Section 1

HEDGING WITH BOND AND NOTE FUTURES

Exhibit 1.2

Futures Hedge Algebra

For a given change in bond yields . . .

$$\boxed{\text{Number of bond futures}} \times \boxed{\text{Change in the value of 1 futures contract}} = -\boxed{\text{Change in the value of the bond portfolio}}$$

Hedge ratio

$$\boxed{\text{Number of bond futures}} = -\boxed{\text{Change in the value of the bond portfolio}} \div \boxed{\text{Change in the value of 1 futures contract}}$$

- The negative sign indicates that you want to offset changes in the value of the bond portfolio. If you want to replicate the change in the value of the bond portfolio, the number of futures contracts would have the same sign as the change in the value of the bond portfolio.

Exhibit 1.3

Calculating Hedge Ratios

Use of DV01 (dollar value of a basis point)

$$\textit{Hedge ratio} = \frac{\textit{Portfolio DV01}}{\textit{Futures DV01}}$$

Use of duration

$$\textit{Hedge ratio} = \frac{[\textit{Portfolio duration} \times \textit{Portfolio market value}]}{\left[\textit{Futures duration} \times \left(\frac{\textit{Futures price}}{100}\right) \times \textit{Par amount}\right]}$$

- For the purposes of reckoning hedge ratios, duration is defined as "modified" or "effective" duration, which measures the percent change in the price or value of the bond, portfolio, or futures price, typically for a 100 basis point change in the underlying yield. The relationship between Macaulay duration, which measures duration in years, and modified duration is

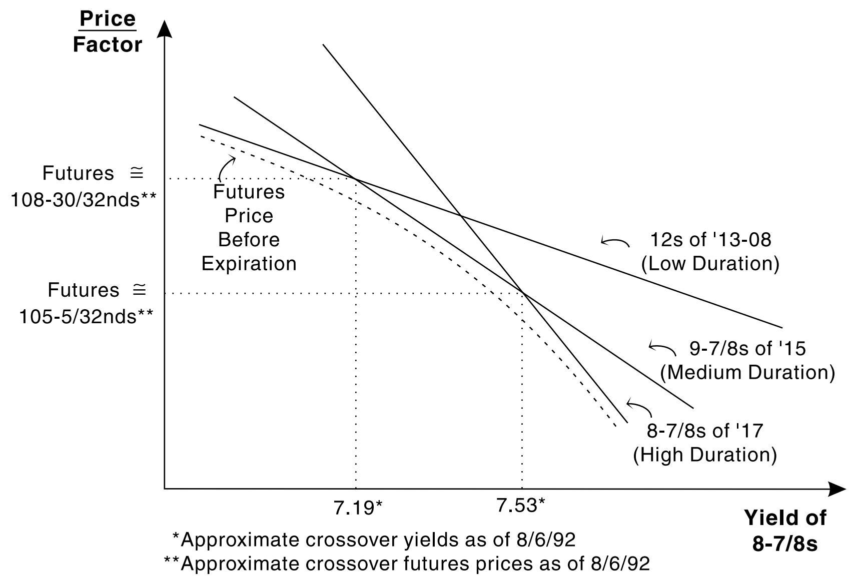
  Modified Duration = Macaulay Duration / [1 + (y/f)]

  where y is the yield on the bond and f is the frequency with which coupon payments are made. If a bond pays coupons twice a year, f = 2.

- The futures price is expressed in decimal form. The par amount for a futures contract is $100,000 for bonds, 10-year notes, and 5-year notes. The par amount for 2-year note futures is $200,000.

