UBS

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Benchmarking and backtesting techniques in model validation

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Model Risk and Model Validation

The rules of the game: Accounting Policies

The Fair Value mirage

- **FAS 115**
  "Debt or equity securities that are bought or sold with the purpose of selling them in the near term are classified as trading and reported at fair value, with unrealised losses and gains included in the earnings"

- **FAS 157**
  "Fair value is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date"
  "Valuation techniques consistent with the market approach, income approach, and/or cost approach shall be used to measure fair value"
  *"If multiple valuation techniques are used to measure fair value, the results shall be evaluated and weighted .... A fair value measurement is the **point** within the range that is most representative of fair value in the circumstances"*

Fair value is the (would-be) exit price: **market is the king**

Even for model-based valuations the fair value is a single number, not a range of possible outcomes: **no error bars**

Strictly speaking conservative(/prudential) valuations are not permitted

Accounting Policies and the myth of the infallible man

The need for error bars

- **FX contingent forward**
  - Description: standard FX fwd which pays only if a pre-specified M&A event takes place. Example: *Chips Corp (US)* wants to buy *The Gravy Ltd (UK)* and fix now the GBPUSD rate

if 

Corp **buys** The 

Ltd

YES → Fwd_{FX}

NO → 0

  - Accounting policies impose that this position is recognized at fair value

$$Fwd_{contingent} = p \cdot Fwd_{\mathrm{FX}}$$ no model for the underlying is needed

  - *p* is the probability that the M&A event is successful, cannot be estimated precisely (especially for non-traded companies or undisclosed M&A events)

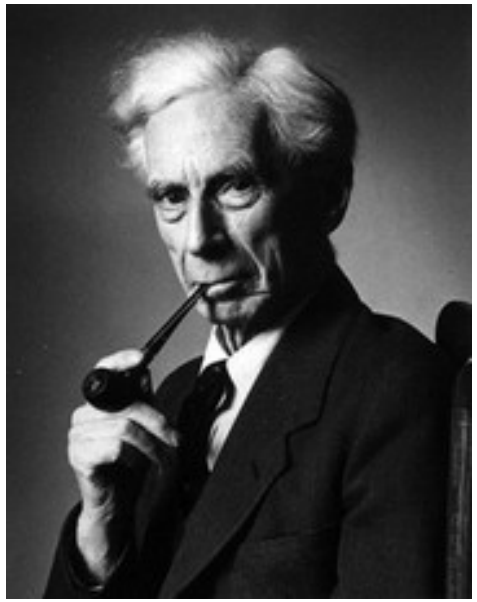
Can we be expected to calculate a reliable single point fair-value? Does actually the fair value concept make sense here?

