The Heston–Hull–White Model Part II: Numerics and Examples

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1 Introduction

This is the second article in a series of three on financial modeling using the Heston-Hull-White model. The aim of this series is to show the full life cycle of model development and implementation.

This article reviews the methods for pricing European options and suggests fast numerical methods for the pricing procedure. In our case we consider the pricing of calls and puts as well as caps and floors using the Carr and Madan (1999) framework. There have been many frameworks suggested in the literature, see for instance Fang and Osterlee (2008), Attari (2004), Lord (2008), Lewis (2000). We review the method of computing stable prices by optimal dampening, see Lord and Kahl (2006). The suggested instruments are liquid traded options and can be used in a calibration procedure. This is a backward problem. We try to infer model parameters from market quotes. Backward problems are known to be unstable but these methods are frequently used to estimate model parameters. Therefore, we show which algorithms may be used to tackle this problem.

Finally, we proceed by considering discretization methods which can then be applied in Monte Carlo simulation to price exotic path dependent payoffs. For the model under consideration we propose to implement a stable numerical scheme based on Andersen (2006). There are several new methods for discretizing a square root dynamic, see for instance Glasserman and Kim (2008), Van Haastrecht (2008) or Chan and Joshi (2010).

For all the algorithms we provide source code which can be obtained by writing to joerg.kienitz(at)gmx.de.

In the third and final article of this series we describe a software framework which implements the discussed methods and can be used to apply the Heston-Hull-White model to financial problems. We show how to combine the discussed numerical methods to create a self-contained pricing, hedging and calibration framework.

2 Numerical Methods for Calibration - Fast Fourier Transform

In this section we compute the characteristic function we use for pricing European options. We review a standard algorithm which we used for pricing and calibration purposes. However, we mention some known problems and show how to overcome such problems. This leads to a stable pricing and calibration procedure.

Even though the characteristic function of the Heston–Hull–White model - derived in the first article of the series - is similar to the Heston characteristic function, the additional factor requires a significant additional amount of computing time. For example the term

$$\mathbb{E}[R_{t,T} \mid \mathcal{F}_t] = B(t,T)(r_t - \psi_t) + \log\left(\frac{P^M(0,t)}{P^M(0,T)}\right) + \frac{1}{2}(V(0,T) - V(0,t))$$

appearing in calculations includes the market discount curve. Low performance computations however are a bottleneck for a calibration procedure since many computations of the price are necessary.

2.1 The Carr Madan method

Despite the fact that at the time of writing many different methods for computing option prices from the characteristic function exist, for instance Fang and Osterlee (2008) or Lewis (2000), we apply the method introduced in Carr and Madan (1999). It offers the possibility of high performance computations. The idea is to compute the prices for a given maturity but across the whole strike range using a single Fast Fourier Transform. That is of particular importance for pricing with smiles and skews.

By a change of numeraire argument, see for instance Brigo and Mercurio (2006), section 2, the price of a call option is given by

$$C_{\mathcal{F}_t}(T,K) = P(t,T)\mathbb{E}^T\Big((S_T - K)^+ \mid \mathcal{F}_t\Big) = P(t,T)C_T(k)$$

$$C_T(k) = \int_k^\infty (\exp(s) - \exp(k)) q_T(s) ds \tag{1}$$

with the log strike price, $k = \log K$. The expectation is taken with respect to the T–forward measure Q^T. Thus q_T denotes the density of S_T with respect to Q^T conditioned on $\mathcal{F}_t$. In order to apply the FFT the non-discounted price (1) has to be square integrable as a function of k. Since a zero strike implies that the payoff is equal to the forward we see that C_T does not tend to zero as $k \to -\infty$. To obtain square integrability Carr and Madan propose to introduce a factor $\exp(\alpha k)$, $\alpha > 0$, and take the Fourier transform with respect to $c_T(k) = \exp(\alpha k) C_T(k)$. One may expect that this function is square integrable for a range of positive values of α. The price below does not analytically depend on α. However, the numerical error that occurs when approximating the continuous Fourier transform by the discrete Fast Fourier Transform relies heavily on the choice of α. As an inverse transform (1) is given by

$$C_T(k) = \frac{\exp(-\alpha k)}{2\pi}\int_{-\infty}^{\infty}\exp(-iuk)\psi_T(u)du = \frac{\exp(-\alpha k)}{\pi}\int_{0}^{\infty}\exp(-iuk)\psi_T(u)du$$

with $\psi_T(u) = \int_{-\infty}^{\infty}\exp(iuk)c_T(k)dk$ denoting the Fourier transform of c_T. The function ψ_T is related to the T−forward characteristic function ϕ_T of S_T:$|$:$\mathcal{F}_t$ via the formula:

$$\psi_T(u) = \frac{\phi_T(u-(\alpha+1)i)}{\alpha^2+\alpha-u^2+i(2\alpha+1)u} \tag{2}$$

Note that α > 0 is also a technical requirement since the FFT algorithm evaluates the integrand in zero. The T−forward characteristic function ϕ_T can be expressed in terms of the ordinary characteristic function ϕ as follows. By means of Bayes' rule we have

$$\begin{aligned}\phi_T(u) &= \mathbb{E}^T(\exp(iuX_T) \mid \mathcal{F}_t)\\ &= \frac{\mathbb{E}(\exp(iuX_T)\frac{dQ^T}{dQ} \mid \mathcal{F}_t)}{\mathbb{E}(\frac{dQ^T}{dQ} \mid \mathcal{F}_t)}.\end{aligned}$$

For the Radon−Nikodym derivative $\frac{dQ^T}{dQ}$ we have

$$\frac{dQ^T}{dQ} = \frac{\exp(-\int_0^T r_s ds)}{P(0,T)} = \frac{D(0,T)}{P(0,T)} = \frac{D(0,t)D(t,T)}{P(0,T)}.$$

Thus, for the Heston−Hull−White model we have

$$\begin{aligned}P(t,T)\phi_T(u) &= \mathbb{E}[D(t,T)\exp(iuX_T) \mid \mathcal{F}_t]\\ &= \mathbb{E}[\exp(-R_{t,T}+iuX_T) \mid \mathcal{F}_t]\\ &= \mathbb{E}[\exp(R_{t,T}i(u+i)+X_{HT}iu) \mid \mathcal{F}_t]\\ &= \phi_R(u+i,t,T)\phi_H(u,t,T)\\ &=: \tilde{\phi}_{\mathrm{HHW}}(u,t,T).\end{aligned} \tag{3}$$

