A BNS-Type Stochastic Volatility Model With Two-Sided Jumps With Applications to FX Options Pricing

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Abstract
We present an extension of the BNS stochastic volatility model, incorporating two-sided jumps in the asset price process. The characteristic function of the log-price process is computed, enabling us to calibrate efficiently to plain vanilla products by means of Fourier pricing methods. Finally, we present as an application of the two-sided BNS model the calibration to FX option prices, where a model with two-sided jumps is more suitable due to the symmetric nature of the FX markets. We find that the two-sided BNS model calibrates better to FX smiles than the classical BNS model with one-directional jumps, even in a setting with equal degrees of freedom.

Keywords
Barndorff-Nielsen–Shephard model, stochastic volatility model, two-sided jumps, Fourier pricing, FX rate modeling, FX options

1. Introduction

Since the rise of risk-neutral valuation, many extensions of the seminal market model of [Black and Scholes 1973] have been published. One direction is the incorporation of jumps into the asset price process, as suggested by, e.g., [Merton 1976, Kou 2002], inspired by market shocks causing the asset price to jump. Another generalization is achieved by substituting the constant Black–Scholes volatility by a stochastic process, leading to stochastic volatility models as in, e.g., [Hull and White 1987, Stein and Stein 1991, Heston 1993], also enhanced by independent jumps in the asset price process by [Bates 1996]. In the model of [Barndorff-Nielsen and Shephard 2001], both approaches are combined, incorporating simultaneous jumps in both the volatility process and the asset price process. The volatility process is driven by a Lévy subordinator, while the diffusion process is superposed with the same driver to account for simultaneous jumps. This model is tractable in the sense that semi-analytic vanilla price calculation is possible via Fourier pricing, as described in [Nicolato and Venardos 2003].

However, the BNS model in its classical variant only supports jumps in the asset price process in one direction, since the driving Lévy subordinator is pathwise non-decreasing. For modeling stock prices, only allowing for one-directional (downward) jumps is not a significant restriction: Empirical studies (e.g. [Bakshi et.al.1997, Eraker 2004]) have highlighted that negative jumps are predominant in risk-neutral stock price dynamics extracted from option prices. Furthermore, the "leverage effect" of observing higher volatility in time periods of falling asset prices is displayed by a BNS model with negative jumps. But in many other applications, e.g. the modeling of FX rates, only allowing for one-directional jumps is a highly unnatural assumption. Restricting to one-directional jumps in FX rates contradict economic intuition, since FX rates are symmetric in nature. This is also underfed by empirical results from time series analysis as in [Jorion 1988], where two-sided jumps are observed for FX rates. Nevertheless, the original BNS model was applied to FX rates modeling by, e.g., [Tompkins 2006]. Thus, incorporating two-directional jumps (upward *and* downward) in the BNS-driven price process may be a desirable feature.

We present an extension of the BNS model which allows for two-sided jumps, but maintains the Lévy subordinator driven Ornstein–Uhlenbeck (OU) structure of the variance process. More precisely, we use a second Lévy subordinator as a driver which is responsible for positive jumps in the asset price process. As an example, we generalize the popular Γ-OU-BNS model and present a two-sided variant with compound Poisson jumps in both directions. Furthermore, it turns out that under mild technical restrictions, a semimartingale decomposition of the asset price process exists and the characteristic function of the log-price can be computed. Hence, Fourier pricing methods can be used for rapid calculation of plain vanilla prices following [Carr and Madan 1999, Raible 2000], ensuring efficient calibration to quoted vanilla prices. Finally, we compare the two-sided Γ-OU-BNS model (in different parameter settings) to the classical Γ-OU-BNS model in an empirical study, scrutinizing the calibration to FX vanilla options. It turns out that the two-sided Γ-OU-BNS model captures the FX smile better than the classical Γ-OU-BNS model, even in a setting with equal degrees of freedom. Hence, our considerations based on stylized statistical facts that a two-sided modification of the BNS model might be more suitable to capture FX smiles than the classical BNS model is supported by empirical evidence.

The remaining paper is organized as follows. In Section 2, we recapitulate the model setup of the classical BNS model and highlight its features via the Γ-OU-BNS example. In Section 3, we discuss shortcomings of the classical BNS model and introduce the two-sided BNS model. We show basic properties of the two-sided BNS model and present a two-sided version of the Γ-OU-BNS model as an example. In Section 4, we discuss Fourier pricing in the two-sided BNS model and compute the characteristic function of the log-price, in the general setting and for the Γ-OU-BNS example. Finally, in Section 5 we calibrate the two-sided Γ-OU-BNS model to FX smiles and compare our result with the classical Γ-OU-BNS model.

2. Review of the Barndorff-Nielsen–Shephard model class

The seminal paper of [Barndorff-Nielsen and Shephard 2001] introduces a new class of stochastic volatility models, the Barndorf-Nielsen–Shephard (BNS) model class. The variance process is modeled by a non-Gaussian Ornstein–Uhlenbeck (OU) process, driven by a Lévy subordinator. Furthermore, the same Lévy subordinator adds jumps to the asset price process, linking jumps in volatility and jumps in the asset price. Altogether, denoting the asset price process with $(S_t)_{t\geq 0}$ and defining $X_t := \log S_t$, $t \geq 0$, the general dynamics of a Barndorff-Nielsen–Shephard model are given by the SDEs

$$\begin{aligned} dX_t &= (\mu + \beta\sigma_t^2)\,dt + \sigma_t\,dW_t + \rho\,dZ_{\lambda t}, \\ d\sigma_t^2 &= -\lambda\sigma_t^2\,dt + dZ_{\lambda t}, \end{aligned}$$

with $(W_t)_{t\geq 0}$ denoting Brownian motion, $\mu, \lambda, \sigma_0^2 > 0$, $\rho < 0$, and $(Z_t)_{t\geq 0}$ being a Lévy subordinator with Laplace exponent ψ_Z independent of the Brownian motion $(W_t)_{t\geq 0}$. We follow [Cont and Tankov 2004] and define the Laplace exponent of Lévy process Y by $\psi_Y(z) := \log \mathbb{E}\,[\exp(zY_1)]$, $z \in \mathbb{C}$, existence provided.

As a volatility driver, one of the most popular choices is a standard compound Poisson process, resulting in the Γ-OU-BNS model, e.g. mentioned in [Barndorff-Nielsen and Shephard 2001, Nicolato and Venardos 2003].