Exhibit 1.4

DV01 or Duration of a Futures Contract



- Because bond futures have neither periodic cash payments nor a yield to maturity, we cannot calculate the duration or DV01 of a futures contract as we do for a cash bond. We can, however, construct a perfectly useful duration or DV01 measure if we link the price sensitivity of the futures contract to changes in the yields of the underlying deliverable bonds.
- In particular, once we have established the relationship between the price of the futures contract and the level of yields for the bonds whose prices drive the contract, changes in the futures price can be tied directly to changes in the level of bond yields.
- Thus, we can calculate the duration of a futures contract directly by dividing the percentage change in the futures price by the change in the yield of one of the underlying bonds. Similarly, we can calculate the DV01 of a futures contract by calculating the change in the market value of the contract for a one basis point change in the yield of one of the underlying bonds.

Exhibit 1.5

Two Methods for Finding Futures Durations and DV01s

Rules of thumb

Rule of thumb #1

$$\text{Futures DV01} = \frac{\text{CTD DV01}}{\text{CTD Factor}}$$

Rule of thumb #2

Futures duration = CTD duration

Option-adjusted

Futures DV01 =	Projected change in the value of one futures contract for a one basis point change in underlying bond yields
Futures duration =	Projected percent change in the futures price divided by the change in underlying bond yields.

Exhibit 1.6

Algebra Behind the Rules of Thumb (DV01)

At futures expiration

$$\textit{CTD price} = \textit{Futures price} \times \textit{CTD conversion factor}\,*$$

Thus, for a given change in the yield of the cheapest to deliver

$$\textit{Futures price change} = \frac{\textit{CTD price change}}{\textit{CTD factor}}$$

As a result,

$$\textit{Futures DV01} = \frac{\textit{CTD DV01}}{\textit{CTD factor}}$$

* Except for a small amount of carry and the value of the switch option. See pages 63 through 68 of *The Treasury Bond Basis* for an explanation of the switch option.

Exhibit 1.7

Algebra Behind the Rules of Thumb (duration)

At expiration, if

$$\textit{Futures price change} = \frac{\textit{CTD price change}}{\textit{CTD factor}}$$

Then the percent change in the futures price is

$$\frac{\textit{Futures price change}}{\textit{Futures price}} = \frac{\textit{CTD price change}}{\textit{Futures price} \times \textit{Factor}}$$

But because the futures price = CTD price / CTD factor at expiration as well,

$$\textit{\% Futures price change} = \frac{\textit{CTD price change}}{\textit{CTD price}}$$

Exhibit 1.8

Hedge Ratio Examples Using Rules of Thumb

- **Bond futures market information for 3/8/95**

  CTD bond = 8-3/4s of 5/15/17

  CTD factor = 1.0771

  CTD DV01 ($/bp per $100,000 par amount) = $113.00

  CTD modified duration = 10.01

- **Futures information**

  Futures DV01 = $113.00 / 1.0771 = $104.91

  Futures duration = 10.01

  Futures price = 102-01/32 (102.03125)

- **Position to hedge**

  Long $100 million par amount of the 7-5/8s of 2/15/25

  Full price (including accrued interest) = 100.3696

  Duration = 11.66

  DV01 (per $100,000 par amount) = $117.03

  Position DV01 = $117,030

Exhibit 1.9

Calculating the Hedge Ratios (continued)

- **DV01 hedge ratio calculation**

$$\text{Hedge ratio} = \frac{\text{Position DV01}}{\text{Futures DV01}}$$

$$= \frac{\$117{,}030}{\$104.91}$$

$$= 1{,}115 \text{ contracts}$$

- **Duration hedge ratio calculation**

$$\text{Hedge ratio} = \frac{(\text{Position duration} \times \text{Position value})}{(\text{Futures duration} \times \text{Portfolio equivalent value of futures})}$$

