# Cutting Edge introduction: SABR rattling

 **The classic approach to the benchmark interest rate options model leads to nonsensical negative probabilities at low strikes – but a new approach promises to fix the puzzle, and allow the pricing of negative-strike options. Laurie Carver introduces this month’s technical articles**

A basic rule of probabilities – leaving aside some esoteric, contrarian lines of thought in academic literature – is that they are positive numbers. But there is a curious defect in the standard way of generating volatility smiles for the stochastic alpha beta rho (SABR) model that is the industry benchmark for interest rate options – it can imply negative probabilities for the underlying to fall below very low strikes.

In the pre-crisis days of 5% Libor, this could be dismissed as an irrelevant quirk. But with three-month US dollar Libor at 31 basis points and euro at just 13bp as Riskwent to press, according to Bloomberg, it is now a very real problem ([RiskNovember 2012, pages 22–24](http://www.risk.net/2218691%22%20%5Ct%20%22_blank)). A negative probability of falling under some strike means a dealer relying on the model should pay counterparties to take the corresponding digital option – even though only the counterparty could benefit. This is an obvious arbitrage opportunity, which has set alarm bells ringing across the industry.

The situation arises because the standard way of implementing the model – the so-called Hagan expansion, named after one of its originators, JP Morgan quant Patrick Hagan – fails when the strike is small and either volatility is high or the trade is long-dated. Now, Jesper Andreasen, universal head of quantitative analysis at Danske Bank in Copenhagen, and Brian Huge, his team’s chief analyst, believe they have a way to solve the puzzle in Expanded forward volatility ([see pages 101–107](http://www.risk.net/risk-magazine/technical-paper/2233952/expanded-forward-volatility)). The article builds on their work in recent years that focuses on discretisations of continuous time models as the basic building block of quantitative finance.

The Hagan expansion is typically used to determine implied volatilities, but Andreasen and Huge show particular forms of the forward volatilities can be derived as well. These have to satisfy the Dupire local volatility equation to be consistent. Using a finite difference scheme to enforce this condition ensures arbitrage is excluded – and that probabilities are therefore necessarily positive.

With the low rate environment spurring demand for negative strike options, one advantage of the method is that such products are easy to price. But the quants were prompted by a more prosaic consideration. “It was partly the need to think about negative rates that motivated us, but it was more the traders’ problems with hedging – once you get down to such a low-rate environment, you can’t really trust the hedging positions the expansion is giving you,” says Andreasen. The pair’s new approach guarantees hedging signals will at least make sense.

The bank implemented the first prototype version for its interest rates business around a year and a half ago, and is looking to apply it to its foreign exchange business. Andreasen is now hoping to expand the method’s scope – in its present form, it can only produce the forward smile for one maturity at a time. But he aims to implement the scheme for an entire term structure simultaneously, using time-dependent forward volatility parameters.

After that, the next logical goal is to implement it for multiple assets – which is bound to make it an order of magnitude trickier. “What we’d like to see is a way to use the expansion to get consistent spread option smiles, using the individual smiles and correlations,” he says.

Also this month, Jon Gregory, a partner at consultancy Solum Financial, and Ilya German, a senior consultant at the firm, look at the interplay between credit and debit valuation adjustments (CVA and DVA), close-out agreements and default correlation in Closing out DVA ([see pages 96–100](http://www.risk.net/risk-magazine/technical-paper/2233949/closing-out-dva)).

The article focuses on the type of close-out agreement used when one of the counterparties defaults – specifically, whether or not to include the CVA and DVA of the replacement trade, dubbed ‘risky’ and ‘risk-free’. The International Swaps and Derivatives Association’s master agreement has nothing to say on the issue, but previous work suggested that for simple contracts – in effect loans – a risky close-out was preferable because it meant the contract value was continuous right through default ([Risk September 2011, pages 94–96](http://www.risk.net/2104946)).

However, things become more complicated when both counterparties have outstanding obligations over the course of the contract. Since each now has a CVA and DVA corresponding to each other, whether to account for the order of any defaults has a great bearing on the risky close-out value – as does the recursive problem of whether the replacement contract will need to be replaced, and its replacement, and so on. The adjustment problem is not going away any time soon.

Cutting Edge introduction: Continuity error

**Some quants discarded the continuous time model when it got in the way of arbitrage-free pricing – but others see a chance to fix the traditional ideal. Laurie Carver introduces this month’s technical section**

Opinion was divided when Jesper Andreasen and Brian Huge – quants at Danske Bank in Copenhagen – unveiled a radical way to generate implied volatilities in two 2011 articles ([RiskMarch 2011, pages 76–79](http://www.risk.net/2029822), and [Risk July 2011, pages 66–71](http://www.risk.net/risk-magazine/technical-paper/2080440/random-grids)). On the one hand, the new method promised arbitrage-free prices, but it did so by abandoning the continuous time model that is traditionally seen as the ideal.

