Limited Price Indexation (LPI) Swap Valuation Ideas

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Abstract
In this article, we will discuss three pricing methods for LPI swap: perturbation method, common factor method, and Monte-Carlo method. The perturbation method can give a clear hedging idea, but not a great result except when the LPI is close to Zero-coupon. The common factor methods are modifications of the common factor method introduced by Brody, Crosby, and Li (2008), the new common factor methods significantly improve the result for long-term LPI pricing. We will also discuss how to simplify and speed up the Monte-Carlo method for LPI pricing. Since the inflation option market also has skew/smile, we will discuss how to add the skew/smile effect into JY model, the idea is tested with real market data.

Keywords
inflation swap, LPI, Jarrow–Yildirim model, market model, common factor, perturbation

I. Introduction
A Limited Price Index (LPI) is a UK inflation index that is used to define typical payout structures of UK pension plans. By definition, LPIs have annual returns that are equal to the corresponding annual UK inflation rates capped at y and floored at x, for some strikes x and y. The most common values for x and y are 0 and 5%, respectively.

LPI swaps are zero-coupon (ZC) swaps where a fixed GBP amount is exchanged at a given maturity T for the LPI return over the interval [0, T]. In general, LPI swaps must be priced with Monte Carlo since the underlying LPI value is defined by non-trivially compounding the embedded caps and floors. However, one may resort to pricing models that allow for analytical approximations. This is the case for instance of Brody et al. (2008), who used a common-factor methodology to derive an LPI swap pricing formula under the Jarrow and Yildirim (JY) model (Jarrow and Yildirim, 2003).

The Brody et al. (2008) approximation method works well for short- to middle-term swaps, but not necessarily for longer maturities. In fact, we will consider the example of a 40-year swap (a traded contract in the inflation swap market) and show cases where its pricing error can be as large as 24 bp, and hence larger than the typical bid-ask spreads observed in the market. Moreover, as already pointed out by Brody et al. themselves, the common-factor method has the major drawback that it cannot reproduce the same maturity ZC swap price in the limit case where the LPI's caps and floors vanish.

In this article, we will modify the Brody et al. method and show that a simple change in their algorithm can already improve the approximation result for long-term swaps. Then, we will introduce a different numeraire, and show that the common factor method presents no bias if applied under this numeraire. In particular, this technique will prove to be very efficient when pricing the more liquid LPI swaps, as it is confirmed by our numerical results below.

In the following, we will first define the LPI swap, introduce a perturbation method for LPI. By using a new numeraire which automatically converge the LPI price into the ZC price in the limit case when cap/floor vanishes. We will then derive out the result for JY model since this is the most popular model in practice. Then we will modify the common factor method introduced by Brody et al. and show that a simple change in their algorithm can already improve the approximation result for long-term swaps. We will also apply the method for the LPI price under the new numeraire and show that the common factor method presents no bias if applied under this numeraire. In particular, this technique will prove to be very efficient when pricing the more liquid LPI swaps, as it is confirmed by our numerical results. We will also discuss how to simplify/speed up MC under JY model. Then we will discuss how to extend the method to include the skew/smile effect. And we will test the real market data. The techniques discussed in this article can be applied to other inflation models; the extension of the method to inflation market model is given in the appendix.

II. LPI swaps
Zero-coupon LPI swap has an inflation leg, there is one payment at the maturity, and the final payment is based on the LPI index times the notional of the swap. There are two types of LPI swaps in current market. The type A N year LPI index is defined as:

$$LPI_N = max\left(min\left(\frac{CPI_N}{CPI_0}, (K_c+1)^N\right), (K_f+1)^N\right) \tag{2.1}$$

where K_c is the cap strike, usually set at 5%, K_f is the floor strike usually set at 0%. This type of LPI can be decomposed into a zero-coupon swap plus a zero-coupon cap and a zero-coupon floor. When $K_c = \infty$ and $K_f = -\infty$ the zero-coupon LPI swap will have to converge to regular zero-coupon inflation swap. So this type of LPI is just vanilla derivative and easy to price. We will not discuss it in this article.

Type B N year term LPI index is constructed as:

$$LPI_N = \Pi_{i=1}^{N} max\left(min\left(\frac{CPI_i}{CPI_{i-1}}, K_c + 1 \right), K_f + 1 \right) \tag{2.2}$$

where CPI_i is the CPI index set at time i, usually the reset frequency is yearly. Again when $K_c = \infty$ and $K_f = -\infty$ the LPI swap has the exactly same payment as regular zero-coupon inflation swap. In more general case, the LPI index has imbedded year-on-year cap/floor option on the inflation rate and is path dependent. In this paper we will discuss how to price this type of LPI.

Denoting by $P_n(t,T_i)$ the (nominal) zero-coupon bond price at time t with maturity T_i and $E^{Q_n^N}$ the expectation under the T_N–forward measure Q^{T_N} whose associated numeraire is $P_n(t,T_N)$, the LPI par swap rate is given by:

$$K = K_N := N\sqrt{\frac{E^{Q_n^N}[LPI_N]}{LPI_0}} - 1$$

Even if we can derive out the forward CPI index from the zero-coupon inflation swap market, to predict LPI index still involves path depended calculation, which is usually done by MC, and is time consuming in general. Blindly using MC does not shed light on the fundamental intuition of this product.

In order to calculate the LPI index in more efficient ways, we will do some calculation first.

Let $I(T_i) = CPI_i$, rewrite the LPI index an in different way:

$$\begin{aligned}
LPI_N &= \prod_{i=1}^{N} max\left(min\left(\frac{I(T_i)}{I(T_{i-1})}, K_c + 1 \right), K_f + 1 \right) \\
&= \prod_{i=1}^{N} \left(\frac{I(T_i)}{I(T_{i-1})} - \left[\frac{I(T_i)}{I(T_{i-1})} - K_c - 1 \right]^+ + \left[K_f + 1 - \frac{I(T_i)}{I(T_{i-1})} \right]^+ \right) \\
&= \prod_{i=1}^{N} \left(\frac{I(T_i)}{I(T_{i-1})} \left(1 - \left[1 - \frac{K_c+1}{\frac{I(T_i)}{I(T_{i-1})}} \right]^+ + \left[\frac{K_f+1}{\frac{I(T_i)}{I(T_{i-1})}} - 1 \right]^+ \right) \right) \\
&= \frac{I(T_N)}{I(T_0)} \prod_{i=1}^{N} \left(1 - \left[1 - \frac{K_c+1}{\frac{I(T_i)}{I(T_{i-1})}} \right]^+ + \left[\frac{K_f+1}{\frac{I(T_i)}{I(T_{i-1})}} - 1 \right]^+ \right) \\
&= \frac{I(T_N)}{I(T_0)} + \frac{I(T_N)}{I(T_0)} \sum_{i=1}^{N} \left(\left[\frac{K_f+1}{\frac{I(T_i)}{I(T_{i-1})}} - 1 \right]^+ - \left[1 - \frac{K_c+1}{\frac{I(T_i)}{I(T_{i-1})}} \right]^+ \right) + \cdots
\end{aligned} \tag{2.3}$$

So the expected value at time t is

$$\begin{aligned}
&E_t\left(exp\left(-\int_t^{T_N} n(s)ds \right) LPI_N \middle| \mathcal{F}_t \right) \\
&= E_t\left(exp\left(-\int_t^{T_N} n(s)ds \right) \frac{I(T_N)}{I(T_0)} \prod_{i=1}^{N} \left(1 - \left[1 - \frac{K_c+1}{\frac{I(T_i)}{I(T_{i-1})}} \right]^+ + \left[\frac{K_f+1}{\frac{I(T_i)}{I(T_{i-1})}} - 1 \right]^+ \right) \middle| \mathcal{F}_t \right) \\
&= E_t\left(exp\left(-\int_t^{T_N} n(s)\,ds \right) \frac{I(T_N)}{I(T_0)} \middle| \mathcal{F}_t \right) \\
&\quad + E_t\left(exp\left(-\int_t^{T_N} n(s)ds \right) \frac{I(T_N)}{I(T_0)} \sum_{i=1}^{N} \left(\left[\frac{K_f+1}{\frac{I(T_i)}{I(T_{i-1})}} - 1 \right]^+ - \left[1 - \frac{K_c+1}{\frac{I(T_i)}{I(T_{i-1})}} \right]^+ \right) \middle| \mathcal{F}_t \right) \\
&\quad + \cdots
\end{aligned} \tag{2.4}$$

Here E_t means under the spot measure, $\mathcal{F}_t$ denotes the information available in the market at time t. We only expand into first order term on the option pricing, and ignored the higher order terms. From the equation, we can see LPI can be decomposed into leading order, which is the regular Zero-coupon swap, plus first order correction of the embedded cap/floor option, and also higher orders which are the compounding corrections. With this decomposition, we can recover easily the trivial zero-coupon result when the cap/floor vanishes.

Let us define the forward CPI at time t for maturity T_i by $\mathcal{I}(t,T_i) = E^{Q_n^i}[I(T_i) | \mathcal{F}_t]$, where $E^{Q_n^i}$ denotes expectation under the T_i–forward measure. Denote by $P_n(t,T_i)$ the (nominal) zero-coupon bond price at time t with maturity T_i, and forward CPI ratio at time t for period from T_0 to T_N as

$$\gamma(t, T_0 : T_N) = E^{Q_n^N}\left[\frac{I(T_N)}{I(T_0)} \middle| \mathcal{F}_t \right] = \frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)} e^{C(t,T_0:T_N)} \tag{2.5}$$

where $C(t,T_0:T_N)$ is the convexity adjust term for the, which can be found in the literature (Mercurio, 2005) and will also be shown in Appendix A.