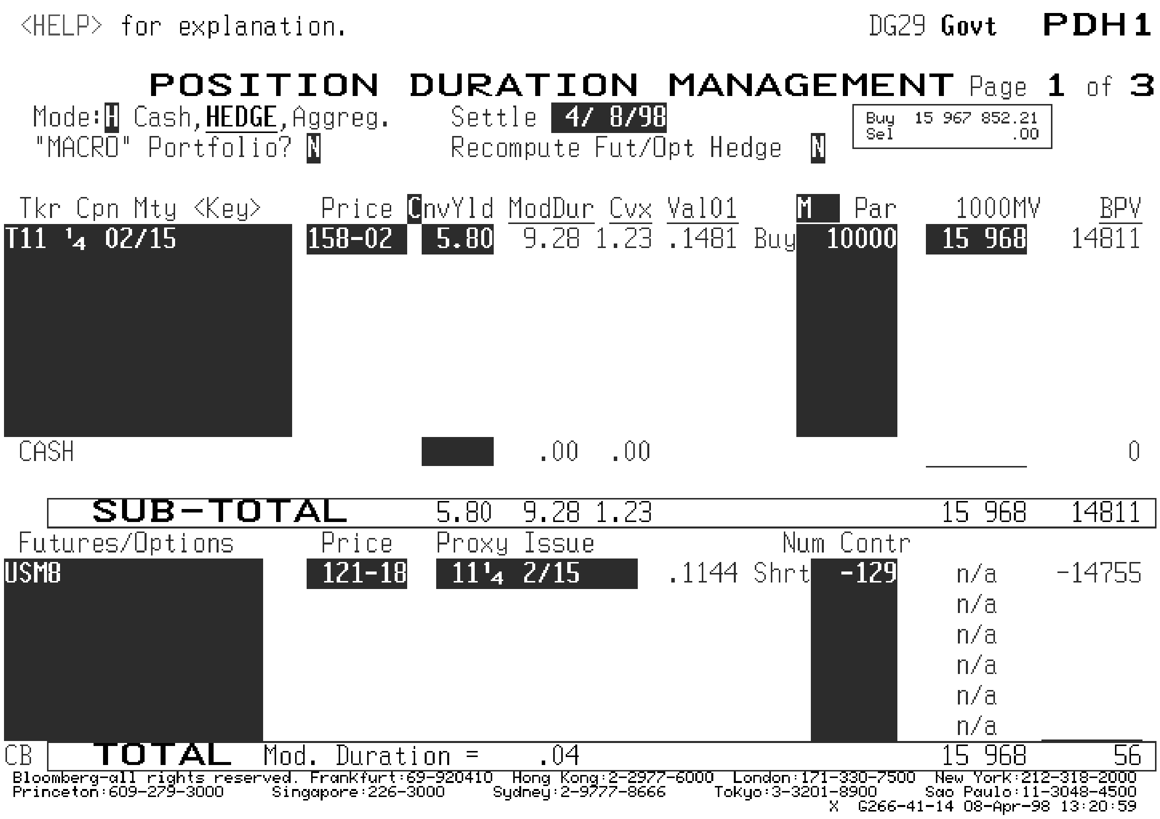
$$= \frac{(11.66 \times \$100{,}369{,}600)}{(10.01 \times 102.03125 \times \$1{,}000)}$$

$$= 1{,}145 \text{ contracts}$$

- Properly applied, duration is multiplied by the full price or full liquidating value of the position.
- Futures have no net liquidating value, but for the purpose of reckoning hedge ratios, a futures contract is treated as if it has a portfolio equivalent value equal to $100,000 x (Futures price / 100), or futures price x $1,000.
- These two rules of thumb will not provide the same answers because of conceptual flaws in the way they are applied. Both rules ignore carry, for example, and the duration rule of thumb ignores accrued interest. For a full reconciliation of these two rules of thumb, see pages 105 and 106 of *The Treasury Bond Basis*.

