Fixed Income Analysis

Mortgage-Backed Securities

The Danish mortgage market
Problems with pricing mortgage-backed bonds
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The Danish mortgage market

- We focus on the Danish market for MBS — but there are many similarities to the US (fixed-rate loan with prepayment option).
- Mortgage-backed bonds are used for real-estate finance (up to 80% of value in Denmark).
- Each MBS is backed by thousands of individual mortgages (widely different sizes — corporate as well as single-family borrowers)
- Fixed coupon, annuity loans, 20–30 years to maturity at issue.
- Except for a small fee to the mortgage institution, all payments are passed through to the investors (pass-through securities).
- Borrowers can prepay their mortgage at any time. They have an **option** to refinance the loan if interest rates drop.
- Payments from a given borrower: **scheduled** payments (interest and principal) and **prepayments** (remaining principal).

Problems with pricing MBS – 1

- MBS: fixed-income derivative with payments, $\{B(t_i)\}_{i=1}^N$ at times $\{t_i\}_{i=1}^N$, depending on the (future) evolution of interest rates.
- General expression for the value today ($t = 0$) of the MBS:

$$V_0 = \sum_{i=1}^{N} E_0^Q \left[e^{-\int_0^{t_i} r_s ds} B(t_i) \right]. \tag{1}$$

- If the mortgage is non-callable, payments are **non-stochastic** and the value is given by:

$$V_0 = \sum_{i=1}^{N} B(t_i) \cdot P(0, t_i). \tag{2}$$

- Danish MBS are callable (borrowers have a prepayment option).

Problems with pricing MBS – 2

- The **actual** payments can be very different from the **scheduled** payments (constant annuity payment) — see figures.
- The actual payments (their size and timing) of the MBS depend on the prepayment behavior of borrowers.
- Because of prepayment, the actual cash flows are shorter (occur earlier) than the scheduled cash flows. This has implications for hedging, for example **duration** measures.
- Main reason for prepayment: interest rates have dropped compared to date of issue (refinancing motive).
- Note that this reduces the value of the MBS (price of the option).
- The main problem is modeling prepayment behavior . . .

Modeling prepayment behavior — how?

- We know the optimal call strategy for a callable bond:
  - Minimize liability by calling the bond (prepaying) when the hold-on value is greater than par (plus any costs/penalties of prepaying).
  - This is the same strategy for exercising an **American option**.
  - Calculating the hold-on value requires a term-structure model, for example a binomial tree.
- Complications for MBS:
  - Many borrowers with different behavior: not all borrowers prepay at the same time
  - One reason: differences in **transaction costs** across borrowers who still behave rationally (borrower heterogeneity)
  - Other motives for prepayment: **liquidity concerns** (increase maturity of the loan in order to reduce the monthly payments), **real-estate turnover** (although Danish mortgages are assumable, contrary to most US mortgages), and (perhaps?) **borrower irrationality**.

Prepayment functions – 1

- Instead of optimal call strategies, we use a **prepayment function** approach for pricing MBS.
- We denote the prepayment function in state s at time n by $\lambda(n, s)$.
- Definition: $\lambda(n, s)$ is the fraction of the remaining principal which is prepaid in state s at time n.
- Alternative definition: the probability of prepayment given that the loan has not been prepaid earlier.
- Specification of $\lambda(n, s)$ using, e.g., the Probit model:

$$\lambda(n, s) = \Phi\left[\beta' z(n, s)\right]. \quad (3)$$

- Here, β is a parameter vector, $z(n, s)$ the explanatory variables at the node (n, s), and $\Phi(\cdot)$ the CDF of the normal distribution.
- We estimate β from historical prepayment data.

Prepayment functions – 2

- Explanatory variables [notation from Jakobsen (1992, 1994)] in a "typical" prepayment model:

1. **C/R:** Coupon rate / market rate (refinancing rate). Positively related to prepayments.
2. **GAIN:** the gain from prepaying — PV(annual gain) / PV(old loan). Positively related to prepayments.
3. **MATURITY:** the maturity of the loan. Negatively related to prepayment.
4. **SPREAD:** the difference between long and short interest rates. Positively related to prepayment (borrowers prepay now because they expect short/medium term rates to increase in the future).
5. **LOAN SIZE:** some prepayments costs are fixed and matter less for large loans. Positively related to prepayment.
6. **BURNOUT:** prepayment slows over time since borrowers with low transaction costs prepay first. Negatively related to prepayment.

MBS valuation – 1

- All explanatory variables, $z(n,s)$, **except burnout**, can be obtained from the yield curve in node (n,s).
- Burnout measures depend on previous levels of prepayment, and hence on the path followed by interest rates (non-Markovian).
- Using binomial trees requires a Markovian model, so we ignore burnout in the following.
- The value of the MBS at node (n,s) is given by:

$$V(n,s) = \lambda(n,s)W(n,s) + (1 - \lambda(n,s))V^{+}(n,s). \quad (4)$$

- The actual price, $V(n,s)$, is a weighted average of two prices:
  - $W(n,s)$ is the value of the MBS if **all borrowers** decide to prepay.
  - $V^{+}(n,s)$ is the value of the MBS if **no borrowers** decide to prepay.

