

Credit Value Adjustment and Funding Value Adjustment All Together

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Credit value adjustment (CVA) and funding value adjustment (FVA) to the derivative contracts have been demonstrated to be important in the credit crisis after Lehman Brothers crash. Accurate valuations of CVA and FVA are essential to reflect the economic values of credit and funding risks. In the present article, we reviewed the concepts leading to definition of funding value adjustment, and discussed various implications due to the existence of CVA and FVA. We argue that FVA is consistent with the replication economics and DVA should not be used. The FVA asymmetry, the impact on no-arbitrage condition and trading competition are discussed. The general model for derivative pricing in the presence of CVA/FVA is also derived. Cross currency collateral posting and option on collateral selection is touched briefly.

Introduction

There are two important lessons that the derivative industry has learned from the financial turmoil around 2008. One lesson is that big size counterparties and AAA rated entities do default, such as Lehman Brothers and super-senior CDOs. Without the government rescue, there could have been numerous bankruptcy filings from big banks. This fact makes the credit value adjustment (CVA) more prevalent in derivative pricing and trading. On the one side, the choice of counterparty credit becomes more important. Behind every trade, there is a credit approval process. On the other side, traded derivative prices will always have CVA included, which can be very different among different dealers, due to differences in credit assignment to the trading entity, self-default considerations, credit support annex (CSA) term differences, among other things.

The second lesson learned is that funding cost embedded in derivative operation is of great importance to the business bottom line. Liquidity squeeze after Lehman Brother crash made the funding especially difficult and costly for an extended period of time. Without Fed's lifeline support in funding, many banks and financial institutions could have run out of capital easily. While banks have centralized funding desks to deal with firm-wide funding needs, the funding cost becomes an essential part to each line of business's profit and loss (P&L).

Given what have happened in the financial turmoil, central clearing and more collateral exchange has become the future trend. While this helps in providing more stability in the overall financial system and gives more transparency into the derivative trading and pricing, it also has implied consequences to the derivative pricing. First of all, fully collateralized derivative value will carry little credit risk, therefore CVA should be close to zero if any. The non-collateralized part will be subject to default, which then carries CVA.

Secondly, when collateral is posted, interests are paid based on some general overnight index (OIS) rate. For example, overnight fed fund rate in US. This implies the collateralized derivative value should be carrying OIS interest. Or the future

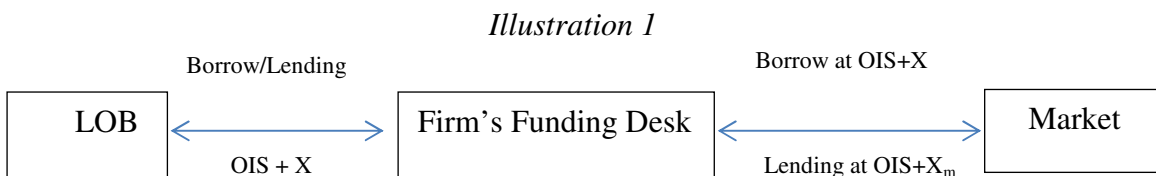
collateralized cashflows should be discounted using OIS discounting curve.¹ In order to post collateral, one has to raise funding, and in practice borrowing from centralized funding desk. While the funding of collateral costs OIS plus some funding spread, say X , it only receives OIS as interest from the trade counterparty. Therefore collateral posting do carry negative economic value in the amount of funding spread X to the business. The extra funding cost associated with derivative operation is termed funding value adjustment (FVA). On the other hand, collateral received from other trading counterparties will carry positive values as funding benefits.

It is now commonly recognized that the true economic value of the derivative security should have both CVA and FVA included, even though the official accounting rule for FVA on marking the book is still to be seen. While there have been numerous literatures on the subject of CVA and its calculations [LJ10], [CZ07], [PD03], FVA has only received serious examinations by some authors recently [FT11], [MP11], [P11]. It is the goal of the present article to discuss the valuations of both CVA and FVA, and more importantly the implications of CVA and FVA.

The article is arranged as the following: First we will discuss the simple concepts in borrowing/lending within a firm and the value of collateral. Then we will use the some simple cases to illustrate the value of secured and unsecured borrowing, as well as CVA and FVA concepts. Simple swap example calculations based on analytic formula are used to show a number of important consequences of FVA. Different CSA situations and their implications in CVA/FVA valuations are presented afterwards. We argue that CVA and FVA reflect the necessary economics involved in the dynamic replication of derivatives, and there is no need for DVA once FVA is properly taken into account. The FVA asymmetry, the effect of FVA on market trading price and competition among the dealers will also be discussed. The general derivative pricing model in the presence of funding and credit will be presented afterwards. We also touched briefly on trades involving multiple currencies and the option on collateral selection at the end of the article.

Borrowing, Lending and the Value of Collateral

Before going further into details to discuss FVA, it is worthwhile to examine the mechanism of unsecured (without collateral) borrowing and lending within a firm, as shown conceptually in below:



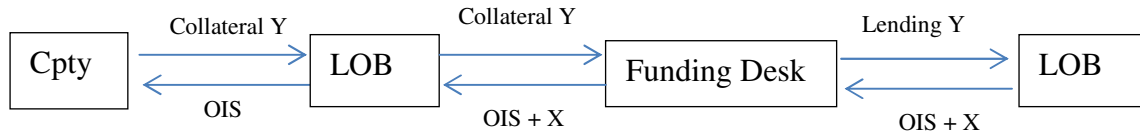
where LOB stands for line of business, and X_m stands for the funding cost of counterparty that Funding desk is lending to. Here LOB will borrow and lend money

¹ See for example [B10] and references therein.

through firm's funding desk at OIS+X, which should be the same for all LOBs firm-wide. Assuming funding desk does not make money from LOBs, its borrowing cost from market place is also OIS+X. Therefore OIS+X is only transfer pricing from the market. For simplicity, we use X to indicate a funding spread, which could be a term structure curve at different maturities in reality.