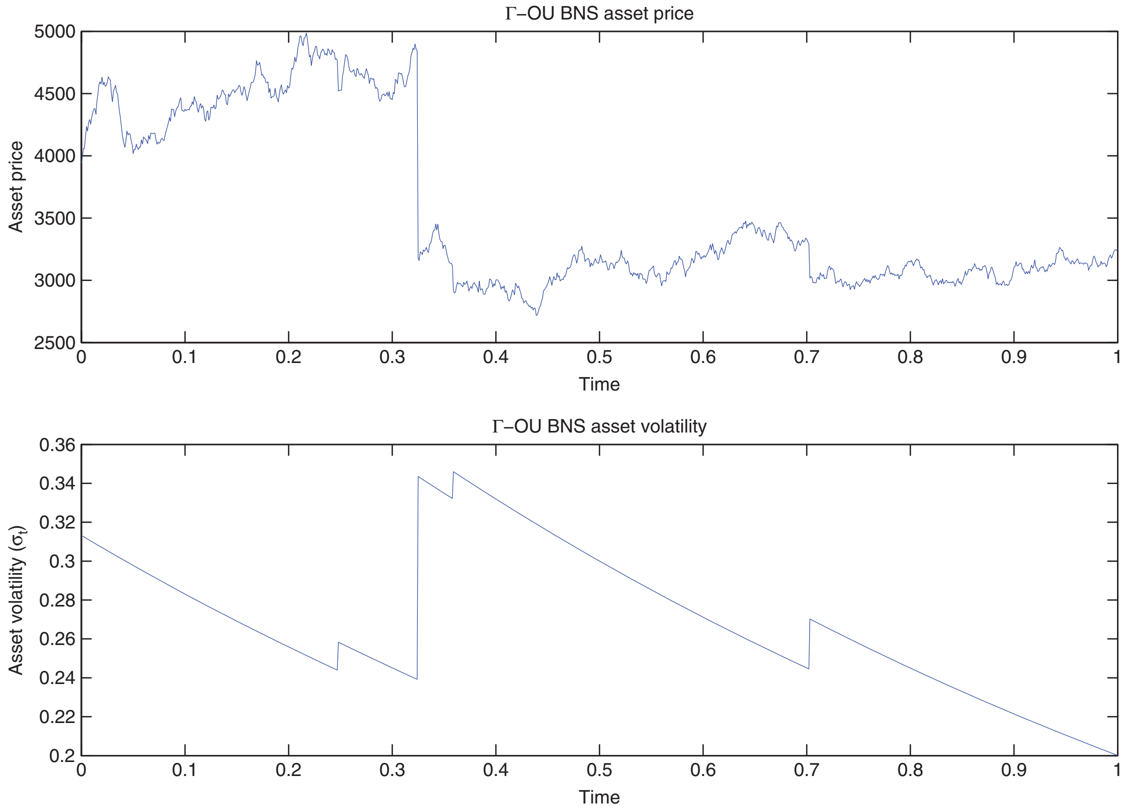
Example 2.1 (Γ-OU-BNS model) *Let $(Z_t)_{t\geq 0}$ be a standard compound Poisson process with exponentially i.i.d. jump sizes, i.e.*

$$Z_t := \sum_{j=1}^{N_t} E_j,$$

denoting by $(N_t)_{t\geq 0}$ a Poisson process with intensity $c > 0$ and by $(E_j)_{j\in\mathbb{N}}$ a sequence of i.i.d. random variables with $E_j \sim \mathrm{Exp}(\eta)$, $j \in \mathbb{N}$. Due to the exponential jump sizes, the process Z has Gamma distributed marginals (cf. [Barndorff-Nielsen and Shephard 2001]). Thus, we call the variance process $(\sigma_t^2)_{t\geq 0}$ a Γ-Ornstein–Uhlenbeck process and the BNS model with compound Poisson driver Z a Γ-OU-BNS model (see Figure 1).

The Γ-OU-BNS model allows for a fruitful interpretation when modeling stock prices: When the compound Poisson process exhibits a jump, volatility jumps up and simultaneously the stock price jumps down, due to $\rho < 0$. Market shocks in stock prices observe a similar behavior, often combining sudden decreases in stock prices with a simultaneous rise in volatility. On the other hand, upward movements of the stock price are rarely induced by large jumps, but are often observed in calm market periods where low volatility is observed. In several empirical studies, it has been shown that stock prices mainly exhibit negative jumps. Thus, the lack of modeling upward jumps by the "classical" BNS model does not fundamentally affect the

Figure 1: A sample path of the asset price and volatility process following the dynamics of a Γ-OU BNS model. The model incorporates downward jumps in the asset price process, always accompanied by upward jumps in the volatility process.



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usability of the model for modeling stock prices. Concluding, the incorporation of downward jumps in the stock price, accompanied by upward jumps in the stock's volatility, and decreasing volatility in times without jumps makes the Gamma-OU-BNS model suitable for the modeling of stock prices. This, however, does not have to hold for other objects, e.g. FX rates.

3. An extension of the BNS model with two-sided jumps

As we have seen, the BNS model can be a sensible choice for modeling stock price dynamics. However, it may not be appropriate to model, e.g., the dynamics of FX rates with a BNS model, since in the FX world, there is no economic reason to restrict to the stylized property of one-sided shocks. Jumps in FX rates are often due to unanticipated economic downturn or prosperity in one monetary area (relatively to another) or unanticipated political decisions (e.g. unilateral interest rate hikes or cuts of one central bank). Furthermore, the way FX rates are quoted (e.g. EUR-USD) is arbitrary from a modeling point of view. Thus, symmetry is a desirable feature for FX modeling. Hence, the common practice of adapting equity models for the FX world like the Garman–Kohlhagen adaption of the Black–Scholes model in a two-interest world (cf. [Garman and Kohlhagen 1983]) may not work for the BNS model. Thus, we modify the original BNS framework and present a BNS-style model which allows for two-sided jumps and may therefore be more suitable for FX rates modeling.

3.1 The two-sided BNS model

Motivated by the shortcoming of the classical BNS model to model symmetric situations with jumps in both directions, we modify the classical BNS setting to allow for two-sided jumps. We start with a very general formulation of the framework, analoguously to the general formulation of the BNS model. Given two independent Lévy subordinators $(Z_t^+)_{t\geq 0}, (Z_t^-)_{t\geq 0}$, we consider the process $(X_t)_{t\geq 0}$ governed by the following SDEs:

$$\begin{aligned} dX_t &= (\mu \ + \beta\sigma_t^2)\,dt + \sigma_t\,dW_t + \rho_+\,dZ_{\lambda t}^+ + \rho_-\,dZ_{\lambda t}^-, \qquad (1)\\ d\sigma_t^2 &= -\lambda\sigma_t^2\,dt + dZ_{\lambda t}^+ + dZ_{\lambda t}^-, \end{aligned}$$

with $(W_t)_{t\geq 0}$ being Brownian motion independent from $(Z_t^+)_{t\geq 0}$ and $(Z_t^-)_{t\geq 0}$, $\mu \in \mathbb{R}$, $\lambda > 0$, $\rho_+ > 0$, $\rho_- < 0$.

The restriction $\rho_-\rho_+ < 0$ is crucial to allow for two-directional jumps in the process $(X_t)_{t\geq 0}$. If we define the process $S_t := \exp X_t$, we obtain a non-negative process jumping bidirectionally, while the variance process follows the same characteristics as in the classical BNS model. Thus, we call a model where the log-asset price process $(X_t)_{t\geq 0}$ follows the dynamics given in (1) a *two-sided BNS model* and abbreviate it with *"BNS2 model"*. If we choose $\rho_- = -\rho_+$ and assume $(Z_t^+)_{t\geq 0}$ and $(Z_t^-)_{t\geq 0}$ to be independent copies of each other, we obtain a two-sided BNS model which has the same number of degrees of freedom as the classical BNS model, but behaves in a symmetric way in the jumps. Thus, we call this special situation of the two-sided BNS model a *"symmetric BNS model"* or *"SBNS model"*.

To justify the name "two-sided BNS model", we give a brief overview of the main properties of a BNS2 model that are immediately clear from the definition.