Then consider the measure Q_r^N associated with the numeraire

$$Z(t) = P_n(t, T_N)\,\gamma(t, T_0 : T_N)$$

And define by

$$\begin{aligned}
\frac{dQ_r^N}{dQ_n^N} &= \frac{Z(T_N)\,P_n(t,T_N)}{Z(t)\,P_n(T_N,T_N)} = \frac{\gamma(T_N, T_0 : T_N)\,P_n(t,T_N)}{P_n(t,T_N)\,\gamma(t, T_0 : T_N)} \\
&= \frac{I_N/I_0}{\gamma(t, T_0 : T_N)} = \frac{I_N/I_0}{\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)} e^{C(t,T_0:T_N)}}
\end{aligned} \tag{2.6}$$

Then the expectation of $\frac{I(T_i)}{I(T_{i-1})}$ under Q_r^N measure is defined as

$$E^{Q_n^N}\left(\frac{I(T_N)}{I(T_0)} \frac{I(T_i)}{I(T_{i-1})} \middle| \mathcal{F}_t \right) = \frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)} e^{C(t,T_0:T_N)} E^{Q_r^N}\left(\frac{I(T_i)}{I(T_{i-1})} \middle| \mathcal{F}_t \right) \tag{2.7}$$

From Equation (2.4), if we can derive out the expectation and volatility of $\frac{I(T_i)}{I(T_{i-1})}$ under Q_r^N measure defined as:

$$\begin{aligned}
&E_t\left(exp\left(-\int_t^{T_N} n(s)\,ds \right) \frac{I(T_N)}{I(T_0)} \frac{I(T_i)}{I(T_{i-1})} \middle| \mathcal{F}_t \right) \\
&= p_n(t,T_N)\,E^{Q_n^N}\left(\frac{I(T_N)}{I(T_0)} \frac{I(T_i)}{I(T_{i-1})} \middle| \mathcal{F}_t \right) \\
&= p_n(t,T_N) \frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)} e^{C(t,T_0:T_N)} E^{Q_r^N}\left(\frac{I(T_i)}{I(T_{i-1})} \middle| \mathcal{F}_t \right)
\end{aligned}$$

then we can easily price LPI swap. In the following we will try to price LPI under JY model. The detail of the derivation is given in Appendix A. The formula for market model result also will be provided in Appendix B.

III. LPI swap price under JY model

The JY model is constructed as follows (Jarrow and Yildirim, 2003; Mercurio, 2009):

$$\begin{aligned}
dn(t) &= [\vartheta_n(t) - a_n n(t)]\,dt + \sigma_n dW_n(t) \\
dr(t) &= [\vartheta_r(t) - \rho_{rI}\sigma_I\sigma_r - a_r r(t)]\,dt + \sigma_r dW_r(t) \\
dI(t) &= I(t)\,[n(t) - r(t)]\,dt + \sigma_I dW_I(t)
\end{aligned} \tag{3.1}$$

(W_n, W_r, W_I) is a three-dimensional Brownian motion whose components have the following instantaneous correlations: $\rho_{nr}, \rho_{rI}, \rho_{nI}$.

So spot CPI will be

$$I(T) = I(t)\, exp\left(\int_t^T [n(s) - r(s)]\, ds - \frac{1}{2}\sigma_I^2 (T - t) + \sigma_I [W_I(T) - W_I(t)\right) \quad (3.2)$$

Then the expectation of $\frac{I(T_i)}{I(T_{i-1})}$ under Q_r^N measure is

$$E^{Q_r^N}\left(\frac{I(T_i)}{I(T_{i-1})}\middle|\mathcal{F}_t\right) = E^{Q_r^N}\left(exp\left\{\int_{T_{i-1}}^{T_i}[n(s) - r(s)]ds - \frac{1}{2}\sigma_I^2(T_i - T_{i-1}) + \sigma_I[W_I(T_i) - W_I(T_{i-1})]\right\}\middle|\mathcal{F}_t\right) \quad (3.3)$$

Since JY model can be analytically solved, after some straightforward but tedious algebra we can get the final answer considering $T_0 \le T_{i-1} < T_i \le T_N$:

$$E^{Q_r^N}\left(\frac{I(T_i)}{I(T_{i-1})}\middle|\mathcal{F}_t\right) = \frac{J(t,T_i)}{J(t,T_{i-1})} exp\left[\int_t^{T_{i-1}} dsX(s,T_{i-1},T_i) + \int_t^{T_0} ds\tilde{Y}(s,T_0,T_{i-1},T_i,T_N) + \int_{max(t,T_0)}^{T_{i-1}} dsY(s,T_{i-1},T_i,T_N) + \int_{T_{i-1}}^{T_i} dsZ(s,T_i,T_N)\right] \quad (3.4)$$

with

$$\begin{aligned}
X(s,T_{i-1},T_i) &= (B_r(s,T_{i-1})\sigma_r - B_r(s,T_i)\sigma_r)(B_r(s,T_{i-1})\sigma_r \\
&\quad - B_n(s,T_{i-1})\rho_{nr}\sigma_n) - (B_r(s,T_{i-1})\sigma_r - B_r(s,T_i)\sigma_r)\rho_{rI}\sigma_I \\
Y(s,T_{i-1},T_i,T_N) &= (B_n(s,T_i)\sigma_n - B_n(s,T_{i-1})\sigma_n)(B_n(s,T_i)\sigma_n \\
&\quad - B_r(s,T_N)\rho_{nr}\sigma_r + \rho_{nI}\sigma_I) - (B_r(s,T_i)\sigma_r \\
&\quad - B_r(s,T_{i-1})\sigma_r)(B_n(s,T_i)\rho_{nr}\sigma_n - B_r(s,T_N)\sigma_r + \rho_{rI}\sigma_I) \\
\tilde{Y}(s,T_0,T_{i-1},T_i,T_N) &= (B_n(s,T_i)\sigma_n - B_n(s,T_{i-1})\sigma_n)([B_n(t,T_i) - B_n(t,T_0)]\sigma_n - [B_r(t,T_N) \\
&\quad - B_r(t,T_0)]\rho_{nr}\sigma_r) - (B_r(s,T_i)\sigma_r - B_r(s,T_{i-1})\sigma_r)([B_n(t,T_i) \\
&\quad - B_n(t,T_0)]\rho_{nr}\sigma_n - [B_r(t,T_N) - B_r(t,T_0)]\sigma_r) \\
Z(s,T_i,T_N) &= B_n(s,T_i)\sigma_n[B_n(s,T_i)\sigma_n - B_r(s,T_N)\rho_{nr}\sigma_r + \rho_{nI}\sigma_I] \\
&\quad - B_r(s,T_i)\sigma_r[B_n(s,T_i)\rho_{nr}\sigma_n - B_r(s,T_N)\sigma_r + \rho_{rI}\sigma_I] \\
&\quad + \sigma_I(B_n(s,T_i)\rho_{nI}\sigma_n - B_r(s,T_N)\rho_{rI}\sigma_r) + \sigma_I^2
\end{aligned} \quad (3.5)$$

$\mathfrak{I}(t,T_i)$ is the forward CPI index. For $x \in \{n, r\}$, $B_x(\cancel{t,T}) = \frac{1-e^{-a_x(T-t)}}{a_x}$

The variance can also be derived out:

$$Var\left(ln\left(\frac{I(T_i)}{I(T_{i-1})}\right)\middle|\mathcal{F}_t\right) = \int_t^{T_{i-1}} ds[(B_n(s,T_i) - B_n(s,T_{i-1}))^2\sigma_n^2 + (B_r(s,T_i) - B_r(s,T_{i-1}))^2\sigma_r^2 - 2(B_n(s,T_i) - B_n(s,T_{i-1}))(B_r(s,T_i) - B_r(s,T_{i-1}))\rho_{nr}\sigma_n\sigma_r] + \int_{T_{i-1}}^{T_i} ds[B_n(s,T_i)^2\sigma_n^2 + B_r(s,T_i)^2\sigma_r^2 + \sigma_I^2 - 2B_n(s,T_i)B_r(s,T_i)\rho_{nr}\sigma_n\sigma_r + 2B_n(s,T_i)\rho_{nI}\sigma_n\sigma_I - 2B_r(s,T_i)\rho_{rI}\sigma_r\sigma_I] \quad (3.6)$$

The covariance can also be derived out for $j > i$, which has also been derived out in Brody et al. (2008):

$$\begin{aligned}
&cov\left(ln\left(\frac{I(T_i)}{I(T_{i-1})}\right), ln\left(\frac{I(T_j)}{I(T_{j-1})}\right)\middle|\mathcal{F}_t\right) \\
&= \int_t^{T_{i-1}} cov(B_n(T_{i-1},T_i)e^{-a_n(T_{i-1}-s)}\sigma_n dW_n(s) \\
&\quad - B_r(T_{i-1},T_i)e^{-a_r(T_{i-1}-s)}\sigma_r dW_r(s), B_n(T_{j-1},T_j)e^{-a_n(T_{j-1}-s)}\sigma_n dW_n(s) \\
&\quad - B_r(T_{j-1},T_j)e^{-a_r(T_{j-1}-s)}\sigma_r dW_r(s)) \\
&\quad + \int_{T_{i-1}}^{T_i} cov(B_n(s,T_i)\sigma_n dW_n(s) - B_r(s,T_i)\sigma_r dW_r(s) \\
&\quad + \sigma_I dW_I(s), B_n(T_{j-1},T_j)e^{-a_n(T_{j-1}-s)\sigma_n dW_n(s)} - B_r(T_{j-1},T_j)e^{-a_r(T_{j-1}-s)}\sigma_r dW_r(s))
\end{aligned} \quad (3.7)$$

Since we know the average and the variance of lognormal process $Y_i = \frac{I(T_i)}{I(T_{i-1})}$ then the first order term of the LPI price is

$$P_n(t,T_N)\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)}e^{C(t,T_0,T_N)}E^{Q_n^N}\left(\left[\frac{K_f+1}{Y_i} - 1\right]^+ - \left[1 - \frac{K_c+1}{Y_i}\right]^+\right) \quad (3.7)$$

Consider Y_i is a lognormal process with drift term of $\mu(t)$ then

$$Y_i(T) = Y_i(t)\, exp\left(\int_t^{T_i}\mu(s)\, ds - \frac{1}{2}\sigma_i^2(T_i - t) + \sigma_i(W_i(T_i) - W_i(t))\right) \quad (3.7)$$

For $w = \pm 1$ we can get

$$\begin{aligned}
&E\left(\left[w\left(\frac{K}{Y_i} - 1\right)\right]^+\right) \\
&= w\frac{K}{Y_i(t)}e^{-\int_t^{T_i}\mu(s)ds + \sigma_i^2(T_i - t)}N\left(w\frac{\ln(\frac{K}{Y_i(t)}) - \int_t^{T_i}\mu(s)ds + \frac{3}{2}\sigma_i^2(T_i - t)}{\sigma_i\sqrt{(T_i - t)}}\right) \\
&\quad - wN\left(w\frac{\ln(\frac{K}{Y_i(t)}) - \int_t^{T_i}\mu(s)ds + \frac{1}{2}\sigma_i^2(T_i - t)}{\sigma_i\sqrt{(T_i - t)}}\right)
\end{aligned} \quad (3.8)$$

Then we can calculate the first-order correction of LPI price.