Andreasen has been disarmingly blunt about this sacrifice. “I don’t care,” he told one audience member at Risk’s Quant Congress Europe in 2011 who pointed it out. “I just want arbitrage-free prices.”

In the Andreasen-Huge view, the continuous time model is a romantic mathematical fiction that only exists on paper. If adherence to that fiction is causing dealers to hand arbitrage opportunities to their competitors, then something is clearly wrong.

But instead of tossing it aside, some quants are trying to fix it. This school of thought may agree that continuous time is a fiction, but insists it is convenient – even essential. Without it, the model’s risk outputs might be incoherent, they say, and the traders that are familiar with the industry-standard stochastic alpha beta rho (SABR) approach won’t know what to do in the long-dated, high-strike wings of the volatility surface.

“The thing is, if you build Greeks using the same continuous time model, then that common assumption forces them to be consistent with one another. On top of that, traders understand SABR dynamics. That intuition is lost if you abandon a continuous time model,” says Philippe Balland, global head of rates, currencies and credit analytics at UBS.

Of course, SABR’s propensity to return arbitrageable prices was well known for years, but was ignored until the low-rate, high-volatility environment meant those mispriced low-strike or long-maturity options became significant. Now, it needs a shake-up – and Balland and his colleague Quan Tran, a quant at UBS, offer one in [SABR goes normal](http://www.risk.net/risk-magazine/technical-paper/2269613/sabr-goes-normal).

The key is the constant elasticity of variance (CEV) parameter, known as beta. This can be set to zero or one – the Gaussian or normal case versus the lognormal or Black-Scholes case, respectively – or anywhere in between. The standard Hagan expansion of SABR is badly behaved for small beta, but Balland and Tran derive a different one that fits exactly when beta is zero, and stays relatively accurate as it moves away. The result is a more stable implementation for the areas where Hagan breaks down.

Even Danske’s Andreasen likes the approach – but he hasn’t converted to the cult of continuity. He maintains the continuous time model is the wrong thing to aim at, and argues an expansion will always be limited. “Maybe now it prices 99% of options correctly rather than 90%. But there will always be some region where the approximation breaks down,” he says.

For his part, Balland thinks modifying SABR is still the way to go. He says the model the Andreasen approach converges to – a complicated pure jump process unfamiliar to traders – is a poor guide for intuition, and is difficult to check against historical data. The major innovation of his and Tran’s paper is perhaps a more subtle form of heresy – the CEV parameter is no longer constant, and is allowed to depend on the spot. A function is pieced together that behaves sufficiently well where it needs to – a pragmatic answer.

Also this month, Stéphane Crépey, professor of financial engineering at the Université d’Evry Val d’Essonne, and Raphaël Douady, a researcher at Centre d’Économie de la Sorbonne, examine the basis between Libor and the overnight indexed swap rates in [Lois: credit and liquidity](http://www.risk.net/risk-magazine/technical-paper/2269615/lois-credit-and-liquidity). A regression model applied over the crisis finds the basis goes through distinct periods when either credit or liquidity risk is its main driver. According to this analysis, the initial panic from August 2007 really was a credit crisis. There was comparative calm from March 2009 until mid-2011, but since then, the basis has seen a much larger liquidity premium component.

# Multi-curve hedge accounting models

**Derivatives pricing practices have undergone huge change since the crisis, including the move to overnight indexed swap discounting. What impact do these changes have on hedge accounting under International Financial Reporting Standards? By Dirk Schubert**

Financial market professionals are still getting to grips with the many changes that have occurred to pricing practices since the financial crisis. Whereas practitioners could once afford to ignore the tiny differences that existed between key market rates – Libor and overnight indexed swap (OIS) rates or three-month and six-month Libor, for instance – the crisis meant that was no longer possible. The basis between Libor and OIS, as well as between different parts of the Libor curve, blew out dramatically. Cross-currency basis – for example, the difference between US dollar Libor and euro Libor – also increased significantly (see figure 1).



This prompted some major changes in valuation methodologies. One key shift was the use of OIS to discount cash-collateralised derivatives, with the correct discount curve determined by the currency of the underlying collateral. This can be hugely complicated, though – especially as each set of counterparties can agree on a list of acceptable collateral they will post to each other when signing a credit support annex. In theory, dealers would need to decide what discount curve is the most appropriate for each of the eligible collateral types, and determine which to use. Far from using a single Libor curve to discount everything, many dealers started to develop multi-curve valuation models.