Properties 3.1 (Stylized facts of the BNS2 model)

- *Positive and negative jumps in the process $(X_t)_{t\geq 0}$ occur independently, due to independence of the drivers $(Z_t^+)_{t\geq 0}, (Z_t^-)_{t\geq 0}$.*
- *A jump in the process $(X_t)_{t\geq 0}$ is always accompanied by an upward jump in the variance process $(\sigma_t^2)_{t\geq 0}$, regardless of the direction of the jump in $(X_t)_{t\geq 0}$.*
- *Since $Z_t := Z_t^+ + Z_t^-$ is again a Lévy subordinator (being a positive linear combination of Lévy subordinators), the variance process $(\sigma_t^2)_{t\geq 0}$ is a Lévy-driven Ornstein–Uhlenbeck process with driver $(Z_t)_{t\geq 0}$ and matches the variance process classification of the classical BNS model.*

Thus, we find it justified to call a model with dynamics as in (1) a two-sided BNS model. Summarizing, the BNS2 model is a quite natural extension of the classical BNS model, allowing for positive and negative jumps, but preserving the process classification (Lévy-driven Ornstein–Uhlenbeck process) in its stochastic variance component.

Figure 2 shows sample paths of the EUR-SEK FX rate and its volatility in the BNS and BNS2 models.

3.2 Asset price dynamics of the two-sided BNS model

Since we want to model the underlying asset price with $S_t := \exp X_t$, the Itô–Döblin theorem immediately yields the following dynamics of the asset price.

Proposition 3.2 (Asset price dynamics) *If the process $(X_t)_{t\geq 0}$ follows the dynamics (1) of a BNS2 model, the process $(S_t)_{t\geq 0}$, $S_t := \exp X_t$, is governed by the following SDEs:*

$$\begin{aligned} dS_t &= S_{t-}\left((\mu + (0.5+\beta)\sigma_t^2)\,dt + \sigma_t\,dW_t + dM_t^+ + dM_t^-\right)\\ d\sigma_t^2 &= -\lambda\sigma_t^2\,dt + dZ_{\lambda t}^+ + dZ_{\lambda t}^- \end{aligned}$$

with

$$M_t^* = \sum_{0<s\leq t}\left(\exp\left(\rho_*\Delta Z_{\lambda s}^*\right) - 1\right), \quad * \in \{+,-\},$$

as usually denoting by $\Delta Z_s := Z_s - Z_{s-}$ the jump height process associated with Z.

Proof

Follows directly from the Itō–Döblin theorem for semimartingales applied to $x \mapsto \exp(x)$. □

For many applications, the following assumption (analog to [Nicolato and Venardos 2003]) is necessary.

Assumption 1 (Domain of Laplace exponents) *Denoting the Laplace exponent of a Lévy subordinator S by ψ_S, we assume $\xi_+ := \sup\{x \in \mathbb{R} : \psi_+(x) := \psi_{Z^+}(x) < \infty\} > 0$ and $\xi_- := \sup\{x \in \mathbb{R} : \psi_-(x) := \psi_{Z^-}(x) < \infty\} > 0$.*

Fortunately, this assumption does not restrict the choice of the subordinators too much (e.g. compound Poisson processes fulfill that property). Thus, in the remaining paper we assume Assumption 1 to hold.

The following variant of the dynamics of $(S_t)_{t\geq 0}$ is helpful for further theoretical considerations.

Corollary 3.3 (Semimartingale decomposition of asset price dynamics) *Let $(S_t)_{t\geq 0}$ be as in Proposition 3.2 and $\rho_+ < \xi_+$. Then $(S_t)_{t\geq 0}$ is decomposed into a local martingale and a process of finite variation by*

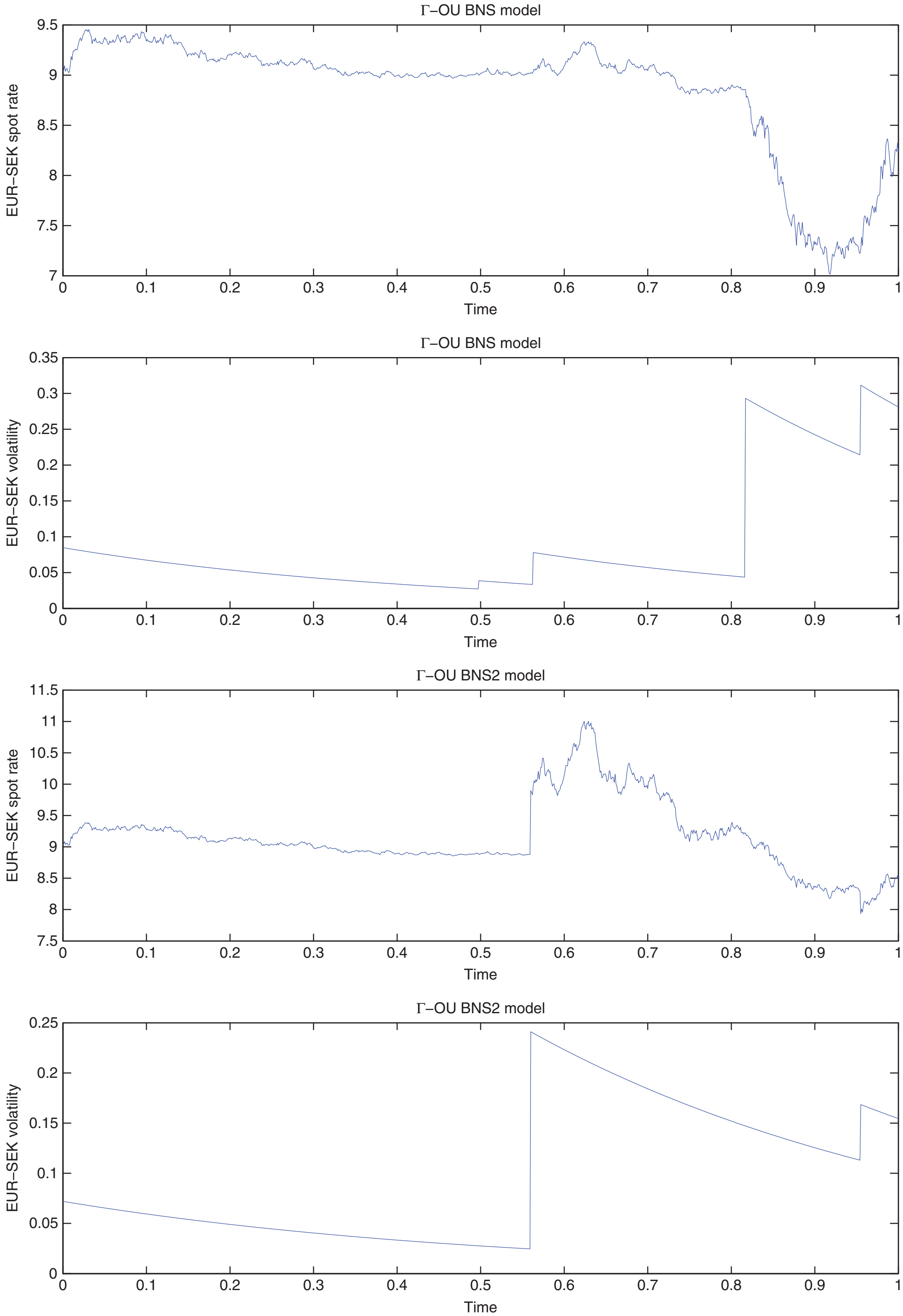
$$dS_t = S_{t-}\left((\mu + \lambda(\psi_+(\rho_+) + \psi_-(\rho_-)) + (0.5+\beta)\sigma_t^2)\,dt + \sigma_t\,dW_t + d\tilde{M}_t^+ + d\tilde{M}_t^-\right)$$

with

$$\tilde{M}_t^* = M_t^* - \lambda t\psi_*(\rho_*) = (\exp(\rho_* x) - 1) \star (\mu_{Z^*} - \nu_{Z^*})_{\lambda t}, \qquad * \in \{+,-\},$$

following [Jacod and Shiryaev 2002, p.66] by denoting μ_Z the random measure of Z's jumps and ν_Z its Lévy measure/compensator and the integral of some function $f: \mathbb{R} \times \mathbb{R}_{\geq 0}$ w.r.t. some random measure μ by

