WHITE PAPER

THINKING FORWARD ABOUT PRICING AND HEDGING VARIABLE ANNUITIES

numerix®
Taking Analytics Further

> *"We can't solve problems by using the same kind of thinking we used when we created them."*
>
> —Albert Einstein

As difficult as the recent market turmoil was to go through, financial companies have emerged stronger, with an intense focus on improving how they measure and manage risk. For variable annuity writers, this has led to innovative thinking about product pricing and hedging practices—thinking that marks a significant shift from previous methods.

In this paper, we discuss recent developments in modeling and product design that address key challenges in the current market:

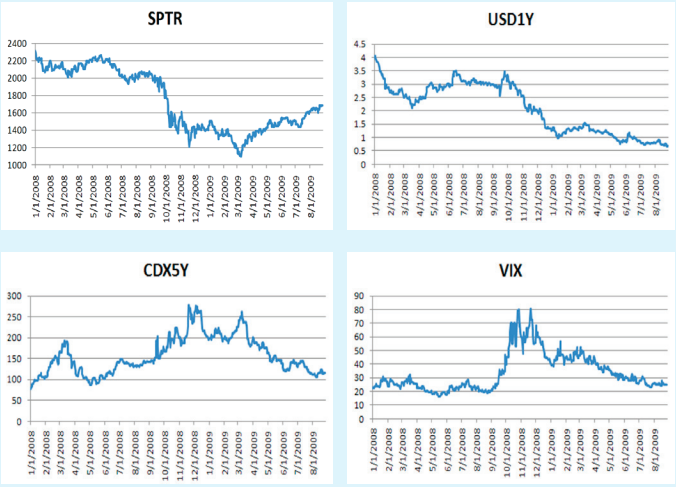
- **Factoring correlation into models** – Achieving market-consistent calibrations using advanced multi-factor hybrid models that bring together multiple risk factors simutaneously
- **Choosing the right model** – In calculating a fair rider premium, is model selection as important as lapse assumptions?
- **Handling large computations** – Evolving the infrastructure to achieve convergence for IR gamma and other high-order Greeks (it doesn't always mean adding CPUs)
- **Numerix "One-Step" Monte Carlo VaR** – A highly efficient approach to simulating VaR on GMXB portfolios that takes about the same amount of time as pricing
- **Rapid product prototyping** – How an object-oriented architecture and simple payoff scripts make it possible to respond quickly to new product requirements

The Danger of Ignoring Correlation

Prior to 2008, the industry standard for modeling VA hedge requirements was one in which only the equity value was stochastic. This overly simplistic practice was deemed "good enough" under most conditions. But as the financial crisis unfolded, historically non-correlated assets quickly moved together in the wrong direction all at once. Since most hedge programs did not consider credit spreads, VA writers were left with significant unhedged exposures.

Figure 1: The "Perfect Storm"

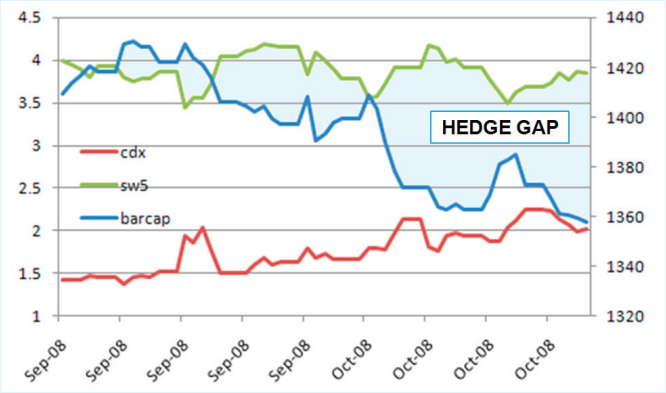
S&P Total Return plunges,1YrTreasury yields hit historic lows, credit spreads (Yr CDX) test highs, and the Volatility Index soars (clockwise from top left)



Source: Bloomberg

Figure 2: Annuity writers that didn't hedge credit risk were left with large unhedged exposures

Even though swap rates moved sideways (green), bond values (blue) took a serious hit due to rising credit spreads (red), leaving a large hedge gap.



