The Danish mortgage market

- We focus on the Danish market for MBS — but there are many similarities to the US (fixed-rate loan with prepayment option).
- Mortgage-backed bonds are used for real-estate finance (up to 80% of value in Denmark).
- Each MBS is backed by thousands of individual mortgages (widely different sizes — corporate as well as single-family borrowers)
- Fixed coupon, annuity loans, 20–30 years to maturity at issue.
- Except for a small fee to the mortgage institution, all payments are passed through to the investors (pass-through securities).
- Borrowers can prepay their mortgage at any time. They have an option to refinance the loan if interest rates drop.
- Payments from a given borrower: scheduled payments (interest and principal) and prepayments (remaining principal).
Problems with pricing MBS – 1

- MBS: fixed-income derivative with payments, \( \{B(t_i)\}_{i=1}^{N} \) at times \( \{t_i\}_{i=1}^{N} \), depending on the (future) evolution of interest rates.

- General expression for the value today \( (t = 0) \) of the MBS:

\[
V_0 = \sum_{i=1}^{N} E_0^Q \left[ e^{-\int_0^{t_i} r_s ds} B(t_i) \right].
\] (1)

- If the mortgage is non-callable, payments are non-stochastic and the value is given by:

\[
V_0 = \sum_{i=1}^{N} B(t_i) \cdot P(0, t_i).
\] (2)

- Danish MBS are callable (borrowers have a prepayment option).

Problems with pricing MBS – 2

- The actual payments can be very different from the scheduled payments (constant annuity payment) — see figures.

- The actual payments (their size and timing) of the MBS depend on the prepayment behavior of borrowers.

- Because of prepayment, the actual cash flows are shorter (occur earlier) than the scheduled cash flows. This has implications for hedging, for example duration measures.

- Main reason for prepayment: interest rates have dropped compared to date of issue (refinancing motive).

- Note that this reduces the value of the MBS (price of the option).

- The main problem is modeling prepayment behavior . . .
Modeling prepayment behavior — how?

- We know the optimal call strategy for a callable bond:
  - Minimize liability by calling the bond (prepaying) when the hold-on value is greater than par (plus any costs/penalties of prepaying).
  - This is the same strategy for exercising an American option.
  - Calculating the hold-on value requires a term-structure model, for example a binomial tree.

- Complications for MBS:
  - Many borrowers with different behavior: not all borrowers prepay at the same time
  - One reason: differences in transaction costs across borrowers who still behave rationally (borrower heterogeneity)
  - Other motives for prepayment: liquidity concerns (increase maturity of the loan in order to reduce the monthly payments), real-estate turnover (although Danish mortgages are assumable, contrary to most US mortgages), and (perhaps?) borrower irrationality.

Prepayment functions — 1

- Instead of optimal call strategies, we use a prepayment function approach for pricing MBS.
- We denote the prepayment function in state $s$ at time $n$ by $\lambda(n,s)$.
- Definition: $\lambda(n,s)$ is the fraction of the remaining principal which is prepaid in state $s$ at time $n$.
- Alternative definition: the probability of prepayment given that the loan has not been prepaid earlier.
- Specification of $\lambda(n,s)$ using, e.g., the Probit model:
  \[
  \lambda(n,s) = \Phi \left[ \beta' z(n,s) \right].
  \] (3)
- Here, $\beta$ is a parameter vector, $z(n,s)$ the explanatory variables at the node $(n,s)$, and $\Phi(\cdot)$ the CDF of the normal distribution.
- We estimate $\beta$ from historical prepayment data.
Prepayment functions – 2

- Explanatory variables [notation from Jakobsen (1992, 1994)] in a "typical" prepayment model:
  
  1. **C/R**: Coupon rate / market rate (refinancing rate). Positively related to prepayments.
  2. **GAIN**: the gain from prepaying — PV(annual gain) / PV(old loan). Positively related to prepayments.
  3. **MATURITY**: the maturity of the loan. Negatively related to prepayment.
  4. **SPREAD**: the difference between long and short interest rates. Positively related to prepayment (borrowers prepay now because they expect short/medium term rates to increase in the future).
  5. **LOAN SIZE**: some prepayments costs are fixed and matter less for large loans. Positively related to prepayment.
  6. **BURNOUT**: prepayment slows over time since borrowers with low transaction costs prepay first. Negatively related to prepayment.

MBS valuation – 1

- All explanatory variables, \( z(n,s) \), **except burnout**, can be obtained from the yield curve in node \((n,s)\).
- Burnout measures depend on previous levels of prepayment, and hence on the path followed by interest rates (non-Markovian).
- Using binomial trees requires a Markovian model, so we ignore burnout in the following.
- The value of the MBS at node \((n,s)\) is given by:

  \[
  V(n,s) = \lambda(n,s)W(n,s) + (1 - \lambda(n,s))V^+(n,s).
  \]  

- The actual price, \( V(n,s) \), is a weighted average of two prices:
  - \( W(n,s) \) is the value of the MBS if **all borrowers** decide to prepay.
  - \( V^+(n,s) \) is the value of the MBS if **no borrowers** decide to prepay.
MBS valuation — 2

• If the **term of notice** is zero, \( W(n, s) \) equals the scheduled payment \( A(n) \) (usually constant), plus remaining principal:

\[
W(n, s) = A(n) + H(n).
\] (5)

• In practice, though, the term of notice is non-zero. It varies between 2 and 11 months (2–5 months for recent issues)

• Conditional upon prepayment, the cash flows are non-stochastic so \( W(n, s) \) is easy to compute (by discounting the payments).

• The MBS value assuming no prepayments, \( V^+(n, s) \), follows from the backward equation:

\[
V^+(n, s) = A(n) + \frac{1}{2} p(n, s) \{ V(n + 1, s + 1) + V(n + 1, s) \}. \] (6)

• Boundary conditions: \( V(N + 1, s) = 0 \) (loan is fully amortized).

Numerical example

• Three-period short-rate tree:

\[
\begin{array}{c}
8.00 \\
10.00 \\
12.50 \\
6.40 \\
8.00 \\
5.12
\end{array}
\]

• MBS security: three-year annuity with 10 percent coupon.

• Prepayment function:

\[
\lambda(n, s) = \begin{cases} 
0 & \text{if } g(n, s) < 0 \text{ or } n = 0 \text{ or } n > 2 \\
25 \cdot g(n, s) & \text{if } 0 \leq g(n, s) \leq 0.04 \\
1 & \text{otherwise.}
\end{cases}
\] (7)

• The gain function \( g(n, s) \) is defined as

\[
g(n, s) = \frac{(PV(n, s) - 1.02 \cdot H(n, s))/PV(n, s)}{}
\] (8)

• \( H(n, s) \) is the remaining principal, and \( PV(n, s) \) is present value of the remaining payments (in node \( (n, s) \) for both variables).

• Note that we assume 2% proportional transactions cost when prepaying.
Modeling burnout

- Consider two scenarios:
  1. Interest rates decrease by 2\% after the first period, and increase by 1\% between first and second period.
  2. Interest rates increase by 1\% after the first period, and decrease by 2\% between first and second period.

- In both cases, interest rates have dropped by 1\% at time $n = 2$, compared to time $n = 0$.
- Should we expect the same level of prepayments then?
  - **Probably not** — in the first scenario there has been some prepayment at time $n = 1$, and the borrowers with the lowest transaction costs are the first to prepay their loans.
- Thus, in scenario 1, the remaining borrowers face higher transaction costs (on average) at $n = 2$.
- This problem is known as **burnout**.