Figure 2: Sample paths of the EUR-SEK FX rate and its volatility in the BNS and the BNS2 model, generated with calibrated parameters. As one can see, the scaling parameter for the jump size in the BNS asset price process is very low, hence, there are hardly any observable jumps in the asset price process. Contrasting, in the BNS2 setting, the asset price exhibits clearly identifyable (positive and negative) jumps.



$$(f(x,s)\star\mu)_t := \int_{\mathbb{R}\times[0,t]} f(x,s)\,\mu(\mathrm{d}x,\mathrm{d}s).$$

Proof
Add $\lambda t(\psi_+(\rho_+)+\psi_-(\rho_-))$ to the process described in Proposition 3.2 and note that the processes $(M_t^*)_{t\geq 0}$ can be represented in terms of random measures via

$$M_t^* = ((\exp(\rho_* x)-1)\star\mu_{Z^*})_t.$$

Furthermore, the Lévy exponent ψ_* of the process Z_* is actually the scaled random measure $t\psi_*(\rho_*) = ((\exp(\rho_* x)-1)\star\nu_{Z^*})_t$ due to the Lévy–Khintchine representation. Hence, general theory delivers that the solution $(S_t)_{t\geq 0}$ of the remaining part

$$\mathrm{d}S_t = S_{t-}(\sigma_t\,\mathrm{d}W_t + \mathrm{d}\tilde{M}_t^+ + \mathrm{d}\tilde{M}_t^-)$$

is a local martingale and the remaining process is of finite variation. □

Analog to the Γ-OU-BNS model presented in Example 2.1, we can easily construct a two-sided BNS model with compound Poisson drivers. We conclude this chapter by presenting a two-sided extension of the Γ-OU-BNS model.

Example 3.4 (A two-sided Γ-OU-BNS model) *Let $(Z_t^+)_{t\geq 0}$, $(Z_t^-)_{t\geq 0}$ be standard compound Poisson processes with exponentially i.i.d. jump sizes, i.e.*

$$Z_t^* := \sum_{j=1}^{N_t^*} E_j^*, \quad t\geq 0,$$

with $(N_t^*)_{t\geq 0}$ being Poisson processes with intensity $c_*>0$ and $(E_j^*)_{j\in\mathbb{N}}$ being a sequence of i.i.d. random variables with $E_j^*\sim \mathrm{Exp}(\eta_*)$, $j\in\mathbb{N}$, $*\in\{+,-\}$. To assure the independence of $(Z_t^+)_{t\geq 0}$ and $(Z_t^-)_{t\geq 0}$, we assume the Poisson processes $(N_t^+)_{t\geq 0}$ and $(N_t^-)_{t\geq 0}$ to be independent. Since the sum of standard compound Poisson processes remains a standard compound Poisson process, our variance process is also described by a Γ-Ornstein-Uhlenbeck process. Hence, we obtain a two-sided modification of the Γ-OU-BNS model of Example 2.1.

To check for the existence of a possible semimartingale decomposition, one has to remark that $\xi_* = \eta_*$ holds for $*\in\{+,-\}$. Hence, we have to assume that $\rho_+ < \eta_+$. Since $1/\eta_+ = \mathbb{E}[E_j^+]$ is the average jump height of the driver Z^+, we obtain an appealing and easy-to-interpret criterion: A semimartingale decomposition of S_t exists, if the average jump height of the positive jumps in the log-price *ρ_+/η_+ does not exceed* 1. *Since jumps of this magnitude in the log-price are rather unrealistic for most processes occuring in mathematical finance, the restriction $\rho_+ < \eta_+$ does not practically affect the construction of a BNS2 model with compound Poisson drivers.*

4. Pricing European claims in the two-sided BNS model

As described in the previous section, the BNS2 model has rich dynamics induced by its driver triplet (W, Z^+, Z^-). Obtaining price estimates by Monte Carlo simulation (involving Euler or Milstein discretization schemes) is straightforward, but it is computationally expensive and typically not suitable for calibration purposes, where prices have to be calculated as a part of an optimization procedure. Thus, we have to find a way to price European contingent claims in an efficient manner, since these calculations are made very often when calibrating to market-quoted vanilla prices. The celebrated semi-analytic pricing approaches based on Fourier/Laplace inversion techniques of [Carr and Madan 1999, Raible 2000] have shown to be an efficient and tractable way to price vanilla options. We therefore compute the characteristic function of the log-price X_t in the BNS2 dynamics, which is sufficient for applying Fourier pricing.

The characteristic function can be computed in a straightforward manner, following the arguments of [Nicolato and Venardos 2003].

Theorem 4.1 (Characteristic function of* log-*price in a BNS2 model) *Let $(X_t)_{t\geq 0}$ be a process following the dynamics given in (1). Then the characteristic function of X_t, t>0, is given by*

$$\log\phi_{X_t}(u) = \mathrm{i}u(X_0+\mu t) + \sigma_0^2\delta(0,t,\mathrm{i}u,\beta,\lambda) + \lambda\left(\int_0^t \psi_+(\delta(s,t,\mathrm{i}u,\beta,\lambda)+\mathrm{i}u\rho_+)\,\mathrm{d}s + \int_0^t \psi_-(\delta(s,t,\mathrm{i}u,\beta,\lambda)+\mathrm{i}u\rho_-)\,\mathrm{d}s\right) \quad (2)$$

with

$$\delta(s,t,z,\beta,\lambda) := \frac{(z^2+2z\beta)\epsilon(s,t,\lambda)}{2},$$
$$\epsilon(s,t,\lambda) := \frac{1-\exp(-\lambda(t-s))}{\lambda},$$

ψ_+ and ψ_- denoting the Laplace exponents of the Lévy processes $(Z_t^+)_{t\geq 0}$ and $(Z_t^-)_{t\geq 0}$.

Proof
The proof follows the lines of [Nicolato and Venardos 2003, Theorem 2.2]. The key step in the proof is the fact that for a Lévy process Y with associated Laplace exponent ψ_Y,

$$\mathbb{E}\left[\exp\left(\int_0^t f(s)\,\mathrm{d}Y_s\right)\right] = \exp\left(\int_0^t \psi_Y(f(s))\,\mathrm{d}s\right) \quad (3)$$

holds for $f:\mathbb{R}_{\geq 0}\to\mathbb{C}$, $\mathrm{Re} f < \xi_Y := \sup\{s\in\mathbb{R}_{\geq 0}:\ \psi_Y(s)<\infty\}$. In particular,

$$\mathbb{E}\left[\exp\left(\int_0^t f(s)\,\mathrm{d}Z_{\lambda s}^*\right)\right] = \exp\left(\lambda\int_0^t \psi_*(f(s))\,\mathrm{d}s\right) \quad (4)$$

holds for $*\in\{+,-\}$. Furthermore, note that the Ornstein–Uhlenbeck shape of the stochastic volatility w.r.t. the Lévy subordinator Z, $Z_t := Z_t^+ + Z_t^-$, delivers the representation

$$\int_0^t \sigma_s^2\,\mathrm{d}s = \frac{1-\exp(-\lambda t)}{\lambda}\sigma_0^2 + \int_0^t \frac{1-\exp(-\lambda(t-s))}{\lambda}\,\mathrm{d}Z_{\lambda s}$$
$$= \epsilon(0,t,\lambda)\sigma_0^2 + \int_0^t \epsilon(s,t,\lambda)\,\mathrm{d}Z_{\lambda s}^+ + \int_0^t \epsilon(s,t,\lambda)\,\mathrm{d}Z_{\lambda s}^- \quad (5)$$

