

Collateral, Funding
and Discounting

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1 Risk free rate

- Traditional derivatives valuation framework starts by assuming an existence of a (credit) risk-free rate
 - Little “ r ” in Black-Scholes
- Everything is discounted off risk-free rate
- Where is the risk-free rate now?
 - Give cash to another bank?
 - Give cash to a government?
- Nothing in modern economy looks like a theoretically-classic money market account
- How a banks funds itself, i.e. what it does with spare cash (and where it gets cash when it needs to), is of critical importance
 - Before: Bank funding rate \approx Libor \approx FedFunds \approx Government rate...
 - Not anymore! That is why we need to revisit the foundations

2 Credit risk mitigation in OTC trading

- Over-the-counter (bilateral) trading is governed by legal documents, primary of which is ISDA Master Agreement
- Part of it, Credit Support Annex (CSA) specifies credit risk mitigation in form of collateral posting
- In broad strokes, it specifies that if party *A* owes money to party *B*, it has to post collateral in that amount, and vice versa
- So if *A* defaults, *B* could take that collateral in lieu of the promise of *A*
- CSA specifies other important credit risk mitigants such as netting – if *A* owes *B* on one contract and *B* owes *A* on some other, they can be offset against each other in the case of default
- CSAs between each two parties are (somewhat) different. CSA specifies
 - Eligible collateral (cash in a number of currencies, bonds)
 - Rates paid on collateral (party holding collateral typically pays certain rate to the collateral "owner")
 - frequency of collateral posting (e.g. daily)
 - Thresholds, minimum transfer amounts, etc

