Hedging with Bond and Note Futures

Exhibit 1.2

Futures Hedge Algebra

For a given change in bond yields . . .

\[
\text{Number of bond futures} \times \text{Change in the value of 1 futures contract} = - \text{Change in the value of the bond portfolio}
\]

Hedge ratio

\[
\frac{\text{Number of bond futures}}{- \text{Change in the value of the bond portfolio}} = \frac{-1}{\text{Change in the value of 1 futures contract}}
\]

- The negative sign indicates that you want to offset changes in the value of the bond portfolio. If you want to replicate the change in the value of the bond portfolio, the number of futures contracts would have the same sign as the change in the value of the bond portfolio.
Hedging with Bond and Note Futures

Exhibit 1.3

Calculating Hedge Ratios

Use of DV01 (dollar value of a basis point)

\[
\text{Hedge ratio} = \frac{\text{Portfolio DV01}}{\text{Futures DV01}}
\]

Use of duration

\[
\text{Hedge ratio} = \frac{\left[ \text{Portfolio duration} \times \text{Portfolio market value} \right]}{\left[ \text{Futures duration} \times \left( \frac{\text{Futures price}}{100} \right) \times \text{Par amount} \right]}
\]

- For the purposes of reckoning hedge ratios, duration is defined as “modified” or “effective” duration, which measures the percent change in the price or value of the bond, portfolio, or futures price, typically for a 100 basis point change in the underlying yield. The relationship between Macaulay duration, which measures duration in years, and modified duration is

  \[
  \text{Modified Duration} = \frac{\text{Macaulay Duration}}{[1 + (y/f)]}
  \]

  where \(y\) is the yield on the bond and \(f\) is the frequency with which coupon payments are made. If a bond pays coupons twice a year, \(f = 2\).

- The futures price is expressed in decimal form. The par amount for a futures contract is $100,000 for bonds, 10-year notes, and 5-year notes. The par amount for 2-year note futures is $200,000.
Because bond futures have neither periodic cash payments nor a yield to maturity, we cannot calculate the duration or DV01 of a futures contract as we do for a cash bond. We can, however, construct a perfectly useful duration or DV01 measure if we link the price sensitivity of the futures contract to changes in the yields of the underlying deliverable bonds.

In particular, once we have established the relationship between the price of the futures contract and the level of yields for the bonds whose prices drive the contract, changes in the futures price can be tied directly to changes in the level of bond yields.

Thus, we can calculate the duration of a futures contract directly by dividing the percentage change in the futures price by the change in the yield of one of the underlying bonds. Similarly, we can calculate the DV01 of a futures contract by calculating the change in the market value of the contract for a one basis point change in the yield of one of the underlying bonds.
Hedging with Bond and Note Futures

Exhibit 1.5

Two Methods for Finding Futures Durations and DV01s

Rules of thumb

RULE OF THUMB #1

\[ \text{Futures DV01} = \frac{CTD \ DV01}{CTD \ Factor} \]

RULE OF THUMB #2

\[ \text{Futures duration} = CTD \ duration \]

Option-adjusted

Futures DV01 = Projected change in the value of one futures contract for a one basis point change in underlying bond yields

Futures duration = Projected percent change in the futures price divided by the change in underlying bond yields.
Hedging with Bond and Note Futures

Exhibit 1.6

Algebra Behind the Rules of Thumb (DV01)

At futures expiration

\[ CTD \text{ price} = \text{Futures price} \times CTD \text{ conversion factor} \]

Thus, for a given change in the yield of the cheapest to deliver

\[ \text{Futures price change} = \frac{CTD \text{ price change}}{CTD \text{ factor}} \]

As a result,

\[ \text{Futures DV01} = \frac{CTD \text{ DV01}}{CTD \text{ factor}} \]

* Except for a small amount of carry and the value of the switch option. See pages 63 through 68 of *The Treasury Bond Basis* for an explanation of the switch option.
Hedging with Bond and Note Futures

Exhibit 1.7

Algebra Behind the Rules of Thumb (duration)