Exhibit 1.10

How Bloomberg Calculates Hedge Ratios


<HELP> for explanation. DG29 Govt PDH1

POSITION DURATION MANAGEMENT Page 1 of 3

Mode: H Cash, HEDGE, Aggreg. Settle 4/ 8/98 | Buy 15 967 852.21 / Sel .00

"MACRO" Portfolio? N Recompute Fut/Opt Hedge N

Tkr Cpn Mty <Key>	Price	CnvYld	ModDur	Cvx	Val01		M	Par	1000MV	BPV
T11 ¼ 02/15	158-02	5.80	9.28	1.23	.1481	Buy		10000	15 968	14811
CASH			.00	.00						0
SUB-TOTAL		5.80	9.28	1.23					15 968	14811

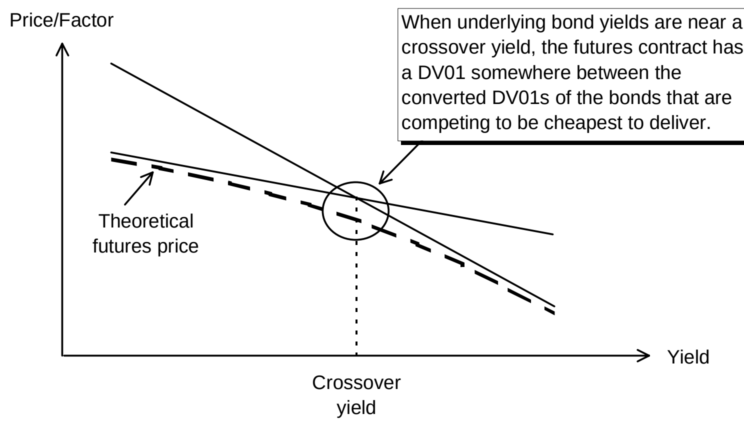
Futures/Options	Price	Proxy Issue			Num Contr		BPV
USM8	121-18	11¼ 2/15	.1144	Shrt	-129	n/a	-14755
						n/a	
						n/a	
						n/a	
						n/a	
						n/a	
CB TOTAL	Mod. Duration =	.04				15 968	56

Bloomberg-all rights reserved. Frankfurt:69-920410 Hong Kong:2-2977-6000 London:171-330-7500 New York:212-318-2000
Princeton:609-279-3000 Singapore:226-3000 Sydney:2-9777-8666 Tokyo:3-3201-8900 Sao Paulo:11-3048-4500
X G266-41-14 08-Apr-98 13:20:59

- Bloomberg invokes the first rule of thumb when calculating hedge ratios. That is, it finds the cheapest to deliver bond, and sets the DV01 or $/bp of the futures contract equal to the CTD DV01 / CTD Factor
- In this example, the bond that we want to hedge also happens to be the cheapest to deliver, which is used as the proxy issue. Its conversion factor isn't shown on this page, but it is 1.2949 in this case. You can see that the hedge ratio is simply the conversion factor.

Exhibit 1.11

Shortcomings of the Rules of Thumb



- The rules of thumb ignore any possibility of a change in the cheapest to deliver.
- As a result, you can get radical changes in hedge ratios when bond yields pass through a crossover point. Also, the rules of thumb produce biased hedge ratios because they either overstate or understate the price sensitivity of the futures contract.
- The rules of thumb ignore carry, which, before expiration, drives a wedge between cash and futures prices. As yields change, carry costs also change so that futures prices will not track converted cash prices one for one.

Exhibit 1.12

Effect of Shifting Deliverables on Rule of Thumb Hedge Ratios

Market data

Date	Cheapest to deliver bond				8-1/2s of '20 DV01 (per $100K)
	Issue	DV01 per $100,000	Factor	DV01/factor	
5/31/90	7-1/2s of '16	$92.960.9453	$98.34	$105.76	
6/1/90	11-3/4s of '14-09	$114.11	1.3649	$83.60	$108.64

If your position contains $100 million par amount of the 8-1/2s, then your hedge ratios would have been...