In the above equation, $R()$ again denotes the integrated short rate. We see that using the T−forward measure is very convenient. All discounting issues are done shifting the integration line of the integrated short rate characteristic function by one imaginary unit. Now, set ψ as in (2) but substituting ϕ_T with $\tilde{\phi}$HHW. Then, what we implement as a first step is the following formula

$$C_{\mathcal{F}_t}(T,K) = \frac{\exp(-\alpha k)}{\pi}\int_0^{\infty}\exp(-iuk)\psi_T(u)du. \tag{4}$$

The following piece of code gives the implementation:

```
void FFT::run()
{
    double SwitchSign, DeltaJ0;
        complex<double> IntegrandValue;  //integrand value
        complex<double> EvaluationPoint(0.0, 0.0);
          //current evaluation point
    for(unsigned int j = 0; j < N; j++)
         //initialize array with integrand values
    {
          SwitchSign = (j % 2) ? (-1.0) : 1.0;
             //minus one to the power of j
          DeltaJ0 = j ? 0.0 : 1.0;     //Kronecker-Delta
          IntegrandValue = itsOption->
                              Integrand(EvaluationPoint)
          * SwitchSign * eta;
          IntegrandValue *= (1.0 / 3.0) * (3.0 -
              SwitchSign - DeltaJ0); // correction term

          itsOption->PricingArray[2*j]
              = IntegrandValue.real();
          itsOption->PricingArray[2*j+1]
              = IntegrandValue.imag();
          EvaluationPoint += eta;
    }
    // Fast Fourier Transform, see GNU Scientific
       Library manual
    gsl_fft_complex_radix2_forward
      (itsOption->PricingArray, 1, N);
}
```

There are two points left to remark. The choice of an optimal value of the parameter α and how we approximate the continuous Fourier transform in (4) as precisely as possible using the FFT algorithm. Finally, we show how to price caplets and floorlets in the Hull−White model. Caplets, **Cpl**, are equivalent to zero bond puts, **ZBP**, via

$$\mathbf{Cpl}(t,t_1,t_2,N,X) = N'\mathbf{ZBP}(t,t_1,t_2,X'). \tag{5}$$

Here t denotes the present time (which will be zero in the implementation), t_1 and t_2 are starting and ending time of the caplet, N is the nominal value and X is the strike interest rate. The modified values X and N are given by

$$X' = \frac{1}{1+X(t_2-t_1)} \quad \text{and} \quad N' = N(1+X(t_2-t_1)). \tag{6}$$

Analogously, we have the formula

$$\mathbf{Fll}(t,t_1,t_2,N,X) = N'\mathbf{ZBC}(t,t_1,t_2,X') \tag{7}$$

which relates floorlets to zero bond calls. This should clarify the source code snippet that computes the caplet or floorlet price.

```
void HullWhite::CapletFloorlet::run()
{
  itsCorP = -itsCorF + 1;
  itsT = itsStartT;
  itsBondMaturity = itsEndT;
  itsN = itsCapletFloorletN*(1.0 + itsCapletFloorletK*
                              (itsEndT - itsStartT));
  itsK = 1.0 / (1.0 + itsCapletFloorletK *
                (itsEndT - itsStartT));
  HullWhite::ZeroBondOption::run();
}
```

The function call in the last command line will compute the zero bond option price using a Black–like formula. The availability of such an analytical formula renders the Hull–White calibration very fast with fast meaning almost instantaneously. The final function call is implemented as follows:

```
void HullWhite::ZeroBondOption::run()
{
      double P_T = getZeroBondValue(itsT);
      double P_S = getZeroBondValue(itsBondMaturity);
      double sigma_p = itsEta * sqrt((1.0 - exp
       (-2.0 * itsLambda * itsT))
           / (2.0 * itsLambda)) * factorB(itsT, itsBond
                    Maturity);
      double h = (1.0 / sigma_p) * log (P_S / (P_T *
              itsK)) + sigma_p / 2.0;

      if(itsCorP == 1)
             itsPrice = itsN * (P_S * CumulativeNormal(h)
        - itsK * P_T * CumulativeNormal(h - sigma_p));
      else
             itsPrice = itsN * (itsK * P_T *
              CumulativeNormal(-h + sigma_p)
        - P_S * CumulativeNormal(-h)); //equation 3.41

      setUp2D(true);
}
```

2.2 The optimal damping factor

The auxiliary parameter α has to be chosen within a certain range $[\alpha_{min,} \alpha_{max}]$. This is to guarantee square integrability. Within the allowed range there are inappropriate choices of α which cause the integrand to be highly oscillating and can lead to significant mispricing of standard options. For instance it can lead to negative prices.

Lord and Kahl (2006) analyzed the problem of computing an optimal dampening parameter. Ideally, an optimization algorithm for α minimizes the total variation of the integrand ψ. This would require more function evaluations than the original problem and thus is mainly useless in practice. Therefore, Lord and Kahl (2006) suggest to minimize the maximal value of the integrand which occurs in zero. They consider

$$\alpha^* = \underset{\alpha \in (0, \alpha_{\max})}{\operatorname{argmin}} |\exp(-\alpha k)\psi(0)|. \tag{8}$$

In general α_{min} will be negative and we assume $\alpha > 0$ there is only one local minimum of this function within the positive reals. So as long as the initial guess the minimization routine starts from is not too large, the minimum will be found without any problems involving the maximal allowed alpha α_{max} which in general is hard to determine.