Next we consider the value of collateral. Normally there can be choices of collateral, such as cash and sovereign debt securities with different currencies. For this article, we will only consider the cash collateral. The collateral choice option could mean extra values for the party posting collateral and we will touch briefly the case where there is a choice of collateral currency at the last section. In general the value of extra collateral in hand of a LOB is shown in the diagram below. It shows that LOB receives collateral Y from counterparty, and then deposit it in funding desk, which in turn receives OIS+X as interest from other LOBs. This means any extra collateral can be monetized from the funding desk by lending to other LOBs within the same firm, and thus receives OIS+X as interest. Therefore collateral in hand generally can be considered generating X spread for the business.

Illustration 2



Collateral Posting, CVA and Funding Cost: Simple One Period Case

The following notations are used for the remainder of this article. For simplicity, we use ' to indicate those properties of counterparty. In addition, we use scalar representation for all quantities, which can be easily replaced by vectors with term structure curves.

SF	Ourselves
CP	Trading counterparty
r_u	Funding rate for SF. $r_u = r_c + u + L$
u	Credit spread for SF
u'	Credit spread for CP
L	Liquidity Spread for SF
X	Unsecured funding spread for SF. $X = u + L$
X'	Unsecured funding cost for CP
P_d	Probability of default for SF
P_d'	Probability of default for CP
R	Recovery rate in the case of SF default
R'	Recovery rate in the case of CP default
V_t	Derivative contract value. So value at $t=0$ will be V_0

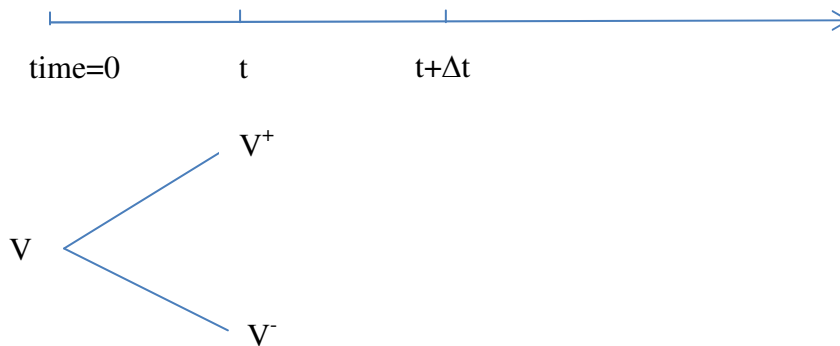
where L is the liquidity cost embedded in funding rate and usually is indicated by bond vs. CDS spreads. As it involves funding to purchase bonds and CDS is non-funded, L is normally positive except for supranationals, whose bonds are treated as sovereigns and could be used as collateral posting.

We also assume the simple CSA in place with no mutual put and other ingredients:

H Collateral threshold above which SF has to post collateral to CP
 H' Collateral threshold above which CP has to post collateral to SF

Without loss of generality, we assume the contract mark to market (MTM) value V_0 is around zero at the time of initial trading. As market moves, derivative MTM moves up and down, giving positive and negative values. It is useful to examine the simple one period process from time 0 to t .

Illustration 3



The following funding situations result:

$$V_t = V^+ > 0, CP \Rightarrow \text{post collateral } \max(V^+ - H', 0) \Rightarrow SF$$

$$V_t = V^- < 0, SF \Rightarrow \text{post collateral } \max(-V - H, 0) \Rightarrow CP$$

So if MTM is positive, then CP would have to post collateral according to CSA definitions, and vice versa. Only the part of MTM that exceeds the threshold definition will be posted. So assuming collateral will be posted for a period Δt from t to $t + \Delta t$ until we re-calculate derivative value, the funding cost in raising collateral for the period will be:

$$V_t = V^+ > 0, \text{ Funding cost } \approx \max(V^+ - H', 0) D_t \times \Delta t > 0$$

$$V_t = V^- < 0, \text{ Funding cost } \approx - \max(-V - H, 0) D_t \times \Delta t < 0$$

where we use \approx to indicate that it is approximately correct for the discrete setting, and we have not considered any default for the situation. Note that this funding cost is not funding value adjustment. We will define FVA later.

Meanwhile, we have the following credit situations:

$$V_t = V^+ > 0, \min(V^+, H^+) \text{ at risk to SF, CVA} = -\min(V^+, H^+) P_d (1-R)$$

$$V_t = V^- < 0, \min(-V^-, H) \text{ at risk to CP, CVA} = \min(-V^-, H) P_d (1-R)$$

The part of CVA with $V < 0$ is also called DVA, with D for debt. DVA is due to self-default and is an asset or benefit for the business. While the concept of benefiting from one's own default is confusing, the practicality of hedging DVA is even more troublesome. Traditionally it has been very difficult for financial institutions to hedge DVA risk because of the fact that one cannot sell its own default risk in any form. As we will show later, FVA is a better representation of market economics, and there is no need for DVA once FVA is properly accounted for.

The Value of Money: Lending to Counterparty

First we will consider the simple cases of lending to counterparty. In all cases below, counterparty will borrow \$1 at $t=0$ and return money back at $t=T$. From replication mechanism, we will derive the value that counterparty has to pay back at T in order to be fair.