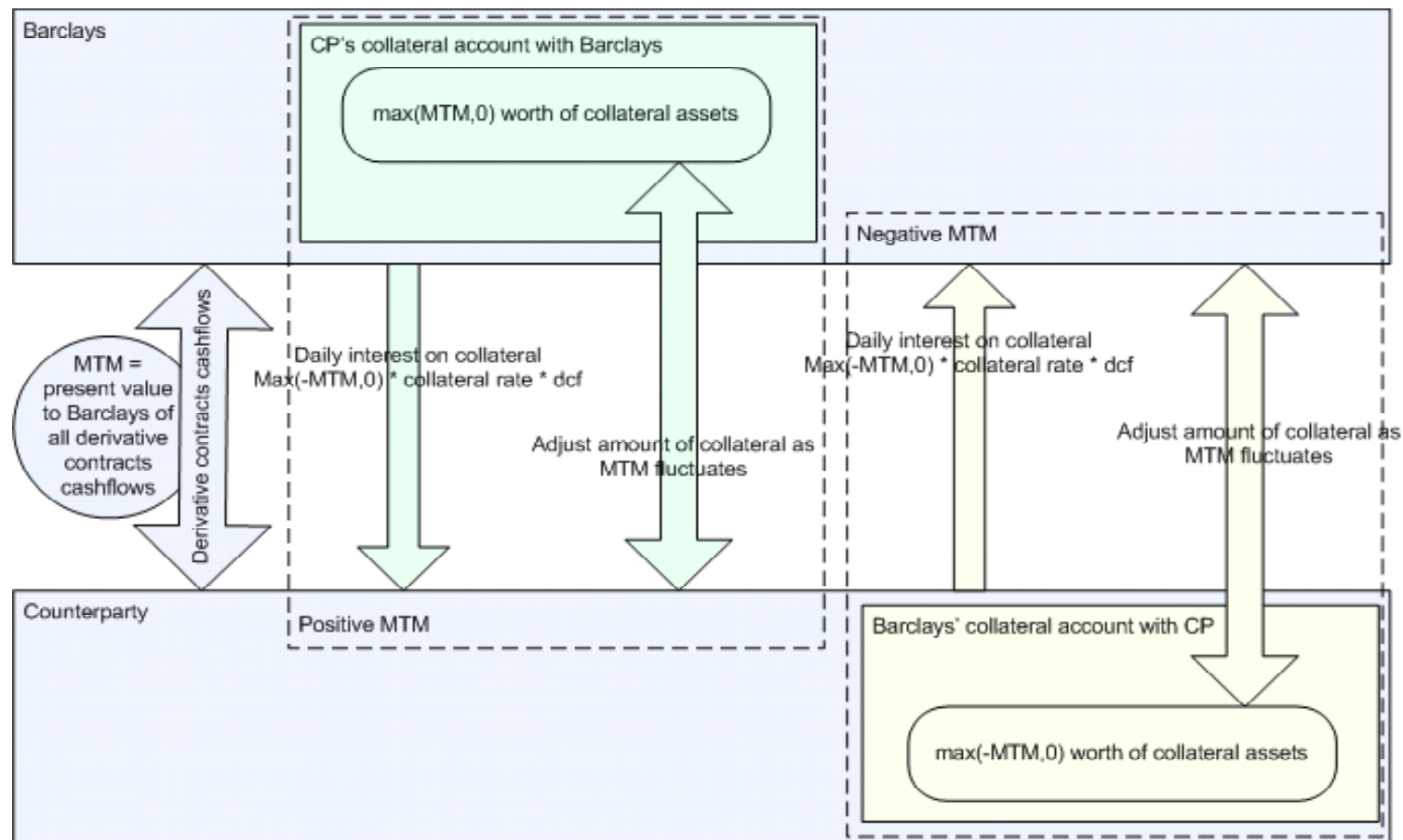
3 Collateralized Assets

- Let us look at the mechanics of collateralized trading
- Party A sells a call option to party B
- B pays $V(0)$ dollars to A
- A promises to pay the payoff of the option at expiry to B
- Any promise needs to be collateralized. A needs to post collateral. How much?
- Well, it is the value of the promise (option) so $V(0)$ dollars! They go right back to B
- During life, the value of the option fluctuates. Depending on the move A will post or claim back collateral
- B will pay an agreed-upon overnight rate on the outstanding collateral to A
- At any point in time the t total collateral posted by A will be $V(t)$ which is the value of the option on that day

4 Collateralized Assets

- No cash exchange at inception
- Pays daily (in math abstraction *continuous*) cashflows
- At any time the option contract could be dissolved and collateral kept – the collateral will exactly offset the market value of the option
- In particular, at option expiry B will just keep the collateral it has and A does not need to pay anything else
- Quite different from a classic picture of “buy and hold”
- Similar to a future. *Future = collateralized forward with collateral rate=0*

5 Collateralized Assets – Picture



6 Hedging instruments

- Trading in hedging instruments (stocks, bonds) fits the same pattern
- When we need to buy stock, where does the bank get money? (How does it fund the shares)
- By borrowing them, with the borrow secured by the shares just bought (repo)
- The rate for this loan is the repo rate
- Borrow the money, buy stock
- Deliver shares as collateral for the loan
- Get collateral back the next day
- Return the loan and the overnight interest (repo rate)
- Repeat for as many days as the shares are needed
- Paying repo rate more efficient than borrowing unsecured – lower rate due to absence of credit risk
- Same type of continuous cashflows as with collateralized derivatives

7 Discounting rate

- Consider fully collateralized counterparty
- Main question: what rate should we use to discount trades with this counterparty?
- In full generality a very difficult question to answer. We try in [Pit12]
- Notations
 - $V(t)$ is price of a collateralized asset between party A and B. If $V(t) > 0$ for A, party B will post $V(t)$ to A.
 - $c(t)$ is a contractually specified collateral rate $c(t)$ on $V(t)$. If $V(t) > 0$, A will pay this rate to B

8 Cashflow Analysis

- Assume A buys some collateralized asset from B (“buying” and “selling” in this context are somewhat meaningless but we keep the terminology for simplicity).
1. Purchase of the asset. The amount of $V(t)$ is paid by A to B
 2. Collateral at t . Since A’s mark-to-market is $V(t)$, the amount $V(t)$ of collateral is posted by B to A
 3. Return of collateral. At time $t + dt$ A returns collateral $V(t)$ to B
 4. Interest. At time $t + dt$, A also pays $V(t)c(t) dt$ interest to B
 5. New collateral. The new mark-to-market is $V(t + dt)$. Party B pays $V(t + dt)$ in collateral to A.

