OIS and Its Impact on Modeling, Calibration and Funding of OTC Derivatives

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Agenda

• Changes in Interest Rate market dynamics after the market crisis
• Impact on OTC Derivatives Valuation
• Curve Construction
• Single vs Cross Currency Situation
• Deterministic Spread Framework
• Stochastic Spread Frameworks
• Risk & Hedging
• Conclusions
Huge Increase in Basis Swaps Spreads

Basis Swap Spreads
EUR
5y swap maturity

3m vs 6m
1m vs 3m
1m vs 6m
Much Wider Libor-OIS Spread

Basis Swap Spread Quotes
FedFunds vs LIBOR 3m Rate
For various swap tenors
Spread Curve Changes Over Time

Source: Anna Barbashova, Numerix Internal Working Paper
Big Increase in Collateral Posted (USD Billions)

Source: ISDA Margin Survey, 2012
Impact on OTC Derivative Valuation

• The “right” discounting curve, depends on Credit quality of self, counterparty, netting and collateral clauses in CSA (valuation & even curve construction is a “portfolio” problem)

• Since collateralization is standard practice in a large portion of the market:
  – Each OTC derivative is really bound to an underlying CSA
  – Standardization around cash collateral, funded at OIS/Compounded Overnight Cash rates
  – Since cost of funding for most large participants is OIS, funding curve is converging to OIS in market quotations

• Even in the absence of collateralization, modeling funding costs explicitly is a must due to divergence among different funding rates (various Libor tenors, OIS, Repo, etc)

• Even in the presence of collateralization, “imperfect” CSAs result in CVA, DVA and FVA exposures that need to be factored in to the valuation

• Explicit modeling of Credit, Liquidity premia, and funding costs
Impact on OTC Derivative Valuation (contd.)

• Interest rate Curves evolve to an Interest Rate “Surface” based on tenor and credit quality
• Curve Construction and Volatility Surface construction need to incorporate OIS discounting
• Model Calibration should incorporate multiple interest rate curves
• The spreads between different types of rates need to be modeled effectively
• Calculating CVA/DVA requires joint calibration and simulation of interest rate and credit factors
• Hedging and Risk Management need to be rethought: Basis risk across maturities and credit qualities is now significantly higher and needs to be measured and managed
• Valuation challenges can turn an ordinary swap into a Level 2 FAS 157 asset

Multi-curve modeling is really one component of the new, post-crisis modeling framework that includes credit costs/benefits (CVA/DVA), funding costs (FVA) and collateral agreements
Swap Pricing under Single vs Dual Curves

- Take swaps with 10y, 15y, 20y, 30y maturity
- Price in single curve and dual curve framework from 2006 to 2012

<table>
<thead>
<tr>
<th>Start</th>
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<tbody>
<tr>
<td>2/7/2006</td>
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<table>
<thead>
<tr>
<th>Maturity</th>
<th>Notional</th>
<th>Swap Rate</th>
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<tbody>
<tr>
<td>10y</td>
<td>10,000,000</td>
<td>3.67510%</td>
</tr>
<tr>
<td>15y</td>
<td>10,000,000</td>
<td>3.85410%</td>
</tr>
<tr>
<td>20y</td>
<td>10,000,000</td>
<td>3.94150%</td>
</tr>
<tr>
<td>30y</td>
<td>10,000,000</td>
<td>3.98450%</td>
</tr>
</tbody>
</table>

Source: Anna Barbashova, Numerix Internal Working Paper
## Steps in Curve Construction