for the integrated variance process (cf. [Barndorff-Nielsen and Shephard 2001]). Hence, we obtain

$$\begin{aligned}\phi_{X_t}(u) &= \mathbb{E}\Bigg[\exp\Bigg(\mathrm{i}u\Bigg(X_0+\mu t+\beta\int_0^t\sigma_s^2\,\mathrm{d}s + \int_0^t \sigma_s\,\mathrm{d}W_s + \int_0^t \rho_+\,\mathrm{d}Z_{\lambda s}^+ + \int_0^t \rho_-\,\mathrm{d}Z_{\lambda s}^-\Bigg)\Bigg)\Bigg]\\ &= \mathbb{E}\left[\exp\left(\mathrm{i}u\left(X_0+\mu t+\beta\int_0^t\sigma_s^2\,\mathrm{d}s + \int_0^t\rho_+\,\mathrm{d}Z_{\lambda s}^+ + \int_0^t\rho_-\,\mathrm{d}Z_{\lambda s}^-\right)\right)\right.\\ &\qquad \left.\mathbb{E}\left[\exp\left(\int_0^t \mathrm{i}u\sigma_s\,\mathrm{d}W_s\right)\Big|(Z_{\lambda s}^+)_{0\leq s\leq t},(Z_{\lambda s}^-)_{0\leq s\leq t}\right]\right]\\ &= \mathbb{E}\Bigg[\exp\Bigg(\mathrm{i}u(X_0+\mu t) + \left(\mathrm{i}u\beta + \frac{(\mathrm{i}u)^2}{2}\right)\int_0^t \sigma_s^2\,\mathrm{d}s + \int_0^t \mathrm{i}u\rho_+\,\mathrm{d}Z_{\lambda s}^+ + \int_0^t \mathrm{i}u\rho_-\,\mathrm{d}Z_{\lambda s}^-\Bigg)\Bigg]\end{aligned}$$

$$= \mathbb{E}\Big[\exp\Big(\mathrm{i}u(X_0+\mu t)+\sigma_0^2\delta(0,t,\mathrm{i}u,\beta,\lambda) + \int_0^t \delta(s,t,\mathrm{i}u,\beta,\lambda)+\mathrm{i}u\rho_+\,\mathrm{d}Z^+_{\lambda s} + \int_0^t \delta(s,t,\mathrm{i}u,\beta,\lambda)+\mathrm{i}u\rho_-\,\mathrm{d}Z^-_{\lambda s}\Big)\Big]$$

$$= \exp\Big(\mathrm{i}u(X_0+\mu t)+\sigma_0^2\delta(0,t,\mathrm{i}u,\beta,\lambda) + \lambda\int_0^t \psi_+(\delta(s,t,\mathrm{i}u,\beta,\lambda)+\mathrm{i}u\rho_+)+\psi_-(\delta(s,t,\mathrm{i}u,\beta,\lambda)+\mathrm{i}u\rho_-)\,\mathrm{d}s\Big)$$

for the characteristic function. The last equality requires both $\mathrm{Re}(\delta(s,t,z,\beta,\lambda)+z\rho_+)<\xi_+$ *and* $\mathrm{Re}(\delta(s,t,z,\beta,\lambda)+z\rho_-)<\xi_-$ *to hold for* $z\in\mathbb{C}$ *on a whole imaginary strip. It immediately follows that*

$$\frac{\epsilon(s,t,\lambda)}{2}\,\mathrm{Re}(z)^2+(\beta\epsilon(s,t,\lambda)+\rho_*)\,\mathrm{Re}(z)-\xi_*<0 \qquad (6)$$

suffices to hold. Due to Assumption 1, $\xi_*>0$*, thus* $\mathrm{Re}\,(z)=0$ *fulfills inequality (6) (if* $\varepsilon(s,t,z)=0$*, the quadratic inequality reduces to a linear inequality and the same argument applies). Hence, choosing* $z\in\mathrm{i}\mathbb{R}$ *assures that (3) can be applied.* □

Theorem 4.1 yields a convenient expression for the characteristic function of X_t, but one has to admit that the general characteristic function in (2) does not have a closed-form solution, yet. The integral of the Laplace exponents of the driving subordinators cannot be expressed in terms of elementary functions for every Lévy subordinator. Fortunately, in case of the two-sided Γ-OU-BNS model that we have introduced in Example 3.4, the characteristic function of X_t has a closed-form solution which can be calculated explicitly.

Example 4.2 (Characteristic function in a two-sided Γ-OU-BNS model)
As described in Example 3.4, the two-sided Γ-OU-BNS model is obtained if the driving Lévy subordinators $(Z_t^+)_{t\geq 0}$, $(Z_t^-)_{t\geq 0}$*, are specified as compound Poisson processes with intensities* $c_*>0$ *and jump size parameters* $\eta_*>0$, $*\in\{+,-\}$*. In this case, following [Nicolato and Venardos 2003], the integrals in (2) can be calculated explicitly.*

Using elementary calculations, we obtain the following closed-form term for the characteristic function in the two-sided Γ-OU-BNS model:

$$\log\phi_{X_t}(u)=\mathrm{i}u(X_0+\mu t)+\sigma_0^2\delta(0,t,\mathrm{i}u,\beta,\lambda)+h(u,t,\beta,\lambda,c_-,\eta_-,\rho_-) + h(u,t,\beta,\lambda,c_+,\eta_+,\rho_+) \qquad (7)$$

with

$$h(u,t,\beta,\lambda,c,\eta,\rho)=\frac{c}{\eta-f_2(u,t,\lambda,\beta,\rho)}\left(\eta\log\left(\frac{\eta-f_1(u,t,\lambda,\beta,\rho)}{\eta-\mathrm{i}u\rho}\right)+f_2(u,t,\lambda,\beta,\rho)\lambda t\right),$$

$$f_1(u,t,\lambda,\beta,\rho)=\mathrm{i}u\rho+\delta(0,t,\mathrm{i}u,\beta,\lambda),$$

$$f_2(u,t,\lambda,\beta,\rho)=\mathrm{i}u\rho-\frac{u^2-2\mathrm{i}u\beta}{2\lambda}.$$

Similar to [Nicolato and Venardos 2003], choosing an IG process for Z^+ or Z^- yields a closed-form characteristic function for the log-price. Hence, a model with, e.g., compound Poisson downward jumps with exponential jump sizes and simultaneously small infinite activity upward IG jumps as a "Γ-IG-OU mixture model" can still be calibrated by means of Fourier methods.

5. The two-sided BNS model applied: Modeling FX rates

Compared to equity models, specific models for FX rates are treated only rarely in academia, which might be due to the less typical quoting conventions. Typically, in practice, equity models are adapted to the FX situation (like the famous Garman–Kohlhagen extension of the Black–Scholes model). A compendium about FX rates modeling in mathematical finance is [Lipton 2002].

