

# An Introduction to the Generalized Marginal Risk

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## Abstract

The concept of generalized marginal risk is presented. This new metric allows a portfolio manager to measure the portfolio risk sensitivity under a broad range of allocations scenarios.

## Keywords

marginal risk, generalized marginal risk, portfolio risk, value-at-risk

## Introduction

Portfolio risk management requires assessing the aggregated risk of a portfolio, such as the value-at-risk (VaR). VaR is the value of the portfolio return such that lower returns will only occur with at most a preset probability level. In order to manage the risks of a portfolio effectively, the risk impact of new trades and/or reallocations within the portfolio must be assessed. The aim of the portfolio risk analysis is to gain insight through the sensitivity of the aggregated risk with respect to the portfolio holdings (see, e.g., Litterman, 1996; 1997a,b).

The *marginal risk* aims at measuring how investment decisions affect the risk profile of the portfolio. Mathematically, this is simply the gradient of the portfolio risk measure with respect to the allocation weights. It is defined as the linear approximation of the change in the portfolio risk when a position is altered while all other positions remain the same. Therefore, it is applicable when a position is levered or when a position is reduced and the proceeds are put in the cash account of the portfolio. However, it leads to flawed results when the adjustments are carried out through capital in- or outflows in the portfolio as well as reallocations within the portfolio, for instance. This is obviously caused by the change in all of the relative positions in the portfolio when there are capital adjustments.

This note introduces an extension of the marginal risk recently proposed by Keel and Ardia (2011) and named *generalized marginal risk*. As for the traditional marginal risk, the new concept allows a portfolio manager to measure the sensitivity of the portfolio to new marginal allocations. However, it ensures that potential effects on the other positions are correctly taken into account. This therefore helps analyzing the risk impact under more general and realistic scenarios.

## Marginal and generalized marginal risk

Let us assume that the portfolio is composed of  $n$  assets whose allocation weights are collected into the vector  $\mathbf{w} \equiv (w_1, \dots, w_n)'$ . We denote the risk measure of the portfolio return by  $\rho(\mathbf{w})$  and assume that  $\rho(\mathbf{w})$  is at least once differentiable.

For the risk measure  $\rho$ , the marginal risk of the  $i$ th asset in the portfolio is defined as the change in the portfolio risk measure for an infinitesimal change in the allocation to the  $i$ th asset:

$$\rho_i^m(\mathbf{w}) \equiv \frac{\partial}{\partial w_i} \rho(\mathbf{w}).$$

For convenience, the  $n$  marginal risks of the portfolio are collected into the vector  $\rho^m \equiv (\rho_1^m, \dots, \rho_n^m)'$ ;  $\rho^m$  is the gradient of  $\rho(\mathbf{w})$ . The marginal risks can be computed explicitly in some cases, otherwise numerical or Monte Carlo methods are required (see, e.g., Hallerbach, 2003).

We emphasize two limitations of the marginal risk. First, the concept is based on a marginal argument, and this must be kept in mind when interpreting the measures. Second, the marginal risk is the linear approximation of the risk impact of *leveraging* the corresponding position in the portfolio. Indeed, the gradient is the linear approximation of the change in the portfolio risk when *a position is altered while all others remain constant*. This point is often neglected in practice; this leads to false conclusions in a sensitivity analysis, where capital might be shifted in the portfolio but the sensitivity measure relies on the leveraged scenario. The differences can be substantial, as illustrated in our empirical analysis.

To overcome the latter deficiency, Keel and Ardia (2011) introduced the generalized marginal risk. This extension of the traditional marginal risk, based on the directional derivative of the portfolio risk, allows to consider allocations' scenarios where the change in a position results in the change of other positions as well. This is typically the case when there are capital in- and outflows in the portfolio since all percentage allocations change in this setting. The generalized marginal risk of the  $i$ th asset in the portfolio can be expressed as

$$\rho_i^{gm}(\mathbf{w}) \equiv \rho^m(\mathbf{w})' \mathbf{a}_i(\mathbf{w}),$$

where  $\mathbf{a}_i(\mathbf{w}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$  describes how an additional investment in the  $i$ th position affects the overall positions; it can be interpreted as an allocation

scheme (examples are given below). Once the marginal risks of the positions have been computed, a portfolio manager can run a generalized sensitivity analysis in a straightforward manner. Note that the global sensitivity indices or derivative-based sensitivity measures presented in Sobol and Kucherenko (2010) could be used to measure the sensitivity of the portfolio risk with respect to subsets of weights. However, these approaches are computationally demanding and do not account for the specific portfolio's reallocation scheme chosen by the portfolio manager.

