Modeling counterparty exposure

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Introduction

In the following, we describe the current framework for deriving the counterparty exposure for a financial product, cf Jorion, *Value-at-Risk*, Section 13.3. For the description of the framework let us assume that we are at time s (usually s = 0) and denote the price of a financial product at any (future) time t (t > s) with maturity T by V(t,T). Products without a clear maturity are usually classified as equity and as such do not possess any counterparty exposure.

This document only covers the general framework for the quantification of counterparty exposure. Please note that counterparty exposure and counterparty risk are in general two different concepts: counterparty exposure models the "exposure at risk", i.e. the potential amount of money which could be lost. However, it does not take into account "loss given default" or "recovery rates", as well as "probability of default" is not considered. Thus, counterparty exposure should be merely seen as one of the three ingredients for counterparty risk measurement.

Potential counterparty exposure

1. Replacement value at time s

For each future time t we define

$$RV(t) = \text{replacement value at time } t = \max(0, \text{market value at time } t) = \max(0, V(t,T))$$

As the market value at time t is a random variable, the *replacement value* (sometimes also called *credit exposure*) at time t is also a random variable. In mathematical terms, note that both random variables (market value and replacement value) are t-measurable, which means they are known at time t, but not before. If there is a negative market value at time t, the *replacement value* is obviously set to 0.

2. Future change of market value at time t seen from time s

If we assume that we are at time s, we know the current market value V(s, T) of the financial product. Then we can decompose the future market value at times t > s as

$$V(t,T) = V(t,T) - V(s,T) + V(s,T) = V(s,T) + \Delta V(s, t, T)$$

with

$$\Delta V(s, t, T) = V(t,T) - V(s,T)$$

which measures the change of the market value from time s to time t. In the same manner, the replacement value may be decomposed as

$$RV(t) = RV(s) + \Delta RV(s, t)$$

with

$$\Delta RV(s, t) = RV(t) - RV(s)$$

3. Expected replacement value at time t observed at time s

Being at time s, the *expected replacement value* ERV(s, t) at a future time t > s can be measured by the conditional expectation (which is now s-measurable and hence known at time s).

$$ERV(s, t) = \mathbf{E}[\, RV(t) \mid s] = \mathbf{E}[\, \max(0, V(t,T)) \mid s]$$

From this formula we see that the forward looking expected replacement value is at least 0, but may also be strictly greater than zero, depending on the distribution of the market value V(0, t) (or, equivalently , the distribution of the change of market value ΔV(s,t,T)). Looking differently at the same formula we see that

$$ERV(s, t) = RV(s) + \mathbf{E}[\, RV(t) - RV(s) \mid s] = RV(s) + \mathbf{E}[\Delta RV(s, t) \mid s]$$

i.e. the excepted replacement value at time t is the replacement value at time s plus the expected change in replacement value (which may indeed become negative).

4. Potential counterparty exposure at time s

Being at time s, we can calculate the ERV(s, t) for all future times t until maturity T. For example, with varying t, for a standard interest rate swap, the ERV will increase up to a maximum until it will come down again and finally ERV(s, T) = C – L (which is coupon minus Libor, in case of a receiver swap), as this is the inherent value of a swap at maturity T. For a bond position, the expected replacement value will finally equal the notional amount plus coupon, i.e. ERV(s, T) = N(1+C). For most investments, the ERV can analytically or approximately be calculated, if the distribution of the underlying risk factor is known. In case that ΔV(s, t, T) is normally or log-normally distributed analytical formulas for the ERV can be given (similar in nature to the Black-Scholes formula).

Having the knowledge about the future behaviour of the expected replacement value, this should be taken into account and a first candidate for counterparty exposure, the *potential counterparty exposure* at time s may be defined as

$$PCE(s) = \max\_t \text{ (with } s<t<T) \; ERV(s, t)$$

or, writing it in different terms,

$$PCE(s) = RV(s) + \max\_t \text{ (with } s<t<T) \; \mathbf{E}[\, RV(t) - RV(s) \mid s]$$

From this we see that the potential counterparty exposure at time s is at least the replacement at time s plus the maximum over all expected future changes of the replacement value. As this maximum is a non-negative number, this means that the potential counterparty exposure is at least as large as the replacement value.

Example:

At time s let us consider a (receiver) swap with nominal N = 1 and annual payments which trades at par, i.e. the fixed rate equals the par rate(s) (e.g at inception). As the future present value V(t,T) is given by

$$V(t,T) = (\text{fixed rate} - \text{par rate}(t)) * A(t)$$

where A(t) = sum_t_i (with t_i > t) D(t_i) represents the sum over all (forward) discount factors over all payment dates t_i of the swap after time t. The future replacement value is then given by

$$RV(t) = \max(0, \text{fixed rate} - \text{par rate}(t)) * A(t).$$

Now the expected replacement value at time s for time t can be calculated if we know the distribution of the par rate at time t. Assuming a log-normal distribution for the par rate (as is done in the Black-Scholes setting), the ERV can be easily computed. Note the similarity to the valuation of a swaption, but this time the expectation is taken in the real-world measure and not in a risk-neutral setting. To finally get the potential credit exposure, the maximum over all these “swaption prices” is taken.

