Validation of CVA models

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Contents

- Some principles of pricing model validation revisited
- Peculiarities of CVA models
- Implications on the CVA model validation
- Critical points

Pricing model validation: Common assumptions

- Additivity of prices
  - Split into types of trades
  - Separate pricing function for each type of trade
  - Restricted number of market factors for each trade
- Liquidity of the market

Most important criteria for pricing function validation

- Theoretical soundness: no arbitrage
- Calibration, meet the market: keep model
  - as complex as needed
  - as simple as possible
- Stability
  - Sensitivity of model parameters to market changes
  - Behavior under extreme market conditions

Criteria of medium importance

- Consistency between trade types
  - Most important, if inconsistencies allow for arbitrage
  - Financial crisis shows segmentation of markets (e.g. discounting for swaps and cross currency swaps)
- Performance
  - Analytical tractability
  - Approximations are not appreciated

Least important

- Is the stochastic model realistic?
  - Real world vs. risk neutral measure
- Are assumptions reasonable?
  - Calibrated model meets market expectations
  - Analytical tractability might be more important
- How is the model used?

CVA: Non additive on trade level

- Netting between trades with single counterpart
- Potentially large portfolio with a counterpart as atomic unit
- May contain plain vanilla as well as exotic trades
- May depend on large number of risk factors

CVA: Not fully tradeable

- Trading CVA via innovation or contingent CDS:
  - (Incremental) CVA for single trade depends on portfolio it is part of
  - Trading CCR for complex portfolio will depend on result of bilateral negotiations
- Replication
  - Determine expected exposure profile and close by CDS
  - Capture changes of the expected exposure via instruments depending on respective market factors
  - Expected exposure conditional on default needed to capture wrong way risk
  - Restricted by availability of CDS on respective counterpart

CVA: Complexity

- Large number of risk factors to be considered simultaneously for multitrade underlying portfolio
- Expected exposure is an option on the underlying portfolio and thus always one step more complex than the latter
  - For each risk factor needed to value the underlying portfolio also an (implied) volatility is needed
  - In addition statistical dependencies (correlations) are needed
- Spread of the counterpart:
  - CDS on counterparts might not be traded on the market
- WWR depends on volatility of spreads and correlation to underlying risk factors

Examples

- One swap only as underlying portfolio

  ⇒ (unconditional) expected exposure at time $t \sim$ price of swaption on remaining swap at t

- Payer and receiver swap with different maturity

  ⇒ Option on spread between swap rates with different tenor

  ⇒ Multifactor interest rate model might be needed to calculate expected exposure

- Swaps in different currencies

  ⇒ Correlation between evolution of IR rates in different currencies, FX rates

  ⇒ Quanto effects

Examples cont.

- Swaptions

  ⇒ Implied volas become a stochastic quantity

  ⇒ Calibration might differ from the one used to explain smiles in stochastic vola models

- For the conditional expected exposure also evolution of credit spreads (hazard rates) and their and correlation to IR rate evolution needed

Margined trades

- Exposure will depend on
  1. Value changes of underlying portfolio in the period between collateral exchange and default
  2. Value changes of collateral
  3. Uncollateralized amounts or overcollateralization due to contract specifications and operational issues
- Based on some assumptions on the length of the respective period 1. may be captured by market risk factors as mentioned above
- Assumptions on future composition of collateral (dependent on default) needed for 2.
- Operational issues are commonly outside the framework of pricing model validation

Common characteristics of CVA calculations

- Usually based on simulation
- Stochastic model underlying the simulation will depend on a large number of parameters
- A fraction only will be fixed by calibration routines
- Historical time series might be used to fix other parameters
- Expert opinion ?
- Parameter choices may be hidden or parameters may be set globally

Simulation

- No (approximate) solution of complex integrals in the main calculation routines
  - However, calibration might be based on complex analytics
  - Analytical solutions for simple portfolios might play a role as benchmark in the validation
- Performance is an issue, i.p. in relation to sensitivity calculations
- Simulation error