Three examples of reallocation schemes are presented in Keel and Ardia (2011). The first assumes that an investor has an additional amount to invest in the portfolio. If the investor adds this capacity to the  $i$ th asset, the allocation scheme reads  $\mathbf{a}_i(\mathbf{w}) \equiv (\mathbf{e}_i - \mathbf{w})$ , where  $\mathbf{e}_i$  denotes the  $i$ th column of the identity matrix. Another example arises when a portfolio manager increases the  $i$ th position through an equal reduction of all other positions. In this case  $\mathbf{a}_i$  is a vector whose components are all equal to  $-1/(n - 1)$  except the  $i$ th position which equals one. Finally, when the increase in the  $i$ th position is financed through leverage,  $\mathbf{a}_i$  becomes  $\mathbf{e}_i$ , and the generalized marginal risk equals the traditional metric.

### Illustration

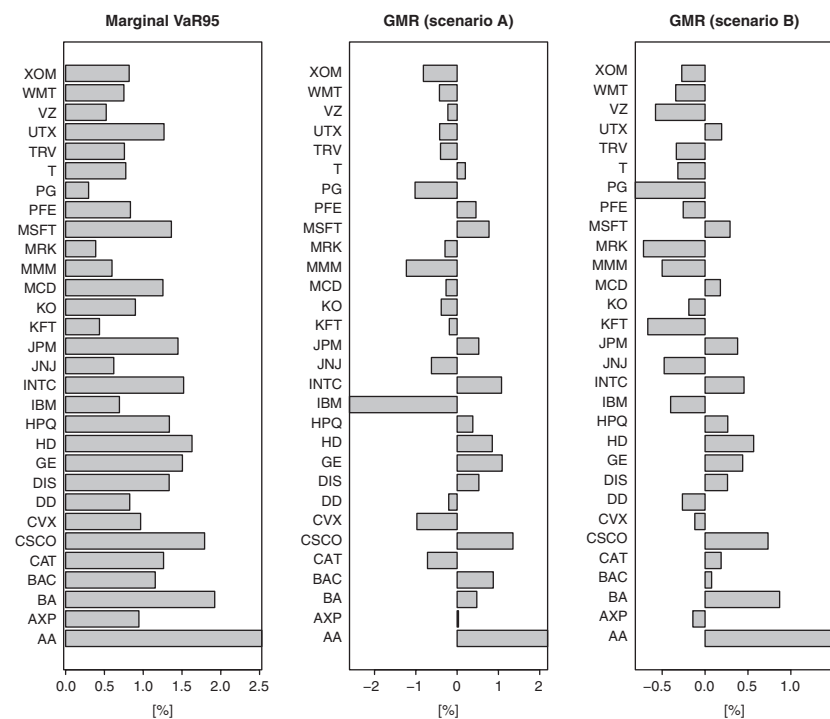
We consider a portfolio of thirty equities whose allocations are chosen to replicate the Dow Jones Index (DJIA) as of January 28, 2011. The value-at-risk is used as the portfolio risk measure. The VaR at the 95th risk level (VaR95) of the portfolio and the marginal VaR95 are estimated using the modified VaR approach of Boudt et al. (2008), which account for coskewness and cokurtosis in the assets' returns. For the estimation, we use monthly closing prices for the DJIA constituents ranging from January 2000 to December 2010 and rely on robust estimators of the location and covariance matrix of the arithmetic returns. All computations are performed in the R statistical computing language (R Development Core Team, 2011); the computer code is available from the authors upon request. The estimated portfolio VaR95 is 4.57%.

The left-hand side of Figure 1 reports the relative marginal VaR95 for the assets in the portfolio. Relative measures are obtained by dividing the sensitivity measures by the portfolio VaR95. From the marginal VaR95 numbers, the portfolio manager can infer that the portfolio risk will increase if any position is levered. Conversely, if the portfolio manager divests from a position and puts the proceeds in the cash account, the portfolio risk is reduced. If the investor wants to decrease the portfolio VaR95, the marginal risk suggests to reduce the allocations in AA first.

Let us now consider the generalized marginal risk as an additional decision tool for the portfolio manager. First, we consider the scenario A, where there are capital inflows in the portfolio. In this case, the sensitivity analysis (Figure 1, middle) indicates that additional capital invested in IBM will have the most effect on decreasing the risk in the new portfolio; an additional one-percent allocation in IBM would reduce the portfolio risk by 2.60%. Second, we consider the scenario B, where a position is increased by an equal decrease in all other positions. Under this scenario, the sensitivity analysis (Figure 1, right) indicates that reallocating capital to Proctor and Gamble will decrease the portfolio VaR95 the most.