IV. Price of LPI with common factor

The LPI price under JY model has been discussed in Brody et al. (2008) and Ryten (2007). The basic idea in those literatures is to replace the covariance matrix $Cov(lnY_i, lnY_j)$ with a rank one matrix. In JY model, Y_i is lognormal distributed under terminal zero coupon bond measure, $Y_i = exp(a_i z_i + b_i)$ where $z_i \sim N(0,1)$ is normal distributed random variable. The key idea of Brody et al. (2008) and Ryten (2007) is to replace Y_i by $\widehat{Y_i} = \exp(b_i + a_i(\hat{\alpha}_i w + \sqrt{(1 - \hat{\alpha}_i^2)}\varepsilon_i)$ where $\{w, \varepsilon_1, ..., \varepsilon_n\}$ are independent normal distributed random variables. And w is the common factor. So conditional on w the individual cap/floors are independent and LPI price can be approximated as

$$\begin{aligned}
P_n(t,T_N)E^{Q_n^N}(LPI_N) &= P_n(t,T_N)E^{Q_n^N}\left(\prod_{i=1}^N max\left(max(Y_i, K_c + 1), K_f + 1\right)\right) \\
&= P_n(t,T_N)E^{Q_n^N}\left(\prod_{i=1}^N\left(Y_i - [Y_i - K_c - 1]^+ + \left[K_f + 1 - Y_i\right]^+\right)\right)
\end{aligned} \quad (4.1)$$

To price LPI becomes one-dimension integration after plug in the individual option value formula.

In reference Brody D et al. (2008), the author approximate

$$\hat{\alpha}_i \approx exp\left[\frac{1}{N-1}\left(\overline{k_\iota} - \frac{\sum_{j=1}^{N}\overline{k_j}}{2\,(N-1)}\right)\right] \tag{4.2}$$

with

$$\overline{k_\iota} = \sum_{j\neq i}^{N} \ln\left[\mathrm{cov}\left(\ln Y_i, \ln Y_j\right)\right] \tag{4.3}$$

(for $N = 1$, $\hat{\alpha}_1 = 1$; for $N = 2$, $\hat{\alpha}_1 = 1$, $\hat{\alpha}_2 = \mathrm{corr}(\ln Y_1, \ln Y_2)$ these are exact results).

Instead of (4.3) using the covariance matrix, we believe it is an error, since the original idea comes from Jäckel (2004), who discussed the approximation of correlation matrix. So it is nature to use the correlation matrix:

$$\overline{k_\iota} = \sum_{j\neq i}^{N} \ln\left[\mathrm{corr}\left(\ln Y_i, \ln Y_j\right)\right] \tag{4.4}$$

In which $\mathrm{corr}(\ln Y_1, \ln Y_2)$ is the correlation. In the following section, we will see this simply modification will improve the result a lot.

As pointed out by the authors in the paper, the above approximation method cannot reproduce the standard zero-coupon price at the limit case when $k_c = \infty$ and $k_f = -\infty$. Their results also show the zero-coupon case is the worst scenario for this approximation method. In order to reproduce the result of zero-coupon, the full correlation matrix is required, not just one common factor.

The most popular LPI swaps have $k_c = 5\%$ and $k_f = 0\%$, which is almost the worst case for the above method under current market condition. In order to improve the result, we will propose different approach as following.

Consider the LPI valuation formula in (2.3) and (2.4):

$$\begin{aligned} &E_t\left(exp\left(-\int_t^{T_N} n\,(s)ds\right) LPI_N \middle| \mathcal{F}_t\right) \\ &= E_t\left(exp\left(-\int_t^{T_N} n(s)ds\right)\frac{I(T_N)}{I(T_0)}\prod_{i=1}^{N}\left(1-\left[1-\frac{K_c+1}{Y_i}\right]^+ + \left[\frac{K_f+1}{Y_i}-1\right]^+\right)\middle|\mathcal{F}_t\right) \\ &= P_n(t,T_N)\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)}e^{C(t,T_0,T_N)}E^{Q_r^N}\left(\prod_{i=1}^{N}\left(1-\left[1-\frac{K_c+1}{Y_i}\right]^+ + \left[\frac{K_f+1}{Y_i}-1\right]^+\right)\middle|\mathcal{F}_t\right) \end{aligned} \tag{4.5}$$

The expectation $E^{Q_r^N}(Y_i)$ has been derived out for JY model, Y_i is a lognormal process, with known average and variance.

If we apply the same common factor technique, let $\tilde{Y}_i = \exp(\tilde{b}_i + a_i(\hat{\alpha}_i w + \sqrt{(1-\hat{\alpha}_i^2)}\varepsilon_i)$ (just remember, now $\hat{Y}_i$ has different drift term $\tilde{b}_i$ due to different measure). We can approximate the pricing of LPI at time t as:

$$\begin{aligned} &E_t\left(exp\left(-\int_t^{T_N} n\,(s)ds\right) LPI_N \middle| \mathcal{F}_t\right) \\ &\approx p_n(t,T_N)\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)}e^{C(t,T_0,T_N)}E^{Q_r^N}\left(\prod_{i=1}^{N}\left(1-\left[1-\frac{K_c+1}{\tilde{Y}_\iota}\right]^+ + \left[\frac{K_f+1}{\tilde{Y}_\iota}-1\right]^+\right)\middle|\mathcal{F}_t\right) \end{aligned} \tag{4.6}$$

It is clear to see that even we approximate the correlation matrix with one common factor load, the above equation will automatically lead to zero-coupon price at the limit case when $k_c = \infty$ and $k_f = -\infty$. For the most popular LPI with $k_c = 5\%$, $k_f = 0\%$, there is clear advantage in this method.

V. Pricing LPI with MC for JY model

Since we can derive the marginal lognormal distribution of Y_i under Q_r^N measure above (the result under terminal measure Q_n^N as been given in Brody et al. (2008), we also know the correlation $corr(\ln Y_i, \ln Y_j)$, which means we analytically have the full covariance matrix of Y_i with $i = 1,\ldots,N$. Then when we do MC calculation, instead of the regular MC method, we can just simulate the full distribution of Y_i for $i = 1,\ldots,N$ with N correlated random variable. With this idea the MC method can be greatly speed up. We also use sobol sequences in the random number generating.

VI. Numerical results

We now examine some numerical examples with the different methods discuss above. For comparison purpose, we will adopt the same model parameters given in Brody et al. (2008), the parameters used there are based on historical estimation of the sterling market data with a four factor JY model, in which the nominal curve was modeled with 2-factor Hull–White model instead of one, and the parameters are given as:

$$\begin{aligned} &a_n^1 = 0.06494565 && \sigma_n^1 = 0.00649825 \\ &a_n^2 = 0.00001557535 && \sigma_n^2 = 0.0063321172 \\ &a_r = 0.032193009 && \sigma_r = 0.006093904 \quad \sigma_I = 0.0104000 \\ &\rho_{nn}^{12} = -0.46296278 && \rho_{rI} = -0.03781752 \\ &\rho_{nr}^{11} = \rho_{nr}^{12} = 0.518100 && \rho_{nI}^{1} = \rho_{nI}^{2} = -0.018398113 \end{aligned}$$

(Notice the correlations ρ_{rI} and ρ_{nI} have different signs from the original Brody et al. (2008), this is because in Brody et al. (2008) they are modeling the bonds, here we are modeling the short rates, so the correlations have different signs). Our results above can be easily extended to the case when the nominal curve is 2-factor Hull–White model instead of one factor. To save the space we will ignore the tedious formulas here. The initial nominal curve is flat 5.0%; real rate is flat 2.5%.

When we look at different maturities with 10 year, 25 year, and 40 year, the swap can have different cap/floor strikes. For comparison purpose we use same strikes as in Brody et al. (2008). The results are shown in Table A, B, C.

In the tables, the MC results from Brody et al. (2008), are quoted, in which paper 130 million paths have been simulated. The one common factor quasi-analytic results in Brody et al. (2008) are also shown as BCL column, the simply modification of the BCL method by using the correlation matrix to derive the common factor load as (4.4) is shown as BCL-Rho. The first order result of LPI described is this article is shown as ZM-o(1). The common factor quasi-analytic result based on the technique in this article is shown as ZM-rho.

If we use MC as benchmark, the error of all the other methods is also shown in the tables. The differences are quoted as basis point. The 40-yr LPI MC are done by our MC method described above. The errors are also shown in Figure 1, the Y-axis is the error, and the X-axis shows the same 11 cases with different cap/floor strikes as used in Brody et al. (2008). Real market usually does not have such variety of strikes, but since we fix the inflation rate around 2.5%, so the variety of strikes will mimic cases when the option is close to the money or far out of the money.

It is interesting to see that all methods have good approximation result for short-term cases, say 10 year. But the result is very different for middle- to long-term, such as 25 year or 40 year. The method provided by BCL has 4 basis points error for the case of strike (0%, 5%) and 25-year term, and the error will be 15 basis points for a 40-year LPI, this error is not ignorable. For the case when the LPI is more close the 0-coupon, the error is even bigger.