The following piece of code implements the optimization suggested in Lord and Kahl (2006). For carrying out the optimization we use the LBFGS optimizer implemented in Bochkanov (2010). This library can be downloaded, see references.

```
void AlphaOptimization::run()
// Computes the optimal alpha
{
       const double epsG = 0.001;  // constant for optimizer
       const double epsF = 0.0;    // constant for optimizer
       const double epsX = 0.0;    // constant for optimizer
       const int MaxIts = 100;    // constant for optimizer
       ap::real_1d_array x;    // alglib real array
       x.setbounds(1,1);             // set dimensionality
       x(1) = 1.5;
       ap::integer_1d_array nbd;// alglib int array
       nbd.setbounds(1,1);
       nbd(1) = 1;     // range for alpha (only lower bound)
       ap::real_1d_array lbound;// alglib real array
       lbound.setbounds(1,1);
       lbound(1) = 0.01;      //lower bound of allowed range
       ap::real_1d_array ubound;// alglib real array
       ubound.setbounds(1,1);
       ubound(1) = 5.0;             //upper bound of allowed
               range. only needed if nbd = 2.
       int info;
   // the lbfgs optimizer
       lbfgsbminimize(MinFunc, itsOption, 1, 1, x, epsG,
               epsF, epsX,
             MaxIts, nbd, lbound, ubound, info);
       itsOption->setAlpha(x(1));     //optimal alpha is
               found
}
```

2.3 FFT approximation of continuous Fourier transforms

The precompiled FFT algorithm we wish to apply is taken from the GNU Scientific Library, GSL (2010). It computes the finite sum

$$x_k = \sum_{j=0}^{N-1} z_j \exp\left(-\frac{2\pi}{N} ijk\right) \tag{9}$$

with k ranging from 1 to N. From (9) we expect the computation time to correspond to an ordinary square matrix multiplication, i.e. to be of order $O(N^2)$. The FFT algorithm improves this computation time to order N times the sum of the prime factors of N. Thus N is chosen to be a power of two resulting in an $O(N \log N)$ performance which is a very considerable improvement.

It is implemented using the function

```
int gsl_fft_complex_radix2_forward (gsl_complex_packed_
                              array data,
                              const size_t stride,
                              const size_t n);
```

of the GSL (2010) library. The latter is available for free.

The integral in (4) has the naive approximation

$$\sum_{j=0}^{N-1} \exp(-ij\eta k)\psi(j\eta)\eta \tag{10}$$

as a Riemann sum from zero to $N\eta$. To compute one single call price we normalize the strike K to one. The spot price has to be divided by the strike and then it is inserted into the model equations. The latter has the advantage that (8) simplifies to

$$\alpha^* = \underset{\alpha\in(0,\alpha_{\max})}{\operatorname{argmin}} |\psi(0)|. \tag{11}$$

For the calibration we write spot and strike as multiples of the forward. Thus, the call prices which can be deduced from one single FFT will range around the at-the-money level just like the market prices do. With these scalings the grid of k should be centered in zero and is of fineness

$$\lambda = \frac{2\pi}{N\eta}. \tag{12}$$

The grid thus lies within $[-b,b]$ with $b = \frac{\lambda N}{2} = \frac{\pi}{\eta}$. Substituting $k \mapsto -b + u\lambda$ with $u = 0,\ldots,N-1$ we obtain

$$\sum_{j=0}^{N-1} \exp\left(-\frac{2\pi}{N} iju\right)(-1)^j \psi(j\eta)\eta\ . \tag{13}$$

For accuracy both a small η (reducing discretization errors) and a small λ (reducing interpolation errors) are desirable. But a simultaneous diminishment is prevented by the relation

$$\lambda\eta = \frac{2\pi}{N}$$

because N is not only limited by computation time but also by the maximal possible power of two integer on the working machine. For this reason we apply the Simpson's rule with a correction factor as in Carr and Madan (1999),

$$\sum_{j=0}^{N-1} \exp\left(-\frac{2\pi}{N} iju\right)(-1)^j \psi(j\eta)\frac{\eta}{3}(3 + (-1)^{j+1} - \delta_{j0}). \tag{14}$$

Note that especially for prices of in-the-money options linear interpolation between two known prices is not completely wrong. That is why a bit contrary to the values recommended by Carr and Madan (1999) we experienced that choosing η according to the rule $\eta = \frac{2^{14}}{N}\cdot 10^{-3}$ works fine in practice.

3 Monte Carlo Heston–Hull–White pricing

With regard to the model dynamic we observe that each time step on the path of the asset price has to be preceded by a time step resulting from simulating the evolution of the variance. To obtain an acceptable computation time we wish to implement a discretization scheme with very small bias. To this end we have modified the Quadratic Exponential scheme introduced in Andersen (2006). Source code in C++ has been presented in Duffy and Kienitz (2009) and a version in Visual Basic can be found in Staunton (2007).

3.1 The drifted Quadratic Exponential Scheme

For the Heston model a discretization scheme has been proposed by Andersen (2006). It discretizes the non-central chi square distributed variance process by a dirac delta with an exponential tail for small values of the variance and by a squared gaussian random variable for larger values of the variance. Again, it is the simplicity of the Gaussian distribution of the short rate Ornstein Uhlenbeck process that allows us to extend the discretization scheme of Andersen (2006) in a bias-free way to include stochastic interest rates. This is possible by sampling just one additional random number for each path since the integrated short rate at maturity is independent and normally distributed with the known parameters. By the Hull-White decomposition the resulting drift is added to the zero short rate final value of the Heston process. We have

$$X_T = R_{0,T} + \hat{X}_T \tag{15}$$

where $\hat{X}_T$ is the ordinary QE-scheme as in Andersen (2006), p. 19, equation (33).

There is the common problem with Monte Carlo methods that both variance and short rate can become negative during the simulation process. In case of the Heston variance this is only due to the discretization since a CIR process has almost sure non zero paths. To compute the asset price the square root of the variance is taken. Thus, negative values cause errors. The QE-scheme overcomes this issue. In case of negative short rates resulting from the Hull-White model the discretization is not problematic since no range of definition is exceeded. It can indeed occur that after a time step some negative drift arises. This is a general feature of the Hull-White model and not of the Monte Carlo simulation.

For the implementation of the drifted QE scheme we choose the following constants with respect to the model parameters:

```
double gamma1 = 0.5;     // see Andersen, section 4.2
double gamma2 = 1.0 - gamma1;
itsConst[0] = -((rho * kappa * vLong) / omega) *
 itsDeltaT;                                          //K_0
itsConst[1] = gamma1 * itsDeltaT * (kappa * rho /
        omega - 0.5) - rho / omega;    //K_1
itsConst[2] = gamma2 * itsDeltaT * (kappa * rho /
        omega - 0.5) + rho / omega;    //K_2
itsConst[3] = gamma1 * itsDeltaT * (1 - rho * rho);
                                            //K_3
itsConst[4] = gamma2 * itsDeltaT * (1 - rho * rho);
```