All these changes can have a significant economic impact, both at the transaction and firm level. However, they also create a number of complexities from an accounting perspective. This article describes the properties of multi-curve valuation models, and analyses the impact on hedge accounting under International Financial Reporting Standards (IFRS).

**Multi-curve valuation models**
Multi-curve valuation models are extremely complex, but the main properties can be summarised as follows:

* They do not change the contractual cashflows involved in a derivatives transaction.
* They distinguish between collateralised and uncollateralised derivatives trades.
* They are also able to support a variety of discount curves, based on OIS and Libor curves.
* They take multiple risk factors into account.

OIS curves are typically used to discount cash-collateralised derivatives, and basis risk – for example, three-month Euribor/euro overnight index average (Eonia) basis – is factored into the multi-curve model using traded (collateralised) basis swaps (for instance, three-month Euribor/Eonia basis swaps). Multi-curve valuation models construct basis-consistent discount and forward curves using the absence-of-arbitrage principle and assuming an integrated market for all traded derivatives.

They essentially synthetically decompose traded derivatives such as three-month Euribor swaps into their risk factors (Eonia and three-month Euribor/Eonia basis risk).

The operation of multi-curve valuation models can be illustrated by an example. Consider a three-month Euribor payer swap discounted with Eonia instead of the three-month Euribor swap curve. Figure 2 illustrates the effects on either side of the interest rate swap separately, comparing the behaviour in a single and a multi-curve framework.



Considering the fixed side of the interest rate swap (payer side – negative present value), the change in discount rate results mainly in a negative shift in the present value, as Eonia is lower than three-month Euribor. The impact of the change in the discount curve represented by the three-month Euribor/Eonia basis spread is also inherent in the floating side of the three-month Euribor interest rate swap, as can be seen at the inception of the derivatives transaction where the present value of both sides adds up to zero. The latter holds regardless of whether single- or multi-curve models are applied. In case of the multi-curve model, however, the three-month Euribor/Eonia basis spread is taken into account and changes on the floating side over time, depending on market conditions. Consequently, volatility in the present value of the three-month Euribor interest rate swap originates from three-month Euribor/Eonia basis risk in a multi-curve model. Similar results hold for currency derivatives involving cross-currency basis risk.

As IFRS requires derivatives to be carried at fair value (present value) through the profit and loss (P&L), the application of multi-curve models leads to increased volatility in earnings/revenues. It’s worth noting, though, that only the periodic P&L of derivatives differs between single- and multi-curve models – not the total P&L over all periods.

**Major sources of IFRS impact**
To repeat, financial institutions are exposed to the risk of higher P&L volatility as a result of changes in the valuation of derivatives and other financial instruments (see figure 3). Derivatives are often used by financial institutions as economic hedges. According to IFRS, the mixed model approach is applied: derivatives are carried at fair value through the P&L, while bonds1/loans are carried at amortised cost. This results in P&L volatility, despite the presence of economically sound hedges.



IFRS enables firms to account for economic hedges by applying the fair-value option or using hedge accounting. In the former, the bond/loan (cash instrument) is designated irrevocably at fair value through the P&L, which at least partly offsets the fair-value changes in the hedging derivative. As a net result, the difference in the valuations of the derivatives and cash instruments is present in the P&L (cash basis). Since basis risks enter into the valuation of derivatives and cash instruments, the likelihood of higher P&L volatility increases.

Meanwhile, there are two major types of hedge accounting model: cashflow and fair-value hedge accounting. The impact on the cashflow hedge accounting models is considered to be limited, as the hypothetical derivatives method is applied.2 Here, the existing derivative is compared with a hypothetical derivative derived from the terms and conditions of the cash instrument. Since both derivatives would be evaluated by multi-curve models, no P&L impact with regard to basis risks is expected, due to their offsetting character.

In the fair-value hedge accounting model, the situation is different because the bond/loan (hedged item) is fair-valued with respect to the hedged risk (for example, interest rate risk). Its fair-value changes are compared with those of the derivative. A hedging relationship is termed effective if the ratio of both fair-value changes is within an 80–125% range. In this case, the fair-value change with respect to the hedged risk of the bond/loan is recognised in the P&L, offsetting the fair-value change of the derivative. In the presence of basis risks, the derivative is exposed to multiple risk factors, which are only fully compensated by the fair-value change in the hedged item if corresponding multiple risk factors are taken into account. This requires a multiple risk factor fair-value model for the hedged item. If such a valuation model is not utilised for the hedged item, then the risk of ineffectiveness for hedging relationships increases – and it might even mean no effectiveness (in terms of IFRS) can be proven for an economically sound hedging relationship.