Regarding the first order perturbation result given in this paper ZM-O(1), it does very good job for the case of (0%, 5%) even up to 40-year term, in which case the error is about 2 basis points. But sometimes it does a bad job, especially when the cap/floor option value is big, for example the case of (2%, 3%). Consider the ATM inflation rate is 2.5315, both cap/floor option values are not ignorable; the error of LPI price can be as high as 17 basis points. This can be easily explained, since the idea of the first order pricing is based on the option value is small, when the option value is not small, the higher order correction must be considered especially when the contract is long term. Fortunately most popular LPI contract is (0%, 5%), and the inflation rate is around 2–3%. So in this normal condition, we believe first order pricing is good enough. Under abnormal economic condition such as hyperinflation of deflation, we should not use the first-order method.

Regarding the modification of the original BCL method by using correlation matrix in common factor calculation, the BCL-rho method greatly improves the original BCL results. The biggest error for 25-year LPI is just 0.7 basis points comparing 5 basis points error in original method. For 40-year LPI, the biggest error is 2 basis points comparing to 25 basis points.

10-yr LPI			Implied Rate (%)					Diff rates (bp)			
Case	Cap	Floor	MC	BCL	BCL-Rho	ZM-o(1)	ZM-Rho	BCL	BCL-Rho	ZM-o(1)	ZM-Rho
1	0.03	0.0	2.288252	2.28746	2.2894	2.2867	2.2895	–0.079	0.115	–0.155	0.125
2	0.03	0.02	2.50856	2.50836	2.5086	2.5078	2.5085	–0.02	0.004	–0.076	–0.006
3	0.032	0.01	2.38549	2.38475	2.3861	2.3850	2.3862	–0.074	0.061	–0.049	0.071
4	0.035	0.005	2.42731	2.42622	2.4279	2.4275	2.4281	–0.109	0.059	0.019	0.079
5	0.04	0.01	2.52303	2.52179	2.5229	2.5231	2.5231	–0.124	–0.013	0.007	0.007
6	0.045	0.0175	2.66821	2.66727	2.6673	2.6665	2.6674	–0.094	–0.091	–0.171	–0.081
7	0.0475	0.0025	2.52878	2.52708	2.5286	2.5289	2.5289	–0.17	–0.018	0.012	0.012
8	0.05	0	2.52978	2.52801	2.5295	2.5299	2.5299	–0.177	–0.028	0.012	0.012
9	0.05	0.005	2.53953	2.53785	2.5392	2.5395	2.5395	–0.168	–0.033	–0.003	–0.003
10	0.06	0	2.53493	2.53311	2.5346	2.5349	2.5349	–0.182	–0.033	–0.003	–0.003
11	0.12	–0.08	2.53145	2.52957	2.5312	2.5315	2.5315	–0.188	–0.025	0.005	0.005

25-yr LPI			Implied Rate (%)					Diff rates (bp)			
Case	Cap	Floor	MC	BCL	BCL-Rho	ZM-o(1)	ZM-Rho	BCL	BCL-Rho	ZM-o(1)	ZM-Rho
1	0.03	0.0	2.19897	2.18018	2.1975	2.1745	2.1982	–1.879	–0.147	–2.447	–0.077
2	0.03	0.02	2.49455	2.49077	2.4941	2.4652	2.4905	–0.378	–0.045	–2.935	–0.405
3	0.032	0.01	2.33336	2.31844	2.332	2.3157	2.3318	–1.492	–0.136	–1.766	–0.156
4	0.035	0.005	2.36778	2.34451	2.3655	2.3576	2.3671	–2.327	–0.228	–1.018	–0.068
5	0.04	0.01	2.50389	2.47889	2.5009	2.498	2.5029	–2.5	–0.299	–0.589	–0.099
6	0.045	0.0175	2.70033	2.68071	2.6972	2.688	2.6982	–1.962	–0.313	–1.233	–0.213
7	0.0475	0.0025	2.5159	2.47651	2.511	2.5146	2.5156	–3.939	–0.49	–0.13	–0.03
8	0.05	0	2.5192	2.47645	2.5138	2.5185	2.5191	–4.275	–0.54	–0.07	–0.01
9	0.05	0.005	2.54103	2.50193	2.536	2.5396	2.5406	–3.91	–0.503	–0.143	–0.043
10	0.06	0	2.5419	2.49511	2.5358	2.5415	2.5417	–4.679	–0.61	–0.04	–0.02
11	0.12	–0.08	2.53145	2.47849	2.5245	2.5315	2.5315	–5.296	–0.695	0.005	0.005

	40-yr LPI		Implied Rate (%)					Diff rates (bp)			
Case	Cap	Floor	MC	BCL	BCL-Rho	ZM-o(1)	ZM-Rho	BCL	BCL-Rho	ZM-o(1)	ZM-Rho
1	0.03	0.0	2.0543	1.9855	2.048	1.9129	2.0449	−6.88	−0.63	−14.14	−0.94
2	0.03	0.02	2.456	2.4455	2.4553	2.2868	2.4394	−1.05	−0.07	−16.92	−1.66
3	0.032	0.01	2.2379	2.1914	2.234	2.1181	2.2271	−4.65	−0.39	−11.98	−1.08
4	0.035	0.005	2.2549	2.1785	2.2485	2.1708	2.2467	−7.64	−0.64	−8.41	−0.82
5	0.04	0.01	2.4348	2.3576	2.4286	2.376	2.4274	−7.72	−0.62	−5.88	−0.74
6	0.045	0.0175	2.6905	2.6318	2.6859	2.6223	2.6825	−5.87	−0.46	−6.82	−0.8
7	0.0475	0.0025	2.4528	2.3181	2.4419	2.4289	2.4494	−13.47	−1.09	−2.39	−0.34
8	0.05	0	2.4608	2.3094	2.4484	2.4434	2.4584	−15.14	−1.24	−1.74	−0.24
9	0.05	0.005	2.4995	2.3669	2.4887	2.4778	2.4964	−13.26	−1.08	−2.17	−0.31
10	0.06	0	2.525	2.3496	2.5103	2.5177	2.5244	−17.54	−1.47	−0.73	−0.06
11	0.12	−0.08	2.5289	2.284	2.5076	2.5314	2.5314	−24.49	−2.13	0.25	0.25

7/15/2010	ZC-Market	LPI-Market	JY-LPI	ZM-o(1)-LPI	ZM-rho-LPI	JY-diff(bp)	ZM-o(1)-diff(bp)	ZM-rho-diff(bp)
2	3.5	3.149	3.1584	3.1291	3.1293	0.94	−1.99	−1.97
3	3.26	3.0625	3.088	3.0693	3.0694	2.55	0.68	0.69
4	3.216	3.044	3.0813	3.0688	3.069	3.73	2.48	2.5
5	3.188	3.067	3.1014	3.0937	3.0938	3.44	2.67	2.68
6	3.1525	3.102	3.1326	3.1335	3.1335	3.06	3.15	3.15
7	3.1725	3.134	3.1558	3.1683	3.1683	2.18	3.43	3.43
8	3.1965	3.167	3.1818	3.204	3.2041	1.48	3.7	3.71
9	3.2245	3.202	3.2113	3.2398	3.2404	0.93	3.78	3.84
10	3.26	3.242	3.2472	3.278	3.2845	0.52	3.6	4.25
15	3.44	3.43	3.4276	3.4588	3.4606	−0.24	2.88	3.06
20	3.565	3.565	3.5598	3.5763	3.5787	−0.52	1.13	1.37
25	3.574	3.611	3.5706	3.611	3.6359	−4.04	0	2.49
30	3.59	3.666	3.5879	3.6505	3.6575	−7.81	−1.55	−0.85
40	3.537	3.679	3.536	3.674	3.689	−14.3	−0.5	1
50	3.525	3.69	3.5248	3.719	3.7474	−16.52	2.9	5.74

Comparing the one common factor method introduced in this paper ZM-rho and the modified BCL-rho method, both do very good job for all cap/floor cases and also long term cases. The biggest error is about 2 basis points for 40-year LPI price, which is less than the market quote bid/ask spread, which is about 6 basis points. We believe either method is good enough in LPI pricing. If we compare these two methods, the biggest error for BCL-rho is when the LPI is close to Zero-coupon case, and do best job at (2%, 3%); instead the ZM-rho method has biggest error at (2%, 3%) and has no error at zero-coupon case. These two methods exactly complement each other. We can use different methods for different cases. Consider the most general case is (0%, 5%), we can see the new method ZM-rho is a better choice.

VI. Skew/smile effect in LPI pricing

We only discussed the JY model, which produces flat implied volatilities in shifted lognormal terms for year-on-year inflation caps/floors. However, the market prices of these caps/floors imply volatility skews/smiles similar to all other option markets. For example Figure 2 shows the implied year on year inflation caplet volatility from Bloomberg on 08/03/10, clear the volatilities are not flat.

