CREDIT SUISSE

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Backtesting of credit exposure models: a sound Basel III compliant framework

6th Annual Pricing Model Validation,

9th – 11th of September 2013

Reference:

A Sound Basel III Compliant Framework for Backtesting Credit Exposure Models

http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2264620

Agenda

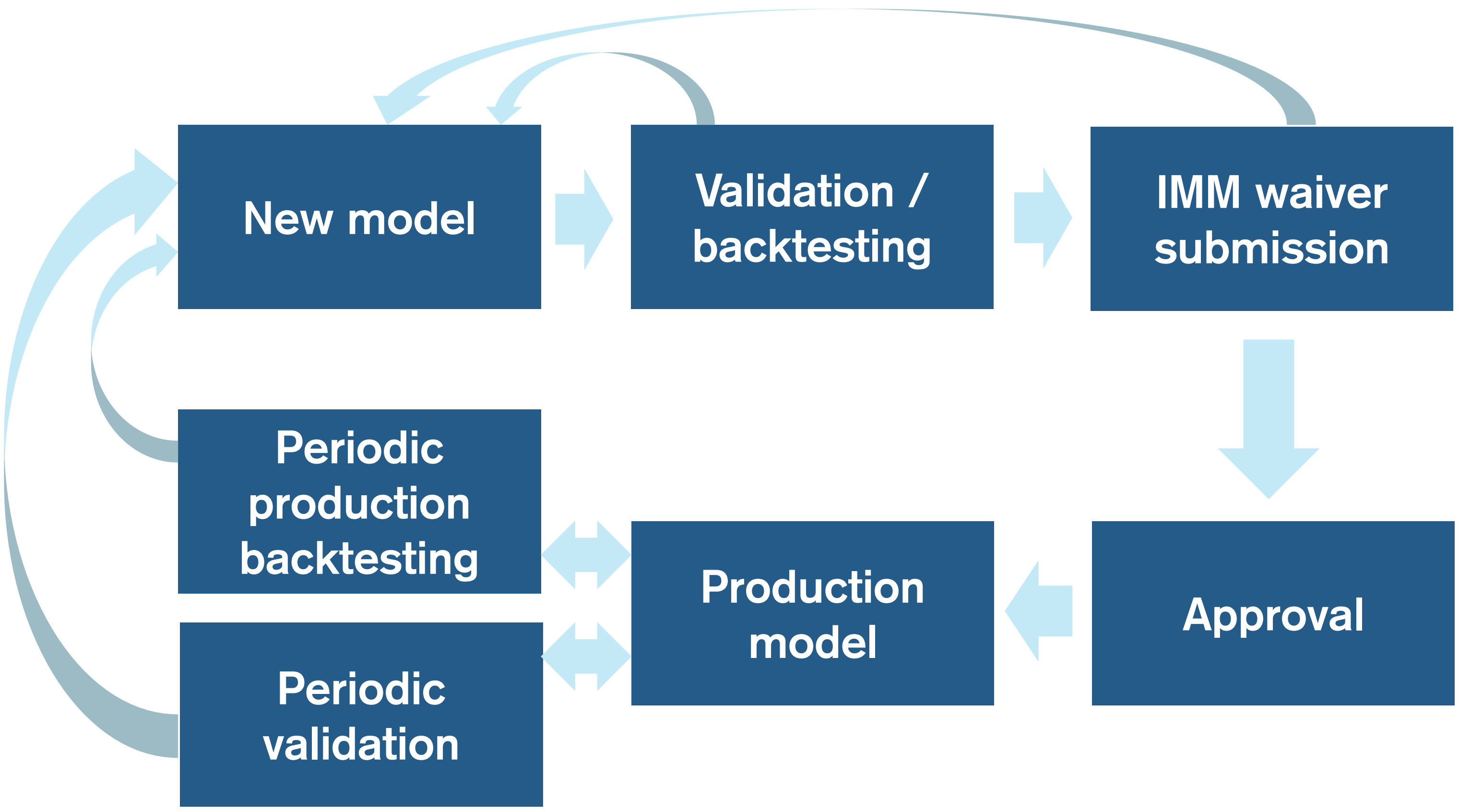
- IMM model governance, CCR, backtesting and Basel III

- A sound B3 compliant framework
  - RF backtesting
  - Correlations backtesting
  - Portfolio backtesting
  - Capital buffers