Case 1

Lending \$1 to counterparty, fully collateralized. We start with 0, borrow \$1 from funding desk, lend to counterparty and receive \$1 as collateral. The following shows all the relevant accounts separately:

	t=0		t=T	
Value of asset:	\$1	-----	?	(a)
Collateral to return:	-\$1	-----	$\$-e^{r_c T}$	(b)
Put Collateral in funding desk:	\$1	-----	$\$e^{r_u T}$	(c)
Borrow \$1 from funding desk:	-\$1	-----	$\$-e^{r_u T}$	(d)

where (a) represents the \$1 lent to counterparty, (b) is the collateral that we need to return to counterparty along with interest r_c , (c) is that one can put the collateral into work by depositing into funding desk, which would become $e^{r_u T}$ at T, (d) is the money borrowed from funding desk.

From (b), (c), (d), we arrive at the money that counterparty needs to return at maturity:

$$? = e^{r_c T} - e^{r_u T} + e^{r_u T} = e^{r_c T}$$

which means that the interest to be charged is r_c , or the discounting rate for collateralized cash flow is r_c .

Alternatively, one can think of (b) and (c) as the value of owning collateral:

$$V_c^{t=T} = e^{r_u T} - e^{r_c T} \text{ or } V_c^{t=0} = e^{(r_u - r_c)T} - 1$$

and for a small time increment dt:

$$dV_c = (r_u - r_c)dt$$

Case 2

Lending \$1 to counterparty, partially collateralized x and no credit considerations. The following shows all the relevant accounts separately:

	t=0		t=T	
Value of asset:	\$1	-----	?	(a)
Collateral to return:	-\$x	-----	\$-x e ^{r_cT}	(b)
Put Collateral in funding desk:	\$x	-----	\$x e ^{r_uT}	(c)
Borrow \$1 from funding desk:	-\$1	-----	\$- e ^{r_uT}	(d)

From (b), (c), (d), we arrive at the money that counterparty needs to return at maturity:

$$? = xe^{r_c T} - xe^{r_u T} + e^{r_u T} = xe^{r_c T} + (1-x)e^{r_u T}$$

which means that the growth rate for collateralized part x is r_c, the growth rate for non-collateralized part (1-x) is r_u.

Case 3

Same as Case 2, but with credit considerations, i.e. counterparty default. The following shows all the relevant accounts separately:

	t=0		t=T	
Value of asset:	\$1	-----	?	(a)
Collateral to return:	-\$x	-----	\$-x e ^{r_cT}	(b)
Put Collateral in funding desk:	\$x	-----	\$x e ^{r_uT}	(c)
Borrow \$1 from funding desk:	-\$1	-----	\$- e ^{r_uT}	(d)
Lost from counterparty default:			-(1-x) e ^{r_uT} (1-R')P _d '	(e)

where (e) gives the loss of value from counterparty default and P_d' is the probability of default of counterparty. The default can also be treated through extra credit discounting:

$$e^{-u'T} = 1 - (1 - R')P_d' \quad (e')$$

From (b), (c), (d), (e), (e'), we arrive at the money that counterparty needs to return at maturity:

$$? = xe^{r_c T} + (1-x)e^{(r_u+u')T}$$

which means that the growth rate for collateralized part x is r_c, the growth rate for non-collateralized part (1-x) with counterparty credit is r_u+u'.

To summarize the above, collateralized value should carry growth rate of r_c and unsecured value should carry growth rate of r_u+u'. For a small time increment, \$1 money with x collateral will grow as:

$$dV = xr_c dt + (1-x)(r_u + u')dt$$

The Value of Money: Borrowing from Counterparty

In the case of borrowing money from counterparty, the same replication mechanism can be used to derive the fair amount that we have to return to the counterparty at maturity. It is easy to confirm that borrowing under full collateral would carry interest of r_c . The non-collateralized value would carry r_u if only funding is considered.

Case 4

For the case with both funding and self-default included, it is a little bit trickier. It is arguable that the non-collateralized borrow would carry r_u as shown below all the relevant accounts:

	t=0		t=T	
Value of asset:	\$-1	-----	?	(a)
Collateral to return:	\$x	-----	$\$x e^{r_c T}$	(b)
Borrow Collateral in funding desk:	$\$-x$	-----	$\$-x e^{r_u T}$	(c)
\$1 premium saved in funding desk:	\$1	-----	$\$ e^{r_u T}$	(d)
Gain from self-default:			$(1-x) e^{r_u T} (1-R)P_d$ or 0	(e)

(e) was listed to compare to Case 3. In reality, one does not gain from self-default economically. Therefore the gain from self-default in our replication should be zero. Then we would have:

$$? = -x e^{r_c T} - (1-x) e^{r_u T} = -x e^{r_c T} - (1-x) e^{(r_c + u + L)T}$$

Note that this is the value from the SF's point of view. From CP's point of view, we know the value that CP is demanding (from Case 3):

$$? = -x e^{r_c T} - (1-x) e^{(r_u' + u)T} = -x e^{r_c T} - (1-x) e^{(r_c + u + u' + L)T}$$

which is different from SF's replication value. This difference also leads to interesting results related to DVA and FVA, as we will show in the later sections.

CVA Calculation

CVA can be calculated through forward simulations and backward valuations. See for example [PZ07], [LJ10] and references therein. We will only discuss briefly the CVA calculation here.

There are in general two different ways of accounting for default behavior in CVA calculations. One way is through simulating default scenarios directly, so that the defaulted value can be collected as time moves on. Conceptually:

$$CVA = -E^Q [(1-R)P_d \min(H, V_t)D]$$

where P_d is the default probability, R is the recovery rate, D is discounting, and E^Q indicates expectation under risk neutral measure. We have used the convention of CVA being negative here. For a two-way CVA with self-default or DVA included:

$$CVA = -E^Q \left[(1 - R_{CP}) P_d^{CP} \min(H_{CP}, V_t^+) D \right] + E^Q \left[(1 - R_{SF}) P_d^{SF} \min(H_{SF}, -V_t^-) D \right] \quad (1)$$

The first term is negative adjustment from counterparty default and the second term DVA is positive, linked to the self-default probability. Later we will present argument to eliminate the second part of the CVA, so only the first part is consistent to the overall CVA/FVA methodology.