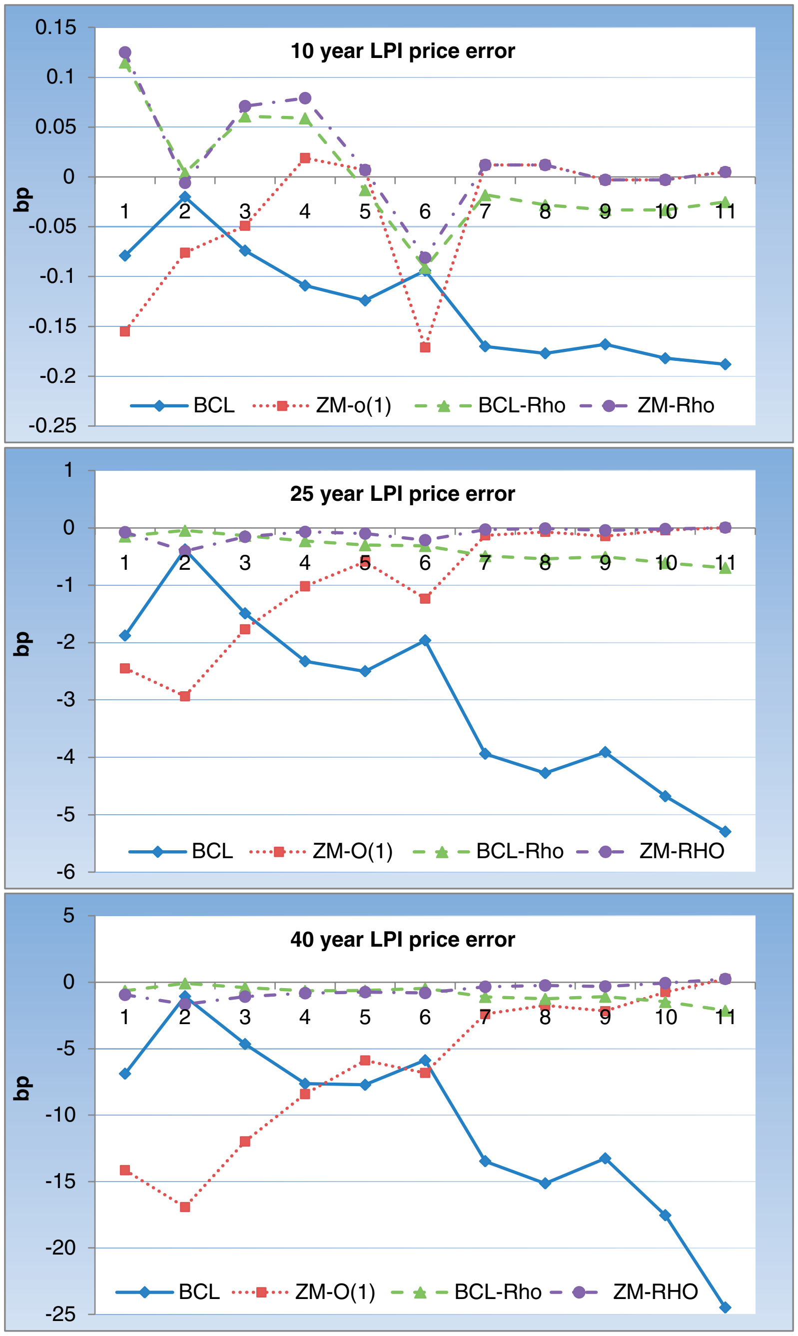
In order to include the skew/smile effect in the LPI pricing, we have to use a more sophisticate model than JY such as stochastic volatility inflation market model (Mercurio and Moreni, 2005). Alternatively, we can resort to a more empirical approach and include skew/smile effects by using the following technique.

The first simply approach is that we can twist the JY model such that the cap/floor price imbedded in the LPI can be reproduced. This is done in the JY calibration: instead of calibration the cap or floor for JY model we will calibrated onto strategy $[K_f + 1 - Y_i]^+ - [Y_i - K_c - 1]^+$. After the calibration we will just use the JY model to price LPI.

Another approach will be described as follows.

The approximate pricing of LPI,

Figure 1: The LPI pricing error with different methods.



$$
\begin{aligned}
&E_t\left(exp\left(-\int_t^{T_N} n(s)ds\right) LPI_N \middle| \mathcal{F}_t\right)\\
&= p_n\left(t,T_N\right)\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)}e^{C(t,T_0,T_N)}\\
&\quad + p_n\left(t,T_N\right)\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)}e^{C(t,T_0,T_N)}E^{Q_T^N}\left(\sum_{i=1}^{N}\left(\left[\frac{K_f+1}{Y_i}-1\right]^+-\left[1-\frac{K_c+1}{Y_i}\right]^+\right)\middle|\mathcal{F}_t\right)+\cdots
\end{aligned}
$$

To value the term $E^{Q_r^N}\left(\left[\frac{K_f+1}{Y_i}-1\right]^+-\left[1-\frac{K_c+1}{Y_i}\right]^+\mathcal{F}_t\right)$ including the skew/smile effect, for any chosen stochastic volatility model, this term can be easily approximated or use replication to price it. Here considering the simplicity of JY model, we still use the JY model to get the correct drift term for Y_i under measure Q_r^N, but we use the market volatility at corresponding strike instead of the JY model implied volatility. The reason for using the pricing formula under the Q_r^N measure is to get option payouts that only depend on Y_i, so that there is no ambiguity on the market volatilities to use in the approximation.

This simple method of handling skew/smile effects gives LPI prices that are very close to those contributed by dealers, this will be shown in the following section. If higher order correction terms are needed, we may add them by using the common factor method.

VII. Real Market Examples

In order to check whether the model described in this paper works in the real market including the skew/smile effect, we will test real market data. In the testing we used the market data on July 15, 22, and 26, 2010. We calculated the LPI price from 2 year to 50 year based on market quoted year-on-year cap/floor option value, inflation zero-coupon curve and swap curve. The option market has price quote only up to 30 year, so we will flat out the implied volatility up to 50 year in order to price 50-year LPI. The results are based on three different methods, first is JY model, second is the first order method including skew/smile effect (ZM-o(1)) and the last is full skew/smile model (ZM-rho). The result is given in Table D, E, and F.

In the tables we also show the LPI market quotes from one dealer, we take that quotes as benchmark and compared the difference with our results. Figure 3 shows the difference between the market quote LPI and our results. The X-axis is the term of LPI swap. Y-axis is the rate difference comparing with the benchmark quotes in basis point.

We can see all three methods priced LPI close to the market quotes up to 20 year, with maximum difference of four basis points. But the behaviors are different for long term above 20 years. The difference with JY method is much bigger. This can be explained that since JY model is fixed parameter model, it cannot fit the cap/floor market on all the terms, and especially we are going to extrapolate the calculation to 50 years LPI.

The methods based on Q_r^N measure, either the first order (ZM-o(1)) or full order (ZM-rho) give closer results to market quotes for long-term LPI. This is because these methods have the freedom to use the real market volatilities and probably the dealer also used similar volatility extrapolation.

For all terms of LPI from 2 to 50 year, we can see most differences are within 4 basis points which are within the current LPI market bid/ask spread. There are a few outliers: the 2-year LPI on 7/22/10 and 7/26/10 is very different from benchmark, this is because the market had a 5 basis point jump in the 2-year zero-coupon, maybe the option market did not update fast enough; the 50-year LPI on 7/22/10 based on the full-order method (ZM-rho), we do not know exactly what's the reason. Intuitively we can try to understand that since the quoted LPI(3.64%) at 50 year is higher than zero-coupon rate (3.505%), which means the floor option has more value than the cap option, when market of the zero-coupon rate move from 3.525% on 7/15 to 3.505% on 7/22. Then the LPI should at most move down by 2 basis points or could move up (when the underline move down the floor option would be even more valuable than the cap option which will push the spread of LPI to zero-coupon even higher). But instead the market quoted LPI moved down by 5 basis points. Thinking in this way maybe the market quote

E.7/22/2010	ZC-Market	LPI-Market	JY-LPI	ZM-o(1)-LPI	ZM-rho-LPI	JY-diff(bp)	ZM-o(1)-diff(bp)	ZM-rho-diff(bp)
2	3.4538	3.212	3.2362	3.1678	3.1679	2.42	−4.42	−4.41
3	3.2537	3.0925	3.1137	3.0895	3.0896	2.12	−0.3	−0.29
4	3.144	3.037	3.0338	3.0537	3.0538	−0.32	1.67	1.68
5	3.091	3.044	3.0232	3.0629	3.0631	−2.08	1.89	1.91
6	3.1255	3.077	3.0498	3.1018	3.1022	−2.72	2.48	2.52
7	3.136	3.11	3.0781	3.139	3.1394	−3.19	2.9	2.94
8	3.143	3.143	3.1099	3.1743	3.1748	−3.31	3.13	3.18
9	3.193	3.179	3.1523	3.2114	3.2116	−2.67	3.24	3.26
10	3.214	3.218	3.1847	3.2507	3.2512	−3.33	3.27	3.32
15	3.3738	3.405	3.3691	3.4317	3.4334	−3.59	2.67	2.84
20	3.529	3.54	3.5059	3.5449	3.5474	−3.41	0.49	0.74
25	3.538	3.585	3.5179	3.5709	3.5763	−6.71	−1.41	−0.87
30	3.544	3.639	3.5354	3.5993	3.6439	−10.36	−3.97	0.49
40	3.511	3.646	3.4874	3.6115	3.6857	−15.86	−3.45	3.97
50	3.505	3.64	3.4834	3.6499	3.7695	−15.66	0.99	12.95

F.7/26/2010	ZC-Market	LPI-Market	JY-LPI	ZM-o(1)-LPI	ZM-rho-LPI	JY-diff(bp)	ZM-o(1)-diff(bp)	ZM-rho-diff(bp)
2	3.44	3.219	3.2024	3.1607	3.1608	−1.66	−5.83	−5.82
3	3.228	3.0985	3.0949	3.0891	3.0894	−0.36	−0.94	−0.91
4	3.151	3.044	3.0245	3.0546	3.0549	−1.95	1.06	1.09
5	3.108	3.05	3.0206	3.0644	3.0648	−2.94	1.44	1.48
6	3.097	3.083	3.0521	3.1043	3.1047	−3.09	2.13	2.17
7	3.138	3.116	3.0853	3.1425	3.1429	−3.07	2.65	2.69
8	3.153	3.149	3.1213	3.1785	3.1788	−2.77	2.95	2.98
9	3.1485	3.185	3.1583	3.215	3.2151	−2.67	3	3.01
10	3.224	3.224	3.1919	3.2528	3.2541	−3.21	2.88	3.01
15	3.41	3.411	3.3806	3.4321	3.433	−3.04	2.11	2.2
20	3.529	3.545	3.5136	3.5463	3.5502	−3.14	0.13	0.52
25	3.536	3.589	3.5243	3.5716	3.5763	−6.47	−1.74	−1.27
30	3.547	3.645	3.5441	3.6032	3.6102	−10.09	−4.18	−3.48
40	3.534	3.674	3.5263	3.6339	3.647	−14.77	−4.01	−2.7
50	3.528	3.661	3.5126	3.668	3.6917	−14.84	0.7	3.07

LPI was not good enough. Still, even the general pricing of long-term LPI is close to the market quotes, we have to be cautious when we trying to price LPI based on extrapolation of option market.