- Final remarks

- Acknowledgments

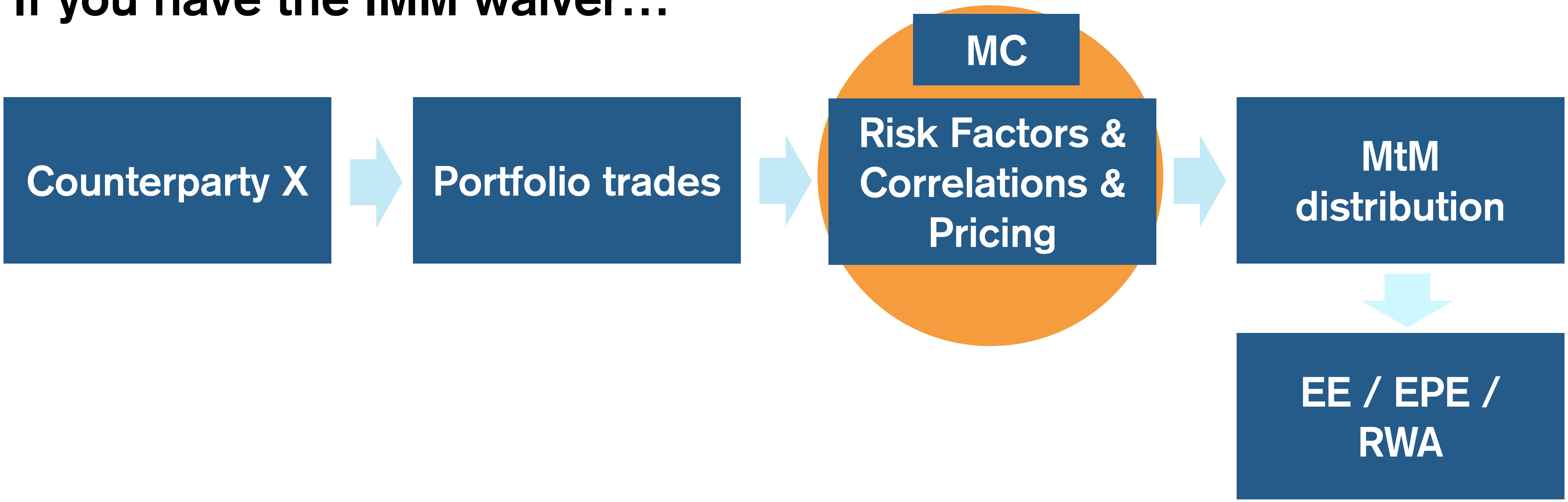
IMM model governance flow diagram



Validation & Backtesting should align their view on assessing model performance. So not to loop endlessy in this diagram…

Case study: Counterparty Credit Risk

If you have the IMM waiver…



- **Model for the RFs →** SDE & historical/market implied calibration
- **Model for the correlations →** historical/market implied calibration
- **Model for the exposure**

Backtesting: how to check your model?

- **Backtesting is based on the real world measure , i.e. we should check the model forecasting performance vs. the realized data history**

- A sufficient amount of history is required for a sensible assessment of the models

- We deal with probabilistic models, i.e. we can accept / reject them only at a certain level of confidence

- Holistic and qualitative, e.g. economical, aspects of the model could be as (if not more) important but they are not in scope

Backtesting: how to check your model and be Basel 3 / CRD4 compliant?

Item	Guidance	Keywords
1	The performance of market risk factor models must be validated using backtesting. The validation must be able to identify poor performance in individual risk factors.	RF BT
2	The validation of EPE models and all the relevant models that input into the calculation of EPE must be made using forecasts initialised on a number of historical dates.	Multiple sampling points
3	Historical backtesting on representative counterparty portfolios and market risk factor models must be part of the validation process. At regular intervals as directed by its supervisor, a bank must conduct backtesting on a number of representative counterparty portfolios and its market risk factor models. The representative portfolios must be chosen based on their sensitivity to the material risk factors and correlations to which the bank is exposed.	Validation & BT
4	Backtesting of EPE and all the relevant models that input into the calculation of EPE must be based on recent performance.	Recent performance BT
5	The frequency with which the parameters of an EPE model are updated needs be assessed as part of the on-going validation process.	Include calibration & BT
6	Firms need to unambiguously define what constitutes acceptable and unacceptable performance for their EPE models and the models that input into the calculation of EPE and have a written policy in place that describes how unacceptable performance will be remediated	Accept / Reject framework
7	IMM firms need to conduct hypothetical portfolio backtesting that is designed to test risk factor model assumptions, e.g. the relationship between tenors of the same risk factor, and the modelled relationships between risk factors	Correlations BT
8	Firms must backtest their EPE models and all relevant models that input into the calculation of EPE out to long time horizons of at least one year	BT horizons at least 1y
9	Firms must validate their EPE models and all relevant models that input into the calculation of EPE out to time horizons commensurate with the maturity of trades covered by the IMM waiver	BT horizons commensurate with the IMM trades maturity
10	Prior to implementation of a new EPE model or new model that inputs into the calculation of EPE a firm must carry out backtesting of its EPE model and all the relevant models that input into the calculation of EPE at a number of distinct time horizons using historical data on movements in market risk factors for a range of historical periods covering a wide range of market conditions	BT for IMM waiver application
11	Backtesting of forecast distributions produced by EPE models and risk factor models should not rely on the assessment of a single risk measure.	Multiple tests
12	The backtesting of EPE models and all the relevant risk factors that input into the calculation of EPE should be performed separately for a number of distinct time horizons. The time horizons considered must include those that reflect typical margin periods of risk.	Include MPR among the BT horizons

BT is a key reg requirement for going IMM and for staying IMM !

Backtesting: our proposed framework

Our framework contains four pillars:

- **(1) The risk factor backtesting**, i.e. the assessment of the forecasting ability of the SDE used to describe the dynamics of the single risk factors
- **(2) The correlations backtesting**, i.e. the assessment of the statistical estimators used to describe the cross-asset evolution
- **(3) The portfolio backtesting**, i.e. the assessment of the complete exposure model (:= SDEs + correlations + pricing)
- **(4) The computation of the capital buffer** , i.e. the extra amount of capital that the firm should hold if the model framework is not adequate

	Diagnostics		Remedy
(1)	RF BT	→	Improve the models
(2)	Correlations BT	→	Improve the models
(3)	Portfolio BT	→	CB & improve the models