In Monte Carlo simulations, or trinomial pricing tree nodes, or pricing lattice points, the defaulted value in equation (1) by either party will be collected and discounted back to present time. Direct simulation of both SF and CP credits, including ratings migrations, can be achieved by using risk neutralized transition matrix, through which the rating based threshold can be handled properly.

Another way to account for default in CVA calculations is to through implicit credit discounting in backward valuations, as credit discounting is essentially equivalent to default value:

$$e^{-(r+u)t} = e^{-rt} [1 - (1 - R)P_d]$$

where r is risk free rate, u is the discounting spread, R is recovery rate and P_d is the probability of default. The CVA is calculated based on valuations under two different settings: one with credit discounting, and another without credit discounting. CVA is defined as:

$$CVA = V_d - V_{nd}$$

As an example, consider one simple cashflow \$1 to be paid by counterparty at time T . The CVA without collateralization is then:

$$CVA = e^{-rt} (1 - e^{-ut})$$

Similarly cashflows with different credit implications will be discounted using different credit spreads with zero spread for fully collateralized values. Applying the concept through backward induction, we would be able to arrive at the CVA for a derivative security in Monte Carlo simulations, trinomial trees and finite difference lattice. More detailed discussions can be found in [LJ10].

FVA Calculation, Phantom DVA and the Proposed Approach

From discussions in previous sections, the funding cost/benefit impact comes from the extra funding cost in raising collateral and received collateral on hand. We therefore define the funding value adjustment as the following:

$$V = V^{FC} + FVA \quad \text{or} \quad FVA = V - V^{FC}$$

where V^{FC} stands for fully collateralized value. So the true derivative value should be equal to the fully collateralized value plus an adjustment from funding. If extra funding cost is incurred, FVA will be negative. For an asymmetric CSA where CP does not post collateral, significant FVA may result. This is indeed the case for dealer trading with supra-national entities.

Making the FVA expression more explicitly using notations above for a small time increment dt:

$$FVA = (-V(t) + C(t))X(t)dt \quad (2a)$$

or for the time period from t=0 to t=T and for all possible values:

$$FVA = \int_0^T \left[\int_{-\infty}^{\infty} e^{\int_0^t -r_c(v)dv} (-V(t) + C(t))P(V, t)dV \right] X(t)dt \quad (2b)$$

where X(t) is the funding spread, T is the maturity of the derivative security, P(V,t) is the risk neutral probability of V(t) at time t, and C(t) is:

$$\begin{aligned} C(t) &= \max(V - H, 0) && \text{if } V > 0 \\ &= -\max(-V - H, 0) && \text{if } V < 0 \end{aligned}$$

C(t) is taken to be positive if counterparty posts collateral and negative if counterparty receives collateral. In (2), the term $-V(t)+C(t)$ would be negative if $V > 0$, representing extra funding costs; and it is positive if $V < 0$, representing extra funding benefits. FVA is the expectation of all future funding impact for all time periods. Note that V(t) can be CVA adjusted value in specific calculations.

As we have shown in the replication scheme, term (d), which is the premium or cash payment for the derivative, is always included in the FVA calculation. This is a quite important point. While derivative value is in principle marking to market, its MTM is the expected fair value to be fetched in the market place. The premium represents part of the funding equation, or can be taken as alternative capital usage and return. Premium plus the derivative creates a zero starting value. The sum of (b), (c), (d) then reflects all the cash account values related to funding situation. For example, the FVA for Case 2 in counterparty lending is the sum of values from (b), (c), (d) subtracting the same terms in fully collateralized Case 1:

$$FVA(\text{Case2}) = -(1-x)(e^{r_u T} - e^{r_c T})$$

or for a small time increment dt:

$$FVA(\text{Case2}) = -(1-x)(r_u - r_c)dt$$

Consider now Case 4, where we borrow from counterparty. From borrower's point of view, DVA for a small time increment dt would be:

$$DVA(\text{Case4}) = (1-x)udt$$

and if we compare to collateralized case, we have the FVA as:

$$FVA(\text{Case4}) = (1-x)(r_u - r_c)dt = (1-x)(u + L)dt$$

Clearly if we include both DVA and FVA, we would have double counted in DVA and FVA the self-default risk u^2 . If the derivative valuation includes DVA, FVA should not have default risk in it. On the other hand, if the FVA calculation includes self-default risk, DVA should not be calculated. We believe calculating FVA with default risk better reflects the real economics of borrowing: one can never realize phantom benefit from its own default, but there is funding benefit as one LOB can provide funding for another LOB. From the firm's point of view, it should be managing its funding cost/benefit or FVA, not its default benefit, even though they are related.

So the fair value of a derivative instrument should be:

$$V_{Fair} = V_{FC} + CVA + FVA$$

where CVA is the one-way CVA without DVA.

Similarly to CVA calculation, FVA from equation (2) can be readily fit into the different numerical valuation methodologies for derivatives. Given the valuations of derivatives at different scenarios, tree nodes and lattice points, it is easy to determine the FVA through time steps. For example, the FVA for the derivative at time t_i , scenario j and for time increment dt is:

$$\begin{aligned} FVA(i, j) &= [-V(i, j) + \min(V(i, j) - H, 0)](r_u(i) - r_c(i))dt \quad \text{if } V > 0 \\ &= [-V(i, j) - \min(-V(i, j) - H, 0)](r_u(i) - r_c(i))dt \quad \text{if } V < 0 \end{aligned}$$

Accumulating discounted FVA backward through time would result the funding value adjustment at present time. In effect, FVA can be easily implemented as a side product of CVA calculations.