Above, we stressed that the original BNS model falls behind in modeling FX rates, since it only allows for one-directional jumps. However, jumps in FX rates have to be bidirectional, since they are typically induced by a shock in one of the two monetary areas, or unanticipated central bank decisions may cause a sudden rise/fall of the FX rate. However, additional activity after a sudden jump makes equal sense for FX rates as for stock prices. Thus, modeling FX rates offers to be a natural application for a two-sided BNS model and an attractive alternative to other models that do not capture the smile and term structure of implied volatility correctly. In this section, we calibrate a classical and a two-sided BNS model to a quoted FX implied volatility surface, where the models are driven by Γ-Ornstein–Uhlenbeck processes. Afterwards, we compare the calibration performances to quantify the benefit from incorporating upward jumps in the BNS model. Furthermore, we discuss possible parameter reductions for the two-sided BNS model in the Γ-OU framework, e.g. using symmetric BNS (SBNS) dynamics.

5.1. FX rates framework

We assume to have deterministic risk-free continuously compounded interest rates in both monetary areas $r_{\mathrm{dom}}, r_{\mathrm{for}}>0$, denoting by r_{dom} the domestic currency interest rate[1] and by r_{for} the foreign currency interest rate[2]. The role of each currency is given by market standards, for details on the quotation of FX rates and options on FX rates we refer to [Reiswich and Wystup 2010].

5.2. Model setup

We propose to model FX rates with a two-sided Γ-OU-BNS model as treated in Example 3.4. We incorporate up- and downward jumps in the FX rate, induced by independent compound Poisson processes with exponential jump sizes Z^+, Z^- with matching Laplace exponents ψ_+, ψ_-.

Furthermore, to ensure risk-neutral valuation in the BNS2 model, our FX rate process S has to fulfill that that the parity-discounted process $(\exp((r_{\mathrm{for}}-r_{\mathrm{dom}})t)S_t)_{t\geq 0}$ is a martingale (cf. [Lipton 2002]). Hence, we assume S_t to follow the dynamics

$$\mathrm{d}S_t=S_{t-}\left((r_{\mathrm{dom}}-r_{\mathrm{for}})\,\mathrm{d}t+\sigma_t\,\mathrm{d}W_t+\mathrm{d}M_t^*\right),$$

$$\mathrm{d}\sigma_t^2=-\lambda\sigma_t^2\,\mathrm{d}t+\mathrm{d}Z_t^++\mathrm{d}Z_t^-$$

with $(W_t)_{t\geq 0}$ being a Brownian motion which is mutually independent of the compound Poisson processes Z^+ and Z^- and

$$\tilde{M}_t^*=M_t^*-\lambda t\psi_*(\rho_*)=(\exp(\rho_*x)-1)\star(\mu_{Z^*}-\nu_{Z^*})_{\lambda t}, \qquad *\in\{+,-\}.$$

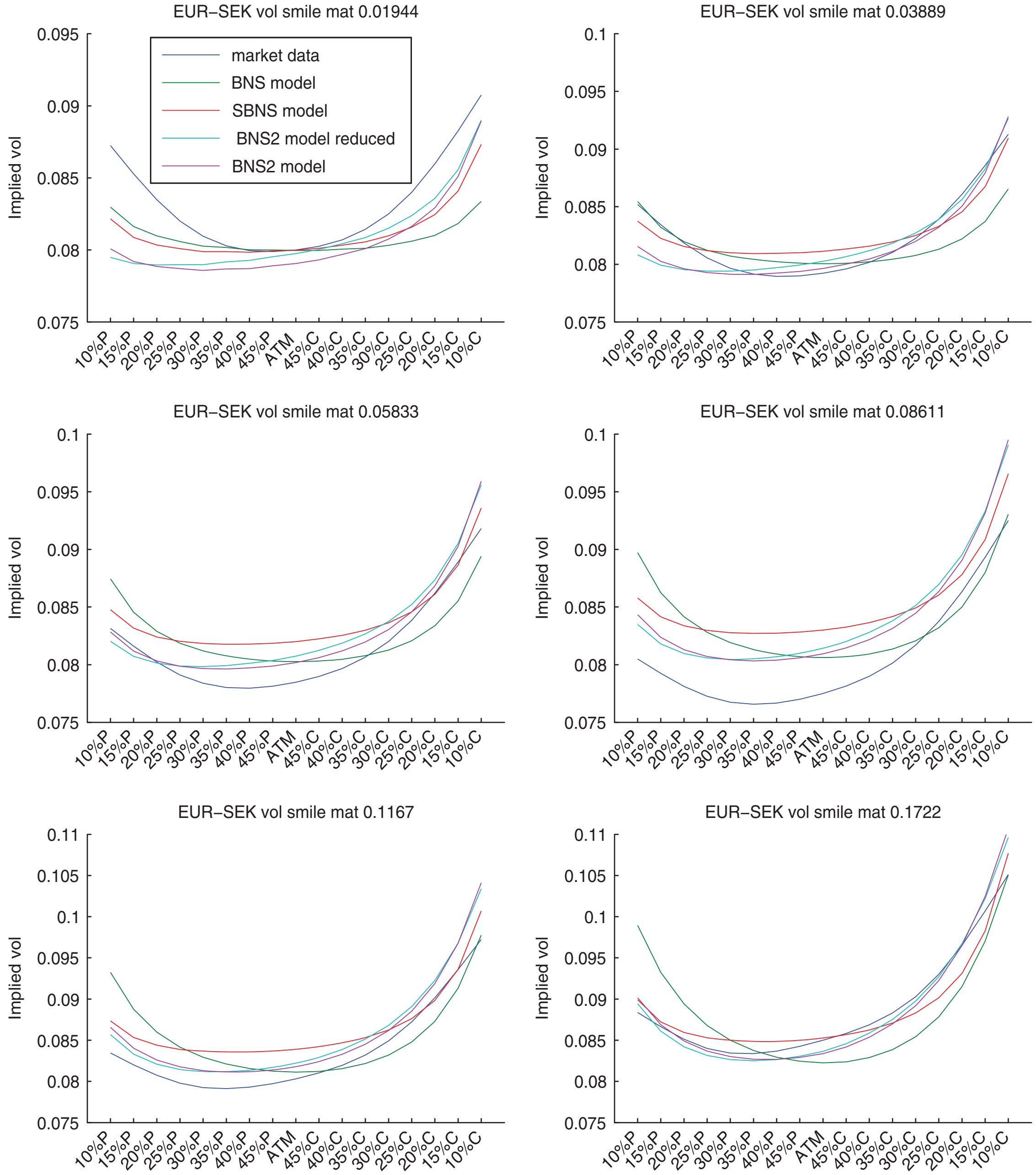
We calibrate to market prices via FFT methods as described in [Carr and Madan 1999, Raible 2000]. Therefore, to ensure that the characteristic function of $X_t:=\log S_t$ exists as in (2), we require that $\rho_+<\xi_+=\sup\{x\in\mathbb{R}:\psi_+(x)<\infty\}$. Risk-neutral valuation implies choosing $\mu=r_{\mathrm{dom}}-r_{\mathrm{for}}-\lambda(\psi_+(\rho_+)+\psi_-(\rho_-))$ and $\beta=-0.5$. As we have discussed in Example 3.4, this requires that the expected upward jump sizes in the log-price should not exceed 1 to guarantee for the finiteness of $\psi_+(\rho_+)$, which does not restrict a reasonable choice of ρ_+ in reality. The characteristic function of the two-sided Γ-OU-BNS model can be found in (7).

For some further investigations, we run our two-sided Γ-OU-BNS model in three different setttings:

1. We use a model with the maximum degrees of freedom and separate intensities, jump sizes, and scaling factors for the compound Poisson drivers.[3] Thus,

>

Figure 3: The market and calibrated short-term volatility smiles from EUR-SEK options. One can clearly see that the classical BNS model has more difficulties to fit the smile compared to the different BNS2 variants. In particular, the OTM put options are overvalued by the classical BNS model. The very short-term smile (5 days) cannot adequatly be captured by neither of the models.