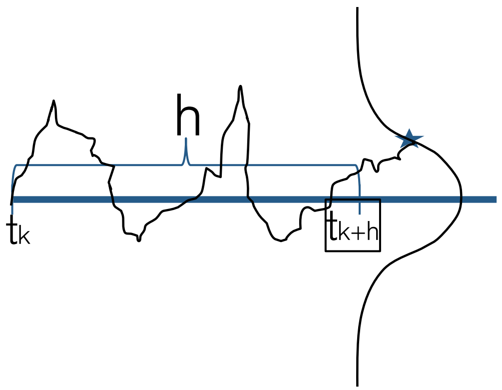
(1) RF backtesting: summary

- **Goal is to assess at distinct horizons the forecasting ability of the SDE (+ calibration) used to describe the dynamics of the single risk factors**
- We propose to backtest both uncollateralized & collateralized models, i.e. one time scales (h) vs. two (h & MPR)
- Our analysis is based on distributional tests (*) on the PIT (see also the book of C. Kenyon & R. Stamm (**))
- For any chosen distributional test, we show how to aggregate results & how to check the discriminatory power given the available data history

(*) Here we refer to Anderson-Darling (AD) and Cramer-Von-Mises (CVM) as working examples
(**) C. Kenyon and R. Stamm, *Discounting, Libor, CVA and Funding: Interest Rate and Credit Pricing*. Palgrave Macmillan, 2012

(1) RF backtesting: the PIT method (realized value)

A)

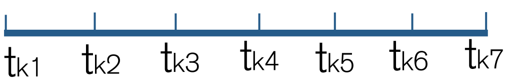


Forecast distribution vs. realized value

The PIT maps the realized value in the [0,1] interval using the forecasted distribution

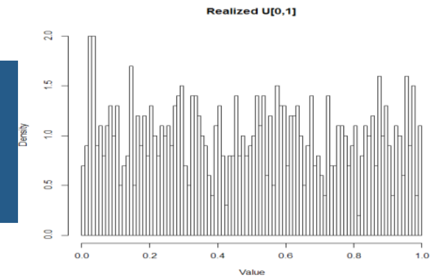
$$u_k = \Phi_{t_k,h}\left(X_{t_{k+h}}\right),\ u_k \in [0,1]$$

B)



Sampling through history

$$u_{k_1}, u_{k_2}, \dots, u_{k_n} \in [0,1]$$



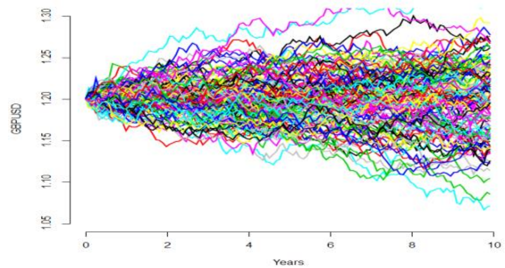
C) CVM $= \int_0^1 (U_n(x) - U(x))^2\, dU(x)$

Distance between exact and realized U(0,1) based e.g. on CVM or AD metrics

Realized test value: Determination of a single value that quantify the distance between exact and realized U(0,1)

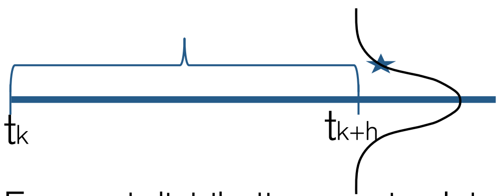
(1) RF backtesting: the PIT method (statistic)

A)



Genaration of simulated paths based on the chosen model

B)

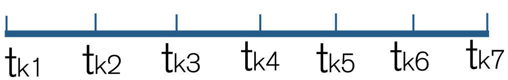


Forecast distribution vs. simulated value

For every path, the PIT maps the simulated value in the [0,1] interval using the forecasted distribution

$$u_k = \Phi_{t_k,h}\left(X_{t_{k+h}}\right), \; u_k \in [0,1]$$

C)



Sampling through history

For every path,

$$u_{k_1}, u_{k_2}, \ldots, u_{k_n} \in [0,1]$$

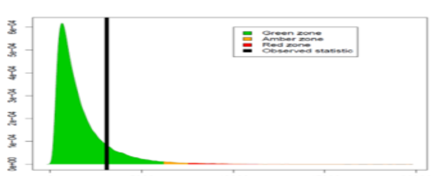
D)

$$\text{CVM} = \int_0^1 (U_n(x) - U(x))^2 \, dU(x)$$

Distance between exact and realized U(0,1)

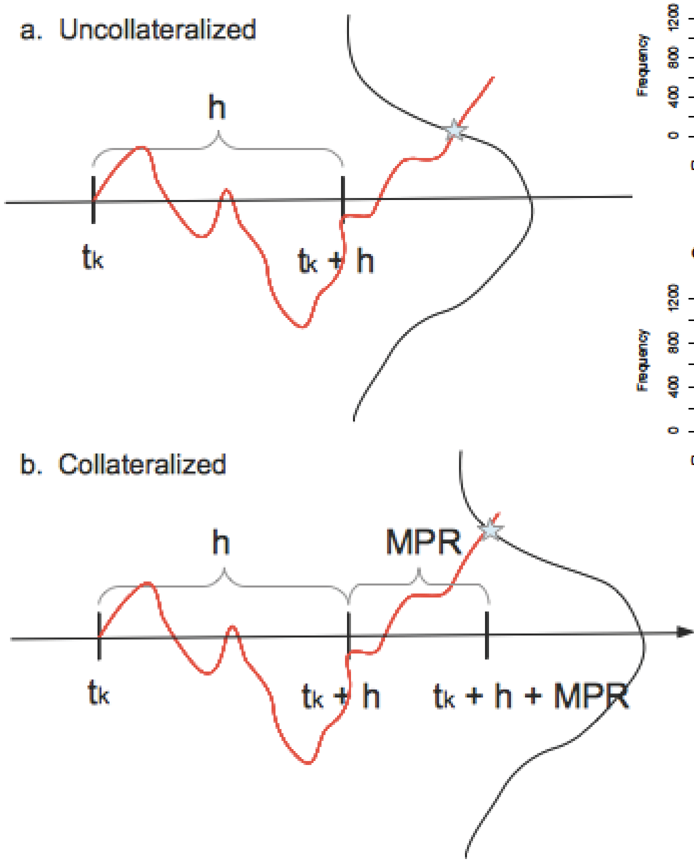
For every path, determination of a single value that quantify the distance between exact and realized U(0,1)