5/31/90

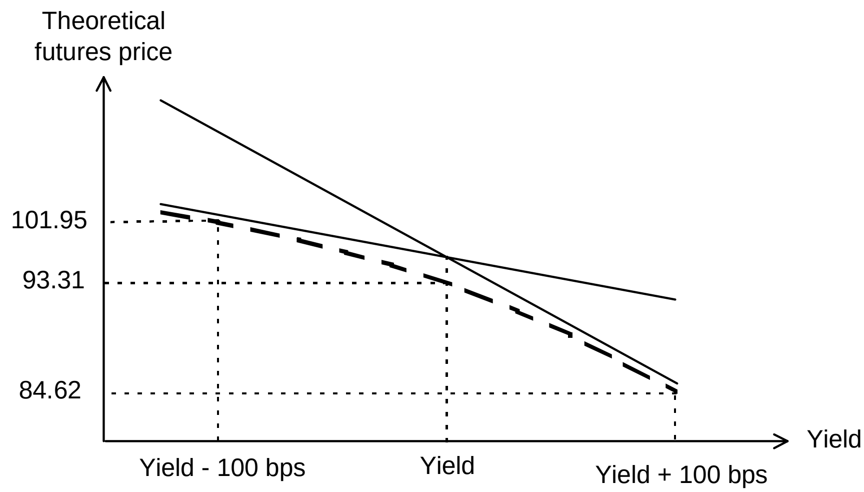
Hedge ratio = $105,760 / $98.34 = 1,075 contracts

6/1/90

Hedge ratio = $108,640 / $83.60 = 1,300 contracts

Exhibit 1.13

Reckoning an Option-Adjusted Futures DV01



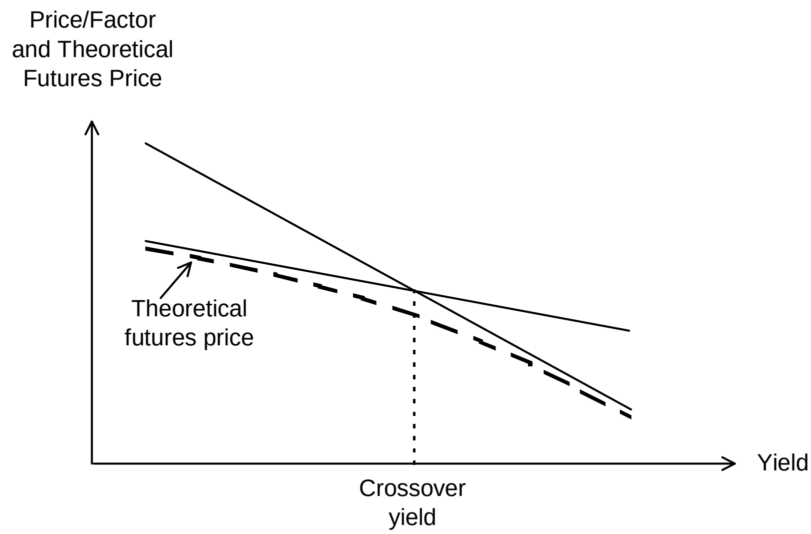
$$\text{Futures duration} = \left[\frac{(101.95 - 84.62)}{93.31}\right] \times \left[\frac{100}{2}\right] = 9.29$$

$$\text{Futures DV01} = \$93{,}310 \times \left(\frac{.0929}{100}\right) = \$86.68$$

- With this approach, you can use either the option-adjusted duration or the option-adjusted DV01 for calculating hedge ratios. Both approaches will give you the same answer.
- In this example, the DV01 was calculated from the option-adjusted duration. One also can calculate the DV01 of the theoretical futures price and calculate the futures duration from this.
- Durations can be calculated for smaller yield changes - for example, 10 basis points up and down. The answers will be slightly different because of the changing curvature of the theoretical price relationship.
- This illustration makes plain the negative convexity in bond futures prices.