Using these constants we implement the drifted QE scheme. To this end we have implemented a function which takes as an input two arrays of uniformly distributed random variables WI1 and WI2 which are then transformed to the desired variates. The following piece of code implements this method:

```
void HestonHullWhite::MonteCarlo::QE(double WI1, double
WI2)
            {
                  // Calculate the solution at time level
                   n+1 in terms of solution at time level n
                  double result[3];
                  // N.B. This assumes that WI1 and WI2
                    are uniformly distributed
                  // 1.Variance - see Andersen's paper
                  double Psi;                // switching
                    parameter
```

```
double Psi_C = 1.5;    // switching
  parameter
//Parameters used in for loop
double a, b2, p, beta, m, s2, c4;

//This becomes a Gaussian variate in the
  for loop.
double GV = 0.0;
double Coeff1 = exp(-
  itsKappa*itsDeltaT);
double Coeff2 = 1.0 - Coeff1;
double Coeff3 = itsOmega * itsOmega /
  itsKappa;
m = itsVLong + (itsVec[2] - itsVLong) *
  Coeff1;
s2 = itsVec[2] * Coeff1 * Coeff2 * Coeff3
      + 0.5 * itsVLong * Coeff2 * Coeff2
         * Coeff3;
// martingale correction is required?
  Store Psi for each time step
Psi = s2 / (m * m);
//Switching Parameter Psi_C
if (Psi <= Psi_C)
{
      //V[t+Delta] approximated as non
         central chi-square
      //with one degree of freedom
      c4 = 2.0 / Psi;
      b2 = max(c4 - 1.0 + sqrt(c4*(c4 -
         1.0)),0.0);  //eq. (27)
      a = m / (1.0 + b2);
             //eq. (28)

      //If martingale correction is
         required store a and b2
      //for each time step since it is
         needed for calculation of S(t).
     GV = (sqrt(b2) + InverseCumulative
         Normal(WI1));
      result[2] = a*(GV*GV);
        //step c
}
else
{
      //Approximation Density with
         Dirac mass and exponential tail
      p = (Psi - 1.0) / (Psi + 1.0);  //
         eq. (29)
      beta = (1-p) / m;      // eq. (30)
      //If martingale correction is
       required store p and beta at each
      //time step since it is required
         to calculate S(t)
      // eq. (25)
            if (WI1 <= p)
                  result[2] = 0.0;
            else
                  result[2] = log((1.0 - p) /
                     (1.0 - WI1)) / beta;
      }
      result[1] =  itsConst[0] + itsConst[1] *
        itsVec[2] + itsConst[2] * result[2] +
sqrt(itsConst[3] * itsVec[2]
+ itsConst[4] * result[2]) * InverseCumulative
        Normal(WI2);
      itsVec[1] += result[1];
      itsVec[2] = result[2];
}
```

The main part of the code is the implementation of the switching rule as suggested in the original work by A06. The final value `result[1]` is a predictor corrector scheme applied to the previously generated values.

Finally, the Monte Carlo estimator can be implemented. It calls the QE scheme and it is given by:

```
void HestonHullWhite::MonteCarlo::run()
{
      setSeed();
      //path variables
      double SumCallValue = 0.0;
      double SumPutValue = 0.0;
      double SumIntegralR = 0.0;
      double SumDiscountFactor = 0.0;
      double WI1, WI2;         //uniformly distributed
                                  randoms
      int seed = (int)time(NULL);// use system time to
                                  produce random seed
      MersenneTwister2 mt;      // init Mersenne Twister
      mt.SetSeed(seed);        // set the seed
      for(int i = 1; i <= itsNSIM; ++i)     // loop over
                                            simulations
      {
            // variables containing current values
            itsVec[1] = log(itsS);          // log of
                                            stock price
            itsVec[2] = itsVInst;           // variance
            for(int nt = 1; nt <= itsNT; ++nt) // loop over
                                                time steps
            {// get random numbers
                  WI1 = mt.GetRandomNumber();
                  WI2 = mt.GetRandomNumber();
                  QE(WI1, WI2);                // Call QE
                                                 scheme
            }
            // Hull White integrated short rate process is
                    normally distributed
            double Z = InverseCumulativeNormal(mt.
                    GetRandomNumber());
```

```
            double IntR = getMeanOfIntegral(itsRInst,
                    0.0, itsT)
         + sqrt(getVarianceOfIntegral(0.0, itsT)) * Z;
      // store integrated short rate process as well as
          discount factor
            SumCallValue += exp(-IntR) * max(0.0,
             exp(itsVec[1] + IntR) - itsK);
            SumPutValue += exp(-IntR) * max(0.0, itsK -
             exp(itsVec[1] + IntR));
            SumIntegralR += IntR;
            SumDiscountFactor += exp(-IntR);
      }
      //compute arithmetic means as an estimator of
             expectations
      itsIntegralR = SumIntegralR / itsNSIM;
      itsDiscountFactor = SumDiscountFactor / itsNSIM;
      itsCall = SumCallValue / itsNSIM;
      itsPut =  SumPutValue / itsNSIM;
      itsPrice = itsCorP ? itsCall : itsPut;

      setUp2D(true);
}
```

The code may be modified such that the `seed` is an argument. Then, using the same seed anytime the simulation starts the random generator produces the same stream of random numbers. We have chosen the system clock to generate a seed. Therefore, for each run you get different random numbers and therefore different realisations for the Monte Carlo estimator.

This presented piece of code is used to price European call and put options. For valuing path dependent options we have to modify the Monte Carlo simulation by storing all the relevant points in the path `itsVec[]`. Then, the payoff corresponding to the path dependent option can be evaluated.

3.2 Numerical Test

We apply the methods to the following sets of parameters:

The following results on the one hand show that the prices calculated via FFT and MC agree and therefore we can be sure that both methods are implemented correctly. On the other hand we observe that the performance of the MC is worse for long maturities. Table 2 summarizes our findings:

To get a better view on the results we have plotted the outcomes, see figure 1.