Note that there is no actual cash exchange at time t . At time $t + dt$, net cash flow to A is given by

$$V(t + dt) - V(t)(1 + c(t)) dt = dV(t) - c(t)V(t) dt.$$

As already noted, at time $t + dt$, the MTM+collateral for each party is 0, meaning they can terminate the contract (and keep the collateral) at no cost.

9 Two Collateralized Assets

- Start by assuming two assets both collateralized with rate $c(t)$
- In real world measure the asset prices follow

$$dV_i(t) = \mu_i(t)V_i(t) dt + \sigma_i(t)V_i(t) dW(t), \quad i = 1, 2. \quad (1)$$

- Note the same Brownian motion. Case of a stock (i.e. a repo transaction with stock) and an option on that stock.
- At time t form a portfolio to hedge the effect of randomness of $dW(t)$ on the cash exchanged at time $t + dt$ (no cash exchange at t)
- Go long asset 1 notional $\sigma_2(t)V_2(t)$ and go short asset 2 notional $\sigma_1(t)V_1(t)$
- The cash exchange at time $t + dt$ is then equal to

$$\sigma_2(t)V_2(t) (dV_1(t) - c(t)V_1(t) dt) - \sigma_1(t)V_1(t) (dV_2(t) - c(t)V_2(t) dt)$$

which, after some manipulation, gives us

$$\sigma_2(t)V_1(t)V_2(t) (\mu_1(t) - c(t)) dt - \sigma_1(t)V_1(t)V_2(t) (\mu_2(t) - c(t)) dt$$

- This amount is known at time t and the contract can be terminated at $t + dt$ at zero cost. Hence, the only way both parties agree to transact on this portfolio (no arbitrage), this cash flow must actually be zero

10 Two Collateralized Assets

- Hence

$$\sigma_2(t) (\mu_1(t) - c(t)) = \sigma_1(t) (\mu_2(t) - c(t)).$$

- Using this we can rewrite (1) as

$$dV_i(t) = c(t)V_i(t) dt + \sigma_i(t)V_i(t) d\tilde{W}(t), \quad i = 1, 2, \quad (2)$$

where

$$d\tilde{W}(t) = dW(t) + \frac{\mu_1(t) - c(t)}{\sigma_1(t)} dt = dW(t) + \frac{\mu_2(t) - c(t)}{\sigma_2(t)} dt.$$

- Now, looking at (2) we see that there exists a measure \mathbb{Q} , equivalent to the real world one, in which both assets grow at rate $c(t)$. This is the analog to the traditional risk-neutral measure.
- In \mathbb{Q} , the price process for each asset is given by

$$V_i(t) = E_t^{\mathbb{Q}} \left(e^{-\int_t^T c(s) ds} V_i(T) \right), \quad i = 1, 2. \quad (3)$$

- Collateralized assets should be priced using collateral, usually OIS (Fed-Funds, Eonia) discounting curve

11 Different Collateral Rates

- Even if two assets can be collateralized at different rates, c_1 and c_2 , and the same result would apply. In particular we would still have the condition

$$\sigma_2(t) (\mu_1(t) - c_1(t)) = \sigma_1(t) (\mu_2(t) - c_2(t))$$

from the cash flow analysis.

- Hence, the change of measure is still possible, and (3) still holds with $c(t)$ replaced by the appropriate c_1 or c_2 :

$$V_i(t) = \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^T c_i(s) ds} V_i(T) \right), \quad i = 1, 2. \quad (4)$$

- In the stock option example, the stock will grow at its repo rate and the option will grow at its collateral rate in the risk-neutral measure

12 Many Collateralized Assets

- Consider more than two assets. Assume that $N + 1$ collateralized (with the same collateral rate c) assets are traded, and their real-world dynamics are given by

$$dV = \mu V dt + \Sigma dW,$$

where dW is an N -dimensional Brownian motion and $dV = (dV_1, \dots, dV_{N+1})^\top$

- By the cashflow/no arbitrage arguments similar to the above we can find a vector λ such that

$$\mu - c\mathbf{1} = \Sigma\lambda.$$

- Thus we can write

$$dV = cV dt + \Sigma (dW - \lambda dt)$$

and define the risk-neutral measure by the condition that $dW - \lambda dt$ is a driftless Brownian motion. In this measure all processes V have drift c .