Exact Sciences: a humbler view on human capabilities

The need for error bars

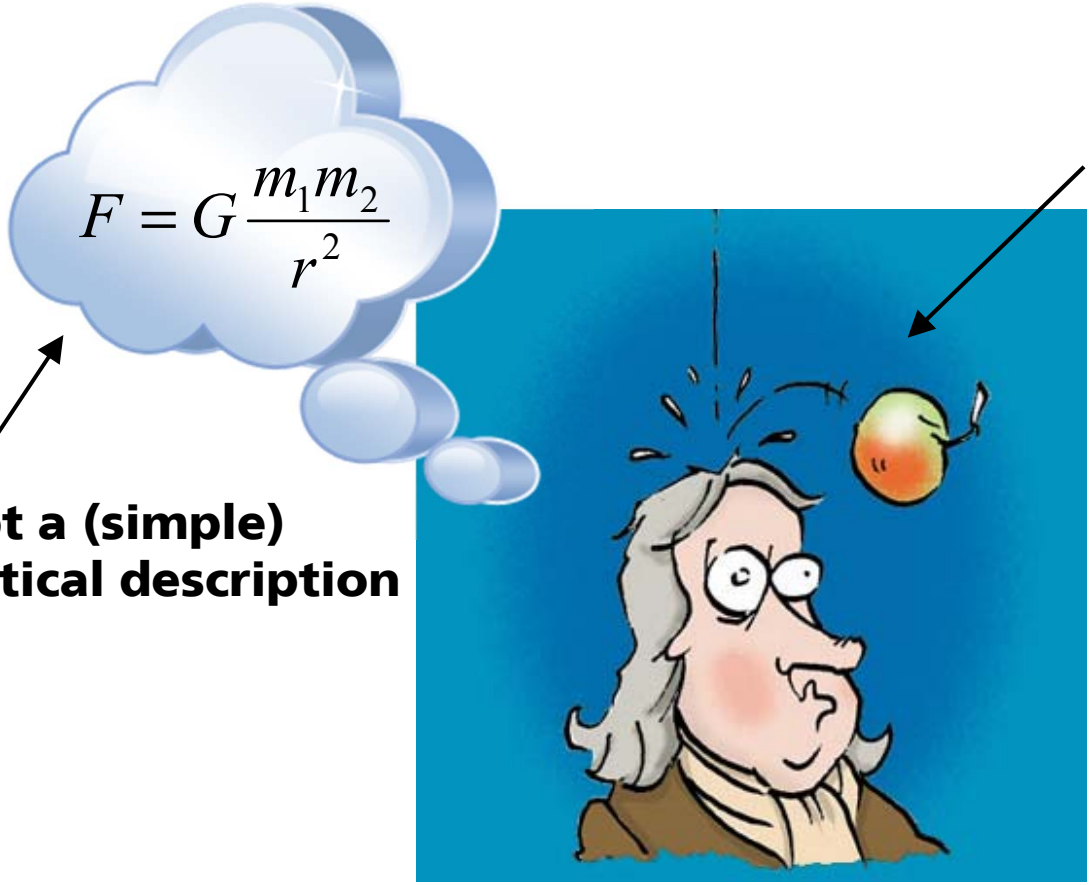
Bertrand Russell, The Scientific Outlook, 1931

"Although this may seem a paradox, all exact science is dominated by the idea of approximation. When a man tells you that he knows the exact truth about anything, you are safe in inferring that he is an inexact man. Every careful measurement in science is always given with the probable error... every observer admits that he is likely wrong, and knows about how much wrong he is likely to be"



Quick digression on models: physics vs finance

Successful models in physics: gravitational law



1) reproducibility (nature doesn't change)

tomorrow apples still fall down (and are equally painful).

2) consider salient features only (simplify)

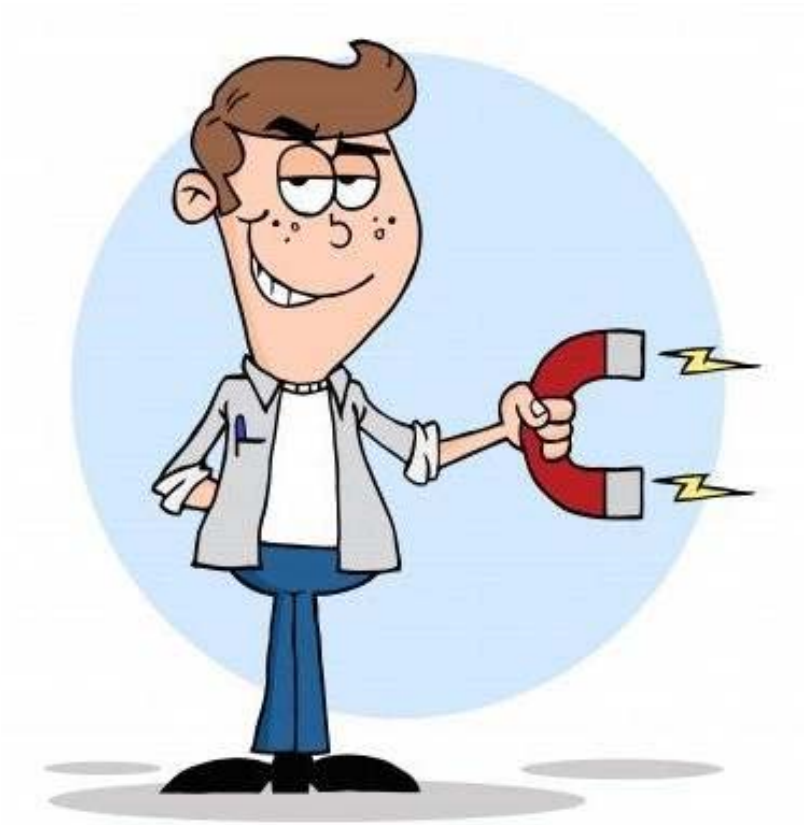
- point-wise objects
- consider vacuum
- remove irrelevant aspects