Conclusion

Even there is a substantial market demand in LPI swap, but there is very little discussion in the literature on how to price this product. In this paper, we discussed a few efficient pricing ideas based on the Jarrow–Yildirim model. The methods not just improve the pricing speed, the simple extension can be used in real market pricing by including the skew/smile effect, and also it could provide a nature hedging for LPI instrument. The method can also be generalized to other model such as the market model, this generalization will be given in Appendix B. Apply the method on more sophisticated stochastic model will be discussed other place.

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Joshua Xingzhi Zhang received his D Phil. in physics from Columbia University in 2001. Before his graduation, he started to work with Bloomberg LP as the key quant programmer in building the interest rate derivative system from scratch on Bloomberg terminal. Later he

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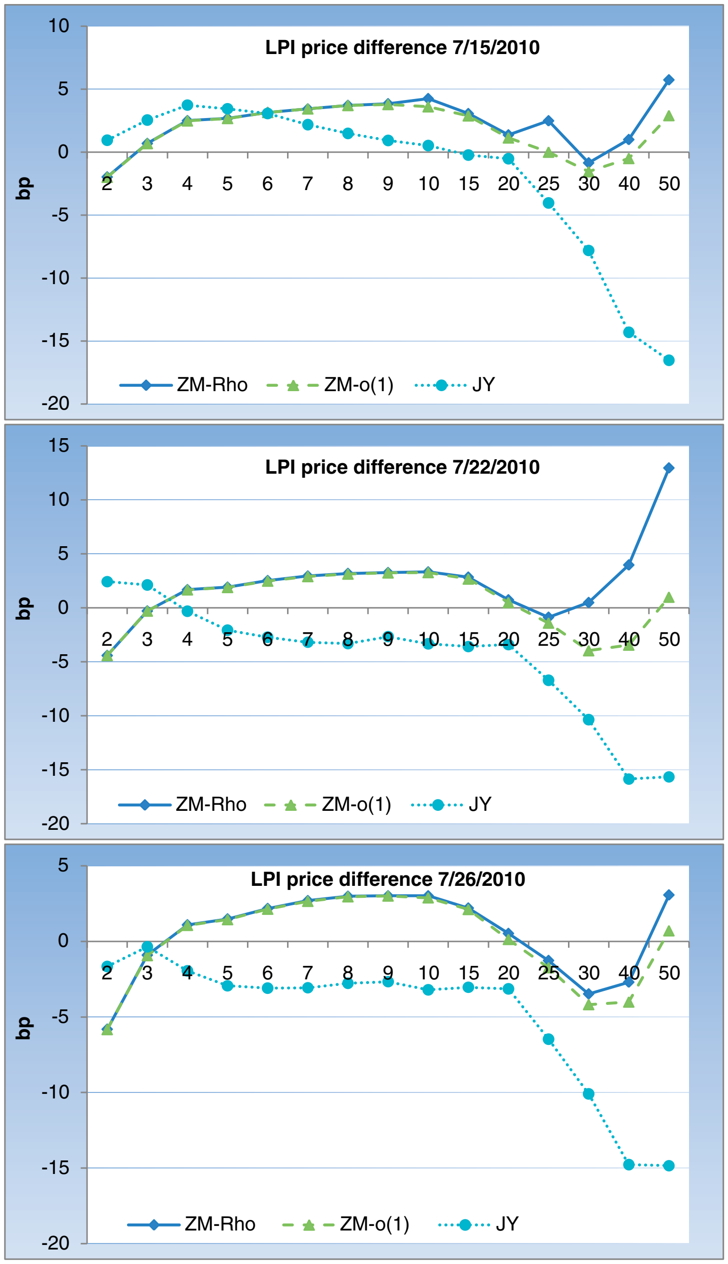
Figure 2: The implied inflation year on year caplet volatility for GBP on 08/03/10.

Tenor	-2.00%	-1.00%	-0.50%	0.00%	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
1 YR	2.82	2.64	2.55	2.46	2.38	2.38	2.28	2.19	1.59	1.64
2 YR	2.63	2.44	2.34	2.25	2.15	1.88	1.81	1.74	1.90	1.82
3 YR	2.25	2.07	1.98	1.89	1.80	1.79	1.71	1.63	1.56	1.50
5 YR	1.94	1.77	1.63	1.48	1.51	1.42	1.34	1.26	1.19	1.13
7 YR	1.62	1.48	1.46	1.43	1.26	1.18	1.12	1.05	0.99	0.93
10 YR	1.49	1.37	1.30	1.24	1.17	0.99	0.95	0.90	0.85	0.80
12 YR	1.32	1.20	1.14	1.08	1.02	0.92	0.86	0.81	0.76	0.70
15 YR	1.25	1.14	1.09	1.03	0.97	0.86	0.81	0.76	0.71	0.66
20 YR	1.26	1.15	1.09	1.04	0.98	1.09	1.00	0.91	0.82	0.73
30 YR	1.16	1.05	1.00	0.95	0.89	0.91	0.83	0.75	0.68	0.60

Figure 3: The pricing difference of LPI comparing with market quotes.



focused on developing pricing/risk models for interest rate derivatives, equity derivatives, currently responsible for inflation derivative products.

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APPENDIX A

Expectation of $\frac{I(T_i)}{I(T_{i-1})}$ under $\mathbf{Q}_r^N$ measure for JY model is presented.

The JY model is constructed as follows:

$$\begin{aligned} dn(t) &= [\vartheta_n(t) - a_n n(t)]\,dt + \sigma_n dW_n(t) \\ dr(t) &= [\vartheta_r(t) - \rho_{rI}\sigma_I\sigma_r - a_r r(t)]\,dt + \sigma_r dW_r(t) \\ dI(t) &= I(t)[n(t) - r(t)]\,dt + \sigma_I dW_I(t) \end{aligned} \tag{A.1}$$

(W_n, W_r, W_I) is a three-dimensional Brownian motion whose components have the following instantaneous correlations: $\rho_{nr}, \rho_{rI}, \rho_{nI}$.

Setting, for $x \in \{n, r\}$,

$$B_x(t,T) = \frac{1 - e^{-a_x(T-t)}}{a_x}$$

So spot CPI will be

$$I(T) = I(t)\,exp\left(\int_t^T [n(s) - r(s)]\,ds - \frac{1}{2}\sigma_I^2(T-t) + \sigma_I[W_I(T) - W_I(t)]\right) \tag{A.2}$$

Let us define the forward CPI at time t for maturity T_i by $\mathcal{I}(t,T_i) = E^{Q_n^i}[I(T_i)\,|\,\mathcal{F}_t]$, where $E^{Q_n^i}$ denotes expectation under the T_i–forward measure and $\mathcal{F}_t$ denotes the information available in the market at time t. Denote by $P_n(t,T_i)$

the (nominal) zero-coupon bond price at time t with maturity T_i. And forward CPI ratio at time t for period from T_0 to T_N as

$$\gamma\left(t,T_0\!:\!T_N\right)=E^{Q_n^N}\left[\frac{I(T_N)}{I(T_0)}\bigg|\mathcal{F}_t\right]=\frac{J\left(t,T_N\right)}{J\left(t,T_0\right)}e^{C(t,T_0:T_N)} \tag{A.3}$$

where $C(t,T_0{:}T_N)$ is the convexity adjust term for the, which can be found in literature Mercurio (2005) and will also be shown later in this section. Then consider the measure Q_r^N associated with the numeraire

$$Z\left(t\right)=P_n\left(t,T_N\right)\gamma\left(t,T_0\!:\!T_N\right) \tag{A.4}$$

and defined by

$$\begin{aligned}\frac{dQ_r^N}{dQ_n^N}&=\frac{Z\left(T_N\right)P_n\left(t,T_N\right)}{Z\left(t\right)P_n\left(T_N,T_N\right)}=\frac{\gamma\left(T_N,T_0\!:\!T_N\right)P_n\left(t,T_N\right)}{P_n\left(t,T_N\right)\gamma\left(t,T_0\!:\!T_N\right)}\\&=\frac{\frac{I_N}{I_0}}{\gamma\left(t,T_0\!:\!T_N\right)}=\frac{\frac{I_N}{I_0}}{\frac{\mathcal{J}(t,T_N)}{\mathcal{J}(t,T_0)}e^{C(t,T_0:T_N)}}\end{aligned} \tag{A.5}$$

Then the expectation of $\frac{I(T_i)}{I(T_{i-1})}$ under Q_r^N measure is defined as

$$E^{Q_n^N}\left(\frac{I\left(T_N\right)}{I\left(T_0\right)}\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)=\frac{\mathcal{J}\left(t,T_N\right)}{\mathcal{J}\left(t,T_0\right)}e^{C(t,T_0:T_N)}E^{Q_r^N}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right) \tag{A.6}$$

$$\begin{aligned}&E^{Q_r^N}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)\\&=E^{Q_r^N}\left(exp\left\{\int_{T_{i-1}}^{T_i}\left[n(s)-r(s)\right]ds-\frac{1}{2}\sigma_I^2(T_i-T_{i-1})+\sigma_I[W_I(T_i)-W_I(T_{i-1})]\right\}\bigg|\mathcal{F}_t\right)\end{aligned} \tag{A.7}$$

Consider for $x\in\{n,r\}$,

$$\begin{aligned}\int_{T_{i-1}}^{T_i}x\left(s\right)ds&=G_x\left(T_{i-1},T_i\right)+\frac{1-e^{-a_x\left(T_i-T_{i-1}\right)}}{a_x}x\left(T_{i-1}\right)\\&\quad+\frac{\sigma_x}{a_x}\int_{T_{i-1}}^{T_i}\left[1-e^{-a_x\left(T_i-s\right)}\right]dW_x\left(s\right)\end{aligned}$$

and

$$x\left(T_{i-1}\right)=H_x\left(t,T_{i-1}\right)+\sigma_x\int_t^{T_{i-1}}e^{-a_x\left(T_{i-1}-s\right)}dW_x\left(s\right)$$

where $G_x(T_{i-1},T_i)$ and $H_x(t,T_{i-1})$ are deterministic functions.