The correct mean of discount factors

When the number of path simulations is exceeded we just set the element variable `itsIntegralR` equal to `SumIntegralR` divided by this number

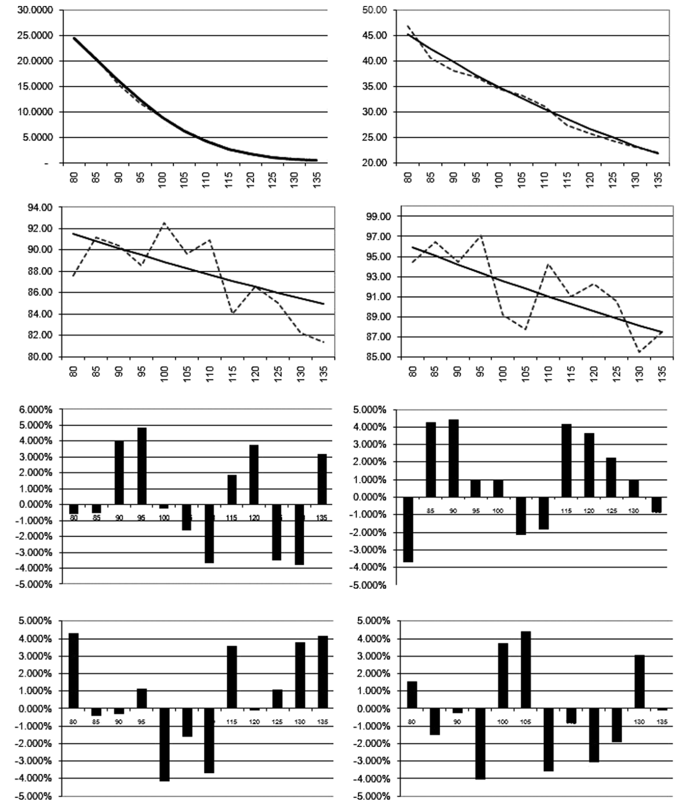
Table 1: Scenarios for testing the FFT and the MC implementations.

T	S_0	λ	σ	$\bar{v}$	v_0	κ	ρ	ω
1	100	0.4	0.2	0.02	0.03	0.2	–0.6	0.5
5	105	0.7	0.2	0.02	0.018	0.4	–0.3	0.2
15	110	0.5	0.3	0.015	0.02	0.5	–0.6	0.3
25	115	0.3	0.1	0.02	0.018	0.8	–0.8	0.2

Table 2: Numerical results for the scenarios using the proposed algorithms, illustrated in figure 1.

K	Monte Carlo	FFT	Relative Error
80	24.6838808189857	24.5461647391366	–0.00561049277199614
85	20.4086129617117	20.3043200086086	–0.00513649080879682
90	15.5326065533633	16.1822548895257	0.0401457238560059
95	11.74953391433	12.3461401987241	0.0483233038659167
100	9.00821317440454	8.98824407233211	–0.00222169112361959
105	6.35638117184006	6.25574110388277	–0.0160876331494649
110	4.3382948035737	4.18517436919134	–0.0365863930328761
115	2.65699991570719	2.70698244194532	0.0184642964297211
120	1.64783416804738	1.71202291887556	0.03749292729699
125	1.10644069067188	1.06877715142072	–0.0352398432181037
130	0.690532315997906	0.665142172627011	–0.0381725056924532
135	0.403395746929655	0.416773266063299	0.0320978340573595
80	47.0256193644734	45.3648403933267	–0.0366093864046961
85	40.7234541153276	42.5401009008153	0.0427043365440835
90	38.0983149600262	39.8623300348626	0.0442526835057979
95	36.9577790432644	37.3299192484043	0.0099689528569205
100	34.6108304998454	34.9400403063771	0.00942213585459595
105	33.3874388981813	32.6888896067872	–0.0213696243523981
110	31.1332097474065	30.5719092871391	–0.0183600067302141
115	27.394455371475	28.5839821655106	0.041615153100357
120	25.7539152336102	26.7195999278723	0.0361414353833468
125	24.4112306876842	24.973005503804	0.0224952825976111
130	23.125412723376	23.3383114921657	0.00912228671132616
135	21.9929296297187	21.8095969842613	–0.00840605379318489
80	87.6337289006257	91.5564151476584	0.0428444718014172
85	91.2226967456762	90.8572004151793	–0.00402275580610864
90	90.436560740868	90.1815178314055	–0.00282810619731789
95	88.5308878906847	89.527679527106	0.0111338933577465
100	92.585557170488	88.8941870620129	–0.0415254386195124
105	89.6574512372581	88.2797027192348	–0.0156066284274326
110	90.8888287825975	87.6830262127688	–0.0365612674230657
115	84.0126210634404	87.1030756004615	0.0354804295452992
120	86.6074096098252	86.5388715029133	–0.000791992150135284
125	85.0807858215454	85.9895239491759	0.0105680097516021
130	82.2414077395726	85.4542213300819	0.037596897385552
135	81.4083407556848	84.9322210578466	0.0414904998158676
80	94.4625701181163	95.9247603320968	0.015243094785104
85	96.466692014291	95.0525847651337	–0.0148771046326769
90	94.4171449232684	94.203904710092	–0.00226360270131688
95	97.1314600284146	93.3774462068146	–0.0402025753979763
100	89.1497862557897	92.5720449629597	0.0369685979016594
105	87.7776117041414	91.7866333094318	0.0436776190687277
110	94.2810055251711	91.020229125284	–0.0358247439192755
115	91.0128565198847	90.2719263749146	–0.00820775821148268
120	92.280279649375	89.5408869752897	–0.0305937629905457
125	90.5208691859644	88.8263337681558	–0.0190769487597174
130	85.4274348176917	88.1275444163673	0.0306386569211394
135	87.493651524381	87.4438460778225	–0.00056957063066701

Figure 1: Prices and Differences using FFT and MC methods. We have plotted the difference between the MC price to the FFT price as the error. The *x*-axes are the strikes and the *y*-axes is the option price, resp. the difference in %.



to obtain the arithmetic mean as an estimator for the expectation. One might then be tempted to set

```
itsDiscountFactor = exp(-itsIntegralR);
```

But since

$$\exp\left(-\frac{1}{N}\sum_{k=1}^{N}\int_0^T r_t^k dt\right) = \sqrt[N]{\prod_{k=1}^{N} P_k(0,T)}$$

this would compute the geometric mean of discount factors. The law of large numbers however suggests that the correct estimator for the mean of a random variable is the arithmetic mean. Therefore, the correct code is

Table 4: The discount curve.

Maturity	Discount
0	1
0.003	0.999884333380315
0.083	0.996803132736937
0.167	0.993568709230647
0.25	0.990285301195274
0.333	0.986945903402709
0.417	0.983557350486521
0.5	0.980185549124449
0.583	0.976782934344041
0.667	0.973361992614499
0.75	0.96997679330522
0.833	0.966616749933289
0.917	0.96291431795816
1	0.959904777446077
2	0.920091903961326
3	0.882870065420196
4	0.847186544281939
5	0.812742515687365
6	0.779459552415061
7	0.747152463119429
8	0.715745016074346
9	0.68513872380846
10	0.655753392359115
11	0.627333845297308
12	0.599226698198774
13	0.572763319281569
14	0.547259133751455
15	0.52344199625308
16	0.499646068368557
17	0.477507905873099
18	0.456481811728753
19	0.436385788738282
20	0.41735025383105
21	0.399187111819286
22	0.381865611666566
23	0.365435617455498
24	0.349786183601181
25	0.334806921914717
26	0.320548897004994
27	0.306983265264429
28	0.29408180091705
29	0.282443547729164
30	0.269929224010243

Table 3: Extensions of the Heston–Hull–White model with jumps, Greek calculation and comparison to other models.