3) attempt a (simple) mathematical description

Very successful models become laws...

Quick digression on models: physics vs finance

Successful models in physics: ferromagnetism and Curie temperature

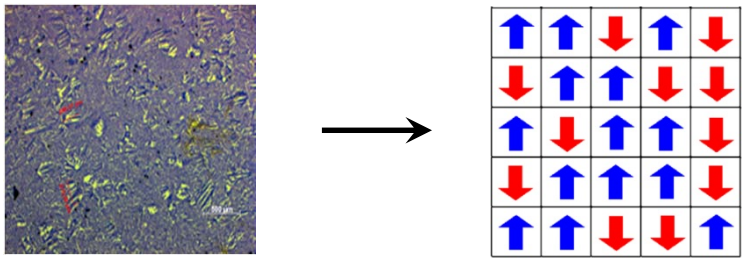


1) reproducibility

2) simplify

neglect: dodgy character, U-shape, colour, exact chemical structure, exact mechanism and type of dipole-dipole interaction, quantum mechanical effects, impurities...

Ising model: magnetic dipoles in a square lattice with attractive nearest-neighbour interaction



3) simple mathematical description

Crude model helps understanding the mechanism of ferromagnetism but does not predict the exact details (e.g. exact Curie temperature)

Model is approximate (but it can be refined)

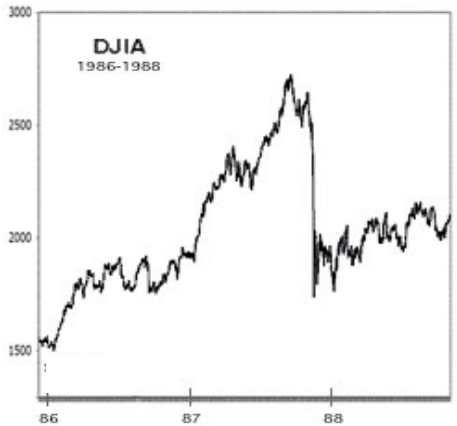
Quick digression on models: physics vs finance

Finance, the softest of the soft sciences (E. Derman)

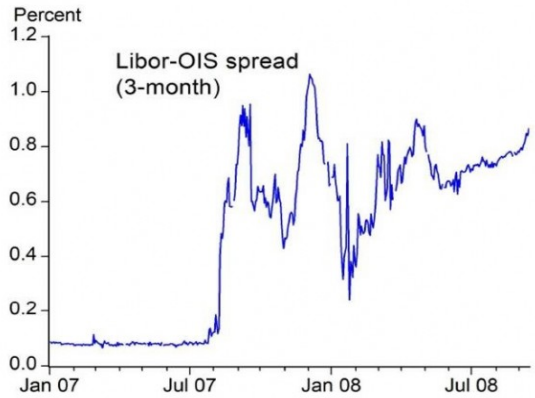
1) reproducibility? No, finance markets keep on changing

- Markets are not governed by universal and constant laws of nature
- Moving target governed by human beings, their (often irrational) behavior, interactions and limited knowledge
- Price discovering is a continuous process, as new info becomes available and new players join in: **no lifetime guarantee**!