At expiration, if

\[ \frac{\text{Futures price change}}{\text{Futures price}} = \frac{\text{CTD price change}}{\text{CTD factor}} \]

Then the percent change in the futures price is

\[ \frac{\text{Futures price change}}{\text{Futures price}} = \frac{\text{CTD price change}}{\text{Futures price} \times \text{Factor}} \]

But because the futures price = CTD price / CTD factor at expiration as well,

\[ \% \text{ Futures price change} = \frac{\text{CTD price change}}{\text{CTD price}} \]
Hedging with Bond and Note Futures

Exhibit 1.8

Hedge Ratio Examples Using Rules of Thumb

- **Bond futures market information for 3/8/95**
  CTD bond = 8-3/4s of 5/15/17
  CTD factor = 1.0771
  CTD DV01 ($/bp per $100,000 par amount) = $113.00
  CTD modified duration = 10.01

- **Futures information**
  Futures DV01 = $113.00 / 1.0771 = $104.91
  Futures duration = 10.01
  Futures price = 102-01/32 (102.03125)

- **Position to hedge**
  Long $100 million par amount of the 7-5/8s of 2/15/25
  Full price (including accrued interest) = 100.3696
  Duration = 11.66
  DV01 (per $100,000 par amount) = $117.03
  Position DV01 = $117,030
Hedging with Bond and Note Futures

Exhibit 1.9

Calculating the Hedge Ratios (continued)

**DV01 hedge ratio calculation**

\[
\text{Hedge ratio} = \frac{\text{Position DV01}}{\text{Futures DV01}} \\
= \frac{\$117,030}{\$104.91} \\
= 1,115 \text{ contracts}
\]

**Duration hedge ratio calculation**

\[
\text{Hedge ratio} = \frac{(\text{Position duration} \times \text{Position value})}{(\text{Futures duration} \times \text{Portfolio equivalent value of futures})} \\
= \frac{(11.66 \times \$100,369,600)}{(10.01 \times 102.03125 \times \$1,000)} \\
= 1,145 \text{ contracts}
\]

- Properly applied, duration is multiplied by the full price or full liquidating value of the position.
- Futures have no net liquidating value, but for the purpose of reckoning hedge ratios, a futures contract is treated as if it has a portfolio equivalent value equal to $100,000 \times (\text{Futures price} / 100), or futures price \times \$1,000.
- These two rules of thumb will not provide the same answers because of conceptual flaws in the way they are applied. Both rules ignore carry, for example, and the duration rule of thumb ignores accrued interest. For a full reconciliation of these two rules of thumb, see pages 105 and 106 of *The Treasury Bond Basis*. 

Carr Futures 9
Hedging with Bond and Note Futures

Exhibit 1.10

How Bloomberg Calculates Hedge Ratios

• Bloomberg invokes the first rule of thumb when calculating hedge ratios. That is, it finds the cheapest to deliver bond, and sets the DV01 or $/bp of the futures contract equal to the CTD DV01 / CTD Factor.

• In this example, the bond that we want to hedge also happens to be the cheapest to deliver, which is used as the proxy issue. Its conversion factor isn’t shown on this page, but it is 1.2949 in this case. You can see that the hedge ratio is simply the conversion factor.
Exhibit 1.11

Shortcomings of the Rules of Thumb

• The rules of thumb ignore any possibility of a change in the cheapest to deliver.

• As a result, you can get radical changes in hedge ratios when bond yields pass through a crossover point. Also, the rules of thumb produce biased hedge ratios because they either overstate or understate the price sensitivity of the futures contract.

• The rules of thumb ignore carry, which, before expiration, drives a wedge between cash and futures prices. As yields change, carry costs also change so that futures prices will not track converted cash prices one for one.
**Hedging with Bond and Note Futures**

### Exhibit 1.12

**Effect of Shifting Deliverables on Rule of Thumb Hedge Ratios**

<table>
<thead>
<tr>
<th>Date</th>
<th>Cheapest to deliver bond</th>
<th>8-1/2s of '20 DV01 (per $100K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Issue DV01 per $100,000</td>
<td>Factor DV01/factor</td>
</tr>
<tr>
<td>5/31/90</td>
<td>7-1/2s of '16 $92.960.9453</td>
<td>$98.34</td>
</tr>
<tr>
<td>6/1/90</td>
<td>11-3/4s of '14-09 $114.11</td>
<td>1.3649</td>
</tr>
</tbody>
</table>

If your position contains $100 million par amount of the 8-1/2s, then your hedge ratios would have been...