Model	Black call	Black put	Heston call	Heston put	Bates call	Bates put	HHW call	HHW put
price	32,7879	4,2053	31,6662	3,0835	32,3931	3,8105	34,9585	5,6393
Delta	0,8471	–0,1530	0,6072	–0,3928	0,6072	–0,3928	0,8723	–0,1277
Gamma	0,0056	0,0056	0,0038	0,0038	0,0038	0,0038	0,0047	0,0047
Vega	70,9523	70,9523	59,3023	59,3023	59,3023	59,3023	55,5321	55,5321
Theta	2,4530	–0,2649	2,5967	–0,1211	2,5967	–0,1211	2,8508	–0,0394

Table 5: Caplet and floorlet data.

caplet(1)/floorlet(0)	starting time	ending time	nominal val.	strike	weight	price
1	2	10	9416.92926877517	0.0342611390640023	1	1200
1	5	7	2314	0.05	1	100
0	20	29	7376.07619599394	0.05	1	200
0	3	6	3613.66436933663	0.0388271087457236	1	205
1	9	11	1427.47352171406	0.05	1	60
1	2	5	7797.16349378825	0.0500211104176346	1	450
0	3	10	6948.13844555887	0.04	1	310
0	18	27	2603.29281202345	0.0534926849722827	1	90
1	12	20	4210.88672094886	0.0414695930901716	1	290
0	1	3	5099.18277084983	0.0330138086463581	1	210
1	9	15	3766.9990581367	0.0467058178022948	1	230
0	3	9	6771.47820565578	0.0374192097111886	1	325
0	3	10	5858.21485694162	0.0470252029744057	1	350
1	8	26	8485.46428280891	0.034125568203124	1	1700
0	21	24	7311.09394688128	0.0553688393773477	1	250
1	2	8	3647.58059225496	0.0500580599519105	1	230
1	6	12	4668.12019649194	0.0508318626408997	1	270
0	12	16	2566.08253901933	0.04	1	95
0	23	25	6686.70889498921	0.0486508139569397	1	170
0	8	10	6689.96339106525	0.0572032238200365	1	360
1	16	28	2700.96396266436	0.046654872802985	1	175
1	12	20	9898.31933931068	0.0347861985610641	1	820
0	4	9	3950.66382263394	0.0493024051944809	1	280
1	2	6	7465.83520847548	0.0482007953378562	1	500
0	3	8	4259.12782546458	0.0415042409900612	1	256
1	8	12	3196.67995524612	0.04	1	210
1	10	14	1433.13517058908	0.030433597650365	1	96
1	19	32	8187.53565283835	0.0422933553887046	1	600
1	20	25	1253.90749171738	0.04	1	52
1	24	28	8067.49557095318	0.0340581988788553	1	260
0	1	3	9772.13354932506	0.0575804248337502	1	609
0	3	8	3415.18336214298	0.0379677887967534	1	180
1	2	9	2665.69872790828	0.0300422228870026	1	315
1	15	18	8468.50986117668	0.0400218226322223	1	350
0	12	19	1767.19304666142	0.0574942505250541	1	90
1	23	27	7237.02270759566	0.0457488713004445	1	210
1	17	20	6732.87947630398	0.0529906465862818	1	212
1	10	12	8192.79766031329	0.0489543137146861	1	326
0	1	16	9948.52606630205	0.0584579025074692	1	525
1	3	7	1200	0.04	1	92
0	9	12	8109.49491096141	0.0581120923493502	1	500
1	2	3	3466.35627699096	0.0409627605133861	1	130
1	5	18	5275.2300620622	0.0556162251880608	1	380
0	4	8	8679.50307369369	0.0333440314849487	1	410
1	18	22	6007.33779722487	0.0432106019483251	1	220
1	2	6	791.972365347071	0.04	1	56
1	28	29	8255.73661136544	0.0455450500767311	1	90
1	14	17	5539.87536936097	0.0509304193428627	1	201
0	10	12	1007.22151059842	0.0590681356060862	1	45

Table 5: Continued.

caplet(1)/floorlet(0)	starting time	ending time	nominal val.	strike	weight	price
0	2	6	7434.19652157982	0.0327893144544669	1	350
1	9	12	2137.23039553082	0.04	1	110
1	22	28	1883.37056498061	0.0431004396686225	1	65
1	23	26	3397.75735905472	0.0374201124784955	1	100
1	1	6	1024.08192796619	0.0372897814209294	1	90
0	4	9	2002	0.04	1	106
1	29	30	3003	0.04	1	55
1	3	10	6198.82309600511	0.0418561395092096	1	535
1	1	9	5200	0.04	1	540
1	1	6	5182.47470948161	0.04	1	420
0	8	12	2000	0.04	1	85
1	5	19	4961.05524517236	0.0501070362095233	1	480
0	6	9	8974.95162564571	0.0486530513238298	1	520
1	10	15	3762.1867184309	0.04	1	220
0	12	19	7131.9444194102	0.0476080485923243	1	260
1	23	27	2802.85918498859	0.0364648442217984	1	100
0	21	26	4024.19745856715	0.0433198817727001	1	95
1	14	18	1922.25317529192	0.04	1	100
0	18	23	8886.66938935406	0.0409556582292757	1	225
1	19	22	7296.9174850853	0.046130857927964	1	240
1	20	29	9717.35166560507	0.0413848302426658	1	450

```
{
  (...)
  SumDiscountFactor += exp(-intR);
}
(...)
itsDiscountFactor = SumDiscountFactor / itsNSIM;.
```

The variable names should explain what they stand for. This completes the discussion of implementation. We will now proceed to some experiments with our model.