As we can see, depending on adjustment scheme chosen by the portfolio manager, the traditional and the new sensitivity metrics can differ substan-

**Figure 1: Left: relative marginal VaR95 for the assets in the DJIA portfolio. Middle: relative generalized marginal VaR95 in the case where additional capital is brought in the portfolio and invested in one position (e.g., the bar in the chart corresponding to AA gives the relative impact on the portfolio VaR95 of an additional allocation to AA). Right: relative generalized marginal VaR95 in the case where the increase in one position is financed by an equal reduction in all other positions. Relative measures are obtained by dividing the sensitivity measures by the portfolio VaR95. Therefore, a positive (negative) value of x percent indicates an increase (decrease) of x percent of the current portfolio risk after an additional one-percent allocation in the corresponding asset.**



tially. This underlines the importance of accurately modeling the way the portfolio is adjusted and choosing the appropriate sensitivity measure.

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## REFERENCES

Boudt, K., Peterson, B. G., Croux, C., 2008. Estimation and decomposition of downside risk for portfolios with non-normal returns. *Journal of Risk* 11(2), 79–103.

Hallerbach, W. G., 2003. Decomposing portfolio Value-at-Risk: A general analysis. *Journal of Risk* 5(2), 1–18.

Keel, S., Ardia, D., 2011. Generalized marginal risk. *Journal of Asset Management* 12, 123–131.

Litterman, R., 1996. Hot spots and hedges. *Journal of Portfolio Management* 22, 52–75.

Litterman, R., 1997a. Hot spots and hedges (i). *Risk* 10(3), 42–45.

Litterman, R., 1997b. Hot spots and hedges (ii). *Risk* 10(5), 38–42.

R Development Core Team, 2011. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. <http://www.R-project.org>.

Sobol, I. M., Kucherenko, S., 2010. A new derivative based importance criterion for groups of variables and its link with the global sensitivity indices. *Computer Physics Communications* 181(7), 1212–1217.

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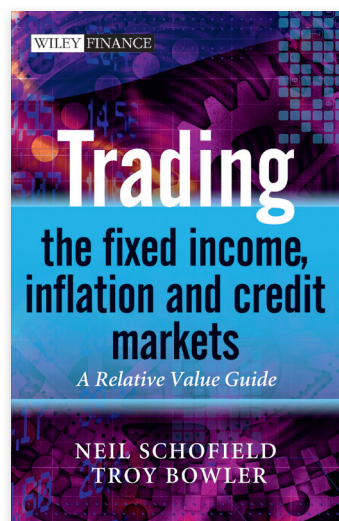
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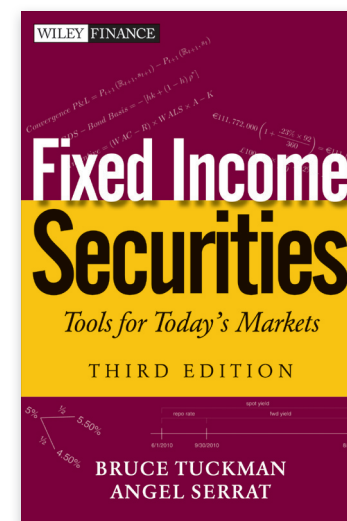
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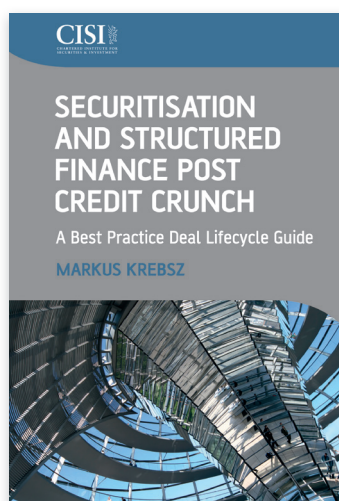
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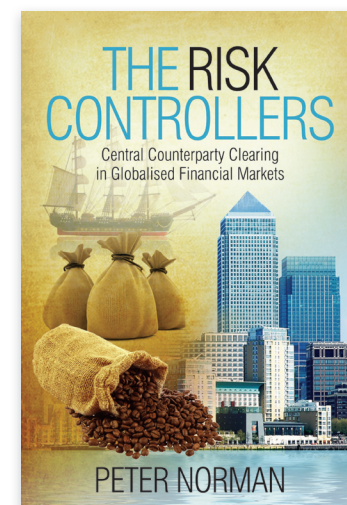
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