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SOME ASPECTS OF THE RANDOM WALK MODEL OF STOCK MARKET PRICES: COMMENT*

By BENOIT MANDELBROT

REGRETTABLY, Mr. C. W. J. Granger's argument (see Section 3 of the preceding paper) does *not* suffice to "save" the central limit theorem with a Gaussian limit. It only shifts the point at which a non-Gaussian limit theorem is required.

To represent price data correctly, the instants of transaction must indeed be such that Granger's function $N(t)$ remains a widely scattered random variable even when t is large. To account for my observation that $Z(t+T)-Z(t)$ has infinite variance, $N(t)$ must even have an infinite expectation. But $N(t)/E[N(t)]$ rapidly tends to one whenever the Gaussian central limit theorem applies to the instants of transaction. Thus, those instants cannot satisfy the Gaussian central limit theorem. A process for which $N(t)$ remains widely scattered even when t is large, studied by Howard M. Taylor and myself [6], is no less "strange" than the infinite-variance random processes.

Economists interested in applying the Gaussian central limit theorem to dependent variables, may ponder the following, still timely, quotation from U. Grenander and M. Rosenblatt [1, (181)]: "... the experimentalist would argue that in most physically realizable situations where a stationary process has been observed during a time interval long compared to time lags for which correlation is appreciable, the average of the sample would be asymptotically normally distributed. ... Unfortunately none of the extensions of the central limit theorem to dependent variables seems to answer this problem in terms well adapted for practical interpretation." I may add that, if the interdependence between addends is sufficiently strong (but not unrealistically strong), their weighted sum will *not* converge to a Gaussian limit. See for example my paper [5], where I describe a process with a Gaussian marginal distribution whose long-term average is not Gaussian but stable Paretian. Such was shown, in [2], to be the case for prices.[1]

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* Manuscript received May 31, 1967.
[1] See also [3] and [4].

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