This issue is of immediate practical relevance beyond hedge accounting, as such models are also required for the determination of transfer prices applied by treasury departments.

**Hedging cost approach and transfer prices**
The basic components of a multiple risk factor model for the hedged item are developed using the example of a fixed-rate bond/loan economically hedged by a collateralised three-month Euribor interest rate swap, both with maturity T and similar notional. Funding is assumed at six-month Euribor3, and does not coincide with the floating side of the interest rate swap.

Three cases are portrayed in figure 4. In case A (separate markets), the three financial instruments are considered individually in the economic hedging relationship – the black lines indicate market segmentation, illustrating that all three financial instruments are referenced to distinct markets (cash and derivatives markets, respectively). Since each financial instrument is traded in a separate market, the market price – termed full fair value – is relevant for case A. If margins are ignored, then the cashflows approximately offset.



In case B (single-curve), the bank decides to measure the economic risk performance of the economic hedging relationship on a present value basis relative to the three-month Euribor interest rate swap curve (benchmark curve). Economically, the question is which part of the fixed-rate bond/loan is exposed to three-month Euribor interest rate risk? The answer is the notional of the fixed-rate bond/loan and the part of interest cashflows corresponding to the three-month Euribor swap rate with maturity T. This result is also termed a ‘(synthetic) risk-equivalent’ bond/loan, as it replicates the interest rate risk of a bond/loan measured by the set of three-month Euribor interest rate swaps, and determines the cost of hedging for a bond/loan (hedging cost approach).

The (synthetic) risk-equivalent bond/loan does not necessarily exactly mirror the fixed leg of the actual three-month Euribor interest rate swap used for economic hedging due to (minor) deviations in terms and conditions of the bond/loan and the interest rate swap. The (synthetic) risk-equivalent bond/loan is derived independently from the funding of the economic hedging relationship. In the hedge accounting relationship – indicated by the light-blue area in case B – only the (synthetic) risk-equivalent bond/loan and the three-month Euribor interest rate swap are of relevance. As a net result, the floating side of the three-month Euribor interest rate swap remains, as funding is not taken into account in the fair-value hedge accounting model.

Given similar terms and conditions of the three-month Euribor interest rate swap and the fixed-rate bond, the hedge will be effective under International Accounting Standard (IAS) 39. Considering the economic hedging relationship – indicated by the grey area in case B – the overall risk and performance is driven by the three-month/six-month basis. Therefore, an economic risk is involved in the hedging relationship, but is not recognised in the P&L under IFRS.4

In case C (multi-curve), interest rate risk is measured by Eonia, which is used as a discount rate for all financial instruments included in the economic hedging relationship. In this case, the three-month Euribor interest rate swap is synthetically decomposed with respect to the two risk factors: Eonia and three-month Euribor/Eonia basis risk. The determination of the (synthetic) risk-equivalent bond/loan in this case depends on Eonia and three-month Euribor/Eonia basis risk, unlike case B.

The coupon of the (synthetic) risk-equivalent bond/loan contains two parts: the fixed part equal to the Eonia swap rate with maturity T, and a variable part (time-dependent) represented by the change in the three-month Euribor/Eonia basis spread. During the life of the hedge, the variable part reflects the dynamic adjustment to the basis spread locked in at the inception of the economic hedge. As a result, an Eonia floater remains, as the basis risk inherent in the three-month Euribor interest rate swap is compensated by the basis risk of the (synthetic) risk-equivalent bond/loan represented by the cashflows originating from the variable part of the coupon.

Similar to case B, the entire economic hedging relationship is exposed to six-month Euribor/Eonia basis risk: the six-month Euribor funding is decomposed in terms of the two relevant risk factors – Eonia and six-month Euribor/Eonia basis risk, where the Eonia part is compensated by the floating leg of the net hedge accounting result. Therefore, an economic risk is involved in the hedging relationship – six-month Euribor/Eonia basis risk – but is not recognised in the P&L under IFRS.

The properties of the hedging cost approach can be defined as follows:

Interest rate risk is an unobservable risk that can only be measured by means of traded financial instruments. Possible qualifying instruments are government bonds, interest rate swaps (derivatives) and so on. Financial market participants typically use derivatives, as they are liquid.