Stochastic model

- Complexity vs. coverage:
  - Simple model will not be able to cover all risk factors needed
  - More complex model will be more difficult to calibrate and generate some degree of arbitrariness
- Approximations to the condition of being arbitrage free may help to formulate a model allowing calibration to a large number of input parameters
- Structure of trading activities to be taken into account

Complexity vs. coverage: Examples

- Portfolio of simple plain vanilla IR swaps:
  - Might need more than one factor for the evolution of IR rates for each currency
  - Alternative point of view: Choosing a one factor model would fix the correlations between swap rates of different maturities
- Portfolio with swaptions:
  - With high probability future interest rates will be such that swaptions are either in the money or out of the money
  - Thus impact of the stochasticity of implied volas may be small
  - Note that a fully consistent model for the simultaneous evolution of interest rates and their volatilities might lead to rather challenging calibration issues

Model uncertainties

- Parameters which can not be calibrated to market data will introduce some ambiguity into the model
- This is particularly challenging for the validation

Implications

- Assessment of the model risk implied will be most essential part of the validation
- It might render no arbitrage conditions and perfect calibration to market data less important
- Renders the simulation error less important
- Questions, whether assumptions are reasonable/realistic become more important
- This might also hold for the non CVA specific risk factors

Example

- We might start with assumptions on the stochastic evolution of some risk factors
- We calibrate to option prices
- For customary pricing functions we might not care for the marginal distributions implied
- CVA calculation might involve modeling the statistical dependency of such risk factors
- No market data are available for the latter
- We want to have a realistic joint distributions
- This might be difficult based on the given marginal distributions

CVA models and pricing

- CVA represents value of losses form counterparty defaults
- If we charge the counterpart for the risk of his default, he might wish to do that with roles exchanged
- This leads to the concept of using a double sided CVA (DVA) for pricing purposes
- Using DVA, we consider the profit we might have from our own default in the pricing
- Thus we trade at prices, which generate losses until we default.

Hedging

- Hedging of default events may be difficult
- Hedging against CVA fluctuations not caused by defaults:
  - The CVA is not tradeable
  - Without default CVA will be zero at maturity of the trade
  - Is there a point in hedging intermediate value changes of such a quantity ?

Roundup

- Rather than asking
  - Is the model arbitrage free?
  - Is it well calibrated to the market?
- We might ask
  - Is the model sufficiently complex?
  - Is it sufficiently close to an arbitrage free model?
  - Besides its ability to explain prices of products traded: Does it forecast reasonable distributions?
  - Quantification of model uncertainties?
  - How is the model used?

Some special issues

- Use of historical data for the calibration
- Credit spread mapping
- Wrong way risk

Historical data: Distribution of increments vs. time lag

- In the absence of market data for the calibration of the stochastic model we might employ historical data
- Basically, the distribution of increments over some time lag might be used to estimate parameters of the stochastic process
- In practice it might turn out, that the results depends on the length of time lag
- While a large sample of independent increments needs short time lags, we are finally interested in the increments over longer time periods
- Typical effect: Correlations increase with the length of the time lag

Historical data: Mean reversion

- Some stochastic models imply mean reversion
- MLE is a standard method to estimate statistical parameters
- However, with a time series of reasonable length, MLE will always significantly overestimate the mean reversion speed
- If you do not believe, try out with a synthetic time series

Credit spread mapping

- For most counterparts, CDS are not traded on the market
- Default probabilities for these counterparts can not be calibrated to the market
- Most CVA models include some mapping to real or synthetic spread curves.
- Differences in this mapping may have a stronger influence on the CVA figures than details of the stochastic model behind the exposure calculation

Wrong way risk

- Statistical dependencies between default and exposure might be covered by including credit spreads into the stochastic model
- When averaging the exposure, the implied default probabilities might then be used to obtain the default conditional expected exposure
- The method will work to the extent correlations between counterparty spreads and market risk factors reflect dependency between market evolution and default probability
- Mapping and poor long term correlation estimates from historical data may have a disturbing effect
- Credit spread mapping and the capturing of WWR will be an important issue in the validation of CVA models