CVA/FVA for A Simple Swap

² The overlapping of DVA and FVA has been discussed recently in [MP11] and it was suggested credit risk should be separated from liquidity risk in the calculation of FVA. Here we propose to use one single FVA calculation to cover both liquidity risk and credit risk.

It is worthwhile to consider different CSAs and the implications in CVA/FVA calculations. We go through the details of the calculation using simple interest rate swap as an example, whose calculations can be done semi-analytically with some assumptions. Through this example, we would be able to see some important consequences due to the existence of FVA, such as asymmetric trading price and market competition.

Assuming that interest rate and default behavior are not correlated, it is well known that CVA for a swap can be calculated semi-analytically through integration. The potential exposure (PE) for a paying fixed K and receiving floating swap at time t is:

$$PE_{pay}(t) = N \int_{-\infty}^{\infty} \max(S(t) - K, 0) A(t) P(S(t)) dS = Payer(K, t)$$

where Payer(K,t) is the payer swaption value with expiry t and strike K, N is the notional, S(t) is the swap rate for time t with maturity T, K is the fixed rate, and A(t) is the accrual for the swap. The CVA for the swap would be:

$$CVA_{pay} = \int_0^T Payer(K, t) P_d(t) (1 - R(t)) dt \approx \sum_i^n Payer_i(K) P_d^i (1 - R_i) \Delta t_i$$

where R(t) is the recovery rate at t, and $P_d(t)$ is the default density, and the second approximate equality with sum over i to n is for chosen discrete time periods. Certainly the payer swaption value should depend on the swaption volatility, which we have dropped to simplify notation.

With collateral, the potential exposure is reduced by the part from collateral above. This means:

$$PE_{pay}(H, t) = Payer(K, t) - Payer(K', t)$$

$$K' = K + \frac{H}{NA}$$

where H is the collateral threshold, K' is the strike that incremental swap value from K to K' will be equal to the collateral level, N is notional and A is the swap accrual. The CVA with collateral is then:

$$CVA_{pay}(K, H) = \int_0^T (Payer(K, t) - Payer(K', t)) P_d(t) (1 - R(t)) dt$$

$$\approx \sum_i^n [Payer_i(K) - Payer_i(K')] P_d^i (1 - R_i) \Delta t_i \quad (3)$$

Practically, the sum can be done over the swap period start dates.

For FVA calculation, we will show the funding benefit (FB) and funding exposure (FE) separately. The funding benefit (FB) at time t, or the part we are not liable to post collateral, is the amount in value that is below the collateral threshold:

$$\begin{aligned}
 FB_{pay}(H, t) &= D(t)N \int_{-\infty}^{\infty} [(\max(K - S(t), 0) - \max(K'' - S(t), 0))] A(t) P(S(t)) dS \\
 &= receiver(K, t) - receiver(K'', t) \\
 K'' &= K - \frac{H}{NA}
 \end{aligned}$$

where K'' is the strike level that incremental swap value from K'' to K will be equal to the self collateral level H. Similarly the amount of collateral we will not receive, or the funding exposure is:

$$\begin{aligned}
 FE_{pay}(t) &= -D(t)N \int_{-\infty}^{\infty} [\max(S(t) - K, 0) - \max(S(t) - K''', 0)] A(t) P(S(t)) dS \\
 &= Payer(K''', t) - Payer(K, t) \\
 K''' &= K + \frac{H'}{NA}
 \end{aligned}$$

with H' being the counterparty collateral threshold. The total FVA over time would then be:

$$\begin{aligned}
 FVA_{pay}(H, H') &= \int_0^T [Payer(K''', t) - Payer(K, t) + Receiver(K, t) - Receiver(K'', t)] X(t) dt \\
 &\approx \sum_i^n [Payer(K_i''') - Payer(K) + Receiver(K) - Receiver(K_i'')] X_i \Delta t_i \quad (4)
 \end{aligned}$$

When H=H' = ∞, it becomes:

$$\begin{aligned}
 FVA_{pay}(H, H') &= \int_0^T [-Payer(K, t) + Receiver(K, t)] X(t) dt \\
 &\approx \sum_i^n [-Payer(K) + Receiver(K)] X_i \Delta t_i
 \end{aligned}$$

where $X(t) = r_u(t) - r_c(t)$ is the funding spread for forward period t to t+dt. When H=H'=0, FVA is obviously 0. Similarly, the FVA formula can be derived for receiver swaps.

In the above derivations, we have used collateral threshold and assumed that this is the only trade with the particular counterparty. This is oversimplification of practical situations and portfolios. In reality, multiple position against the same counterparty has to be netted to be compared against the collateral threshold in determining the collateral

posting. Therefore the application of (3) and (4) may be limited. However, at any given time, one can approximately consider H as a marginal concept, which is the amount that is left to reach the defined threshold in the CSA. With that mind, (3) and (4) can be used as an easy tools for quick back of envelop type of calculations.

Simple Swap Example: Symmetric CSAs, One-way CSAs and No-CSAs

Now let us take a 4% 10y semi-annual payer swap as an example, meaning we will be paying fixed 4% coupon and receiving floating coupon. For simplicity, we apply the constant 4% interest rate and 20% swaption volatility throughout. We examine 4 different extreme CSA situations, the first one with no CSA or no collateral exchange, the second one with the symmetric CSA and both sides exchanging collateral with zero threshold, the last two with asymmetric or one sided CSAs, one with counterparty posting collateral, and the other with ourselves posting collateral. The results are shown in the table below.

Table 1: CVA/FVA Calculations for a Simple 10yr Payer Swap. The results are in basis points of notional. Pd is the annual default probability and X(SF) is the assumed constant funding cost.