Black Monday (1987)



Credit Crunch (2007-2008)



Karl Popper, The Logic of Scientific Discovery, 1934

"*No matter how many instances of white swans we may have observed, this does not justify the conclusion that all swans are white.*"

Quick digression on models: physics vs finance

Finance, the softest of the soft sciences (E. Derman)

2) simplify (model assumptions)

- **model choice**: depending on the assumption
  - Eg, Black-Scholes: no vol smile, deterministic rates, no jumps, no transaction costs,…
- **calibration choice**: choice of calibration instruments and calibration routines
  - Eg. stoch vol model: which points in the smile to calibrate to,…

3) simple mathematical description

$$dS_t = \mu_t S_t dt + \sigma_t S_t dW_t$$

It looks beautiful but it builds on many (some unrealistic) assumptions (cf. Ising model)

- Not possible to obtain a unique price with 100% confidence, despite of FAS 157
- Models are useful only as long as the users are aware of their limitations
- Blindly trusting any model, despite of its mathematical beauty, is a dangerous practice
- **Math can create unjustified overconfidence/legitimization**

The beauty of simpler things

Beware of the Math

J.M. Keynes, The General Theory of Employment, Interest and Money, 1935

"Too large a proportion of recent `mathematical' economics are mere concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols."

- Recent developments have painfully (re)-opened our eyes to this fundamental point
  - The run for most complex and over-engineered products seems to be over (for now)
  - Focus back on simpler and allegedly better understood derivative structures
- Is it the end of quants era? No, rather a paradigm shift in derivative modeling

Derivative (over)-engineering → **Management of model risk**

What regulators say about model risk

R: Please do something. Q: Sure, Sir, but what?

- Regulatory pressure on model risk management is growing
- Accounting principles make the implementation of conservative valuations not easy

  – **FED/OCC**: Supervisory Guidance on Model Risk Management, 2011

  *"**Model risk should be managed like other types of risk. Banks should identify the sources of risk and assess the magnitude**."*
  *"**It can be prudent for banks to account for model uncertainty** by explicitly adjusting model inputs or calculations to produce more severe or adverse model output in the interest of conservatism"*
  *"However, **conservatism can become an impediment to proper model development**"*
  Footnote*: "To the extent that models are used to generate amounts included in public financial statements, **any adjustments for model uncertainty must comply with generally accepted accounting principles**."*

  – **FSA**: Dear CEO letter on Valuation and Product Control principles, 2008

  *"...firms must be prepared to take extra steps to deal with less liquid, complex and/or modeled products. The increased valuation uncertainty that can be associated with these products is likely to precipitate the need for **increased focus on the regulatory principles on prudent valuation**.... **Adjustments to Tier One capital are required** where prudent value is assessed as being materially below fair value as a result of issues as illiquidity, concentration, and model risk. **This situation may arise as accounting standards for the measurement of fair value may not permit firms to reflect fully these factors in the valuations** in their financial statements"*

From Model Risk to Model Validation

Independent Verification Unit and Model risk governance

1. Assessment of model choice

- Understand impact of approximations involved in the model definition
- Estimate the uncertainty coming from different models/calibrations
- `Effective challenge'

2. Performance review

- Implementation tests + stress-tests
- Back-testing analysis
- Price replication vs optimal hedging ratios/strategy

3. Model Reserves

- Definition and Analysis
- Prudential valuations (aka AVA)
- PnL suspension or capital charge?

4. Ongoing Monitoring

- 5y review cycle (1y for `high risk' models)
- calibration frequency
- portfolio reviews

A lot is left to us to do - even if the product types are simpler than in the past

FX case studies

Model risk for (relatively) simple FX derivatives

Does a transparent exotic derivative market imply a low model risk?

Single dimensional FX options

- Vanilla options
  - vol interpolation methods
  - definition of risk sensitivities
- Barrier options
  - transparent broker market

Tough test: FX models must hit both vanillas and barriers...