4 Numerical Example and Extensions

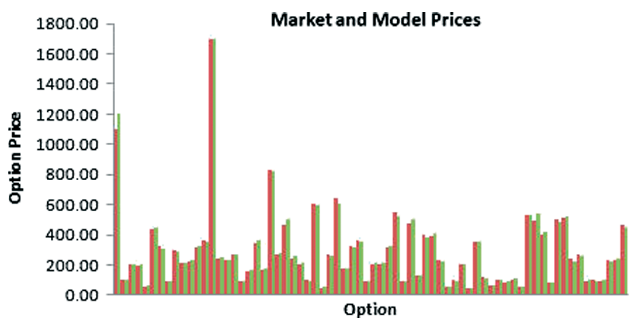
4.1 Numerical Example

Finally, we show the result of a calibration of the Heston–Hull–White model to market prices. The market prices are summarized in section 5. To this end we calibrate the Hull–White model to an initial yield curve, Table 4, as well as to caps and floors, Table 5. The other parameters are then calibrated using the previously calibrated Hull-White parameters. We show the calibration results and compare the results to a calibration of a Heston model. We use tables 6 and 7. First, we consider the calibration of the Hull–White model to caplet and floorlet prices as well as the given yield curve. We show the calibration results in Figure 2.

Second, we consider the calibration of the Heston–Hull–White model and a Heston model to the same call and put prices. Figure 3 shows the calibration error of both models and also the prices from both models with respect to the market prices are given.

Summarizing we remark that for long dated options, especially if they are out–of–the–money, the Heston–Hull–White model shows superior results to the Heston model.

Figure 2: Calibration error for Hull–White model to market prices from Table 5. The x-axes are the used options and the y-axes are the option prices, resp. the error in %



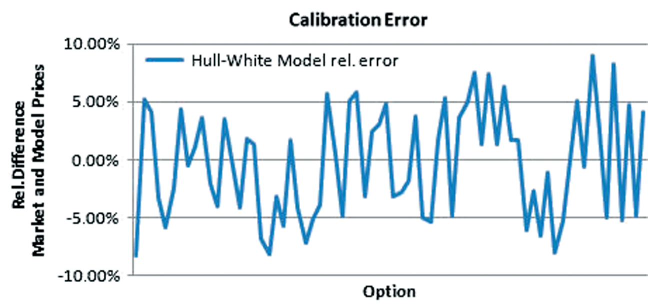


Table 6: Equity option data I.

Call(1)/put(0)	Maturity	Strike	Spot	Weight	Price
1	1.1944	1081.82	2461.44	0.1	1422.3792940646
1	1.1944	1212.12	2461.44	0.2	1300.25370868542
1	1.1944	1272.73	2461.44	0.3	1243.99130024807
1	1.1944	1514.24	2461.44	0.4	1024.24909532335
1	1.1944	1555.15	2461.44	0.5	987.873233682635
1	1.1944	1870.3	2461.44	0.6	718.504165558505
1	1.1944	1900	2461.44	0.7	694.553484851185
1	1.1944	2000	2461.44	0.7	616.07786707975
1	1.1944	2100	2461.44	0.7	541.352430949431
1	1.1944	2178.18	2461.44	0.8	485.476234822808
1	1.1944	2200	2461.44	0.8	470.355265362432
1	1.1944	2300	2461.44	0.8	403.619865578245
1	1.1944	2400	2461.44	0.9	341.329392723369
1	1.1944	2499.76	2461.44	0.9	284.662130191257
1	1.1944	2500	2461.44	1	284.551671409552
1	1.1944	2600	2461.44	1	234.394879670288
1	1.1944	2800	2461.44	0.9	150.466033702423
1	1.1944	2822.73	2461.44	0.9	142.703361496322
1	1.1944	2870.83	2461.44	0.8	126.795377812929
1	1.1944	2900	2461.44	0.8	117.874621567444
1	1.1944	3000	2461.44	0.7	90.6649710320428
1	1.1944	3153.64	2461.44	0.7	61.1561444384128
1	1.1944	3200	2461.44	0.6	54.0033253306561
1	1.1944	3360	2461.44	0.5	34.3977917525318
1	1.1944	3400	2461.44	0.4	30.5318701182539
1	2.1916	1081.82	2461.44	0.1	1461.57788048036
1	2.1916	1212.12	2461.44	0.1	1347.09382355315
1	2.1916	1272.73	2461.44	0.2	1294.64533887627
1	2.1916	1514.24	2461.44	0.2	1091.46437642577
1	2.1916	1555.15	2461.44	0.3	1058.03626462066
1	2.1916	1870.3	2461.44	0.4	811.975310510654
1	2.1916	1900	2461.44	0.5	789.914122496853
1	2.1916	2000	2461.44	0.6	717.153979232596
1	2.1916	2100	2461.44	0.7	648.130294235522
1	2.1916	2178.18	2461.44	0.7	596.11622818202
1	2.1916	2200	2461.44	0.7	581.90113391976
1	2.1916	2300	2461.44	0.8	518.724821472442
1	2.1916	2400	2461.44	0.8	458.76104598328
1	2.1916	2499.76	2461.44	0.9	402.277440098405
1	2.1916	2500	2461.44	0.9	402.168322491237
1	2.1916	2600	2461.44	1	352.800681959238
1	2.1916	2800	2461.44	1	265.29815191738
1	2.1916	2822.73	2461.44	0.9	256.258466145666
1	2.1916	2870.83	2461.44	0.9	237.788657548345
1	2.1916	2900	2461.44	0.8	227.112201880144
1	2.1916	3000	2461.44	0.8	193.737434726345
1	2.1916	3153.64	2461.44	0.7	149.226004850452
1	2.1916	3200	2461.44	0.6	137.377308320527
1	2.1916	3360	2461.44	0.5	101.043834423851
1	2.1916	3400	2461.44	0.4	93.1568497899167

Table 7: Equity Option data II.