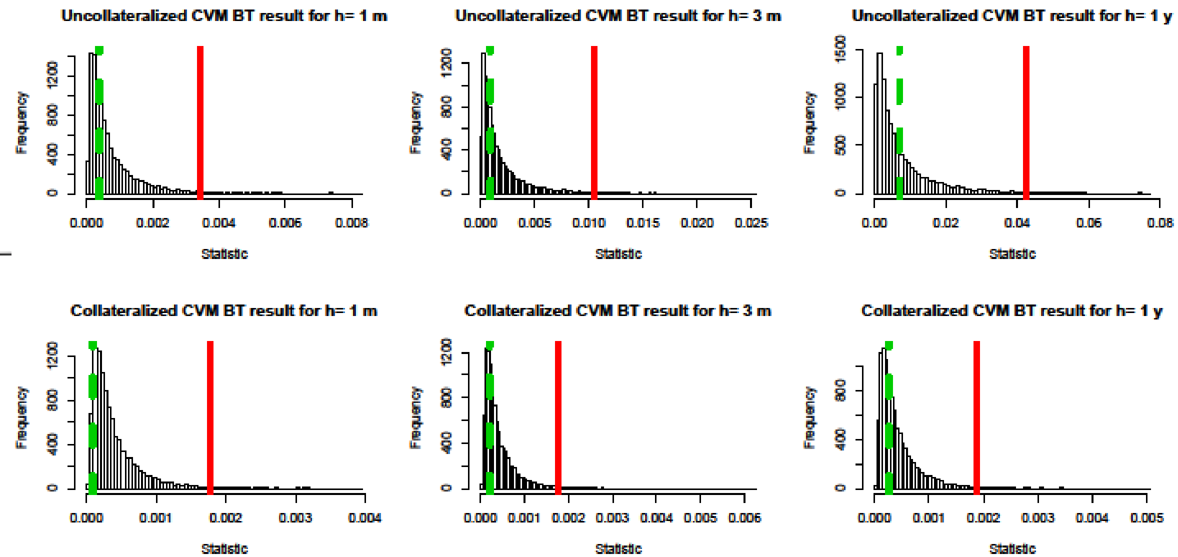
E)



All the values determined in D) define the statistic. The p-value is the fraction of paths that generate a lower distance than the historical path (see realized test value in the slide before)

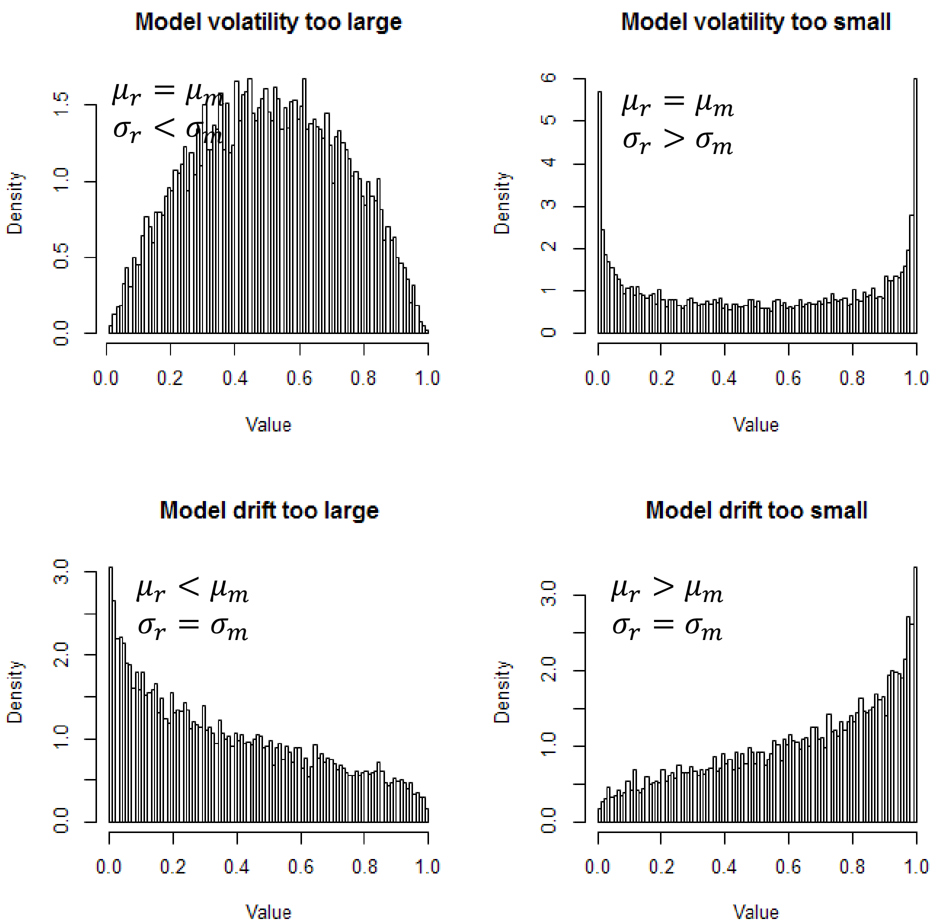
(1) RF backtesting: collateralized vs. uncollateralized





- Uncollateralized RF BT: forecast distribution of the RF at horizon h
- Collateralized RF BT: forecast distribution of the RF variation between h & h+MPR

(1) Qualitative interpretation of the PIT framework



Reality → $\Phi_{norm}(\mu_r, \sigma_r)$

Model → $\Phi_{norm}(\mu_m, \sigma_m)$

A simple BT interpretation framework is key for an efficient feedback loop with model developers!