**5/31/90**

Hedge ratio = \( \frac{105,760}{98.34} = 1,075 \) contracts

**6/1/90**

Hedge ratio = \( \frac{108,640}{83.60} = 1,300 \) contracts
Exhibit 1.13
Reckoning an Option-Adjusted Futures DV01

\[
\text{Futures duration} = \left[ \frac{(101.95 - 84.62)}{93.31} \right] \times \left[ \frac{100}{2} \right] = 9.29
\]

\[
\text{Futures DV01} = \$93,310 \times \left( \frac{.0929}{100} \right) = \$86.68
\]

- With this approach, you can use either the option-adjusted duration or the option-adjusted DV01 for calculating hedge ratios. Both approaches will give you the same answer.
- In this example, the DV01 was calculated from the option-adjusted duration. One also can calculate the DV01 of the theoretical futures price and calculate the futures duration from this.
- Durations can be calculated for smaller yield changes - for example, 10 basis points up and down. The answers will be slightly different because of the changing curvature of the theoretical price relationship.
- This illustration makes plain the negative convexity in bond futures prices.
Key Conclusions About Hedge Ratio Calculations

- Option-adjusted DV01s and durations can differ substantially from the DV01 and duration of the cheapest to deliver. When yields are high, the option-adjusted measures will tend to be smaller than those given by the rules of thumb. When yields are low, the option-adjusted measures will tend to be higher.
- Option-adjusted risk measures will converge to the CTD risk measures as futures trading approaches expiration and uncertainty decreases about which bond will be cheapest to deliver.
- Option-adjusted hedge ratios can be much less costly to maintain, especially when yields are trading around crossover points.
What are you Assuming About the RP Rate?

- What you assume about the RP rate has an effect on the relationship between a change in the spot price and the corresponding change in the forward or futures price.

- If the RP rate rises when the bond yield rises, the effect is to decrease carry, which in turn decreases the amount by which the forward price falls.

- As a rule of thumb, for each month remaining to delivery on a futures contract, a 1 basis point change in the RP rate is worth about $0.83 given a notional amount of $100,000 per futures contract. With 3 month to expiration, the difference between assuming the RP rate is constant and assuming that the RP rate shifts in parallel with the bond yield is worth about $2.50 per basis point.
Hedging with Bond and Note Futures

Exhibit 1.16

Practical Consequences of RP Assumptions for Hedge Ratios

Effect of a one basis point drop in yield per $100,000 face value*

August 6, 1992

<table>
<thead>
<tr>
<th>Treasury futures contract (CTD)</th>
<th>Change in value of spot (per $100 par value)</th>
<th>Change in carry</th>
<th>Change in value of forward position</th>
<th>Ratio of change in forward value to change in spot value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>2-year (8-1/2s of 6/94)</td>
<td>18.89</td>
<td>-0.09</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>5-year (6-7/8s of 3/97)</td>
<td>41.55</td>
<td>-0.20</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>10-year (7-1/2s of 11/01)</td>
<td>70.77</td>
<td>-0.34</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>Bond (9-7/8s of 11/15)</td>
<td>132.20</td>
<td>-0.63</td>
<td>1.20</td>
<td></td>
</tr>
</tbody>
</table>

(4) = (1) - (2)  (5) = (1) - (3)  (6) = (4) / (1)  (7) = (5) / (1)