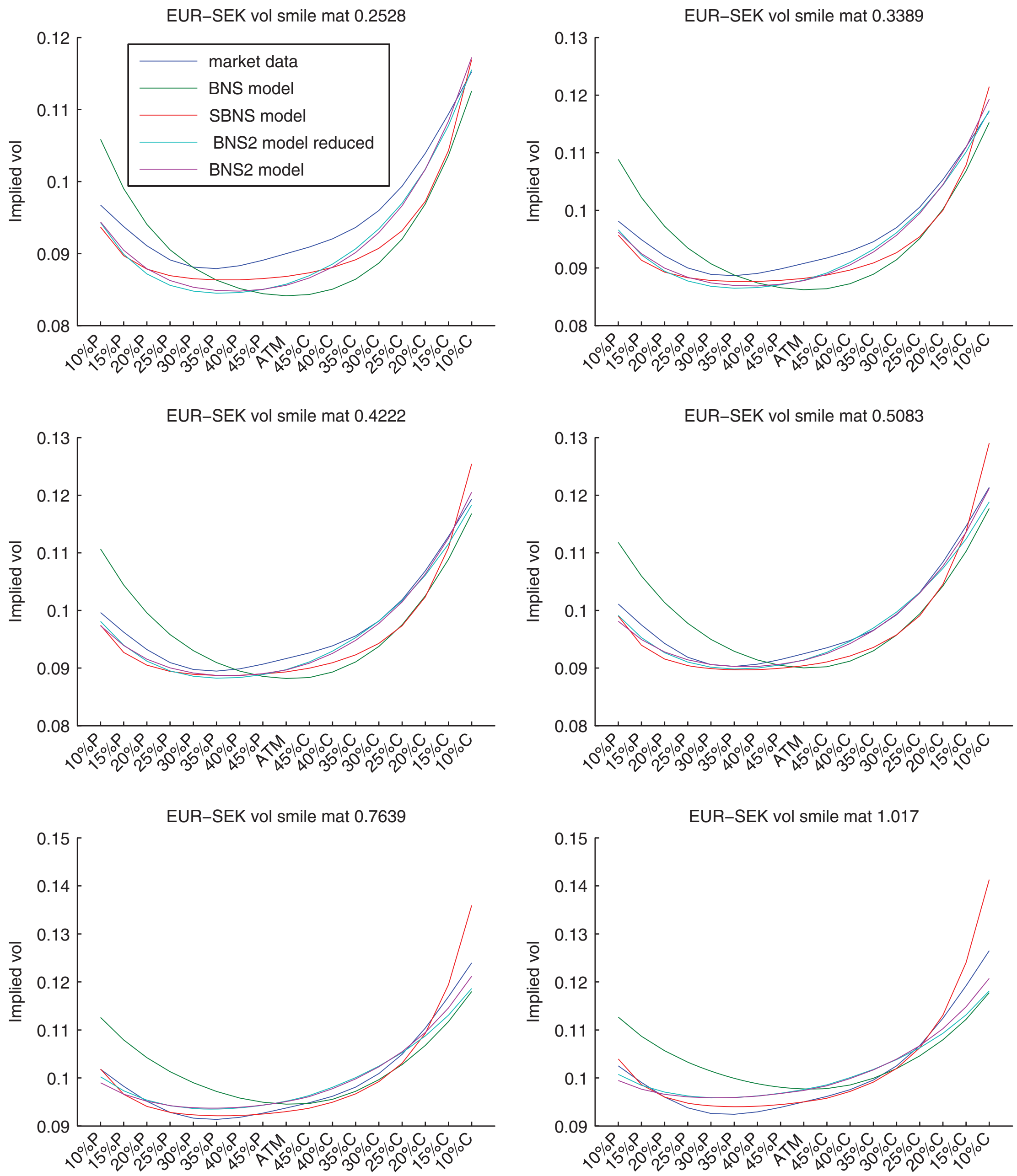
our parameter vector in the full BNS2 setting is the octuple $(\sigma_0^2, c_-, c_+, \eta_-, \eta_+, \lambda, \rho_-, \rho_+)$.

2. We reduce the parameter space and assume that the compound Poisson drivers Z^+ and Z^- are independent copies of each other: Only the scaling factors ρ_+, ρ_- vary independently within their domain.[4] So, our parameter vector is the sextuple $(\sigma_0^2, c, \eta, \lambda, \rho_-, \rho_+)$.
3. To ensure a fair comparison between the original Γ-OU-BNS model, we further reduce the parameters to an SBNS setting where the scaling factors additionally fulfill $\rho_- = -\rho_+$.[5] Hence, our parameter vector in the SBNS setting is the quintuple $(\sigma_0^2, c, \eta, \lambda, \rho)$ – similar to the classical BNS setting.
4. As a reference model, we use the standard one-sided Γ-OU-BNS model, only accounting for negative jumps.

5.3. Data and calibration methodology

We calibrate our two-sided Γ-OU-BNS model and the reference Γ-OU-BNS model to a set of 204 European options on the EUR-SEK foreign exchange rate[6] with

Figure 4: The market and calibrated longer term volatility smiles from EUR-SEK options. Regarding the long-term smiles, the BNS2 variants catch the smile even better than in the short term, in particular the OTM put prices are fitted better by the BNS2 models than by the reference BNS model. The SBNS model has some difficulties to fit the OTM call prices for longer maturities, while the reduced BNS2 model (having one more free parameter) captures the right wing of the smile much better.



different strikes (10%–45% delta and ATM calls and puts) and maturities. For calibration, we minimize the absolute distance of the model implied Black–Scholes volatilities to the market implied volatilities, with equal weights on every option, i.e. we minimize the error function

$$\theta \mapsto \frac{1}{204}\sum_{i=1}^{204}\left|\sigma_{\mathrm{impl}}(\mathrm{Call}^{(i)}_{\mathrm{model}}(\theta)) - \sigma^{(i)}_{\mathrm{market}}\right|,$$

denoting by σ_{impl} the function mapping a call price to its implied Black–Scholes volatility, by $\mathrm{Call}^{(i)}_{\mathrm{model}}$ the function mapping the resp. parameter vector θ to the resp. model call price of the ith call, and by $\sigma^{(i)}_{\mathrm{market}}$ the market-quoted implied volatility for the ith call, $i = 1, \ldots, 204$.

5.4. Calibration results

After calibrating the different model settings to market prices, we obtain the following results:

Model	# of parameters	Error[7]
original BNS model	5	0.40%
SBNS model	5	0.25%
reduced BNS2 model	6	0.20%
full BNS2 model	8	0.19%

Being the most general model, it is not surprising that the full BNS2 model with eight degrees of freedom calibrates best. The more striking result is that the reduced BNS2 model calibrates almost equally well as the full BNS2 model, although it is less flexible due to the additional assumption of identically distributed compound Poisson drivers. Also, one can see that the SBNS model's calibration performance is significantly better than the original BNS model's calibration performance (even without adding more degrees of freedom) and not much worse than the results in the full and reduced BNS2 settings. Overall, this indicates that our two-sided extension of the BNS model, even in a parsimonious setting (SBNS model), actually fits to FX option prices better than the original BNS model. We conclude that a symmetric model incorporating two-directional jumps does not only match to the economic intuition of FX rates, but actually fits to option data significantly better than the asymmetric classical BNS model. The detailed calibration performance for each maturity is displayed in Figures 3 and 4: One observes that particularly the long-term volatility surface is captured better by the BNS2 model variants. The OTM put prices are overestimated by the classical BNS model, while the two-sided variants (even the SBNS model) capture the smile wings more accurate.