Rate = 4%	Pd(CP)	Pd(SF)	X(SF)	H(SF)	H(CP)	CVA	DVA	FVA
NO CSA	1%	1%	1.2%	None	None	-19.1	19.1	0.0
	2%	1%	1.2%	None	None	-36.9	19.1	0.0
	4%	1%	1.2%	None	None	-68.6	19.1	0.0
Symmetric	1%	1%	1.2%	0	0	0	0	0.0
	2%	1%	1.2%	0	0	0	0	0.0
	4%	1%	1.2%	0	0	0	0	0.0
Asymmetric(CP)	1%	1%	1.2%	None	0	0.0	19.1	22.9
	2%	1%	1.2%	None	0	0.0	19.1	22.9
	4%	1%	1.2%	None	0	0.0	19.1	22.9
Asymmetric(SF)	1%	1%	1.2%	0	None	-19.1	0	-22.9
	2%	1%	1.2%	0	None	-36.9	0	-22.9
	4%	1%	1.2%	0	None	-68.6	0	-22.9

Here DVA is listed for comparison purpose. For the first case when no CVA is posted by either party, the funding cost and benefit cancels. For the second case, when both party are fully collateralized, there is neither credit risk or funding value adjustment. For the case when only CP posts collateral, SF enjoys significant funding benefit when borrow money from CP. On the other hand, when only SF posts collateral, SF is exposed to not only CP's credit risk (u' term), but also significant funding cost (r_u term) due to unsecured lending. Clearly, asymmetric CSA could have significant impact on the potential credit exposure and create significant funding burden for the party posting one side collateral.

When market moves in one way or another, the credit and funding situations change accordingly. In Table 2 below, we show the situation when rate moves higher from 4% to 5%. Expectedly, the credit exposure of counterparty paying floating rate increases in this situation. Similarly, the funding cost of not receiving the collateral also increases. For the

first case without any collateral posting, CVA is much larger than DVA. Meanwhile FVA is also significant because of the asymmetric value of receiver and payer swaptions. For the last case where only SF posts collateral, CVA and FVA have both increased significantly.

Table 2: CVA/FVA Calculation for the Simple 10yr Payer Swap when market moves from 4% to 5%.

Rate = 5%	Pd(CP)	Pd(SF)	X(SF)	H(SF)	H(CP)	CVA	DVA	FVA
NO CSA	1%	1%	1.2%	None	None	-40.3	8.3	-38.4
	2%	1%	1.2%	None	None	-78.0	8.3	-38.4
	4%	1%	1.2%	None	None	-146.5	8.3	-38.4
Symmetric	1%	1%	1.2%	0	0	0	0	0
	2%	1%	1.2%	0	0	0	0	0
	4%	1%	1.2%	0	0	0	0	0
Asymmetric(CP)	1%	1%	1.2%	None	0	0.0	8.3	10.0
	2%	1%	1.2%	None	0	0.0	8.3	10.0
	4%	1%	1.2%	None	0	0.0	8.3	10.0
Asymmetric(SF)	1%	1%	1.2%	0	None	-40.3	0	-48.4
	2%	1%	1.2%	0	None	-78.0	0	-48.4
	4%	1%	1.2%	0	None	-146.5	0	-48.4

It is rather clear that the funding effect can be quite significant. For a 10yr swap having potential a few running basis points effect, it can affect the trading price significantly. For example, traditionally dealers have one sided CSAs with sovereigns and supra-nationals entities, with dealers posting collateral only. This creates tremendous funding cost on the dealer side. The estimate of such cost can be, not surprisingly, in the billions of dollars. After the European sovereign crisis in 2010, more and more sovereigns are more open to two sided CSAs.

CVA Symmetry and FVA Asymmetry: Is the No-Arbitrage Rule Broken?

CVA/DVA calculations can be in general uniform among the trading partners, meaning they generally agree on the CVA/DVA value given same credit and market input:

$$CVA_{SF} \approx DVA_{CP} \quad (5a)$$

This means the CVA due to CP's credit risk from SF's point of view is the same as the DVA from CP's point of view. The same is true for the CVA due to SF's credit risk:

$$CVA_{CP} \approx DVA_{SF} \quad (5b)$$

We use \approx to indicate that they could differ in practice slightly due to different credit views. We call equation (5) the CVA symmetry. This makes the day one mark to markets between the two parties to be consistent:

$$V_{SF} + V_{CP} \approx 0 \quad (6)$$

In terms of accounting, there is no net value generated during the trade. From accounting and regulator's point of view, this is consistent: you do not create money with two parties simply trading with each other, therefore Equation (6) should hold at all times. This is A good story.

The nice consistent situation will be broken after considering the different funding situations for the two trading parties. The existence of FVA can make the day one prices significantly different between the two. In another words, the sum of the two marks from two trading partners after CVA/FVA will not be zero:

$$V_{SF} + V_{CP} \neq 0$$

It may generate the situation that both are marking a profit or loss at the same time. While this is intuitively difficult to comprehend, this is the real market economics of derivative business in the market place. Essentially, gain or loss in values is generated from the different funding costs.

The question is then, if both parties can generate profits by trading with each other, is this creating arbitrage opportunity that will break the rules of derivative pricing?

Not really. The value is created through fact that the better credit having less funding cost. However, this value is not unlimited and the funding gain/loss can affect the overall funding for the firm as well as the balance sheet situation. Unsecured funding is always limited and would create pressure in credit assessment of the firm. The more you borrow, the less credit you have. More importantly, return on capital is a critical parameter investor community and firm management always watches for. Therefore, the arbitrage opportunity is really competitive advantage due to credit quality and is not to be extended in general. The opposite side of gaining funding value adjustment is the loss in intangible credit. Hence there should be no worry about the no-arbitrage pricing rule being broken due to the seemingly arbitrage opportunity.