- If we now consider the assets to be (collateralized) ZCBs, we obtain a model of interest rates that looks exactly like the standard HJM model
- Each (collateralized) zero coupon bond grows at the collateral rate $c(t)$ (or its own collateral rate if different). A zero-coupon bond

$$P(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^T c(s) ds} \right)$$

13 Domestic and Foreign Collateral

- Important example of different collateral rates: cross-currency markets
 - On LCH, single-currency swaps are collateralized in the currency of the trade
 - Cross-currency swaps are collateralized in US dollars.
- Also major dealers normally have a choice of the currency to post as collateral
- We need to consider zero coupon bonds (ZCBs) collateralized in the domestic, and as well as some other (call it foreign) currency
- Economy with domestic and foreign assets and an (cash, T0) FX rate $X(t)$ expressed as a number of domestic (\mathcal{D}) units per one foreign (\mathcal{F})
- The domestic collateral rate is $c_d(t)$ and the foreign rate is $c_f(t)$
- Domestic ZCB collateralized in domestic currency by $P_{d,d}(t, T)$. This bond generates the following cashflow at time $t + dt$,

$$dP_{d,d}(t, T) - c_d(t)P_{d,d}(t, T) dt. \quad (5)$$

14 Foreign Bonds with Domestic Collateral

- Now consider a foreign ZCB collateralized with the domestic rate. Let its price, in foreign currency, be $P_{f,d}(t, T)$. Cashflows:
 1. Purchase of the asset. The amount of $P_{f,d}(t, T)$ is paid (in foreign currency \mathcal{F}) by party A to B.
 2. Collateral at t . Since A's MTM is $P_{f,d}(t, T)$ in foreign currency, the amount $P_{f,d}(t, T)X(t)$ of collateral is posted in domestic currency \mathcal{D} by B to A
 3. Return of collateral. At time $t + dt$ A returns collateral $P_{f,d}(t, T)X(t)\mathcal{D}$ to B
 4. Interest. At time $t + dt$, A also pays $c_d(t)P_{f,d}(t, T)X(t) dt$ interest to B in \mathcal{D}
 5. New collateral. The new MTM is $P_{f,d}(t + dt, T)$. Party B pays $P_{f,d}(t + dt, T)X(t + dt)$ collateral to A in \mathcal{D}

The cash flow, in \mathcal{D} , at $t + dt$ is

$$d(P_{f,d}(t, T)X(t)) - c_d(t)P_{f,d}(t, T)X(t) dt. \quad (6)$$

15 Drift of FX Rate

- The equations (5), (6) are insufficient to determine the drift of X
- From (6) we can only deduce the drift of the combined quantity $X P_{f,d}$ and the drift of $P_{f,d}$ is in general *not* c_f (nor it is c_d , for that matter)
- To understand the drift of $X(\cdot)$, we need to understand what kind of (domestic) cash flow we can generate from holding a unit of foreign currency
- Suppose we have $1\mathcal{F}$. If it was a unit of stock, we could repo it out (i.e. borrow money secured by the stock) and pay a repo rate on the stock
- In FX, having $1\mathcal{F}$, we can give it to another dealer and receive its price in domestic currency, $X(t)\mathcal{D}$. The next instant $t + dt$ we would get back $1\mathcal{F}$, and pay back $X(t) + r_{d,f}(t)X(t)dt$, where $r_{d,f}(t)$ is a *rate agreed on this domestic loan collateralized by \mathcal{F}* . As we can sell $1\mathcal{F}$ for $X(t + dt)\mathcal{D}$ at time $t + dt$ the cash flow at $t + dt$ would be

$$dX(t) - r_{d,f}(t)X(t) dt$$

- This is an “instantaneous” FX swap, with a real-life equivalent an overnight (aka tom/next) FX swap
- Importantly, the rate $r_{d,f}(t)$ has no relationship to collateralization rates in two different currencies

16 Cross-Currency Model under Domestic Collateral

1. Market in instantaneous FX swaps allows us to generate cash flow $dX(t) - r_{d,f}(t)X(t) dt$
 2. Market in $P_{d,d}$ generates cash flow $dP_{d,d}(t, T) - c_d(t)P_{d,d}(t, T) dt$
 3. Market in $P_{f,d}$ generates cash flow $d(P_{f,d}(t, T)X(t)) - c_d(t)P_{f,d}(t, T)X(t) dt$
- Assume real world measure dynamics (μ , dW are vectors and Σ is a matrix)