Multi-dimensional FX options

- Basket and Worst-of Options
  - cross volatility smile vs model implied correlation structure
  - consistent extension from 1-dim model

FX Barriers: the quest for the Fair Value

Market inference analysis

- FX barrier options (knock-out, reverse knock-out, single or double one/no touches, ...)
  - Very liquid broker market
  - Uncommonly transparent exotic market
  - Stringent constraints on model choice

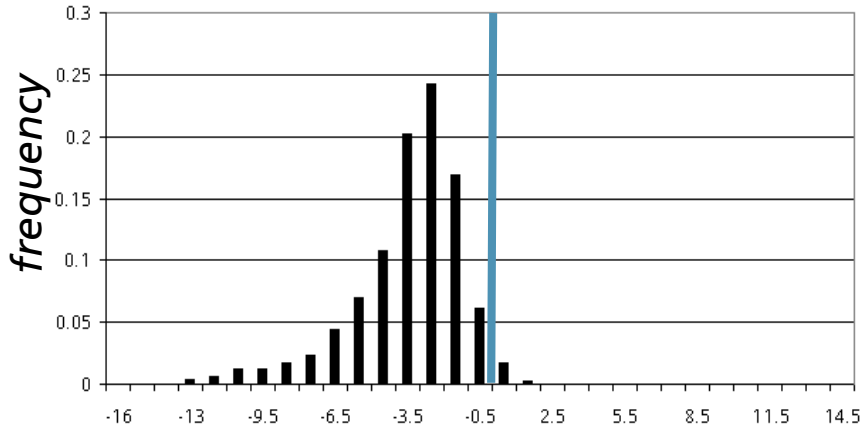
- Back-testing analysis of different models

$$E_i = \frac{\text{model}_i - \text{midprice}}{\text{bid/ask spread}}$$

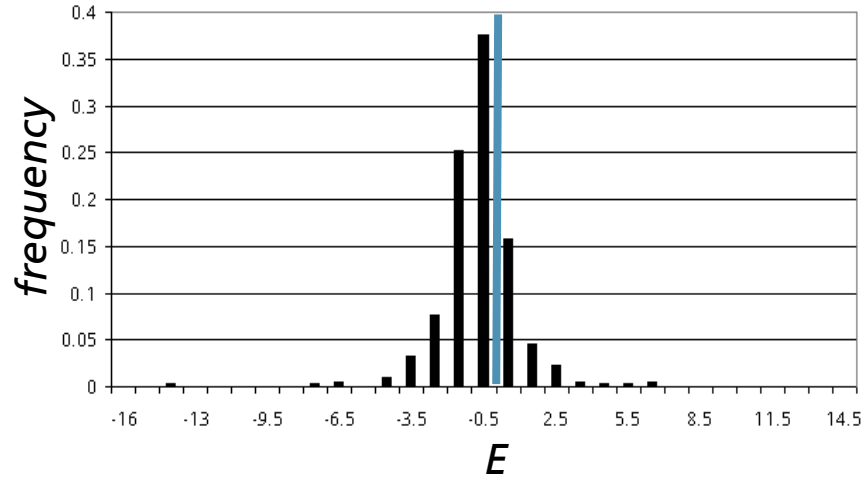
How to read the histogram:

- Good model: narrow peak around zero
- Bad model: bias and large stdev
- Note that outliers are present

Bad model



Good model



The market is the king (FAS 157)

Fair Value: is the market a good king?

- Market inference analyses are important model tests whenever market data is available
  - Data is often owned by the trading desk. Auditors might not accept it
  - Needs to be complemented with standard Markit submissions
- Past performance is not an indication of future one: a regular review process is needed

Market standard choices will be able to hit the market:
Are they good quantitative predictors or just self-fulfilling prophecies?