Then

$$\begin{aligned}&E^{Q_r^N}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)\\&=F(t,T_i,T_{i-1})E^{Q_r^N}\left(\begin{array}{c|c}\begin{array}{l}exp\left\{\frac{1-e^{-a_n(T_i-T_{i-1})}}{a_n}\sigma_n\int_t^{T_{i-1}}e^{-a_n(T_{i-1}-s)}dW_n(s)\right.\\\qquad\left.+\frac{\sigma_n}{a_n}\int_{T_{i-1}}^{T_i}[1-e^{-a_n(T_i-s)}]dW_n(s)\right\}\cdot\\exp\left\{-\frac{1-e^{-a_r(T_i-T_{i-1})}}{a_r}\sigma_r\int_t^{T_{i-1}}e^{-a_r(T_{i-1}-s)}dW_r(s)\right.\\\qquad\left.-\frac{\sigma_r}{a_r}\int_{T_{i-1}}^{T_i}[1-e^{-a_r(T_i-s)}]dW_r(s)\right\}\cdot\\exp\left\{\sigma_I\int_{T_{i-1}}^{T_i}dW_I(s)\right\}\end{array}&\mathcal{F}_t\end{array}\right)\end{aligned} \tag{A.8}$$

where $F(t,T_i,T_{i-1})$ is deterministic function. We already know the answer of

$$\begin{aligned}&E^{Q_n^i}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)\\&=F(t,T_{i-1},T_i)E^{Q_n^i}\left(\begin{array}{c|c}\begin{array}{l}exp\left\{\frac{1-e^{-a_n(T_i-T_{i-1})}}{a_n}\sigma_n\int_t^{T_{i-1}}e^{-a_n(T_{i-1}-s)}dW_n(s)\right.\\\qquad\left.+\frac{\sigma_n}{a_n}\int_{T_{i-1}}^{T_i}[1-e^{-a_n(T_i-s)}]dW_n(s)\right\}\cdot\\exp\left\{-\frac{1-e^{-a_r(T_i-T_{i-1})}}{a_r}\sigma_r\int_t^{T_{i-1}}e^{-a_r(T_{i-1}-s)}dW_r(s)\right.\\\qquad\left.-\frac{\sigma_r}{a_r}\int_{T_{i-1}}^{T_i}[1-e^{-a_r(T_i-s)}]dW_r(s)\right\}\cdot\\exp\left\{\sigma_I\int_{T_{i-1}}^{T_i}dW_I(s)\right\}\end{array}&\mathcal{F}_t\end{array}\right)\end{aligned} \tag{A.9}$$

where Q_n^i means the under the zero coupon $P_n(t,T_i)$ measure. When we do measure change from Q_n^i to Q_r^N, we are really just change the drift term for any process W_x. Consider the drift change based on measure change is

$$Drift\left(W_x;Q_r^N\right)=Drift\left(W_x;Q_n^i\right)+d\left\langle W_x,\ln\left(\frac{P_n\left(\cdot,T_N\right)\gamma\left(\cdot,T_0\!:\!T_n\right)}{P_n\left(\cdot,T_i\right)}\right)\right\rangle_t\Big/dt$$

And W_x is normally distributed under either measure, and Let M is the term within the bracket in equation (A.8) then

$$E^{Q_r^N}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)=F\left(t,T_{i-1},T_i\right)exp\left[E^{Q_r^N}\left(M|\mathcal{F}_t\right)+\frac{1}{2}Var_r^N\left(M|\mathcal{F}_t\right)\right] \tag{A.10}$$

$$E^{Q_n^i}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)=F\left(t,T_{i-1},T_i\right)exp\left[E^{Q_n^i}\left(M|\mathcal{F}_t\right)+\frac{1}{2}Var_n^i\left(M|\mathcal{F}_t\right)\right] \tag{A.11}$$

Consider the measure change does not change the variance $Var_r^N(M|\mathcal{F}_t)=Var_n^i(M|\mathcal{F}_t)$ we get

$$E^{Q_r^N}\left(\frac{I(T_i)}{I(T_{i-1})}\bigg|\mathcal{F}_t\right)=E^{Q_n^i}\left(\frac{I\left(T_i\right)}{I\left(T_{i-1}\right)}\bigg|\mathcal{F}_t\right)exp[E^{Q_r^N}(M|\mathcal{F}_t)-E^{Q_n^i}(M|\mathcal{F}_t)] \tag{A.12}$$

Since we already know the result $EQ_n^i\left(\frac{\mathbf{I(T_i)}}{\mathbf{I(T_{i-1})}}\bigg|\mathcal{F}_t\right)$ then we only need to calculate the drift terms

Consider

$$\begin{aligned}&\left(d\left\langle W_n,ln\left(\frac{p_n\left(\cdot,T_N\right)\gamma\left(\cdot,T_0\!:\!T_N\right)}{p_n\left(\cdot,T_i\right)}\right)\right\rangle t\right)\Big/dt\\&=\begin{cases}-B_r\left(t,T_N\right)\rho_{nr}\sigma_r+B_n\left(t,T_i\right)\sigma_n+\rho_{nI}\sigma_I,t\geq T_0\\-[B_r\left(t,T_N\right)-B_r\left(t,T_0\right)]\rho_{nr}\sigma_r+[B_n\left(t,T_i\right)-B_n\left(t,T_0\right)]\sigma_n,t<T_0\end{cases}\\&\left(d\left\langle W_r,ln\left(\frac{p_n\left(\cdot,T_N\right)\gamma\left(\cdot,T_0\!:\!T_N\right)}{p_n\left(\cdot,T_i\right)}\right)\right\rangle t\right)\Big/dt\\&=\begin{cases}-B_r\left(t,T_N\right)\sigma_r+B_n\left(t,T_i\right)\rho_{nr}\sigma_n+\rho_{rI}\sigma_I,t\geq T_0\\-[B_r\left(t,T_N\right)-B_r\left(t,T_0\right)]\sigma_r+[B_n\left(t,T_i\right)-B_n\left(t,T_0\right)]\rho_{nr}\sigma_n,t<T_0\end{cases}\\&\left(d\left\langle W_r,ln\left(\frac{p_n\left(\cdot,T_N\right)\gamma\left(\cdot,T_0\!:\!T_N\right)}{p_n\left(\cdot,T_i\right)}\right)\right\rangle t\right)\Big/dt\\&=\begin{cases}-B_r\left(t,T_N\right)\rho_{rI}\sigma_r+B_n\left(t,T_i\right)\rho_{nI}\sigma_n+\sigma_I,t\geq T_0\\-[B_r\left(t,T_N\right)-B_r\left(t,T_0\right)]\rho_{rI}\sigma_r+[B_n\left(t,T_i\right)-B_n\left(t,T_0\right)]\rho_{nI}\sigma_n,t<T_0\end{cases}\end{aligned} \tag{A.13}$$

We can add in the convexity adjust term for the YOY swap, which can be found in the literature (Mercurio, 2005):

$$E^{Q_n^i}\left(\frac{I(T_i)}{I(T_{i-1})}\middle|\mathcal{F}_t\right)=\frac{\mathcal{J}(t,T_i)}{\mathcal{J}(t,T_{i-1})}e^{C(t,T_{i-1}:T_i)}$$

with

$$C(t,T_{i-1}:T_i)=\int_t^{T_{i-1}}ds\big[(B_r(s,T_{i-1})\sigma_r-B_r(s,T_i)\sigma_r)(B_r(s,T_{i-1})\sigma_r-B_n(s,T_{i-1})\rho_{nr}\sigma_n)$$
$$-(B_r(s,T_{i-1})\sigma_r-B_r(s,T_i)\sigma_r)\rho_{rI}\sigma_I\big] \tag{A.14}$$

We get the final answer considering $T_0 \le T_{i-1} < T_i \le T_N$:

$$E^{Q_r^N}\left(\frac{I(T_i)}{I(T_{i-1})}\middle|\mathcal{F}_t\right)=\frac{\mathcal{J}(t,T_i)}{\mathcal{J}(t,T_{i-1})}\,exp\left[\int_t^{T_{i-1}}dsX(s,T_{i-1},T_i)+\int_t^{T_0}ds\tilde{Y}(s,T_0,T_{i-1},T_i,T_N)\right.$$
$$\left.+\int_{max(t,T_0)}^{T_{i-1}}dsY(s,T_{i-1},T_i,T_N)+\int_{T_{i-1}}^{T_i}dsZ(s,T_i,T_N)\right] \tag{A.15}$$

with

$$X(s,T_{i-1},T_i)=(B_r(s,T_{i-1})\sigma_r-B_r(s,T_i)\sigma_r)(B_r(s,T_{i-1})\sigma_r-B_n(s,T_{i-1})\rho_{nr}\sigma_n)$$
$$-(B_r(s,T_{i-1})\sigma_r-B_r(s,T_i)\sigma_r)\rho_{rI}\sigma_I$$
$$Y(s,T_{i-1},T_i,T_N)=(B_n(s,T_i)\sigma_n-B_n(s,T_{i-1})\sigma_n)(B_n(s,T_i)\sigma_n-B_r(s,T_N)\rho_{nr}\sigma_r+\rho_{nI}\sigma_I)$$
$$-(B_r(s,T_i)\sigma_r-(B_r(s,T_{i-1})\sigma_r)(B_n(s,T_i)\rho_{nr}\sigma_n-B_r(s,T_N)\sigma_r+\rho_{rI}\sigma_I)$$
$$\tilde{Y}(s,T_0,T_{i-1},T_i,T_N)=(B_n(s,T_i)\sigma_n-B_n(s,T_{i-1})\sigma_n)([B_n(t,T_i)$$
$$-B_n(t,T_0)]\sigma_n-[B_r(t,T_N)-B_r(t,T_0)]\rho_{nr}\sigma_r)$$
$$-(B_r(s,T_i)\sigma_r-B_r(s,T_{i-1})\sigma_r)([B_n(t,T_i)-B_n(t,T_0)]\rho_{nr}\sigma_n-[B_r(t,T_N)$$
$$-B_r(t,T_0)]\sigma_r)$$
$$Z(s,T_i,T_N)=B_n(s,T_i)\sigma_n[B_n(s,T_i)\sigma_n-B_r(s,T_N)\rho_{nr}\sigma_r+\rho_{nI}\sigma_I]$$
$$-B_r(s,T_i)\sigma_r[B_n(s,T_i)\rho_{nr}\sigma_n-B_r(s,T_N)\sigma_r+\rho_{rI}\sigma_I]$$
$$+\sigma_I(B_n(s,T_i)\rho_{nI}\sigma_n-B_r(s,T_N)\rho_{rI}\sigma_r)+\sigma_I^2 \tag{A.16}$$