Liquid derivatives are used to derive the relevant risk factors for measuring interest rate risk and basis risk, and define the only relevant valuation factors. This is indicated in cases B and C, where market segmentation is resolved (the black lines of case A have vanished) and the impact of the cash basis is eliminated.

The relevant discount curves characterising the hedged risk are derived from credit risk-free and collateralised OIS curves. Depending on the choice of collateral currency and the selection of products, the cross-currency basis has to be taken into account. Therefore, the cross-currency basis is part of the discount curve.

By construction, the (synthetic) risk-equivalent loan/bond defines the portion of the bond/loan that is exposed to the hedged risk depending on the relevant discount curve and the remaining risk factors (for example, Eonia as a discount rate and three-month Euribor/Eonia basis risk).

The hedging cost approach is independent from the funding model used, as it relies on the set of derivatives used to measure relevant market risks.

The applied method is similar to transfer pricing techniques applied in bank treasury departments. In contrast to traditional transfer pricing models, transfer prices in a multi-curve model framework involve time-dependent prices subject to the used hedging derivatives, if a plain OIS discount curve is used.

The net result of the hedge accounting relationship always equals a floater (three-month Euribor or Eonia floater) corresponding to the applied discount factor (three-month Euribor or Eonia curve).

**Hedge accounting and economic performance**
The derived results can be further analysed by specifying the P&L components of the examples above. Figure 5 lists the economic and IFRS results in each case. In the table, the interest result shows the economic result (measured on an accrual basis) and the IFRS result, neglecting the impact of fair-value valuation under the assumption of a buy-and-hold strategy without possible impairments, etc.



Case A reveals the impact of the IFRS mixed model approach, indicating P&L volatility resulting from the fair-value changes of the interest rate swap despite an economically sound hedging relationship. In the full fair-value model (fair-value option), the changes in the cash basis (difference between cash and derivatives prices) are entirely reflected in P&L.

The results of the remaining two cases are straightforward, providing an important conclusion concerning the relationship between the overall economic risk/performance result and the recognised IFRS result. In cases B and C, the sum of funding measured by the defined discount curve and net result from hedge accounting under IAS 39 equals the overall economic risk/performance. This is an immediate consequence of the applied hedging cost principle that defines a consistent relationship between the IFRS result and the overall economic risk/performance.

But this one-to-one relationship can also be used to justify the application of a multiple risk factor valuation model for the fixed-rate bond/loan. If the basis risk is not taken into account in order to measure economic risk and performance of the fixed-rate bond/loan, then the economic result is not adequately reflected. Furthermore, the consistency between IFRS and overall economic risk/performance would not be met. The derived results can be extended to economic hedges involving foreign exchange and cross-currency basis risk.

**Hedge accounting in a multi-curve model set-up**
Following the economic rationale outlined above, hedge accounting under IAS 39 within a multi-curve valuation framework is expected to be very complex. A comprehensive and detailed description of such models is beyond the scope of this article, but the main ideas are sketched out.

A justification of a multi-curve hedge accounting model under IAS 39 is mainly based on two requirements:

Since multi-curve valuation models use OIS as a discount factor, the IAS 39.AG99F requirements concerning the designation of an OIS discount curve (benchmark curve) have to be met.

Economically, the risk-equivalent bond/loan requires variable cashflows, so the hedge accounting model requires the designation of changing portions of legal cashflows. This requirement is met according to IAS 39.81.

The description of an IFRS multi-curve hedge accounting model can be portrayed by an example. Figure 6 displays the net hedge result – the sum of fair-value changes of the risk-equivalent bond/loan and the interest rate swap – for three hedge accounting strategies. The blue line shows the result in a single-curve valuation model set-up (similar to case B), which represents the hedge accounting model before the financial markets crisis. The second case (orange line) shows the full impact of the three-month Euribor/Eonia basis risk being reflected in the derivative but not in the risk-equivalent bond/loan (without adjustment). The analysis reveals an increased ineffectiveness and higher IFRS P&L volatility. Similar to case C, the green line displays the results if the three-month Euribor/Eonia basis risk is taken into account in the risk-equivalent bond/loan (with adjustment). Higher effectiveness and lower IFRS P&L volatility is achieved if the hedge accounting model framework follows the economics of multi-curve valuation models. Similar examples can be evaluated for hedges involving foreign exchange and cross-currency basis risk.



**Conclusion**
The analysis focused on two core banking business processes that are affected by multi-curve valuation models: hedge accounting (IAS 39) and transfer pricing. The coherence of both models illustrates the extent of banking business being exposed to alterations in the course of the financial crisis, as well as solutions to avoid unreasonable IFRS P&L volatility from economic hedging activities.