Exhibit 1.14

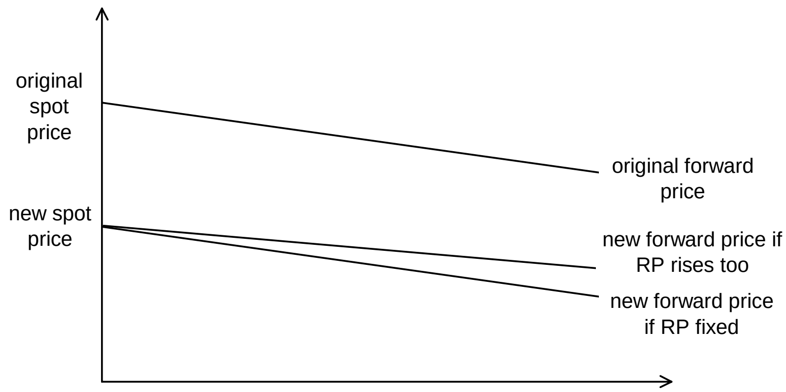
Key Conclusions About Hedge Ratio Calculations



- Option-adjusted DV01s and durations can differ substantially from the DV01 and duration of the cheapest to deliver. When yields are high, the option-adjusted measures will tend to be smaller than those given by the rules of thumb. When yields are low, the option-adjusted measures will tend to be higher.
- Option-adjusted risk measures will converge to the CTD risk measures as futures trading approaches expiration and uncertainty decreases about which bond will be cheapest to deliver.
- Option-adjusted hedge ratios can be much less costly to maintain, especially when yields are trading around crossover points.

Exhibit 1.15

What are you Assuming About the RP Rate?



- What you assume about the RP rate has an effect on the relationship between a change in the spot price and the corresponding change in the forward or futures price.
- If the RP rate rises when the bond yield rises, the effect is to decrease carry, which in turn decreases the amount by which the forward price falls.
- As a rule of thumb, for each month remaining to delivery on a futures contract, a 1 basis point change in the RP rate is worth about $0.83 given a notional amount of $100,000 per futures contract. With 3 month to expiration, the difference between assuming the RP rate is constant and assuming that the RP rate shifts in parallel with the bond yield is worth about $2.50 per basis point.

Exhibit 1.16

Practical Consequences of RP Assumptions for Hedge Ratios

Effect of a one basis point drop in yield per $100,000 face value*

August 6, 1992

Treasury futures contract (CTD)	Change in value of spot (per $100 par value)	Change in carry		Change in value of forward position		Ratio of change in forward value to change in spot value	
		RP rate fixed	RP rate change equals yield change	RP rate fixed	RP rate change equals yield change	RP rate fixed	RP rate change equals yield change
	(1)	(2)	(3)	(4) = (1) - (2)	(5) = (1) - (3)	(6) = (4) / (1)	(7) = (5) / (1)
2-year (8-1/2s of 6/94)	18.89	-0.09	1.45	18.98	17.44	1.005	0.923
5-year (6-7/8s of 3/97)	41.55	-0.20	1.33	41.75	40.22	1.005	0.968
10-year (7-1/2s of 11/01)	70.77	-0.34	1.19	71.11	69.58	1.005	0.983
Bond (9-7/8s of 11/15)	132.20	-0.63	1.20	132.83	131.00	1.005	0.991

* Source: Burghardt, et. al., *The Treasury Bond Basis*, Irwin, 1994.

Exhibit 1.17

Horse Race

EXHIBIT 5.10 Comparison of Hedge Effectiveness*
(June 1989 to March 1990)