- Market is king, but the king can change his mind…
- Ideally, a derivative model is able to predict the total costs of holding and hedging a position
- Future hedging costs vs Fair Value (exit value):
  - For short maturity portfolios, FO can decide to deviate from the exit price, if it believes in a better model
  - Initial PnL hit is then regained before maturity through trading profits
  - Differences between 'Pricing' (quotes to clients) and 'Valuation' (official PnL reporting)

Model validation should include realistic back-testing hedging analysis
(including an assessment of definition of the risk sensitivities)

Hedging Simulation: Vanilla Options

- The initial value is the best estimated of future hedging costs, is it so?
  (e.g. long gamma, one buys low and sell high while delta hedging)
- FX Vanilla options:
  - price is quoted (semi)-directly in the market
  - correct risk sensitivities are model dependent and affect the hedging strategy and its cost
- E.g. sticky strike or sticky delta?

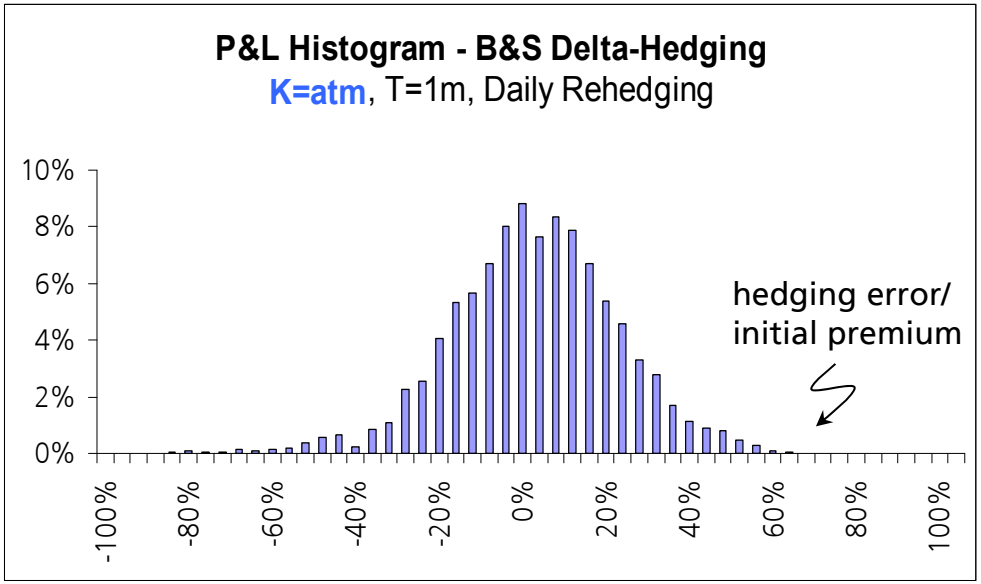
  […] a foreign exchange smile is typically a 'U' shape with the bottom of the 'U' at-the-money. If we modeled our smile by a sticky smile model then we would be saying that at-the-money would be up the sides of the 'U' if spot changed. Clearly this would be a poor model for market behavior. We therefore conclude that for foreign exchange we need a floating smile model. [M. Joshi, 2003]

  - *Implied volatilities are in foreign exchange quoted over delta, in contrast to how they are quoted in the stock market.*

Hedging Simulation: Vanilla Options

Historical hedging simulation of a single FX option

sell one option, hedge it over its life-time using historical market data, and repeat the experiment



Portfolio

- short an option, e.g. USDJPY
- domestic cash, e.g. JPY
- foreign cash (delta), e.g. USD

Simulation (over historical data)

