of the derivatives contract to buy assets yielding more than the risk-free rate on a risk-adjusted basis, and this extra yield was enough to cover the funding costs over the risk-free rate of the bank's bonds, then it could effectively hedge the DVA. There is nothing special or a funding benefit here, just reduced liabilities produce smaller funding costs compensating for the increased DVA cost.

Contract		Time	
		0	T
Forward	Cash	–	$+F$
	Asset	–	–1
Repo	Cash	S	$-Se^{\lambda T} = -F$
	Asset	–1	1
Spot	Cash	$-S$	–
	Asset	1	–
Net	Cash	0	$F - F < 0$
	Asset	0	0

The problem with naively considering the DVA as a funding benefit can also be illustrated with a forward contract. For simplicity, we restrict discussion to the case of a risk-free close-out. In these kinds of contract, starting with zero value for both parties, the DVA can be either paid immediately to the counterparty (in which case there is no way to treat it differently from a cost), or it can be embedded in the value of a contract by modifying the fair forward or swap price so that it is worse for the bank than the risk-free equivalent. In this second case, the bank could include the value of the contract on the balance sheet without separating the DVA component or treating it as a cost but still rely on the funding benefit argument that it is replicable by a buy-back of its own bonds.

But it is easy to see that in this case, the DVA cannot be a funding benefit. Assume the bank sold a forward contract on an asset S maturing at time T to a risk-free counterparty with a fair risk-free forward price $F = Se^{\lambda T}$. Only the default risk of the bank has to be included in the valuation, so the new forward price that would make the value of the contract at inception zero, taking into account the DVA, will be some $\hat{F} < F$. Table A shows the outcome of the forward contract and related positions in the underlying asset and in cash at the start and at expiry. The replication is attained by the bank selling and buying back the asset S in a repo transaction expiring at T , which we assume accrues the risk-free rate, a reasonable approximation. The net result is that the bank loses an amount of money $\hat{F} - F$ equal to the initial DVA, compounded up to the expiry T .

If the bank strictly followed the dynamic strategy indicated in Burgard & Kjaer (2010, 2011), it should also buy back some quantity of its own bonds, but since in a forward contract no cash is received at the inception by the bank, the purchase can be financed only by resorting to a loan in the market, in effect replacing B 's debt for some other equivalent bank's debt.

Conclusion

Given the arguments above, it would be better to simply deduct the DVA from the firm's net equity at the inception of the contract. If positive cashflows are received by the bank in the trade, it can shrink the balance sheet by buying back outstanding debt and paying less interest on its liabilities. In this case, if assets generate an extra yield covering the funding costs of the bonds, this can be used to cover the DVA of the derivatives and counteract the initial deduction. If no positive cashflows are received, then the balance sheet cannot be shrunk and both DVA and funding costs are to be paid in the future. This is perfectly consistent with the new Basel regulation, which forbids accounting of the initial DVA and its subsequent variations as a reduction of liabilities.

A bank that fails to recognise DVA as a cost when booking its derivatives because it believes it can be replicated is implicitly using the margins it can charge on other products – typically those in the banking book – to cover costs by derivatives desks. The simple buy-back of its own bonds is not enough to justify the argument that DVA is a funding benefit without this.

If it is just an investment bank, the derivatives desk would rely on the profits of other desks to cover the DVA, and the bank is destroying any franchise business gained. If the institution operates as a retail bank as well, the derivatives desk would rely on the ability of the desks dealing contracts of the banking book to cover its margins above the risk-free rate in the pricing. If a bank's spread is volatile, these margins should be reviewed frequently to align them to the current funding spread paid by the bank. This would be impractical in reality, so it is more likely that the bank would end up destroying value, or just covering the total funding costs, in this case.

If the bank allows – or requires – its traders to implement replication strategies for the DVA and does not recognise this quantity as a cost immediately, there are two immediate negative consequences. First, the bank is using margins generated in profitable businesses to cover losses generated by the derivatives business. In some cases, this loss can be compensated by shrinking the balance sheet by positive cashflows received. So at best the bank is not increasing its franchise, and at worst is actually destroying it. This could be a very long and opaque process, especially when long-dated contracts are involved, for instance in a swap book, but the bleeding will be inexorable.

Second, the traders (and possibly salespeople), thinking they can hedge the DVA, will not consider it as a cost they paid and will not try to transfer it to other clients when dealing with them. If the bank is unable to find sufficient yield elsewhere to cover the cost, then the derivatives business is a losing one, and it is better to close it. It is the same if the bank keeps on lending money without being able to transfer its funding spreads to clients – sooner or later, the bank ends up losing money. ■

Antonio Castagna is a senior consultant at Iason in London and Milan. He would like to thank two anonymous referees for helpful comments. An extended version of this article is available at <http://iasonltd.com/FileUpload/files/DVA%20Dynamic%20Replication.pdf>. Email: antonio.castagna@iasonltd.com

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