Call(1)/put(0)	Maturity	Strike	Spot	Weight	Price
1	4.2056	1081.82	2461.44	0	1537.89780938587
1	4.2056	1212.12	2461.44	0.1	1436.15374265053
1	4.2056	1272.73	2461.44	0.2	1389.76521576543
1	4.2056	1514.24	2461.44	0.3	1210.94634749796
1	4.2056	1555.15	2461.44	0.3	1181.7432801964
1	4.2056	1870.3	2461.44	0.4	966.218070719351
1	4.2056	1900	2461.44	0.4	946.834481174701
1	4.2056	2000	2461.44	0.5	882.766031799261
1	4.2056	2100	2461.44	0.5	821.909315912906
1	4.2056	2178.18	2461.44	0.6	775.824412354643
1	4.2056	2200	2461.44	0.6	763.157788414055
1	4.2056	2300	2461.44	0.6	706.535835262685
1	4.2056	2400	2461.44	0.7	652.067710023196
1	4.2056	2499.76	2461.44	0.7	599.881006918677
1	4.2056	2500	2461.44	0.8	599.777598336201
1	4.2056	2600	2461.44	0.8	550.994727629446
1	4.2056	2800	2461.44	0.8	460.521491020263
1	4.2056	2822.73	2461.44	0.9	450.727847025402
1	4.2056	2870.83	2461.44	0.9	430.599084642388
1	4.2056	2900	2461.44	1	419.429846883366
1	4.2056	3000	2461.44	1	382.44604273531
1	4.2056	3153.64	2461.44	0.9	329.938079701859
1	4.2056	3200	2461.44	0.9	315.074022307316
1	4.2056	3360	2461.44	0.8	267.585053775738
1	4.2056	3400	2461.44	0.8	256.544421518989
1	5.1639	1081.82	2461.44	0.1	1575.14545916331
1	5.1639	1212.12	2461.44	0.1	1479.03073519054
1	5.1639	1272.73	2461.44	0.2	1435.24541401566
1	5.1639	1514.24	2461.44	0.2	1266.44917918489
1	5.1639	1555.15	2461.44	0.3	1238.84605922055
1	5.1639	1870.3	2461.44	0.4	1034.69682580327
1	5.1639	1900	2461.44	0.4	1016.27541291196
1	5.1639	2000	2461.44	0.4	955.220133255394
1	5.1639	2100	2461.44	0.5	897.202261867494
1	5.1639	2178.18	2461.44	0.5	852.900221803726
1	5.1639	2200	2461.44	0.5	840.811112260794
1	5.1639	2300	2461.44	0.6	786.201108526362
1	5.1639	2400	2461.44	0.6	733.379712207143
1	5.1639	2499.76	2461.44	0.6	682.650495178919
1	5.1639	2500	2461.44	0.7	682.550238143673
1	5.1639	2600	2461.44	0.8	634.54817806271
1	5.1639	2800	2461.44	0.8	544.647852148374
1	5.1639	2822.73	2461.44	0.8	534.950590253353
1	5.1639	2870.83	2461.44	0.9	514.523135676212
1	5.1639	2900	2461.44	0.9	503.194830396318
1	5.1639	3000	2461.44	1	465.409173492743
1	5.1639	3153.64	2461.44	1	411.19105063511
1	5.1639	3200	2461.44	0.9	395.583928285715
1	5.1639	3360	2461.44	0.9	345.05333748364
1	5.1639	3400	2461.44	0.8	333.137714853179

Figure 3: Calibration error of the Heston–Hull–White and the Heston model (left) and the model and market prices (right). The x-axes are the used options and the y-axes is the error in %, resp. the option price. The market data is given by Tables 6 and 7.

4.2 Extensions

We stress the point that the model is easily extendable to a Bates–Hull–White model since it is commonly assumed that the jumps in the Bates model are independent of the volatility and the interest rates. Therefore, the implementation corresponds to a multiplication of the characteristic function.

Furthermore, it is possible to compute greeks very efficiently. Table 3 summarizes the possible extensions we have implemented. The results have been produced using the following parameters $T = 10$, $K = 100$, $S(0) = 95$, $r =$ 4.09% and $\tilde{v} = 0.02$, $v(0) = 0.02$, $\kappa = 0.2$, $\rho = -0.6$, $v = 0.5$ for the Heston dynamic as well as $\sigma = 0.05$, $\kappa = 0.3$ for the Hull–White model and finally, for the jump component $\lambda = 0.2$ (jump intensity), $\sigma_J = 0.05$ (jump volatility) and $\mu_J = 0.2$ (average jump size).

5 Market Data

This section contains the used market data for the yield curve, caplets, floorlets, and the index options.

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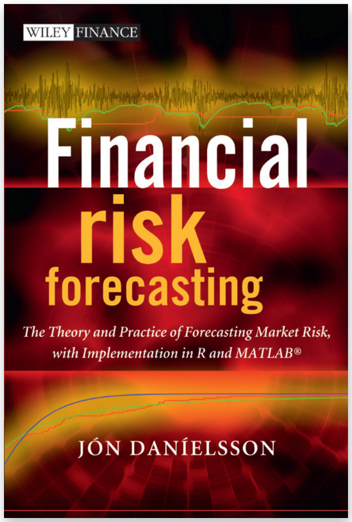

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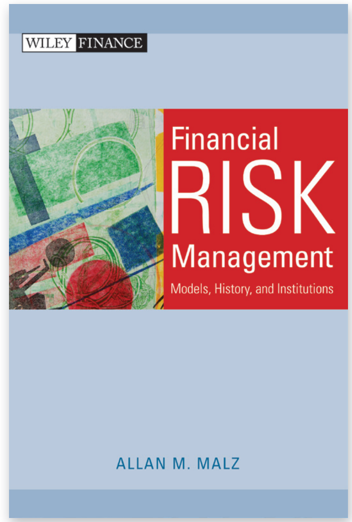

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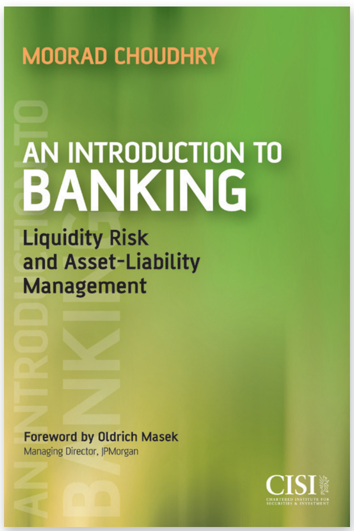

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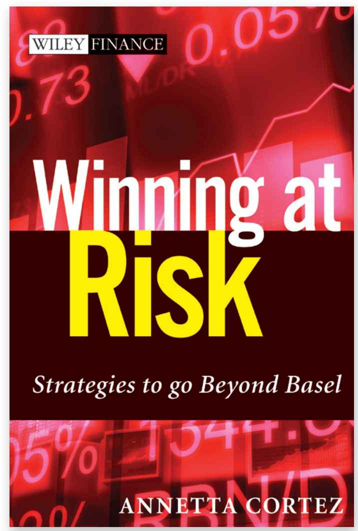

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