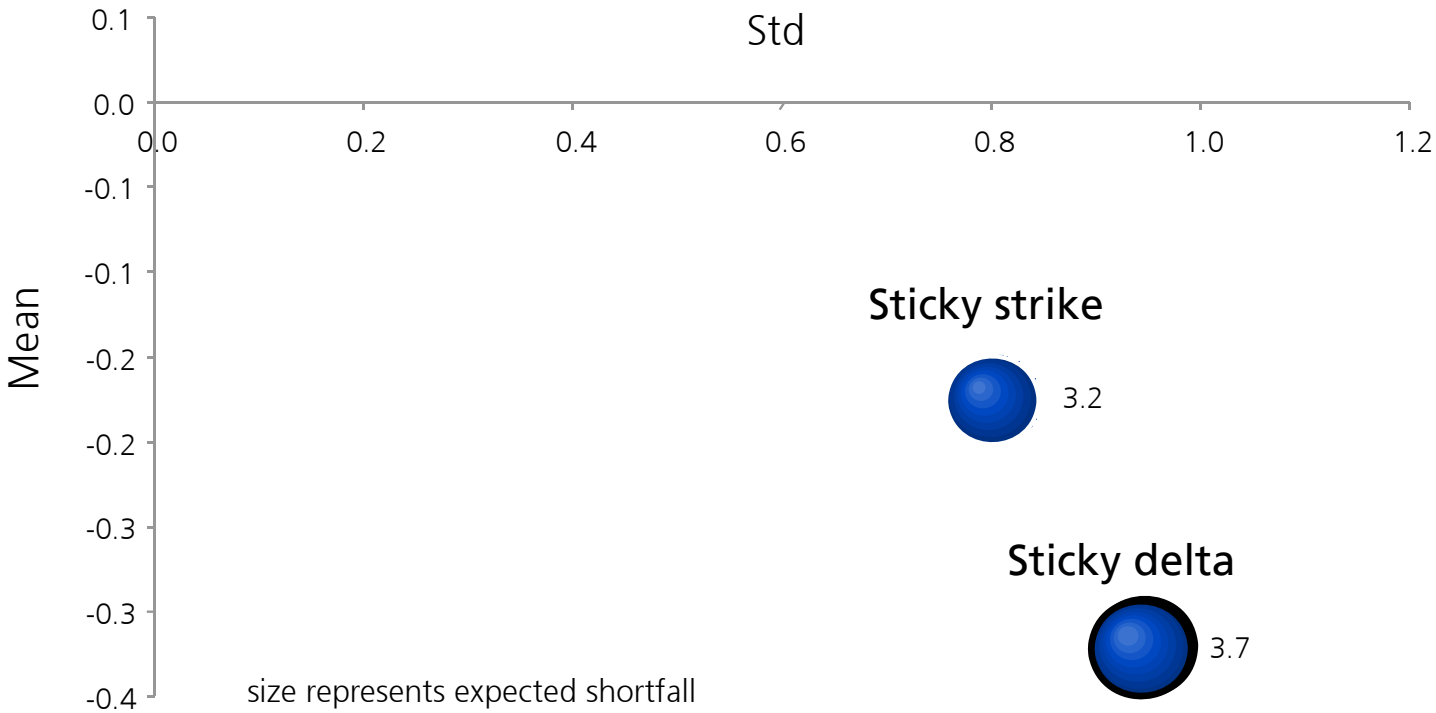
- option value is known at each time
- <u>delta amount is model dependent</u>
- record final PnL and repeat

Why PnL is not zero? (aka why don't we live in an ideal world?)

- implied vol is an estimate of the future realised vol (cannot be eliminated by using a diff model)
- joint vol-spot dynamics (model prediction of volatility change)
- discrete hedging frequency (hedging strategy)
- minor effects: transaction costs and interest rates changes

Hedging Simulation: Vanilla Options

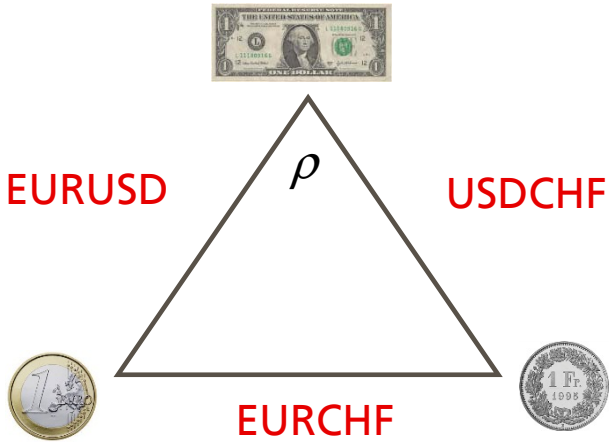
Single FX option – PnL distribution



Sticky strike performs better than sticky delta!? Don't trust rumors

Mathematically inconsistent sensitivities can yield superior results in practical applications (e.g. sticky strike on local vol)