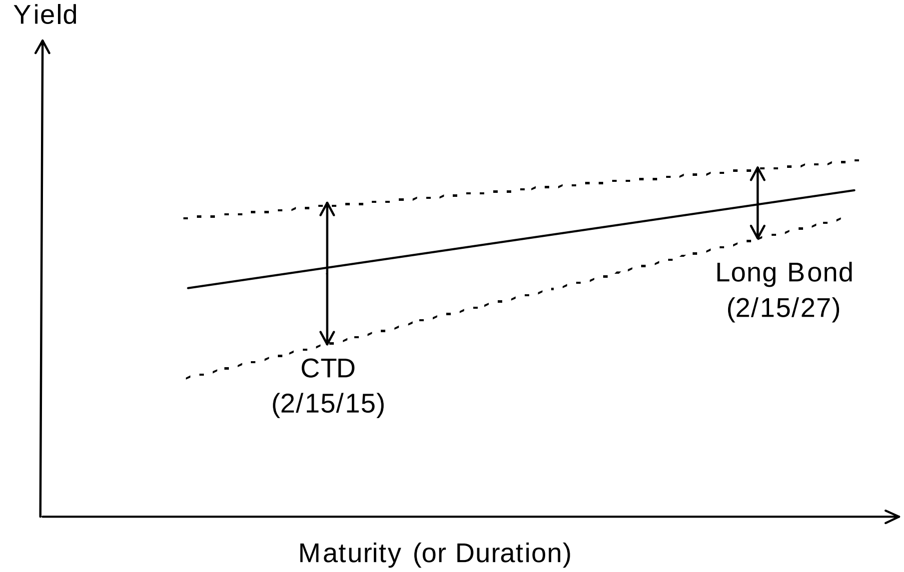
Portfolio	Average hedge ratio using:[1] Cheapest to deliver duration	Option adjusted duration	Average hedge error using:[2] Cheapest to deliver duration	Option adjusted duration
10.75 of ’05	11.9	11.0	9.0	4.5
14.0 of ’11-06	14.7	13.5	5.6	6.3
12.5 of ’14-09	14.9	13.7	7.7	2.6
7.25 of ’16	11.7	10.8	10.9	6.0
8.875 of ’19	14.0	12.9	16.2	9.7
Equally Wtd. Portfolio	13.4	12.4	9.9	5.8

1 Number of contracts to short per $1 million holdings of bonds
2 Absolute value of gain or loss on hedged portfolio in (32nds).
1/32 equals $31.25 on $100,000 portfolio

* Source: Burghardt, et. al., *The Treasury Bond Basis*, Irwin, 1994.

Exhibit 1.18

Yield Betas



- **Adjusting the hedge ratio for yield betas**

$$\#\ \text{of futures} = \frac{\text{Position DV01}}{\text{Futures DV01}} \times \frac{\text{Position yield change}}{\text{CTD yield change}}$$

- **Hedging the current long bond with futures**

$$\frac{\text{Long bond yield change}}{\text{CTD yield change}} = \frac{1}{1.09}$$

so reduce the number of futures by 8.25% (1/1.09)

Yield Betas serve to scale the hedge ratio by the relative movement of position yields and CTD yields.

Exhibit 1.19

Competing Hedge Ratios

Conventional

$$\frac{DV01_{30}}{DV01_5}$$

Yield Beta

$$\frac{DV01_{30}}{DV01_5} \times \frac{\sigma_{30}}{\sigma_5} = \frac{DV01_{30}}{DV01_5(\sigma_5 / \sigma_{30})}$$

Yield Delta

$$\frac{DV01_{30}}{DV01_5} \times \frac{\sigma_{30}}{\sigma_5 \rho_{5,30}} = \frac{DV01_{30}}{DV01_5(\sigma_5 / \sigma_{30})\rho_{5,30}}$$

Minimum Variance

$$\frac{DV01_{30}}{DV01_5} \times \frac{\sigma_{30} \rho_{5,30}}{\sigma_5} = \frac{DV01_{30} \rho_{5,30}}{DV01_5(\sigma_5 / \sigma_{30})}$$

Exhibit 1.20

Different Hedges for $100 Mill of the OTR 30-Year Treasury

(January 12, 1998)

Hedge ratio	Short position in the OTR 5-year Treasury	Hedge characteristics: Daily standard deviation*	Net DV01	Net expected DV01*
	($ mill)	($ thds)		
unhedged	0	770.3	149.1	149.1
	200.0	383.4	61.7	53.2
min. variance	240.7	365.9	43.9	33.6
yield beta	273.6	377.4	29.5	17.9
yield delta	310.9	415.8	13.2	0.0
conventional	341.2	462.5	0.0	-14.6

**based on standard deviations and correlations for 1990-1997*