(*) Similar picture in C. Kenyon and R. Stamm, *Discounting, Libor, CVA and Funding: Interest Rate and Credit Pricing*. Palgrave Macmillan, 2012

(1) RF backtesting: discriminatory power

Results AD 15y 1000 paths

h=1m

Vola / Drift	-5.0%	-2.5%	0.0%	2.5%	5.0%
5.0%	100.00%	100.00%	100.00%	100.00%	100.00%
7.5%	99.83%	99.60%	99.45%	99.59%	99.82%
10.0%	83.22%	62.91%	50.10%	61.36%	83.02%
12.5%	98.05%	96.42%	95.35%	95.96%	97.73%
15.0%	99.96%	99.94%	99.92%	99.93%	99.95%

h=3m

Vola / Drift	-5.0%	-2.5%	0.0%	2.5%	5.0%
5.0%	100.00%	100.00%	100.00%	100.00%	100.00%
7.5%	97.24%	93.40%	91.30%	93.92%	97.65%
10.0%	83.31%	63.23%	49.42%	60.63%	82.76%
12.5%	91.64%	83.53%	78.04%	81.13%	89.59%
15.0%	97.94%	96.33%	95.24%	95.59%	97.10%

h=1y

Vola / Drift	-5.0%	-2.5%	0.0%	2.5%	5.0%
5.0%	98.81%	97.17%	96.60%	97.67%	99.24%
7.5%	90.37%	77.51%	70.19%	78.57%	91.91%
10.0%	83.34%	63.07%	49.60%	60.70%	82.48%
12.5%	84.58%	68.45%	57.07%	63.42%	80.37%
15.0%	88.68%	78.73%	71.47%	73.61%	83.24%

- We simulate synthetic histories and check the average p-value for given mis-parametrizations of the model
- Useful complementary information to assess how much you can(not) assess…

(2) Correlations backtesting: summary

- **Goal is to assess at distinct horizons the forecasting capability of the statistical estimators used to describe the cross-asset evolution**
- For a given pair of assets, we generate a synthetic risk factor where the correlation has the highest possible impact
- We backtest the synthetic risk factor with the PIT framework (see RF BT)
- We show how to produce aggregated results across the different entries of the correlation matrix

(2) Correlations backtesting: the synthetic RF

- The example of GBM:

$$RF_1(t) = RF_1(t=0)e^{\mu_1 t + \sigma_1 W_1(t) - \sigma_1{}^2 \mathrm{t}/2}$$
$$RF_2(t) = RF_2(t=0)e^{\mu_2 t + \sigma_2 W_2(t) - \sigma_2{}^2 \mathrm{t}/2}$$
$$dW_1(t)dW_2(t) = \rho dt$$

- We define a synthetic RF Z as a drift-less GBM

$$Z_{1,2}(t) = RF_1(t)^{1/\sigma_1} RF_2(t)^{1/\sigma_2} \times e^{\frac{(\sigma_1+\sigma_2)t}{2} - \frac{(2+2\rho)t}{2} - \left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right)t}$$

- The volatility of Z is a function of the correlation $\sigma_Z = \sqrt{2(1+\rho)}$ and we can backtest Z as a single RF

- If the RF 1 & 2 fail BT, also Z will most likely fail BT. The opposite is not true

(2) Correlations backtesting: discriminatory power and aggregation

Aggregation

AD 15 years correlation backtesting for h=1y

	GBPUSDFX	CHFUSDFX	JPYUSDFX	USD1y	EUR1y	GBP1y	CHF1y	JPY1y	S&P500	ESX50
EURUSDFX	50.5%	59.9%	61.3%	94.8%	97.5%	94.9%	94.6%	77.2%	73.0%	54.9%
GBPUSDFX		57.8%	54.1%	96.1%	96.2%	92.3%	94.8%	60.4%	73.6%	54.9%
CHFUSDFX			83.4%	93.4%	97.8%	91.6%	94.2%	60.5%	88.6%	82.7%
JPYUSDFX				93.1%	98.2%	92.8%	96.8%	40.1%	84.9%	73.1%
USD1y					99.9%	99.7%	99.8%	99.6%	99.3%	99.4%
EUR1y						99.7%	99.8%	99.6%	98.3%	99.3%
GBP1y							99.6%	98.0%	92.9%	97.5%
CHF1y								99.4%	98.9%	99.6%
JPY1y									48.8%	82.5%
S&P500										82.2%