6. Conclusion

We have constructed a generalization of the Barndorff-Nielsen–Shephard model class incorporating bidirectional jumps in the asset price process by introducing a second Lévy subordinator. As an important parametric example, we enhance the classical Γ-OU-BNS model to a two-sided Γ-OU-BNS model. In case of mildly restricted upward jump height in the log-price, we compute the characteristic function w.r.t. the general two-sided BNS model in a semi-closed form, analogously to the one-sided case. In particular, we are able to compute the characteristic function of the two-sided Γ-OU-BNS model in closed form. As an application, we calibrate our two-sided Γ-OU-BNS model to market prices of FX options, where a model with two-sided jumps is more natural. As a result, we obtain that the two-sided BNS model calibrates considerably better to FX options prices than the classical BNS model, even when assuming i.i.d. Lévy drivers and incorporating identically distributed up- and downward jumps. Concluding, we think that the two-sided BNS model is a useful extension of the classical BNS model due to its higher flexibility, being particularly suitable for FX rates modeling where symmetric behavior is more natural and intuitive than the model-implied assymetry of the classical BNS model.

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ENDNOTES

1. In some literature, it is called accounting interest rate.
2. Sometimes also called underlying interest rate.
3. In the following, we refer to this approach as the *"full BNS2" model*.
4. In the following, we refer to this approach as the *"reduced BNS2" model*. Here, we denote the intensity of the compound Poisson processes by $c := c_+ = c_-$ and the jump size by $\eta := \eta_+ = \eta_-$.
5. In the following, we refer to this approach as the *"SBNS" model*. In this setting, we denote the scaling factor by $\rho := \rho_+ = -\rho_-$.
6. It is a valid question why we did not choose the most liquid currency pair EUR-USD. At the time these calibrations were done, most of the EUR-nonEUR exchange rates and their derivatives markets (and, particularly, EUR-USD) were governed by the Eurozone crisis. We chose the EUR-SEK exchange rate, since the Swedish krona is not pegged with the Euro, but the economy is also closely connected to the Eurozone economy. Hence, the Eurozone crisis which distorts most of the EUR-nonEUR exchange rate markets, has less effect on the EUR-SEK exchange rate. For the EUR-USD exchange rate during the current Eurozone crisis, one should scrutinize whether a model as the presented two-sided BNS model actually makes sense. A pattern which can be observed in the EUR-USD exchange rate is that hits in the Eurozone crisis cause downward jumps on the EUR-USD exchange rate, while upward jumps on the EUR-USD exchange rate are mainly caused by signals of relaxation in the crisis. Hence, it is questionable that the postulated relationship between the exchange rate and the volatility (i.e. jumps in the exchange rate – regardless of the direction – cause upward jumps in the volatility process) holds for the EUR-USD exchange rate during the current crisis.
7. The error to market prices is measured by the average absolute deviation of implied volatilities.

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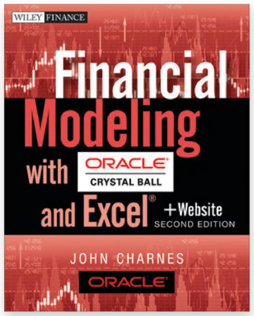

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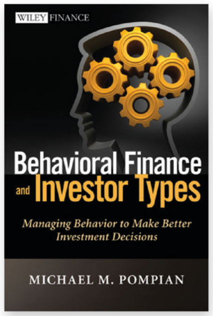

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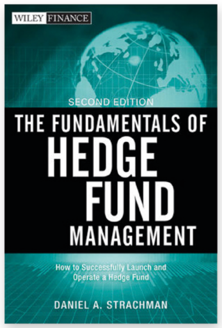

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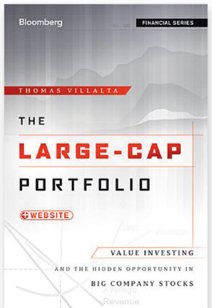

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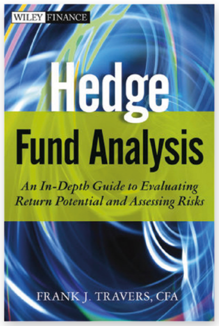

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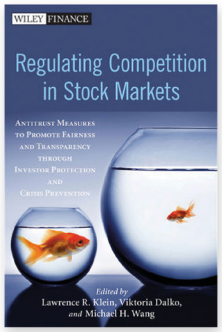

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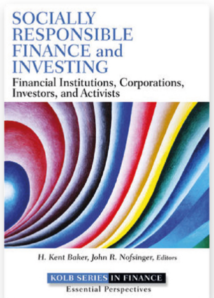

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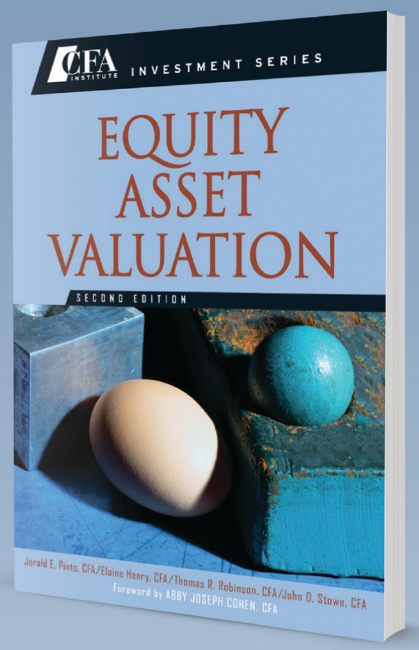

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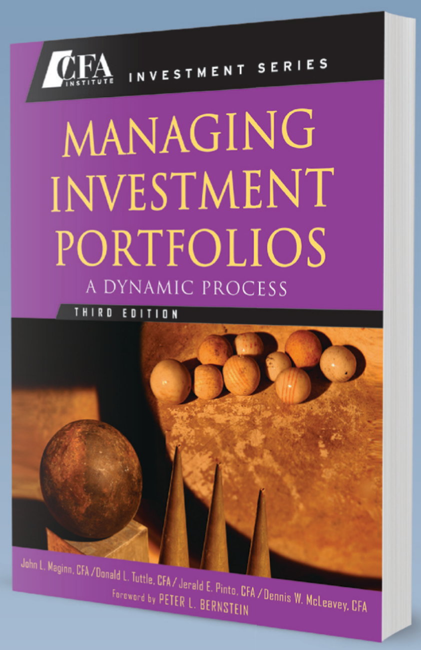

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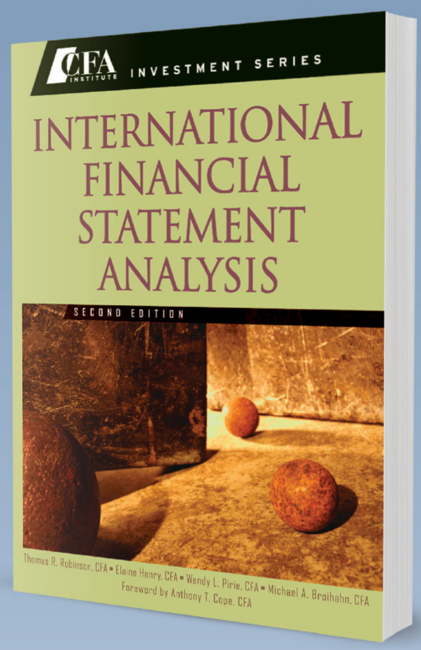

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