<table>
<thead>
<tr>
<th>Currency</th>
<th>OIS Curve</th>
<th>Benchmark Curve</th>
<th>Forward Curve</th>
<th>Basis Curve (Single Currency)</th>
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</thead>
<tbody>
<tr>
<td>EUR</td>
<td>Eonia→</td>
<td>Euribor6m→</td>
<td>Euribor1m</td>
<td>Euribor3m vs 6m</td>
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<td>Euribor3m</td>
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<td>Euribor12m</td>
<td>Euribor12m vs 6m</td>
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<tr>
<td>GBP</td>
<td>Sonia→</td>
<td>GBP Libor6m→</td>
<td>Libor1m</td>
<td>Libor3m vs 6m</td>
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<td>Libor3m</td>
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<td></td>
<td>Libor12m</td>
<td></td>
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<tr>
<td>JPY</td>
<td>Tonar→</td>
<td>JPY Libor6m→</td>
<td>Libor1m</td>
<td>Libor3m vs 6m</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Libor3m</td>
<td></td>
</tr>
<tr>
<td>USD</td>
<td>FedFund *→</td>
<td>USD Libor3m→</td>
<td>MuniSwap</td>
<td>Libor1m vs 3m</td>
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<td>Libor6m vs 3m</td>
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<td>Libor12m vs 3m</td>
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<td>Munivs 3m</td>
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<td>CPvsLibor3m</td>
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<td>FedFund vs Libor3m**</td>
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<td>Primevs Libor3m</td>
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<td>T-Bill vs Libor3m</td>
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Note:
- The list in the table is not a complete list and new instruments are keep evolving.
- OIS curve in USD is a bit more complex. We will discuss more in the following session.
- Traditionally (one curve), FedFund vs. Libor3m basis swap is used to strip FedFund curve given Libor3m curve; Now (under two curve), FedFund curve could be OIS curve itself.
### Steps in Curve Construction (USD)

<table>
<thead>
<tr>
<th>Fed Fund Basis Swap</th>
<th>Synthetic Fed Fund Swap</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pay</strong></td>
<td><strong>Receive</strong></td>
</tr>
<tr>
<td>LiborLeg</td>
<td>FedFundLeg</td>
</tr>
<tr>
<td>-3mLibor</td>
<td>+WeightedAvgFedFund + BasisSprd</td>
</tr>
<tr>
<td><strong>IRS</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Receive</strong></td>
<td><strong>Pay</strong></td>
</tr>
<tr>
<td>LiborLeg</td>
<td>FixedLeg</td>
</tr>
<tr>
<td>+3mLibor</td>
<td>-FixedSwapRate</td>
</tr>
</tbody>
</table>

Source: Dan Li, Internal Numerix Working Paper
Cross Currency Case: Foreign OIS CC-Basis Curve

Using USD 3m Libor vs JPY 3m Libor as an example:

1. Construct Domestic OIS discounting curve (e.g. FF OIS Curve as described earlier)
2. Construct Domestic projection curve (e.g. USD 3m Libor curve)
3. Construct Foreign OIS discounting curve (e.g. JPY OIS Curve from Tonar)
4. Construct Foreign Benchmark projection curve (e.g. JPY 6m Libor)
5. Construct Foreign projection curve (e.g. JPY 3m Libor from Libor Basis swap 3m vs 6m)
6. Solve Implied Foreign Basis Curve (Foreign OIS CC-Basis Curve) from Cross Currency basis swaps given 1, 2 and 5 above

Source: Dan Li, Internal Numerix Working Paper
Modeling: Deterministic Spread Approach

\[ L(t, T_1, T_2) = \frac{1}{\delta T} \left( \frac{P(t, T_1)}{P(t, T_2)} - 1 \right) + s(T_1, T_2) \]

The model calibration reduces to a swaption payment modification w.r.t. the single-modeling. Namely, the swap price can be rewritten as

\[ S(t) = \sum_{i=1}^{M-1} \delta T_i P(t, T_{i+1}) \left( L_{th}(t; T_i, T_{i+1}) + s(T_i, T_{i+1}) - K \right) \]

leading to

\[ S(t) = P(t, T_1) - P(t, T_M) - \sum_{i=1}^{M-1} \delta T_i P(t, T_{i+1}) (K - s(T_i, T_{i+1})) \]

Thus, the multiple-curve model calibration with deterministic spread is equivalent to calibration of the single-curve model to bespoke swaptions

\[
\mathbb{E} \left[ \left( P(t, T_1) - P(t, T_M) - \sum_{i=1}^{M-1} \delta T_i P(t, T_{i+1}) (K - s(T_i, T_{i+1})) \right)^+ \right] \frac{1}{N(t)}
\]

Source: Alexander Antonov, Numerix Internal Working Paper
Dynamics of basis spreads