FX	66.1%
EQ	85.1%
IR	100.0%
ALL	100.0%

CVM 15 years correlation backtesting for h=1y

	GBPUSDFX	CHFUSDFX	JPYUSDFX	USD1y	EUR1y	GBP1y	CHF1y	JPY1y	S&P500	ESX50
EURUSDFX	47.5%	62.3%	58.9%	59.0%	74.9%	81.2%	72.5%	63.1%	74.1%	57.1%
GBPUSDFX		62.0%	63.3%	61.6%	74.2%	72.4%	72.1%	45.7%	75.6%	55.4%
CHFUSDFX			83.9%	59.1%	80.1%	74.3%	76.1%	58.7%	91.4%	86.9%
JPYUSDFX				71.6%	81.4%	75.6%	81.6%	43.6%	88.4%	78.3%
USD1y					96.4%	89.9%	91.8%	93.3%	90.2%	83.6%
EUR1y						89.5%	93.9%	86.5%	81.9%	87.5%
GBP1y							90.1%	84.9%	71.7%	82.3%
CHF1y								90.7%	90.0%	89.6%
JPY1y									18.4%	69.1%
S&P500										83.3%

FX	69.0%
EQ	85.3%
IR	96.0%
ALL	95.0%

- We produce p-values by blocks of the correlation matrix
- We aggregate the PITs across the Zs corresponding to the different elements of a given block
- We generate the corresponding test statistics distribution based on correlated paths generated by the model correlation matrix

Discriminatory power

CVM 15 years Correlation BT

rho \ horizon	1m	3m	1y
-0.99	100.00%	100.00%	99.95%
-0.5	99.98%	97.94%	81.18%
0	88.38%	71.70%	59.17%
0.5	51.10%	51.07%	52.01%
0.99	76.24%	62.32%	54.18%

AD 15 years Correlation BT

rho \ horizon	1m	3m	1y
-0.99	100.00%	100.00%	100.00%
-0.5	100.00%	99.86%	91.12%
0	95.58%	80.26%	63.79%
0.5	51.41%	51.34%	51.98%
0.99	81.97%	65.14%	53.82%

(3) Portfolio backtesting: summary

- **Goal is to assess at distinct horizons the complete exposure model (:= SDEs + correlations + pricing (*))**
- We backtest both uncollateralized & collateralized representative portfolios
- We use the PIT framework to convert the forecast MtM distributions and realized MtM values in a sequence of U[0,1] variables
- We introduce a weighted distributional test CPT based on the exposure metrics relevant for RWA
- Notice that in the case of portfolio BT, a notion of conservatism can be applied, i.e. the model should not fail BT if the forecasted EE is higher than the correct one → uni-directional distance

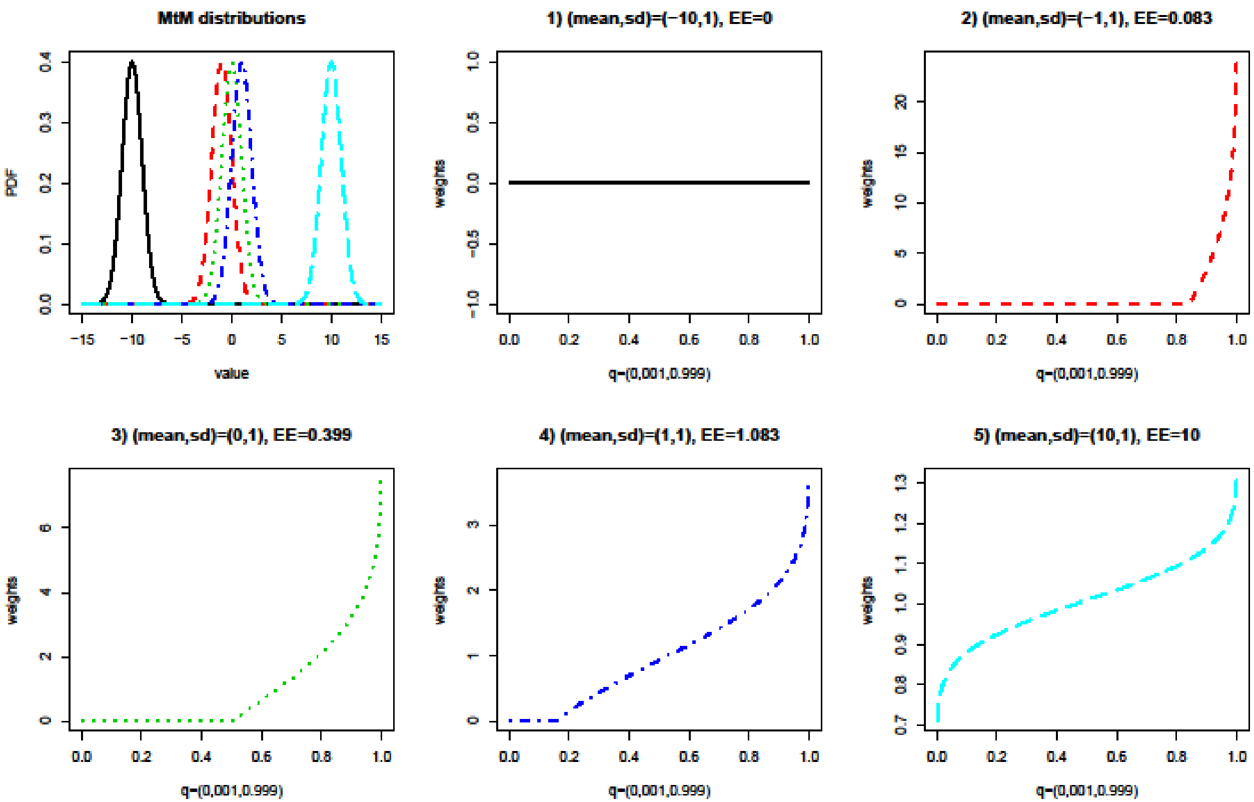
$$CPT = \int_0^1 \left(max(F(x) - F_n(x), 0)\right)^2 \omega(x) dF(x)$$