Worst case counterparty exposure

Although the potential counterparty exposure may seem to be a sufficiently conservative measurement for the counterparty exposure over the lifetime of a financial instrument, it only considers expected values. As the replacement value is a random variable, it can be safely assumed that the potential counterparty exposure is not a very conservative approximation, i.e. the future replacement value may exceed the PCE with non-negligible probability:

$$\mathbf{P}[\, RV(t) > PCE(s, t) \mid s] = \mathbf{P}[RV(t) - RV(s) > \max\_u\ \mathbf{E}[RV(u) - RV(s) \mid s]] >> 0$$

especially at the time t = t* which represents the time where the maximum of the ERV is attained. In general, we can assume that the probability of exceeding the expected value is in fact about 50% (at least at t*, under the assumption of normal distribution). Taking these fluctuations into account, a more conservative measure should be based on percentiles of the distribution of the replacement value.

1. Worst case replacement value

Let us start again with the decomposition of the replacement value in current exposure and future changes:

$$RV(t) = RV(s) + \Delta RV(s, t)$$

Instead of using the (s-measurable) expected replacement value

$$ERV(s, t) = RV(s) + \mathbf{E}[\Delta RV(s, t) \mid s]$$

we introduce the (s-measurable) *worst case replacement value* at (confidence) level L defined as

$$WCRV(s, t) = RV(s) + \mathbf{Percentile}(\Delta RV(s, t), L)$$

where now the percentile of the distribution of the future changes of the replacement value is used. We can equivalently rewrite this to

$$WCRV(s, t) = \textbf{Percentile}(RV(s) + \Delta RV(s, t), L)$$

or, using the conditional distribution of RV(t) given the information of time s, as

$$WCRV(s, t) = \textbf{Percentile}(RV(t) \mid s, L)$$

As this is the percentile of a distribution, at time s this is not a random variable any more, but just a real number. Taking values like L = 99%, it should exceed the expected replacement value by far.

Let us analyze the above equation a bit further:

$$WCRV(s, t) = \textbf{Percentile}(RV(t) \mid s, L) = \textbf{Percentile}(\max(0, V(t,T) \mid s, L).$$

This can be rewritten by taking the percentile inside the max operator:

$$WCRV(s, t) = \max(0, \textbf{Percentile}(V(t,T) \mid s, L)) = \max(0, V(s,T) + \textbf{Percentile}(\Delta V(s,t,T) \mid s, L))$$

2. Worst case counterparty exposure

In a similar fashion to above, we can now measure the *worst case counterparty exposure* by taking the maximum over all future worst case replacement values:

$$WCCE(s) = \max\_t \text{ (with } s<t<T) \ WCRV(s, t).$$

As the maximum over time can be taken inside the above maximum formula, the worst case counterparty exposure can also be stated as

$$WCCE(s) = \max(0, V(s,T) + \max\_t \text{ (with } s<t<T) \ \textbf{Percentile}(\Delta V(s,t,T) \mid s, L))$$

From this we see that the counterparty exposure at time s can be calculated by adding

$$\max\_t \text{ (with } s<t<T) \ \textbf{Percentile}(\Delta V(s,t,T) \mid s, L)$$

to the current market value V(s, T) and then taking the maximum with 0. The above additional term thus defines the potential upward move of the financial instrument which will not be exceeded with a probability of 1 – L.

3. Add-on factors

The above additional term is called add-on factor, as it is added to the current market value, i.e the worst case counterparty exposure is defined as

$$WCCE(s) = \max(0, V(s,T) + \text{Add-On}(s,T))$$

where

$$\text{Add-On}(s,T) = \max\_t \ (\text{with } s<t<T)\ \textbf{Percentile}(\Delta V(s,t,T) \mid s, L).$$

In general, this add-on depends on the current market value V(s, T). However, as it is a computationally demanding task to calculate the add-on for each instrument on a stand-alone basis, a generic add-on is derived on the assumptions that V(0,T) = 0, i.e. RV(0) = 0 and that Add-On(s,T) = Add-On(0, T-s), i.e. the add-on factor only depends on the remaining time to maturity.

The specific models which are used to derive the add-on factor for certain types of financial instruments is not covered in this document. This document only provides the general framework and concept of the counterparty risk methodology; details on the add-on methods can be found in the corresponding documents.

4. Usage of add-on factors for modelling counterparty risk

At the moment, add-on factors are derived for each instrument type. Instrument types are distinguished by the kind of instrument (i.e. interest rate swap, credit default swap, total return swap, …), by currency, by time to maturity, by credit rating (if applicable), etc.
For all these instruments, a table with add-on factors is calculated, i.e. for each instrument type at the values Add-On(0, 1), Add-On(0, 2), etc. are calculated. Each existing instrument is then mapped on one of these “default” instruments (or a interpolation methodology is applied) and the counterparty exposure of this instrument is calculated as

$$\text{Counterparty exposure} = \text{MtM} + \text{Add-On}$$

where MtM (mark-to-market) represents the current market value and Add-On is the nominal adjusted add-on factor.