However this could create opportunity for cash rich and better credit firms to enhance their cash return. Assuming we have the following funding costs:

$$X_{SF} > X_{CP}$$

it would be beneficial for the costly funding party SF to pay effectively something between the two:

$$X_{SF} > X_{effective} > X_{CP}$$

by a virtual transfer pricing embeded in derivative price quote. For example, if the funding cost is 20bps for a deal for SF, and only 5bps for CP, it would be beneficial for

CP to bear the funding part and for SF to pay in the value buried in the derivative price, say 15bps. This way, both party win from the deal: from SF's point of view, it paid 15bps to raise funding, cheaper than his normal 20bps funding cost; and from CP's point of view, it is receiving 15bps for something it is going to pay 5bps. A win-win situation.

The existence of FVA may also change the landscape of market competition. Traditionally, the existence of DVA makes the market maker having better credit quality to be less competitive in pricing, since higher credit quality will benefit less from the self-default given the same customer trade. Unless the customer is selective in credit quality, the less quality market maker could have an upper-hand in grabbing more deals. With everyone including CVA/DVA in their prices, more often you would hear from the marketing people talking about the deal prospects due to the different DVA calculations.

Now the existence of FVA will tilt the balance of trading as well. Higher credit quality market maker will have less funding cost, but also enjoy less funding benefit in receiving collateral. Therefore depending on the trade situation, FVA considerations could drive the business from one type of trade to another type of trade. One dealer may be more competitive in one type of trades and less competitive in another type of trades.

FVA Management vs Traditional DVA Hedging

In the previous sections, we have discussed the relationship between DVA and FVA. If we calculate FVA with self-funding cost, DVA should not be included in the valuation. Since DVA is un-hedgeable in the market place, as one simply cannot sell its own default protection, it has always been a difficult issue to deal with by financial institutions. The change of DVA into FVA as in (2) turns the problem of DVA hedging into overall FVA management: balance of funding costs and benefits.

This could translate into another significant factor in business decision making process for financial institutions. For example, different LOBs may have their own intrinsic funding cost/benefit structure. A financial firm may decide to pursue more trades with sovereign and supra-nationals in overall FVA management. Since the trading with these counterparties with one-sided CSAs could mean significant funding cost, offsetting the firm-wide FVA gain (or DVA in the traditional sense) somewhere else. In another words, funding benefits are used to pay the funding costs of one-sided CSAs.

Moreover, FVA can also be hedged through selective trading, for example some derivative securities carrying significant funding cost, and some may carry more funding benefits because of its asymmetric nature. A simple example could be swaps traded with steep yield curve. One can construct a swap so that the future funding cost is significant, which would then offset the self-default. While you could owe a lot in the future and have to raise collateral, the funding costs will offset the funding benefits from other places.

The General Derivative Pricing Model in the Presence of CVA/FVA

In reference [P11], Piterbarg gave a detailed prescription on the impact of funding effect on derivative pricing through portfolio replications. In that derivation, a collateral amount of $C(t)$ is assumed to be held against the derivative. Then secure and unsecured cash accounts with funding desk are used for collateralized and non-collateralized values in calculating roll over interests/charges, with funding value adjustment derived for simple no CSA and fully CSA cases. Our conclusions are consistent with his results in the presence of funding and without credit default considerations.

In this section, we will try to extend the derivative pricing with both credit and funding effect included. As we have seen, the existence of CSA with various collateral threshold definitions would change the economics of the replication strategy because of credit and funding situations. For example, the forming of a risk neutral portfolio would involve certain collateral moves. Uncollateralized values would have credit implications, while collateral posting will involve funding cost, in addition to the OIS interest. An explicit treatment for funding and credit would be needed for a general pricing model.

Assuming the stochastic process for S :

$$\frac{dS}{S} = r_s dt + \sigma_s dz_s$$

where r_s is the drift for S . From Ito's Lemma, the change of derivative value $V(t)$:

$$dV(t) = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} dS^2$$

This equation states that to replicate the derivative value, we can create a portfolio by dynamic hedging the dS risk by taking $-\frac{\partial V}{\partial S}$ units of S . The value of the portfolio will be:

$$\Pi = V - \frac{\partial V}{\partial S} S$$

The change of this portfolio in time should be:

$$d\Pi = \left(\frac{\partial V}{\partial t} + \frac{\sigma_s^2}{2} \frac{\partial^2 V}{\partial S^2} \right) dt$$

While the stochastic uncertainties are hedged away, the change in Π should only involve the interest carry and funding carry over time. For example, collateralized value should carry r_c as interest, and positive uncollateralized value will carry credit enabled interest $r_u + u'$ or for negative un-secured values. Summarize all these together, we have $d\Pi$ as:

$$d\Pi = (Vr_\gamma - \frac{\partial V}{\partial S} Sr_s) dt$$

where we have used Vr_γ to indicate the carrying of value V through dt and r_γ is the interest/funding/credit carry we are looking for.

Given what we have discussed in previous sections, we have:

$$Vr_\gamma = r_c C + (V - C)r_u + (V^+ - C)u'$$

where C is the collateral posted, and the third term is only for $V > 0$. Or more specific,

$$\begin{aligned} Vr_\gamma &= r_c \max(V^+ - H', 0) + (r_u + u') \min(H', V^+) && \text{if } V = V^+ > 0 \\ &= -r_c \max(-V^- - H, 0) - r_u \min(H, -V^-) && \text{if } V = V^- < 0 \end{aligned} \quad (7)$$

The general stochastic differential equation becomes:

$$\frac{\partial V}{\partial t} + \frac{\sigma_s^2}{2} \frac{\partial^2 V}{\partial S^2} = Vr_\gamma - \frac{\partial V}{\partial S} Sr_s$$

Solving the above stochastic differential equation would lead to a value with both CVA and FVA adjustment included. Clearly from (7), the value of the derivative would be different if it is valued by different trading entity because of difference in u, L and H. The values would also be different for SF and CP, if their funding costs are different. The complexity of (7) makes the derivation of analytic solution difficult. However, the numerical implementation is straightforward.