Appendix B

Expectation of $\frac{I(T_i)}{I(T_{i-1})}$ under Q_r^N measure for market model

Let's define the forward CPI ratio: $Y_i=\frac{I(T_i)}{I(T_{i-1})}$, and assume the dynamics under the forward zero-coupon $P_n(t,T_i)$ measure is

$$\frac{dY_i(t)}{Y_i(t)}=\cdots dt+\sigma_i^Y(t)\,1_{\{t<T_i\}}dW_i \tag{B.1}$$

The CPI index

$$I_N=I(T_N)=\frac{CPI_N}{CPI_0}=\prod_{i=1}^N Y_i \tag{B.2}$$

And the dynamic is

$$dI_N(t)=\cdots dt+I_N(t)\sum_{i=1}^N\sigma_i^Y(t)\,1_{\{t<T_i\}}dW_i(t)=\cdots dt+I_N(t)\sum_{i=\beta(t)}^N\sigma_i^Y(t)dW_i(t) \tag{B.3}$$

$\beta(t)$ is the index which is just later than time t. Under $P_n(t,T_N)$ measure I_N is martingale, so the $\ldots dt$ term is 0.

Let's assume the forward Libor rates is defined as

$$F_i(t)=(P_n(t,T_{i-1})-P_n(t,T_i))/(\tau_iP_n(t,T_i)) \tag{B.4}$$

And the dynamic is

$$\frac{dF_i(t)}{F_i(t)}=\sigma_i^F(t)dW_i^F \tag{B.5}$$

Then under $P_n(t,T_N)$ measure

$$\frac{dY_i(t)}{Y_i(t)}=\cdots dt-\sum_{j=i+1}^N\frac{\tau_jF_j(t)\sigma_j^F(t)}{1+\tau_jF_j(t)}\rho_{ji}^{F,Y}\sigma_i^Y(t)\,1_{\{t<T_i\}}dt+\sigma_i^Y(t)\,1_{\{t<T_i\}}dW_i \tag{B.6}$$

Then under Q_r^N measure

$$\frac{dY_i(t)}{Y_i(t)}=\cdots dt-\sum_{j=i+1}^N\frac{\tau_jF_j(t)\sigma_j^F(t)}{1+\tau_jF_j(t)}\rho_{ji}^{F,Y}\sigma_i^Y(t)1_{\{t<T_i\}}dt+\sum_{j=\beta(t)}^N\sigma_j^Y(t)\sigma_i^Y(t)1_{\{t<T_i\}}\rho_{i,j}dt$$
$$+\sigma_i^Y(t)\,1_{\{t<T_i\}}dw_i \tag{B.7}$$

To derive the $\ldots dt$ term in the above equation, we need to derive the dynamic of

$$\frac{dY_i(t)}{Y_i(t)}=\cdots dt+\sigma_i^Y(t)\,1_{\{t<T_i\}}dW_i \tag{B.8}$$

Consider

$$dI_{i-1}(t)=I_{i-1}(t)\sum_{j=\beta(t)}^{i-1}\sigma_j^Y(t)\,dW_j(t) \tag{B.9}$$

So under $P(0,T_i)$ measure

$$\frac{dI_{i-1}(t)}{I_{i-1}(t)}=-\sum_{j=\beta(t)}^{i-1}\frac{\tau_i\sigma_i^F(t)F_i(t)}{1+\tau_iF_i(t)}\sigma_j^Y(t)\,\rho_{i,j}^{F,Y}dt+\sum_{j=\beta(t)}^{i-1}\sigma_j^Y(t)dW_j(t) \tag{B.10}$$

$$\frac{dY_i(t)}{Y_i(t)}=\frac{d\frac{I_i(t)}{I_{i-1}(t)}}{\frac{I_i(t)}{I_{i-1}(t)}}=\frac{dI_i(t)}{I_i(t)}-\frac{dI_{i-1}(t)}{I_{i-1}(t)}-\frac{dI_i(t)}{I_i(t)}\frac{dI_{i-1}(t)}{I_{i-1}(t)}+\frac{dI_{i-1}(t)}{I_{i-1}(t)}\frac{dI_{i-1}(t)}{I_{i-1}(t)}$$
$$=\frac{\tau_i\sigma_i^F(t)F_i(t)}{1+\tau_iF_i(t)}\sum_{j=\beta(t)}^{i-1}\sigma_j^Y(t)\rho_{i,j}^{F,Y}dt-\sigma_i^Y(t)\sum_{k=\beta(t)}^{i-1}\sigma_k^Y(t)\rho_{i,k}dt+\sigma_i^Y(t)\,dW_i(t) \tag{B.11}$$

Finally we can get the dynamic of $\frac{I(T_i)}{I(T_{i-1})}$ under Q_r^N measure

$$\frac{dY_i(t)}{Y_i(t)}=\frac{\tau_i\sigma_i^F(t)F_i(t)}{1+\tau_iF_i(t)}\sum_{j=\beta(t)}^{i-1}\sigma_j^Y(t)\rho_{i,j}^{F,Y}dt-\sigma_i^Y(t)\sum_{j=i+1}^N\frac{\tau_jF_j(t)\sigma_j^F(t)}{1+\tau_jF_j(t)}\rho_{ji}^{F,Y}1_{\{t<T_i\}}dt$$
$$+\sigma_i^Y(t)\sum_{j=1}^N\sigma_j^Y(t)1_{\{t<T_i\}}\rho_{i,j}dt+\sigma_i^Y(t)\,1_{\{t<T_i\}}dW_i \tag{B.12}$$

From here we can see the measure change just have impact on the expectation but not on variance in market model.

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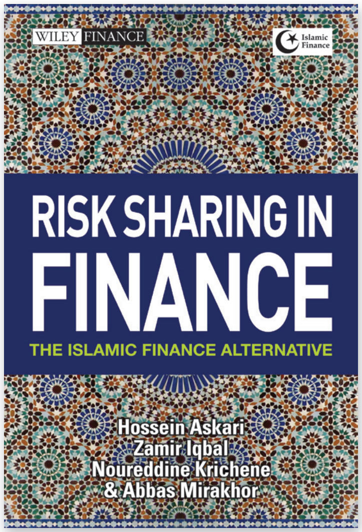

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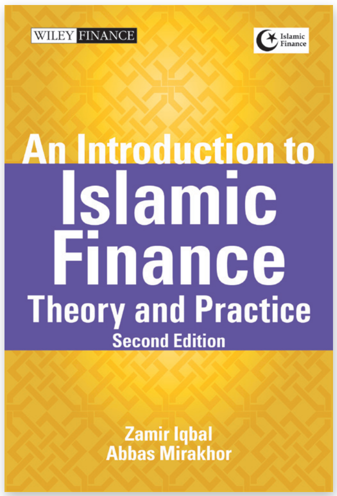

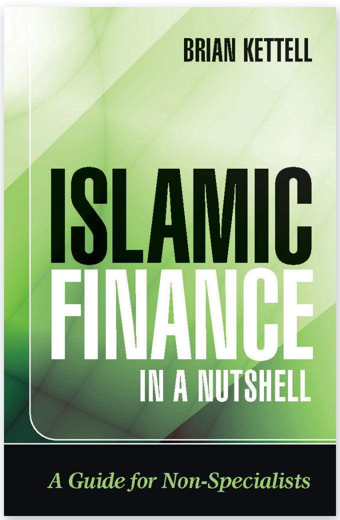

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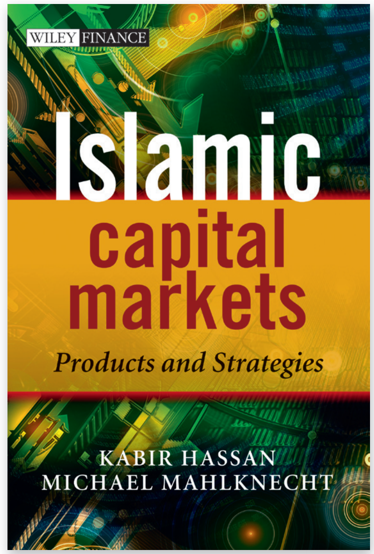

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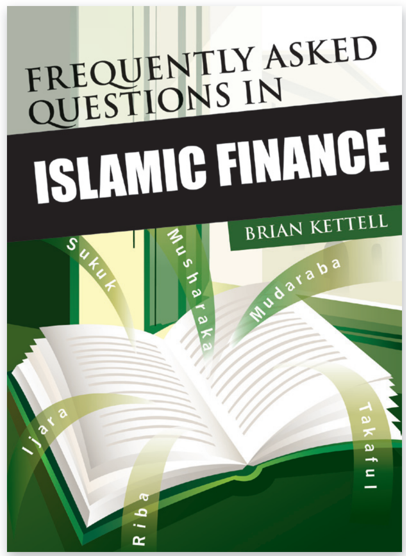

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