USD 3m OIS rates vs 3m LIBORs

Vol(3m OIS) = 68%
Vol(3m LIBOR) = 17%
Vol(3m Basis) = 59%
Corr(3m OIS, 3m LIBOR) = 1%
Corr(3m OIS, 3m Basis) = −62%
Corr(3m LIBOR, 3m Basis) = 60%

USD 6m OIS rates vs 6m LIBORs

Vol(6m OIS) = 75%
Vol(6m LIBOR) = 15%
Vol(6m Basis) = 39%
Corr(6m OIS, 6m LIBOR) = −1%
Corr(6m OIS, 6m Basis) = −65%
Corr(6m LIBOR, 6m Basis) = 64%

Corr(3m Basis, 6m Basis) = 66%

Source: Fabio Mercurio, Calibration of multi-curve models, Global Derivatives 2012, Bloomberg
The Case for Stochastic Spread Modeling

- CSA agreements have embedded options, particularly “Cheapest to Deliver”
- Longer dated, complex structures like CMS Spread Bermudan Callables, etc can be sensitive to the spread correlation and vol dynamics
- Some markets trade single currency interest rate spread options
- OIS-Libor basis is a key, independent risk factor in bank stress scenarios and options on these instruments might trade in the future
“Theoretically, it is not difficult to put together a model – it would be an extension of a stochastic basis model where you have more than one basis,” says Vladimir Piterbarg, global head of quantitative research at Barclays Capital in London. “The huge question is whether you are able to execute the hedging strategy required.”

(Quoted in “Multi-currency CSA chaos behind push to standarized CSA, RISK, 2011, Nick Sawyer”)

Hedging and accurate calibration require a liquid market in:
• OIS Cross Currency Basis Swaps
• Options on OIS and OIS CC Basis Swaps

Resulting complexity might create confusion and be a further challenge to valuation & liquidity, therefore push to standardized CSAs with no optionality
## Model Construction: Cross Currency Analogy

<table>
<thead>
<tr>
<th>Cross Currency World</th>
<th>Dual Curve World</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic USD</td>
<td>Foreign JPY</td>
</tr>
<tr>
<td>Discounting OIS</td>
<td>Forward XIBOR</td>
</tr>
</tbody>
</table>

### 2 Currency Curves
- USD Yield Curve
- JPY Basis-adjusted Curve
- OIS Curve
- XIBOR (adjusted for OIS)

### 1 Currency Curve
- JPY Yield Curve
- XIBOR Forward Curve

Source: Dan Li, Internal Numerix Working Paper
### Model Construction: Credit Risky Model Analogy

<table>
<thead>
<tr>
<th>Credit Risky Model World</th>
<th>Dual Curve World</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounting</td>
<td>Foreign</td>
</tr>
<tr>
<td>Risky</td>
<td>Risk Free</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Credit Risky model</th>
<th>Obligor Credit curve</th>
<th>XIBOR</th>
<th>OIS Curve</th>
<th>XIBOR (adjusted for OIS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Currency Curve</td>
<td>JPY Yield Curve</td>
<td>XIBOR Forward Curve</td>
<td></td>
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</tr>
</tbody>
</table>

- Since funding spreads are usually non-negative, this approach may be better to constrain the dynamics so that the OIS curve is below XIBOR.
- Also, spreads modeled directly as a distinct risk factor allows more direct measurement of sensitivities and hedge ratios.
- Default risk is not the only source for stochastic spreads, so default modeling may be confusing.

Source: Christian Fries, Funded Replication: Valuing with Stochastic Funding
Risk Sensitivities

XIBOR risk
Single Curve

Spread Risk
Dual curve

OIS Rate Risk
Dual Curve
Conclusions

• Valuation and Risk methodologies fundamentally changed after 2008
  • Convergence around “OIS” discounting as a market standard
  • CVA, DVA & FVA measures included as part of trade valuation
  • Curve Construction, Model Calibration, Valuation & Risk methodologies updated
  • Stochastic spread modeling being discussed but not widely used yet
• Consistency of valuation techniques throughout a firm and broadly in the market still a challenge
• Aside from major markets, “OIS-like” curves not available
• Increasing push towards Standardized CSAs* will help to create consensus around methodology

* Source: Nick Sawyer, Isda working group close to finalising standard CSA, RISK
Thank you

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