By changing the formula for Vr_γ , one may also get the equation for the case without credit and funding risk:

$$Vr_\gamma = r_c V \quad (8)$$

The case with credit and CVA, but without funding risk:

$$\begin{aligned} Vr_\gamma &= r_c \max(V^+ - H', 0) + (r_c + u') \min(H', V^+) && \text{if } V = V^+ > 0 \\ &= -r_c \max(-V^- - H, 0) - r_c \min(H, -V^-) && \text{if } V = V^- < 0 \end{aligned} \quad (9)$$

The difference in value between (9) and (8) is the CVA, and the difference between (7) and (9) is the FVA.

Again the above equation only applies to one single derivative, which does not have netting effect in place.

Cross Currency Collateral Posting and Option on Collateral Selection

In this section, we briefly touch on the case where collateral currency is different from the asset/derivative currency. Again, we start from the replication scheme.

Assuming the derivative value is in currency A and collateral posted is in currency B. The exchange rate is: $1A = YB$. Meaning 1 unit of currency A equals Y number of units of currency B. In the following we will use superscript A for quantities related to currency A and superscript B to indicate quantities related to currency B.

Before considering the collateralization and derivative value, let us consider the collateralized lending for both A and B currencies at t=0 and t=T below:

t=0		t=T
1A	-----	$e^{r_c^A T}$ A
1B	-----	$e^{r_c^B T}$ B

and from $1A=Y_0B$ at t=0, we have the exchange rate at t=T from no-arbitrage:

$$Y_T = Y_0 e^{(r_c^B - r_c^A)T} \quad (10)$$

Notice (10) represents an over-simplified situation. In reality, FX forward and cross currency market determines the forward FX rate, which has both credit and liquidity premiums embedded in it. The consistent treatment of OIS discounting and cross currency market is a non-trivial issue, which we leave for future exploration.

Case 5: Fully Collateralized in Currency B

	t=0		t=T	
Value of asset in A:	1A	-----	?	(a)
Collateral to return:	-YB	-----	$-e^{r_c^B T}$ B	(b)
Put Collateral in funding desk:	YB	-----	$e^{r_u^B T}$ B	(c)
Borrow 1A from funding desk:	-1A	-----	$\$-e^{r_u^A T}$ A	(d)

Summarizing the accounts, we would have:

$$? = e^{r_u^A T} A + (e^{r_c^B T} - e^{r_u^B T}) B = \left[e^{r_u^A T} + (e^{r_c^B T} - e^{r_u^B T}) Y_T \right] A$$

For a small time increment dt, the value change would be:

$$\begin{aligned} dV &= r_u^A dt - (r_u^B - r_c^B) dt - (r_c^A - r_c^B)(r_u^B - r_c^B) dt^2 \approx r_u^A dt - (r_u^B - r_c^B) dt \\ &= (r_u^A - r_u^B + r_c^B) dt \end{aligned}$$

This means with the assumption of (10), the rate of carry for 1A collateralized using currency B is $r_u^A - r_u^B + r_c^B$. The same for the rate of carry for borrowing 1A collateralized using B.

This interesting result implies that different choices of currency in collateral posting may imply different growth rate or discount rate for the borrower. To borrower posting collateral, the choice of collateral currency has real economic values. For example, we know

$$r_u^B - r_c^B = u^B + L^B$$

Borrower may choose an optimized currency with the highest liquidity or the lowest L^B so that the value is optimized for the borrower. This can be particularly useful when liquidity hit one or some currencies under stress situations. This intuitive result about collateral choice option may be quantified by modeling stochastic liquidity in various currencies, reflecting the difficulty of raising capitals in them.

Taking

$$Q^{AB} = L^A - L^B$$

as the difference in liquidity premium between the currency pairs, and assuming the simplest normal process for Q:

$$dQ^{AB} = \sigma^{AB} dW_Q$$

One can effectively estimate the value of collateral selection option roughly. Using currency A collateral as a base case, the option of being able to use currency B as collateral, represented in funding carry rate for dt at time t, would be:

$$dr = NormalModel^{Call}(Q_0^{AB}, K = 0, \sigma^{AB}, t)$$

where Q_0^{AB} is current liquidity differential, K is the strike. Here we have assumed the credit risk premiums are the same for the two currencies. Applying the extra funding carry through all time periods would result in the extra collateral option value for the borrower.

For a more general situation where more assets can be used as collateral, such as US treasury and other sovereign issues, the option of collateral selection is quite similar: every asset has a different embedded liquidity premium, and therefore when the liquidity premiums fluctuates, it creates option values for the borrower. At a given time, there is an optimal asset that the borrower can use to post collateral, which optimizes the overall derivative value. This option value can be quantified by modeling around the liquidity premiums for each different collateral asset.

Conclusion

We have reviewed various CVA and FVA related concepts in this article. From collateral process and cash flow replication, we propose a consistent approach in dealing with CVA and FVA together. We argue that DVA concept should be dropped, which is instead replaced as part of FVA. The existence of FVA would imply different market trading mechanism and broken symmetry in counterparty pricing. Furthermore, the change of competitive landscape in derivative pricing could mean important business decision making.

When different assets are allowed in collateral posting, borrower can optimize the derivative value by choosing the asset with lowest liquidity cost. The option of collateral selection can be valued by modeling around the liquidity. We leave the topic for future exploration.

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