Interest rate swap

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In this note, we shall give a formula for the value of a interest rate swap agreement. Let $n \geq 1$ be a natural number. Suppose party A in a interest rate agreement on notional principal Q

- pays floating rate $r_{f,t}\%$ at time $t = 1, 2, 3, \ldots, n$,
- receives fixed rate $r_x\%$ at time $t = 1, 2, 3, \ldots, n$.

Let $0 \leq t_0 \leq n$ be given. We shall give a formula for the value of the swap agreement to party A at time t_0, denoted by $V(t_0)$. Before we proceed any further, we need some more notation.

- Let C be the payment, at time $t = 1, 2, \ldots, n$, associated with r_x. Note that $C = Q\dfrac{r_x}{100}$.
- Let C^* be the payment, at $t = [t_0] + 1$ [1] , associated with $r_{f,[t_0]+1}$. Note that $C^* = Q\dfrac{r_{f,[t_0]+1}}{100}$.
- For $i = 1, 2, \ldots, n - [t_0]$, let $t_i = [t_0] + i - t_0$ and r_i be the continuously compounded interest rate from $t = t_0$ to $t = t_i$.
- Let $BX(t)$ be the value of the fixed rate bond underlying the swap at time t.
- Let $BF(t)$ be the value of the floating rate bond underlying the swap at time t.

It can be shown that (see [1, Chapter 14])

$$BX(t_0) = \sum_{i=1}^{n-[t_0]} C \cdot e^{\frac{-r_i t_i}{100}} + Q \cdot e^{\frac{-r_{n-[t_0]} t_{n-[t_0]}}{100}} \tag{1}$$

$$BF(t_0) \quad = \quad (Q + C^*)e^{\frac{-r_1 t_1}{100}} \tag{2}$$

It is clear that

$$V(t_0) = BX(t_0) - BF(t_0) \tag{3}$$

Without out loss of generality we may assume $0 \leq t_0 < 1$.

Example ([1, Table 14.3] , [2, Example 5.1]) Suppose that a financial institution has agreed to pay six-month LIBOR and receive 8% per annum (with semi-annual compounding) on a notional principal of \$100m. The swap has a remaining life of 1.25 years . LIBOR rates with continuous compounding for 3−month, 9−month, and 15−month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2% (with semi-annual compounding).

In the notation defined earlier, we have the following.

[1] $[t]$ is the smallest integer not exceeding t.

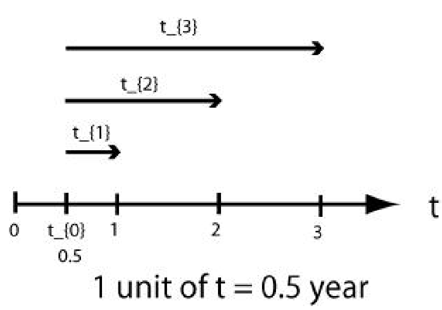
- 1 unit of time $t = 0.5$ year
- We may assume without loss of generality that $n = 3$ and $t_0 = 0.5$.
- $r_x = \frac{8}{2} = 4$
- $r_{f,[t_0]+1} = \frac{10.2}{2} = 5.1$
- $Q = 10^8$
- $C = Q\frac{r_x}{100} = 10^8\frac{4}{100} = 4 \cdot 10^6$
- $C^* = Q\frac{r_{f,[t_0]+1}}{100} = 10^8\frac{5.1}{100} = 5.1 \cdot 10^6$

Also after rescaling,

$$t_1 = 0.5, t_2 = 1.5, t_3 = 2.5$$

and

$$r_1 = 10/2 = 5, r_2 = 10.5/2 = 5.25, r_3 = 11/2 = 5.5$$



$$\begin{aligned} BX(t_0) &= BX(0.5) && (4)\\ &= \sum_{i=1}^{3} Ce^{\frac{-r_i t_i}{100}} + Qe^{\frac{-r_3 t_3}{100}} && (5)\\ &= 9.824 \cdot 10^7 && (6)\end{aligned}$$

$$BF(t_0) = BF(0.5) = (Q + C^*)e^{\frac{-r_1 t_1}{100}} = 1.0251 \cdot 10^8 \quad (7)$$

Value of swap at $t = t_0 = 0.5$

$$V(t_0) = V(0.5) = BX(0.5) - BF(0.5) = -4.27 \cdot 10^6 \quad (8)$$

(End of Example)

References

[1] K Cuthbertson and D Nitzsche, Financial Engineering- Derivatives and Risk Management, Wiley

[2] J Hull, Options, Futures and other Derivatives, Prentice-Hall International