MBS valuation – 2

- If the **term of notice** is zero, $W(n,s)$ equals the scheduled payment $A(n)$ (usually constant), plus remaining principal:

$$W(n,s) = A(n) + H(n). \quad (5)$$

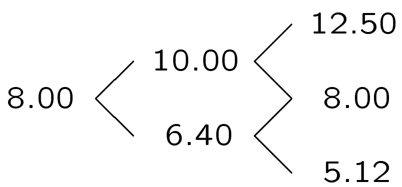
- In practice, though, the term of notice is non-zero. It varies between 2 and 11 months (2–5 months for recent issues)
- Conditional upon prepayment, the cash flows are non-stochastic so $W(n,s)$ is easy to compute (by discounting the payments).
- The MBS value assuming no prepayments, $V^+(n,s)$, follows from the backward equation:

$$V^+(n,s) = A(n) + \tfrac{1}{2}p(n,s)\left\{V(n+1,s+1) + V(n+1,s)\right\}. \quad (6)$$

- Boundary conditions: $V(N+1,s) = 0$ (loan is fully amortized).

Numerical example

- Three-period short-rate tree:



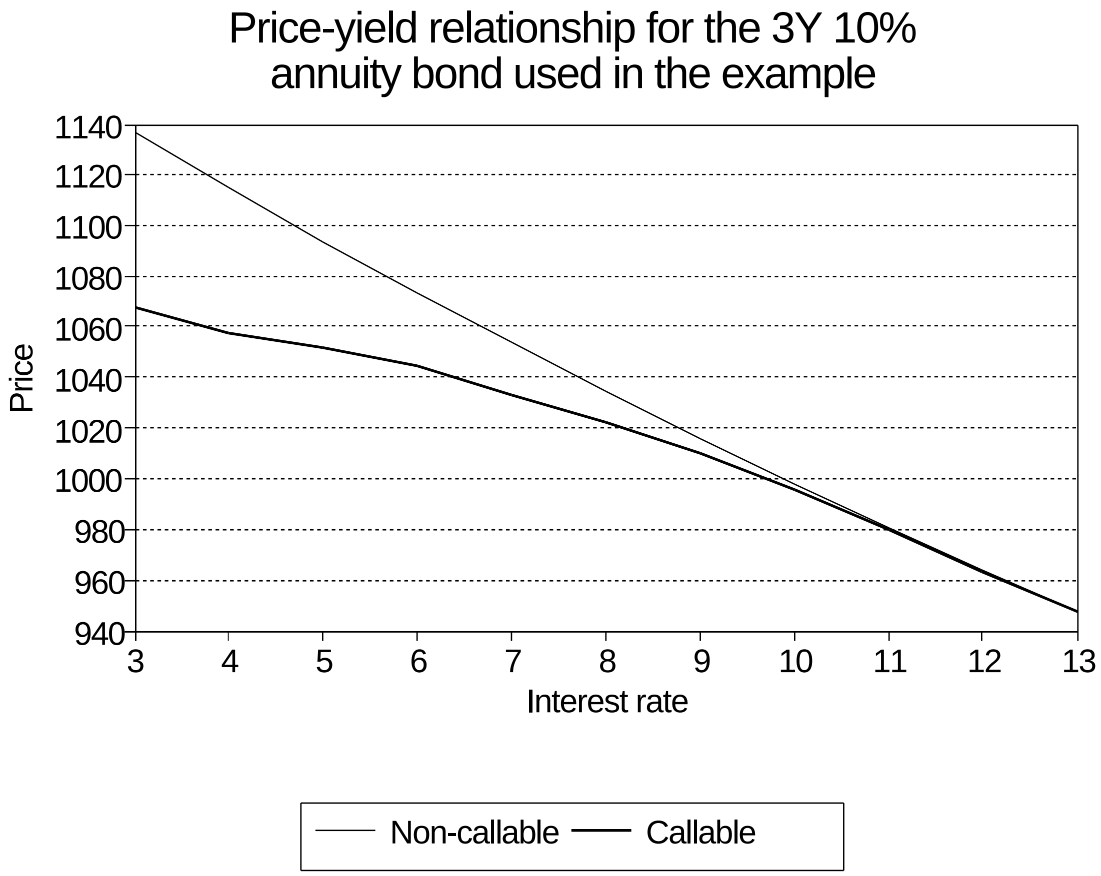
- MBS security: three-year annuity with 10 percent coupon.
- Prepayment function:

$$\lambda(n,s) = \begin{cases} 0 & \text{if } g(n,s) < 0 \text{ or } n = 0 \text{ or } n > 2 \\ 25 \cdot g(n,s) & \text{if } 0 \le g(n,s) \le 0.04 \\ 1 & \text{otherwise.} \end{cases} \quad (7)$$

- The gain function $g(n,s)$ is defined as

$$g(n,s) = (PV(n,s) - 1.02 \cdot H(n,s))/PV(n,s) \quad (8)$$

- $H(n,s)$ is the remaining principal, and $PV(n,s)$ is present value of the remaining payments (in node (n,s) for both variables).
- Note that we assume 2% proportional transactions cost when prepaying.



Modeling burnout

- Consider two scenarios:
  1. Interest rates decrease by 2% after the first period, and increase by 1% between first and second period.
  2. Interest rates increase by 1% after the first period, and decrease by 2% between first and second period.
- In both cases, interest rates have dropped by 1% at time $n = 2$, compared to time $n = 0$.
- Should we expect the same level of prepayments then?
- **Probably not** — in the first scenario there has been some prepayment at time $n = 1$, and the borrowers with the lowest transaction costs are the first to prepay their loans.
- Thus, in scenario 1, the remaining borrowers face higher transaction costs (on average) at $n = 2$.
- This problem is known as **burnout**.