(*) Here we assume that the pricing is exact

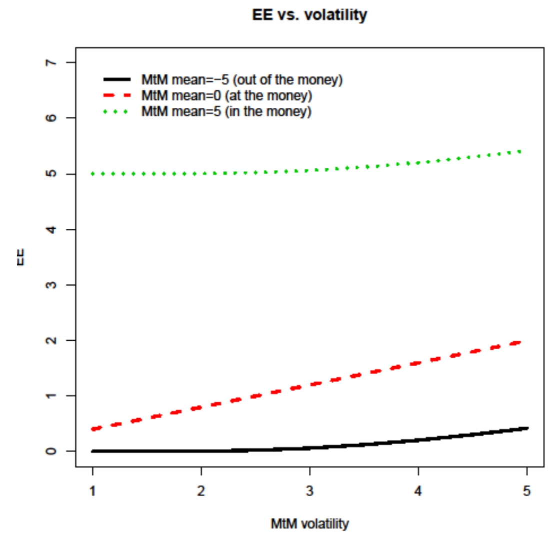
(3) Portfolio backtesting: the metrics $\omega(x)$

How much a given quantile contributes to the EE?

$$\omega(q) = \lim_{\delta q \to 0} \frac{\max\left(\frac{1}{\delta q}\int_{\Phi^{-1}(q)}^{\Phi^{-1}(q+\delta q)} x\varphi(x)dx, 0\right)}{EE} = \frac{\max(\Phi^{-1}(q), 0)}{EE}$$



How important is the volatility for the EE?



(3) Portfolio backtesting: the CPT test

- We consider the case of MtM distributions centered at 0 and a Gaussian

- The weight function can be calculated and is independent on the MtM volatility (*)

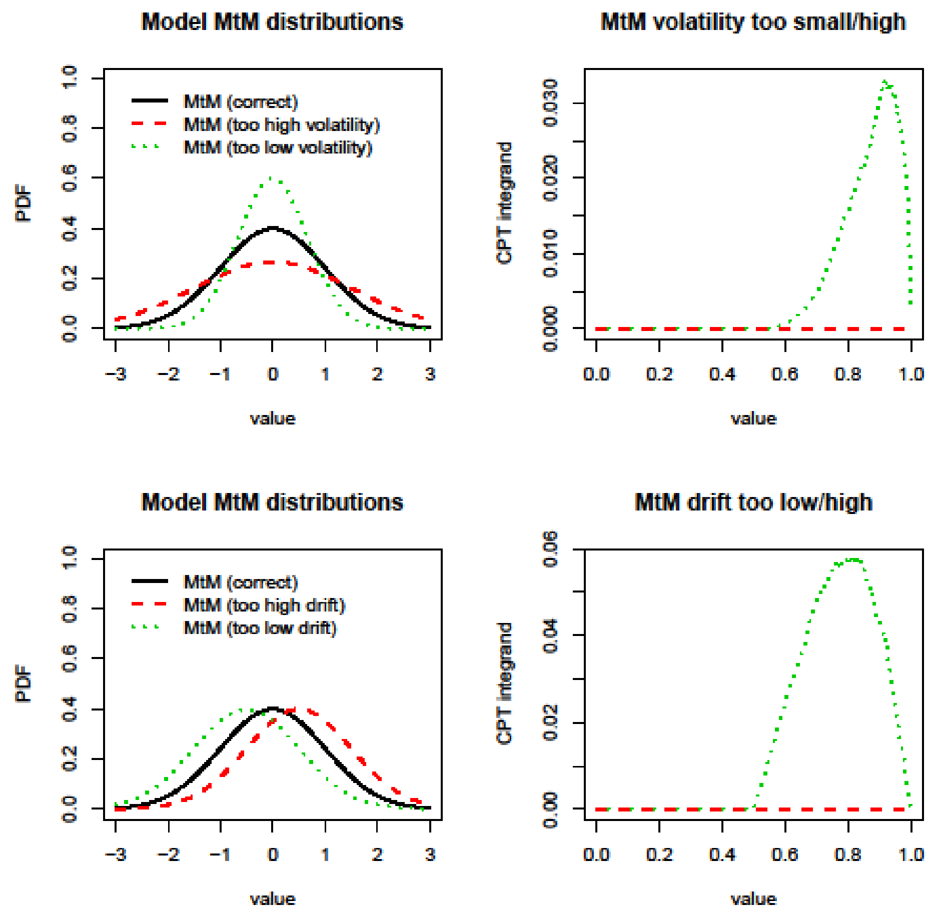
$$\omega(x) = \frac{\max(\Phi^{-1}(x), 0)}{EE} = \frac{\max(\sqrt{2}EF^{-1}(2x-2), 0)}{\varphi_{N(0,1)}(0)}$$

- We define CPT as

$$CPT = \int_0^1 \left(max(F(x) - F_n(x), 0)\right)^2 \omega(x) dF(x)$$

- The max function ensures the uni-directionality of the test, i.e. no distance accrued when the MtM distribution is conservative

- The weight function depends on the typical moneyness. This information can be e.g. extracted/customized based on the average MtM level of the counterparties in the portfolio