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d(P_{f,d}X)/(P_{f,d}X) \end{pmatrix} = \mu dt + \Sigma dW,$$

- By the same cashflow arguments as before, we can find a measure (“domestic risk-neutral”) Q^d under which the dynamics are

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d(P_{f,d}X)/(P_{f,d}X) \end{pmatrix} = \begin{pmatrix} r_{d,f} \\ c_d \\ c_d \end{pmatrix} dt + \Sigma dW^d \quad (7)$$

17 Cross-Currency Model under Domestic Collateral

With

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d(P_{f,d}X)/(P_{f,d}X) \end{pmatrix} = \begin{pmatrix} r_{d,f} \\ c_d \\ c_d \end{pmatrix} dt + \Sigma dW^d,$$

we have

$$\begin{aligned} X(t) &= \mathbb{E}_t^d \left(e^{-\int_t^T r_{d,f}(s) ds} X(T) \right), \\ P_{d,d}(t, T) &= \mathbb{E}_t^d \left(e^{-\int_t^T c_d(s) ds} \right), \\ P_{f,d}(t, T) &= \frac{1}{X(t)} \mathbb{E}_t^d \left(e^{-\int_t^T c_d(s) ds} X(T) \right). \end{aligned} \tag{8}$$

18 Cross-Currency Model under Foreign Collateral

- Same model under foreign collateralization.
- Foreign bonds $P_{f,f}$ and domestic bonds collateralized in foreign currency $P_{d,f}$.
- By repeating the arguments above we can find a measure Q^f under which

$$\begin{pmatrix} d(1/X)/(1/X) \\ dP_{f,f}/P_{f,f} \\ d(P_{d,f}/X)/(P_{d,f}/X) \end{pmatrix} = \begin{pmatrix} -r_{d,f} \\ c_f \\ c_f \end{pmatrix} dt + \tilde{\Sigma} dW^f \quad (9)$$

- In particular

$$P_{d,f}(t, T) = X(t) E_t^f \left(e^{-\int_t^T c_f(s) ds} \frac{1}{X(T)} \right). \quad (10)$$

- Not all processes in (7) and (9) can be specified independently. In fact, with the addition of the dynamics of $P_{f,f}$ to (7), the model is fully specified, as the dynamics of $P_{d,f}$ can then be derived.
- Risk-free rates [FT11] have no economic meaning, their differences (for different currencies) do – they are rates quoted for instantaneous FX swaps and define the rate of growth of the FX rate

19 Forward FX

- A forward FX contract pays $X(T) - K$ at T (in \mathcal{D}). The price process of the *domestic-currency-collateralized* forward contract is

$$\mathbb{E}_t^d \left(e^{-\int_t^T c_d(s) ds} (X(T) - K) \right) = X(t)P_{f,d}(t, T) - KP_{d,d}(t, T)$$

- The *forward FX rate*, i.e. K that makes the price process have value zero is given by $X_d(t, T) = \frac{X(t)P_{f,d}(t, T)}{P_{d,d}(t, T)}$.

- We can also view a forward FX contract as paying $1 - K/X(T)$ in \mathcal{F}

- Then, with *foreign collateralization*, the value would be

$$\mathbb{E}_t^f \left(e^{-\int_t^T c_f(s) ds} (1 - K/X(T)) \right) = P_{f,f}(t, T) - KP_{d,f}(t, T)/X(t)$$

and the forward FX rate collateralized in c_f is given by $X_f(t, T) = \frac{X(t)P_{f,f}(t, T)}{P_{d,f}(t, T)}$

- In the general model, there is no reason why $X_f(t, T)$ would be equal to $X_d(t, T)$, and the forward FX rate would depend on the collateral used. It appears, however, that in current market practice FX forwards are quoted without regard for the collateral arrangements