Source: Bloomberg

Implementing a Hybrid Model Framework

Realizing that a more sophisticated treatment of correlation was required, insurance companies have begun to look at ways to hedge risk in a way that is consistent with market-observed behavior, bringing together interest rate, equity, volatility, credit and other factors within a unified, efficient model framework.

To accomplish this, **it is necessary to incorporate stochastic processes across multiple asset classes and factors.** This requires a simultaneous calibration process to accurately capture correlation between volatility factors. Numerix has developed a hybrid model framework that provides a structure for designing and calibrating such a model.

The concept behind the hybrid model framework is that the best model is selected for each underlying. These "component" models are individually calibrated and then linked together through a correlation matrix that defines the "hybrid" model. A joint calibration is applied, allowing, volatility to be represented in a market-consistent manner. This approach offers a high degree of flexibility in the selection of various single- and multi-factor models across "n" economies for baskets of arbitrary size.

This is a nontrivial achievement, generally known as the "hybrid problem." Consider a typical GMWB policy that requires a four-factor cross-currency equity model, consisting of a Hull White one-factor model for domestic and foreign rates, an FX process between the two currencies using a Black Scholes drift with a stochastic driving force, and an equity process in a domestic economy:

$$dr_d(t) = [\theta_d(t) - a_d(t)r_d(t)]\,dt + \sigma_d(t)dW_d(t), \quad (1)$$

$$dr_f(t) = [\theta_f(t) - a_f(t)r_f(t)]\,dt + \sigma_f(t)dW_f(t), \quad (2)$$

$$\frac{dX(t)}{X(t)} = [r_d(t) - r_f(t)]\,dt + \sigma_X(t)dW_X(t), \quad (3)$$

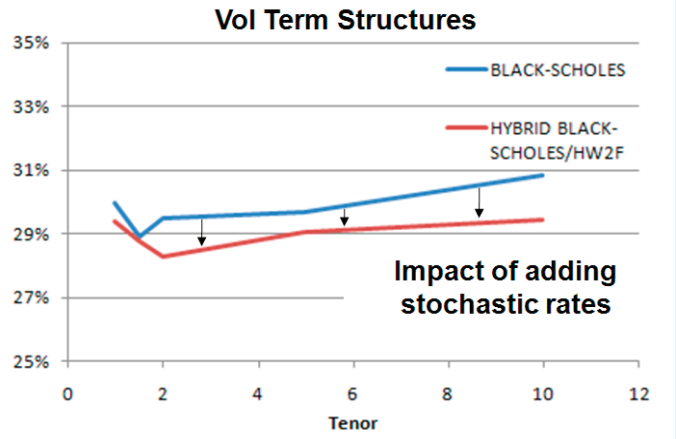
$$\frac{dS(t)}{S(t)} = [r_d(t) - q_d(t)]\,dt + \sigma_S(t)dW_S(t), \quad (4)$$

In calculating the implied volatilities observed in an FX option (Sigma-squared x), **this is not just a function of FX volatility, but also of the other volatilities, plus a correlation structure among them.**

$$\sigma^2_{X,imp}T = \int_0^T \left(\sigma_X(t)^2 + \Sigma_d(t,T)^2 + \Sigma_f(t,T)^2\right) dt + 2\int_0^T \sigma_X(t)\Sigma_d(t,T)\rho_{dX}dt - 2\int_0^T \sigma_X(t)\Sigma_f(t,T)\rho_{fX}dt - 2\int_0^T \Sigma_d(t,T)\Sigma_f(t,T)\rho_{df}dt$$

To illustrate this relationship, the figure below shows the impact on equity volatility term structures when going from a deterministic treatment of rates (Black-Scholes) to stochastic rates (hybrid Black-Scholes/Hull-White 2-Factor). To fit with market-observed prices, equity volatility is adjusted downward to factor in interest-rate volatility. By defining the correlations and performing a joint calibrabtion, it is possible to accurately account for the volatilities of each factor.

Figure 3: Market-consistent equity volatility is affected when a stochastic interest-rate model is used



Source: Numerix

Capital Market Model Risk vs. Lapse

With the hybrid model framework in place, it is possible to measure the impact of model selection on policy pricing. The results may be quite surprising: our results show that changing the model can have just as large of an impact on estimated hedge costs as varying the lapse assumptions.