(*) The independence on the volatility holds beyond the Gaussian assumption

(4) The calculation of the capital buffer: summary

- **The capital buffer is the extra amount of capital that the firm should hold if the model framework is not adequate**
- The capital buffer, defined as multiplicative factor (1+CB) ≥ 1, has the following features:
  - It should be punitive, i.e. the firm should not spare RWA keeping the wrong model and paying a buffer
  - It should be 0 for the correct model (*)
  - (1+CB) x RWA should be capped by the RWA determined by standard rules
- Given the above, we determine the CB based on the portfolio BT performance and on the EE forecasting capability of the models for the current firm portfolio through different historically realized market conditions
- Observe that the RF & correlations BT is a tool to identify issues with the current methodology but does not enter the computation of the CB

(*) Given the finite amount of data history, any model (also the correct one, if any) could potentially cause a small buffer

(4) The calculation of the capital buffer: stress-test based on available history

- For any counterparty c of the N in scope for portfolio BT, the error in exposure estimation $\Delta E_{c,t_1}$ is calculated as the average over the deviation between the realised and the expected exposure as forecasted at t1over the incoming year

$$\Delta E_{c,t_1} = \langle \max(0, MTM_{c,t_2,\mathrm{real}}) - EE_{c,t_1,t_2} \rangle_{t_2 \in [t_1, t_1+1y]}$$

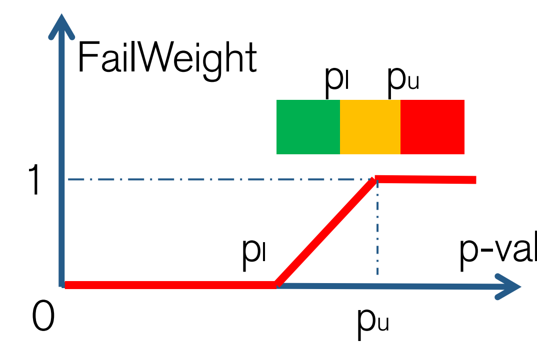
- Define any counterparty c a weight for the contribution to the capital buffer. The weight is dependent on the results of the portfolio BT according to RAG

$$\tau(\mathrm{c}) = min\left(\frac{max(p_{value,c} - p_l; 0)}{p_u - p_l}; 1\right)$$

- We aggregate by multiplying $\Delta E_{c,t}$ with $\tau(c)$ and summing through c. The denominator adjusts for the level of capital (that may vary over the years) and produces a % figure. We obtain the capital buffer factor K_t year t as $K_t = \frac{\sum_{c=1}^{N} \tau(\mathrm{c}) \cdot \Delta E_{t,c}}{\sum_{c=1}^{N} EEPE_{t,c}}$

- We average K_t over the available history and floor it to zero

$$CB = \max(\langle K_t \rangle_t, 0)$$



(4) The calculation of the capital buffer: is it punitive (and meaningful) ?

Capital Buffer (15 years, 1000 paths)

Volatility / drift	-20%	-10%	0%	10%	20%
50%	20.62%	13.87%	8.63%	4.88%	2.51%
60%	14.99%	9.83%	5.95%	3.25%	1.59%
70%	10.95%	6.90%	3.96%	2.00%	0.93%
80%	7.86%	4.66%	2.44%	1.16%	0.51%
90%	5.43%	2.93%	1.44%	0.64%	0.26%
100%	3.52%	1.77%	0.81%	0.33%	0.13%
110%	2.18%	1.02%	0.43%	0.17%	0.06%
120%	1.31%	0.56%	0.23%	0.08%	0.03%
130%	0.75%	0.31%	0.12%	0.04%	0.01%
140%	0.43%	0.17%	0.06%	0.02%	0.00%
150%	0.25%	0.10%	0.03%	0.00%	0.00%

EEPE total – EEPE correct (15 years, 1000 paths)

Volatility / drift	-20%	-10%	0%	10%	20%
50%	14.95%	9.15%	4.96%	4.01%	4.38%
60%	9.94%	5.72%	2.91%	2.94%	4.00%
70%	6.54%	3.43%	1.60%	2.33%	3.99%
80%	4.13%	1.88%	0.84%	2.23%	4.32%
90%	2.42%	0.90%	0.66%	2.52%	4.91%
100%	1.26%	0.54%	0.92%	3.14%	5.71%
110%	0.72%	0.66%	1.53%	3.98%	6.65%
120%	0.71%	1.16%	2.40%	4.97%	7.71%
130%	1.09%	1.96%	3.44%	6.08%	8.85%
140%	1.80%	2.94%	4.59%	7.28%	10.06%
150%	2.73%	4.06%	5.83%	8.54%	11.33%

- We consider 1000 synthetic histories for a toy model of 20 counterparties with MtM described as a BM process
- For every history, we realize the full cycle, i.e. EE forecast, portfolio backtesting, CB calculation
- We consider a series of model mis-parameterizations and we compare the adjusted EEPE (i.e. wrong model EEPE + CB) with the correct EEPE. In all cases the difference is positive (i.e. punitiveness)
- We see also that, where the model is more conservative, the CB is only due to residual noise

Final remarks

- **Backtesting is a key requirement for IMM and his importance is highly emphasized with a devoted set of guidances in Basel III**
- **We propose a simple coherent approach to reach Basel III compliance and meaningfully backtest credit exposure models**
- The central component of the framework is the PIT that accounts for the time dependent changes of the forecasted distributions across the three diagnostic pillars
- Based on the results of the portfolio BT (that uses an EE inspired metrics), we show a possible way to calculate the capital buffer

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  - **Dr. Andreas Nawroth**, Senior Analyst in the IB CCR backtesting methodology team at CS AG

For more details:
A Sound Basel III Compliant Framework for Backtesting Credit Exposure Models
Reference: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2264620