20 Choice Collateral

- Consider a domestic asset, with price process $V(t)$, that can be collateralized either in the domestic (rate c_d) or the foreign (rate c_f) currency.
- Common case for CSA agreements between dealers
- From previous analysis it follows that the foreign-collateralized domestic ZCB grows (in the domestic currency) at the rate $c_f + r_{d,f}$
- It can be shown rigorously that the same is true for any domestic asset
- When one can choose the collateral, one would maximize the rate received on it, so the choice collateral rate is equal to

$$\max(c_d(t), c_f(t) + r_{d,f}(t)) = c_d(t) + \max(c_f(t) + r_{d,f}(t) - c_d(t), 0)$$

- The simplest extension of the traditional cross-currency model that accounts for different collateralization would keep the spread

$$q_{d,f}(t) \triangleq c_f(t) + r_{d,f}(t) - c_d(t)$$

deterministic (intrinsic)

21 Choice Collateral

- In this case the collateral choice will not generate any optionality although the discounting curve for the choice collateral rate will be modified
- Anecdotal evidence suggests that at least some dealers do assign some value to the option to switch collateral in the future

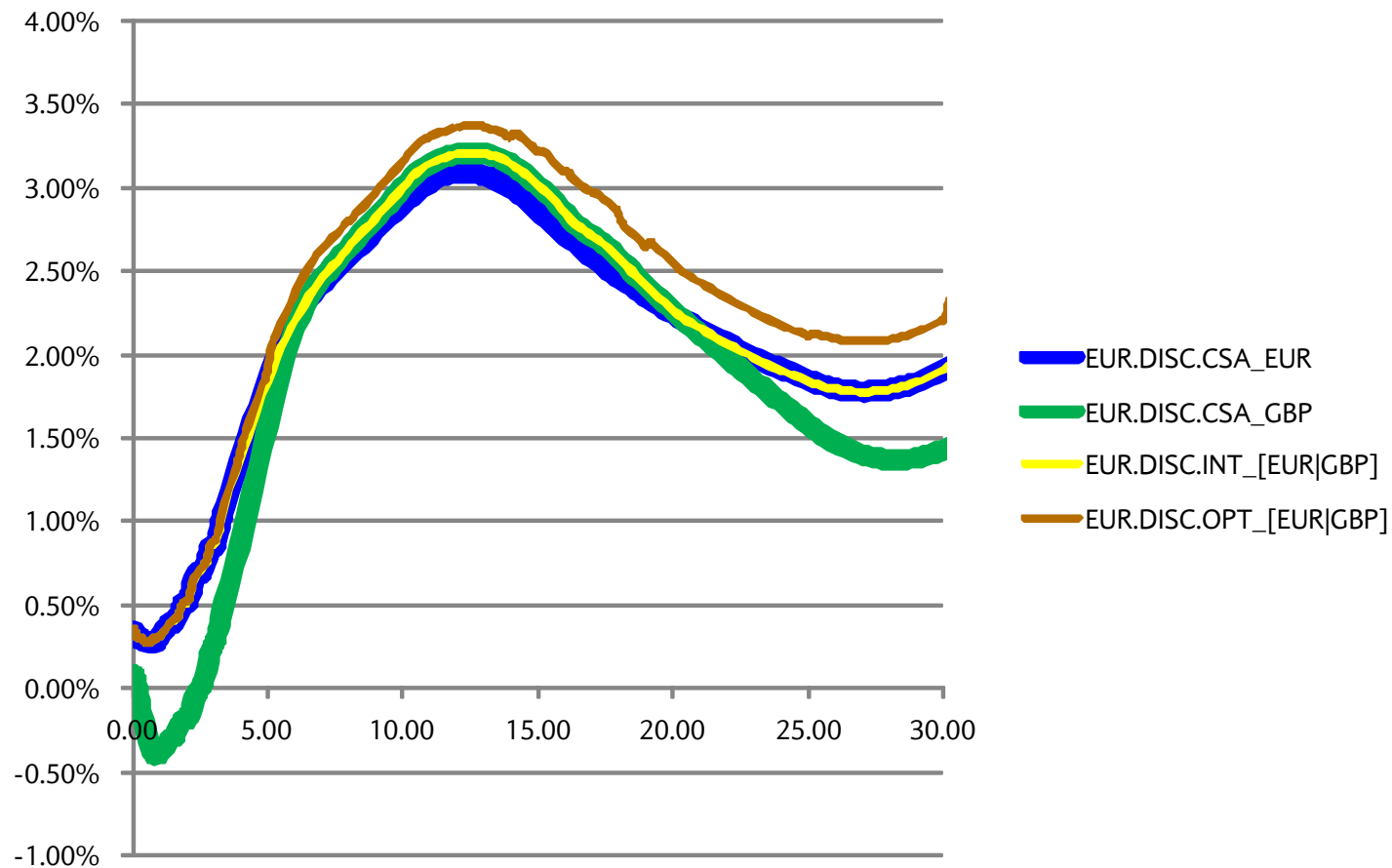
- Full collateral choice model:

$$V(t) = E_t^d \left(e^{-\int_t^T c_d(s) ds} e^{-\int_t^T \max(q_{d,f}(s), 0) ds} V(T) \right)$$

- At least 4 factors: one for each of c_d , c_f , X , $q_{d,f}$. “Standard” XC model recovered with $q_{d,f} \equiv 0$.

22 Choice Collateral

Example. For Intrinsic take FX-adjusted overnight forward curves and form a maximum.



23 Issues with Full Collateral Choice Model

- Large number of unobserved parameters (volatilities, correlations of $q_{d,f}$)
- Uncertain horizon – collateral choice may go away with developments in the industry (more clearing, standard CSA)
- Assumes that instantaneous replacement of collateral from one currency to another is possible
- More realistic assumptions (?)
 - Only *change* in collateral balance can be posted in a choice currency
 - Only currency previously posted can be recalled, not exceeding the total amount posted (and change in MTM)
- This results in a path-dependent, non-linear dynamic optimization problem
- All in all, swaps pricing is getting quite complicated!

24 Sticky Collateral Model

- Consider an asset that we sold, with a positive MTM throughout the life, like a ZCB
- Price process is given by V_n (value to us is $-V_n$), $n = 0, \dots, N + 1$.
- We post collateral. Suppose we can post two types, with rates r_n and p_n . At time t_n we post A_n of type 1 and B_n of type 2.
- Basic restrictions are

$$A_n + B_n = V_n \quad (11)$$

$$A_n, B_n \geq 0. \quad (12)$$

- Path dependent restrictions. Having fixed A_0, \dots, A_{n-1} , we have the following on A_n
 - If $V_n > V_{n-1}$ then the difference $\Delta V_n \triangleq V_n - V_{n-1}$ can be added to A or to B . So we must have

$$0 \leq A_n - A_{n-1} \leq \Delta V_n$$

- If $V_n < V_{n-1}$ we can subtract up to $V_{n-1} - V_n$ from either A or B . But we cannot violate the restriction (12). After some manipulations

$$\max(-A_{n-1}, \Delta V_n) \leq A_n - A_{n-1} \leq \min(V_n - A_{n-1}, 0)$$

25 Bellman-Jacobi Equation

- Total interest on collateral account

$$\sum_{n=0}^N (r_n A_n + p_n B_n) = \sum_{n=0}^N (r_n A_n + p_n (V_n - A_n))$$

- Let $q_n = r_n - p_n$ then the total accrual is equal to

$$\sum_{n=0}^N p_n V_n + \sum_{n=0}^N q_n A_n$$

and the first term is independent of the strategy. Let us focus on the second one.

- Let $J_k(a_k)$ be the time- t_k the expected value of the optimal strategy for terms from k onwards, assuming $A_k = a_k$. Let $(A)_k$ be a particular strategy from k onwards, i.e. $(A)_k = \{A_k, \dots, A_N\}$. Denote the allowed set of values for A_{k+1} given $A_k = a$ by $I_k(a)$, and the set of allowed strategies $(A)_k$ assuming $A_k = a$ by $\theta_k(a)$

26 Bellman-Jacobi Equation

- From the optimal control condition

$$\begin{aligned} J_k(a) &= \mathbb{E}_k \left(\max_{(A)_k \in \theta_k(a)} \left(\sum_{n=k}^N q_n A_n \right) \right) \\ &= q_k a + \mathbb{E}_k \left(\max_{A_{k+1} \in I_k(a)} \left(\max_{(A)_{k+1} \in \theta_{k+1}(A_{k+1})} \sum_{n=k+1}^N q_n A_n \right) \right) \\ &= q_k a + \mathbb{E}_k \left(\max_{\tilde{a} \in I_k(a)} J_{k+1}(\tilde{a}) \right) \end{aligned}$$