To demonstrate, the charts below represent the "fair" rider premium for a typical GMWB policy: paying 5% with a 5-year wait period for a 60-year-old male, using the A2000 mortality table and a 50/50 debt/equity allocation rebalanced annually. Under base lapse assumptions, market dynamics do not play a factor in policyholder decisions of whether or not to lapse. Under dynamic lapse assumptions, policyholders are less likely to lapse when their option is in the money, and more likely to lapse when it is out of the money.

By charting results using varying capital market dynamics, we see that the premium generally increases as you go from a Gaussian interest rate model to a lognormal one, and that the increase is roughly the same proportion for both base and dynamic lapse. By using the Bates model (stochastic volatility with jumps) in place of Black-Scholes for equities, we see an increase in premium ranging from 18-27bp. In comparison, the average increase going from base lapse assumptions to dynamic lapse is only 17bp.

Figure 4: Impact on Fair Rider Premiums

Model selection (green) can be just as significant as lapse assumptions (red)

Base Lapse

RATES	EQUITIES: BLACK	HESTON	BATES	Avg
HW2F	0.46%	0.49%	0.70%	0.55%
LMM2F	0.51%	0.56%	0.69%	0.59%
Avg	0.49%	0.52%	0.69%	0.57%

20bp

Dynamic Lapse

17bp

RATES	EQUITIES: BLACK	HESTON	BATES	Avg
HW2F	0.61%	0.64%	0.88%	0.71%
LMM2F	0.69%	0.73%	0.87%	0.76%
Avg	0.65%	0.69%	0.87%	0.74%

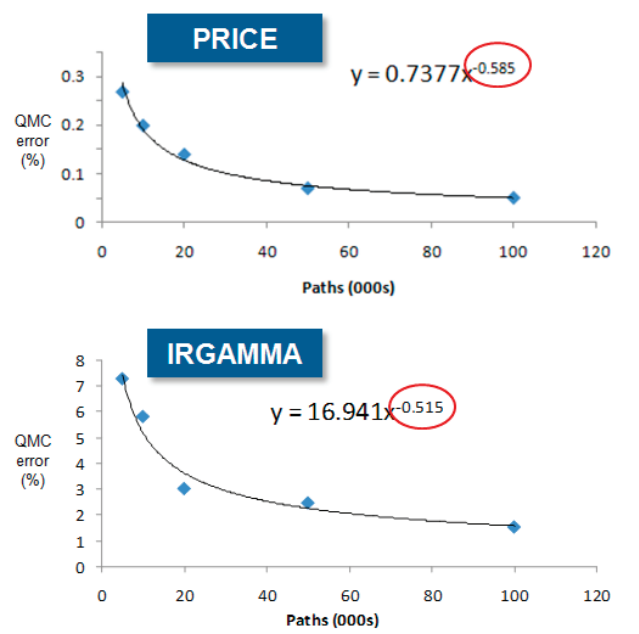
Source: Numerix

Convergence in High-Order Greeks

One of the biggest concerns in using more advanced models is computation time, especially for hedging higher-order Greeks (such as IR gamma) that may require millions of paths to achieve convergence. Numerix uses low-discrepancy sequences—proprietary extensions of Sobol sequences that have better clustering properties in higher dimensions. With this resource, we have found that IR gamma converges almost as rapidly as the valuation.

Figure 5: Convergence of Price and IR Gamma

Low-discrepancy methods in Numerix enable fast convergence of higher-order Greeks



Source: Numerix

Numerix "One-Step" Monte Carlo VaR for GMXBs

The use of Monte Carlo Value at Risk (MC VaR) for variable annuities is the application of functionality based on Monte Carlo methods that were designed to quantify market risk through Value at Risk. It is also applicable to counterparty risk measurement through counterparty credit exposure, with potential future exposure (PFE) as one popular exposure measure among several, and credit valuation adjustment (CVA).

