Swap options (swaptions)

Definition

A swaption is an option on an interest rate swap. Distinction is made between payer swaptions and receiver swaptions. More than 90% of swaptions have European exercise.

Description

- Payer swaptions: the right but not the obligation to pay fixed rate and receive floating rate in the underlying swap.
- Receiver swaptions: the right but not the obligation to receive fixed and pay floating rate in the underlying swap.

Why use a swaption?

Suppose that there is uncertainty about whether interest rates will increase or decrease in the future. Instead of using an interest rate swap, a swaption can be used to protect a firm against the risk of higher borrowing costs, but without giving up the possible benefit of lower interest rates.

Theory

European swaptions are normally priced by using the forward swap rate as input in the Black-76 option-pricing model. The Black-76 value is multiplied by a factor adjusting for the tenor of the swaption. This model is arbitrage-free if a lognormal swap rate is assumed.

The following symbols are used:

t_1 = tenor of swap in years

F = forward rate of underlying swap

X = strike rate of swaption

r = risk-free interest rate

T = time to expiration in years

s = volatility of the forward-starting swap rate

m = compoundings per year in swap rate

If c indicates a payer swaption and p indicates a receiver option, swaption prices are given by

$$c = \left[\frac{1 - \frac{1}{\left(1 + \frac{F}{m}\right)^{t_1 \times m}}}{F} \right] \cdot e^{-rT} [FN(d_1) - XN(d_2))$$

$$p = \left[\frac{1 - \frac{1}{\left(1 + \frac{F}{m}\right)^{t_1 \times m}}}{F} \right] \cdot e^{-rT} [XN(-d_2) - FN(-d_1))$$

where

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \left(\frac{s^2}{2}\right)T}{s\sqrt{T}} \text{ and } d_2 = d_1 - s\sqrt{T} .$$

Example

Consider a 2-year payer swaption on a 4-year swap with semi-annual compounding. The forward swap rate of 7% starts 2 years from now and ends 6 years from now. The strike is 7.5%; the risk-free interest rate is 6%; the volatility of the forward starting swap rate is 20% p.a.

Hence,

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \left(\frac{s^2}{2}\right)T}{s\sqrt{T}}$$

$$= \frac{\ln\left(\frac{0.07}{0.075}\right) + \left(\frac{0.2^2}{2}\right)(2)}{0.2\sqrt{2}}$$

$$= -0.1025$$

$$d_2 = d_1 - s\sqrt{T}$$

$$= -0.1025 - 0.2\sqrt{2}$$

$$= -0.38535$$

$$N(d_1) = 0.4592; \quad N(d_2) = 0.35$$

$$\therefore c = \left[\frac{1 - \frac{1}{\left(1 + \frac{F}{m}\right)^{t_1 \times m}}}{F}\right] \cdot e^{-rT}[FN(d_1) - XN(d_2)]$$

$$= \left[\frac{1 - \frac{1}{\left(1 + \frac{0.07}{2}\right)^{4\times 2}}}{0.07}\right] \cdot e^{-0.06\times 2}[0.07(0.4592) - 0.075(0.35)]$$

$$= 0.017967$$

$$= 1.7967\% \text{ of the notional}$$