- Only numerical solution, done at a portfolio level (not easy)
- Solution can be below intrinsic collateral choice (when $q_{d,f}$ deterministic)
- Strong dependence of solution on average daily (absolute) move in portfolio value
- No non-trivial $\Delta t \rightarrow 0$ limit as BM has infinite absolute variation (so full collateral choice formula is recovered)

27 Non-linear features

- Some CSA features prevent even an approximate “curve” valuation
 - One-way CSA
 - Material thresholds and minimum transfer amounts
 - Ratings triggers
 - Complicated netting rules not available in FO systems
- All in all, swaps pricing is getting quite complicated!

28 Beyond Collateral

- Derivatives trading desk:
 - Trades derivatives
 - Hedges with underlying assets
 - Lends/borrows money
- Money comes from different sources
 - Unsecured
 - Secured by assets
 - Collateral (daily MTM paid/received in cash or high quality bonds)
- Historically do not differentiate between different rates
- But now large differences and volatility in *funding spread*
- How does it affect pricing (from funding prospective)?
- For inclusion of credit risk see [BK11]

29 Uncollateralized Derivatives

- Assume imperfect (allowing for none) collateralization
- Money generated/required by derivatives trading and not posted/received as collateral should come from somewhere
- For a trading desk, it comes from the funding desk
- Funding desk typically funds cash balances at the overnight funding rate determined by the cost of funds the banks can get externally
- Overnight funding can be included into our framework by considering it as another type of “collateral”, i.e. cashflows that generate funding rate
- Have a model as before except “collateral account” pays a mixture of the external collateral and internal funding rate

30 Notations

- Asset $S(t)$
- Various rates
 - Risk-free overnight rate $r_C(t)$, paid on (true) collateral accounts (under CSA)
 - Asset dividend rate $r_D(t)$
 - Rate on borrowing secured by asset $r_R(t)$
 - Unsecured bank funding rate $r_F(t)$
 - funding spread $s_F(t) \triangleq r_F(t) - r_C(t)$
- Let $C(t)$ be the collateral held at time t against this derivative (allow to be different from $V(t)$). Pays rate $r_C(t)$
- The balance $V(t) - C(t)$ pays rate $r_F(t)$

31 Valuation with Partial Collateral

- Considering cashflows as before, we obtain that there exists a (risk-neutral) measure in which the price process grows as

$$r_C(t)C(t) dt + r_F(t) (V(t) - C(t)) dt = r_F(t)V(t) dt - (r_F(t) - r_C(t)) C(t) dt$$

while the stock grows at rate $r_R(t) - r_D(t)$

- The solution is given by

$$V(t) = \mathbb{E}_t \left(e^{-\int_t^T r_F(u) du} V(T) + \int_t^T e^{-\int_t^u r_F(v) dv} (r_F(u) - r_C(u)) C(u) du \right) \quad (13)$$

in measure where

$$dS(t)/S(t) = (r_R(t) - r_D(t)) dt + \sigma_S(t) dW_S(t) \quad (14)$$

- Another useful formula

$$V(t) = \mathbb{E}_t \left(e^{-\int_t^T r_C(u) du} V(T) \right) - \mathbb{E}_t \left(\int_t^T e^{-\int_t^u r_C(u) du} (r_F(u) - r_C(u)) (V(u) - C(u)) du \right) \quad (15)$$

- Same results obtained from the Black-Scholes extension in [Pit10]

32 Conclusions

- Careful considerations of the details of bank funding (through CSAs and externally) are critical to derivatives valuation
- Resulting framework is rather complex
 - Many unobservable parameters
 - Significant uncertainty in evolution of market structure
 - “Small” details of legal agreements (e.g. if collateral can be substituted) lead to significant differences in prices
- Full realism is probably unattainable. Need simplifications that still capture main effects
- Not only discounting is affected – forward curves are counterparty and CSA dependent
- A lot more work remains

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