Simple multi-currency FX derivatives



Black-Scholes: vol triangulation

$$\sigma^2_{\text{EURCHF}} = \sigma^2_{\text{EURUSD}} + \sigma^2_{\text{USDCHF}} + \rho\sigma_{\text{EURUSD}}\sigma_{\text{USDCHF}}$$

What about smiles? Extension from 1-dim to N-dim

No standard model choice

Valuation of FX baskets

- Liquid cross pairs (EUR vs USD, JPY, and CHF)
  - modelling of the cross skew is relevant
  - standard copula/const correlation approaches cannot include all available market info
- Illiquid cross pairs (USD denominated BRIC)
  - similar to Equities: correlation is an unobservable parameter

Same product, but different models and different model risk

Simple multi-currency FX derivatives

No standard choice of multi-dim models with skew

Validation (liquid case)

- Consistency with
  - all vanilla surfaces and basket market prices (if available)
  - 1-dim modelling
  - typical FX symmetries

inversion	$S_{\mathrm{EURCHF}} = 1 / S_{\mathrm{CHFEUR}}$
triangulation	$S_{\mathrm{EURCHF}} = S_{\mathrm{EURUSD}} \cdot S_{\mathrm{USDCHF}}$

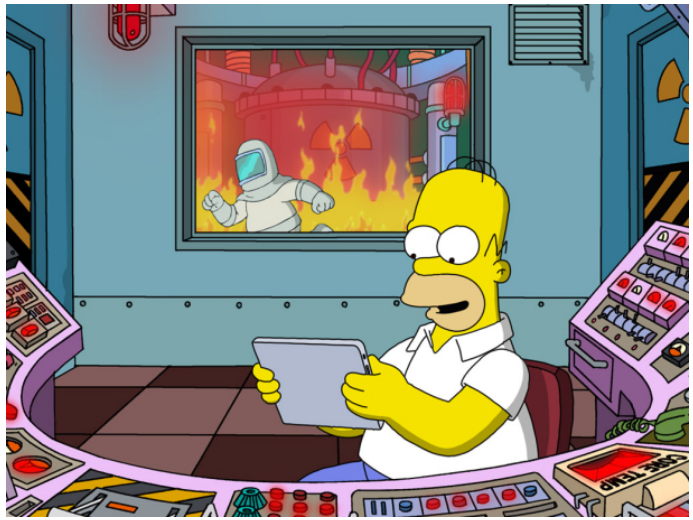
- Assessment of model risk
  - correlation modelling: implement and test against different extension from 1-dim
  - re-value simple and more complex transaction (trade approvals)

Estimation of model risk is not easy - need an independent analytical library

Get an independent benchmark library, now!

Without a benchmark library

- weak test procedure (only simple limiting cases and internal consistency check)
- completely dependent on FO
- independent assessment of model risk difficult/impossible
- reactive approach



With a benchmark library

- large upfront investment
- small incremental effort
- automation/scalability/reproducibility of tests
- team-based approach vs one-off testing solutions
- possibility to investigate alternative approaches/models
- pro-active approach
- validation job is more creative/`fun' (talent retention)

Conclusions

- Model risk
  - Increasing regulatory pressure, but no strict guidelines to follow (yet)
  - Assessment of model risk is a key component of model validation process
- Shift to simpler products
- Invest in an independent benchmark library

- Simple FX options: validation is not trivial
  - strict market requirements: vanilla and barrier prices
  - hedging effectiveness and estimate of hedging costs: hedging simulation
  - consistency between 1-dim and multi-dim FX options

keep a critical mindset - don't delegate your responsibility to mathematics