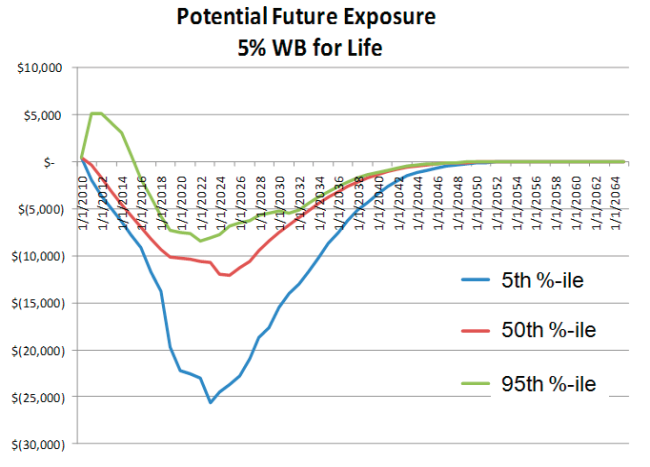
Using arbitrage-free scenarios calibrated to current market data, MC VaR alows the user to project market-consistent scenarios through time, calculate expected NPV at each point in time on each path, and then analyze the distribution.

Normally, this computation is similar in nature to nested-stochastic (stochastic-on-stochastic) projection for capital calculations like VACARVM, which can require thousands of cores to run overnight VaR. However, Numerix has developed an alternative "One-Step" method that greatly simplifies the computation by eliminating the need for a second Monte Carlo process on the outer loop.

Under the Numerix One-Step MC VaR method, the outer real-world loop is made to be fully market-consistent and arbitrage-free, allowing the computation to recycle the Monte Carlo paths for both scenarios and instrument pricing. This process offers a significant boost in performance, enabling intra-day VaR on an entire GMXB portfolio or real-time incremental VaR—without using approximations.

Figure 6: On-demand intra-day MC VaR and PFE using the "One-Step" method

Probability distribution of GMWB value over 1000 scenarios and 50 years, **computation time: ~1sec**



Source: Numerix

Rapid Product Prototyping

Risk reports aren't the only place where speed is needed. Numerix provides a highly efficient object-oriented architecture and a unique scripting language that makes it possible to quickly design new products. Within this architecture, object components that comprise a deal (i.e., market data, indices, events, deal description) can be either locked down or exposed to the end user, including the cashflow logic that defines the underlying payoff structure of the policy. Different parts of this logic can be modularized, assigned to various groups and reused, ensuring consistency across a product line.

For example, one group can be responsible for defining capital markets components, such as fund modeling (how the account value is rolled forward from period to period) and rebalancing strategy (target volatility, CPPI, target date funds). Another team can define the product components, including rider fees and guarantee roll-forwards, while an actuarial team controls the policyholder behavior assumptions, like mortality, lapse and withdrawal components.

Prototyping a new product is now as simple as changing one line of code from an existing product:

Change a fixed withdrawal rate (5% for life)...

```
WITHDRAWALRATE = 0.05
```

...to an Indexed WB (payoff is linked to a reference rate such as a 10-yr swap rate)

```
WITHDRAWALRATE = MIN(FLOOR + PARTICIPATION *
MAX(REFERENCERATE – FLOOR, 0), CAP)
```

Conclusion

Sweeping changes in risk management practices have emerged as companies realize the need to implement valuation methods that capture real-world volatility dynamics, including correlation between asset classes. Fortunately, analytic capabilities have improved making it possible to execute sophisticated hedging strategies and meet today's regulatory and capital requirements.

If you would like more information on the Numerix hybrid model framework, One-Step Monte Carlo VaR, or other ALM solutions, please contact info@numerix.com or visit us on the web at www.numerix.com/ALM.

www.numerix.com

Numerix is the leading provider of cross-asset pricing and risk solutions for derivatives and structured products. Since its inception in 1996, over 700 clients and 50 partners across more than 25 countries have come to rely on Numerix analytics for speed and accuracy in valuing and managing the most sophisticated financial instruments. With offices in New York, London, Tokyo, Hong Kong, Singapore and Dubai, Numerix brings together unparalleled expertise across all asset classes and engineering disciplines.

New York | London | Tokyo | Hong Kong | Singapore | Dubai

Copyright © 2011 Numerix LLC. All rights reserved. Numerix, the Numerix logo, and CrossAsset are either registered trademarks, or trademarks, of Numerix LLC in the United States and/or other countries.