### EXHIBIT 5.10  Comparison of Hedge Effectiveness*
(June 1989 to March 1990)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Average hedge ratio using: 1</th>
<th>Average hedge error using: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cheapest to deliver duration</td>
<td>Option adjusted duration</td>
</tr>
<tr>
<td>10.75 of '05</td>
<td>11.9</td>
<td>11.0</td>
</tr>
<tr>
<td>14.0 of '11-06</td>
<td>14.7</td>
<td>13.5</td>
</tr>
<tr>
<td>12.5 of '14-09</td>
<td>14.9</td>
<td>13.7</td>
</tr>
<tr>
<td>7.25 of '16</td>
<td>11.7</td>
<td>10.8</td>
</tr>
<tr>
<td>8.875 of '19</td>
<td>14.0</td>
<td>12.9</td>
</tr>
<tr>
<td>Equally Wtd. Portfolio</td>
<td>13.4</td>
<td>12.4</td>
</tr>
</tbody>
</table>

1 Number of contracts to short per $1 million holdings of bonds
2 Absolute value of gain or loss on hedged portfolio in (32nds).
1/32 equals $31.25 on $100,000 portfolio

Exhibit 1.18

Yield Betas

- Adjusting the hedge ratio for yield betas

\[
\text{# of futures} = \frac{\text{Position DV01}}{\text{Futures DV01}} \times \frac{\text{Position yield change}}{\text{CTD yield change}}
\]

- Hedging the current long bond with futures

\[
\frac{\text{Long bond yield change}}{\text{CTD yield change}} = \frac{1}{1.09}
\]

so reduce the number of futures by 8.25% (1/1.09)

Yield Betas serve to scale the hedge ratio by the relative movement of position yields and CTD yields.
Hedging with Bond and Note Futures

Exhibit 1.19

Competing Hedge Ratios

Conventional
\[
\frac{DV01_{30}}{DV01_5}
\]

Yield Beta
\[
\frac{DV01_{30}}{DV01_5} \times \frac{\sigma_{30}}{\sigma_5} = \frac{DV01_{30}}{DV01_5 (\sigma_5 / \sigma_{30})}
\]

Yield Delta
\[
\frac{DV01_{30}}{DV01_5} \times \frac{\sigma_{30} \rho_{5,30}}{\sigma_5} = \frac{DV01_{40}}{DV01_5 (\sigma_5 / \sigma_{30}) \rho_{5,30}}
\]

Minimum Variance
\[
\frac{DV01_{30}}{DV01_5} \times \frac{\sigma_{30} \rho_{5,30}}{\sigma_5} = \frac{DV01_{30} \rho_{5,30}}{DV01_5 (\sigma_5 / \sigma_{30})}
\]
**Exhibit 1.20**

**Different Hedges for $100 Mill of the OTR 30-Year Treasury**

(January 12, 1998)

<table>
<thead>
<tr>
<th>Hedge ratio</th>
<th>Short position in the OTR 5-year Treasury</th>
<th>Daily standard deviation*</th>
<th>Net DV01</th>
<th>Net expected DV01*</th>
</tr>
</thead>
<tbody>
<tr>
<td>unhedged</td>
<td>0 ($ mill)</td>
<td>770.3 ($ thds)</td>
<td>149.1</td>
<td>149.1</td>
</tr>
<tr>
<td>min. variance</td>
<td>200.0 ($ mill)</td>
<td>383.4 ($ thds)</td>
<td>61.7</td>
<td>53.2</td>
</tr>
<tr>
<td>yield beta</td>
<td>240.7 ($ mill)</td>
<td>365.9 ($ thds)</td>
<td>43.9</td>
<td>33.6</td>
</tr>
<tr>
<td>yield delta</td>
<td>273.6 ($ mill)</td>
<td>377.4 ($ thds)</td>
<td>29.5</td>
<td>17.9</td>
</tr>
<tr>
<td>conventional</td>
<td>310.9 ($ mill)</td>
<td>415.8 ($ thds)</td>
<td>13.2</td>
<td>0.0</td>
</tr>
<tr>
<td>conventional</td>
<td>341.2 ($ mill)</td>
<td>462.5 ($ thds)</td>
<td>0.0</td>
<td>-14.6</td>
</tr>
</tbody>
</table>

*based on standard deviations and correlations for 1990-1997*