The Term Structure of Interest Rates and Its Risks

- Term Structure Theories
- Yield Curve Risks
- Valuing Embedded Options
- The Binomial Model
- Spreads including Option Adjusted Spread
- Effective Duration and Convexity
- Derivatives
- Option Pricing Models

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The Yield Curve

- Shifts
  - parallel
  - nonparallel
  - twist in slope
  - flattening
  - steepening
  - butterfly shift
- Treasury Returns
  - level of rates
  - changes in slope
  - changes in curvature

Theoretical Spot Rate

- Select securities, then the methodology
  - On-the-run treasuries
    - bootstrapping
  - On-the-run treasuries and other off-the-run treasury issues
    - bootstrapping
  - Strips
    - yield = spot rate
  - All treasury coupon securities and bills
    - elaborate statistical techniques necessary

Term Structure Theories

- Pure Expectation Theory
  - Drawbacks
  - Interpretations
  - Forward Rates and Market Consensus
- Biased Expectations Theory
  - Liquidity Theory
  - Preferred habitat Theory
- Market Segmentation Theory

The Yield Curve

or Spot Yield Curve or Term Structure

- Spot yields vary (rise) with maturity
    - expectation that future short rates will rise
    - investors unwilling to invest long term unless rewarded by increased yield
- Pure expectation theory
    - – forward rates of interest embodied in the term structure are unbiased estimates of expected future spot rates of interest
        - forward rates are biased (future spot rates) due to autocorrelation and by the amount of “term premium”
    - – expected returns arising from different maturity strategies are equal
        - expected HPR are equal only for specific period, not all future holding periods

Pure Expectations Theory

- Investors are indifferent between each of the strategies below:
- Strategy I: Buy 1 year bond today and roll over
- Strategy II: Buy 2 year bond and hold to maturity
- Strategy III: Buy 3 year bond and sell after year two.

- Ignores Uncertainty

Two More Theories

- Long term premium (Liquidity Preference): provides an upward tilt to long end to compensate for uncertainty of longer term investments.
  - borrowers borrow long, lenders lend short
  - uncertainty induces "term premium"
- Segmentation modification : supply and demand for bonds in various term spectrum segments dictates the current market rate.
  - legal and behavioral restrictions create "preferred" maturity demands

- *Compare and contrast three term structure theories.*

Measuring Yield Curve Risk

- Rate Duration and Key Rate Duration
  - *D(1): (50/100)*2 + (0/100)*0+ (50/100)*0 = 1*
  - *D(2): (50/100)*0 + (0/100)*16+ (50/100)*0 = 0*
  - *D(3): (50/100)*0 + (0/100)*0+ (50/100)*30 = 15*
    - Effective: Portfolio III: (50/100)*2 + (0/100)*16+ (50/100)*30 = 16
  - *D(1): (0/100)*2 + (100/100)*0+ (0/100)*0 = 0*
  - *D(2): (0/100)*0 + (100/100)*16+ (0/100)*0 = 16*
  - *D(3): (0/100)*0 + (100/100)*0+ (0/100)*30 = 0*
    - Effective: Portfolio III: (0/100)*2 + (100/100)*16+ (0/100)*30 = 16

Volatility

- Measuring Historical Yield Volatility
  - Determining Number of Observations
  - Annualizing Standard Deviation
  - Interpreting the Standard Deviation
- Historical versus Implied Volatility
- Forecasting Yield Volatility Alternatives
  - expected value equals zero
  - weighted observations

Valuing Bonds with Embedded Options

- What is an interest rate model?
  - one factor (binomial), two factor
  - IE. MV bond $89. Model valuation $90 (based on 12% vol.). Model valuation $86 (based on 15% vol.)
- Option Free Bond Valuation
  - Benchmark Curve
  - Using Spot Curve or Forward Rates

The Binomial Model

- Binomial Interest Rate Tree
- Determining the Value of a Node
- Constructing the Binomial Interest Rate Tree
- Valuing an Option-Free Bond with the Tree
  - value using binomial tree should be identical to bond value found when discounting at spot rates or 1 year forward rates.

Valuing the Callable Bond

- Determining the Call Option Value
  - value of call option = value of option free bond - value of callable bond
- Volatility and Arbitrage-Free Value
  - option value increases with volatility; option spread decreases with volatility.
  - value of callable bond decreases with volatility.
- Option Adjusted Spread (OAS)
  - constant spread that when added to rates (1yr) will make value (arbitrage-tree) equal to market price.
  - Volatility dependent
  - OAS attempts to remove from the nominal spread the amount that is due to the option risk
    - a spread
    - adjusts cash flows for option when computing spread to benchmark

Valuing the Callable Bond

- Callable Bond = Non-callable bond - call option.
  - the greater the value of the call feature, the lower the value of callable bond relative to that of non-callable bonds.
  - the level and volatility of interest rates are key factors in giving value to call feature.
  - value of option greater the greater the expected volatility
- Generally, the greater the variance of expected future interest rates, the higher value placed on call options; hence, higher spreads.

Facts About Spreads and OAS

- Static spread considers only one interest rate path.
- OAS contains MODEL risk. Must estimate call rule and volatility. Different analysts will arrive at different OASs.
- OAS uses averaging. OAS in not the spread the investor will earn over tsy.
- OAS should be used in conjunction with other measurements.

Valuing and Analyzing a Callable Bond

- Modified Duration
  - Macaulay Duration/[1+ytm/2]
  - dP/P = -(MD)*(dR)
  - only good for “small” changes in rates
- Convexity
  - (1/P)(second derivative price/yield)
- dP/P = -(MD)(dR) + 1/2(Cvx)(dR)^2
  - be able to compute approximate price change for interest rate changes.
  - suppose MD = 3.95 and convexity = 19.45, compute price change if rates fell 1%.

Effective Convexity and Duration

- Effective Duration (vs Modified Duration)
  - effective used to evaluate bond's with embedded options
  - approximation of modified duration
  - $= V_{-} - V_{+} / (2V_0 \text{ (delta r))}$
  - valid bond pricing model
  - high interest rate levels, two measurements similar
- Effective Convexity (vs Modified Convexity)
  - $=(V_{+} + V_{-} - 2V_0) / 2V_0 \text{ (delta y)\^{}2}$
  - considers cash flow stream changes - ie. price compression
  - lower than standard convexity for callable bonds

Factors that Affect Duration

- Lower coupon => greater duration
- Longer maturity => greater duration
- Duration is less than the time to maturity for coupon bonds
- The Macaulay duration of a zero coupon bond is its time to maturity
- When yields are high, durations are lower than when the general level of yields is low.

Factors that Affect Convexity

- Maturity: the longer the maturity, the greater the convexity
- Coupon: the lower the coupon rate the greater the convexity. Zeros have greater convexity than coupon bonds <u>of the same maturity</u>.
- Coupon: the lower the coupon rate the smaller the convexity. Zeros have smaller convexity than coupon bonds <u>of same duration</u>.
- As yield increases, convexity decreases.

Valuing a Other Bonds

- A Putable Bond
- A Step-Up Callable Note
  - when more than one embedded option, determination made at each node
- A Capped Floater
  - coupon rate is set at begin period and paid at end period; affects cash flows at nodes

A Convertible Bond

- Basic Features
- Minimum Value
- Market Conversion Price
- Current Income vs Common Stock
- Downside Risk
- Investment Characteristics

Valuation of Interest Rate Derivative Instruments

- Interest Rate Futures Contracts
  - The Cash and Carry Trade
  - The Reverse Cash and Carry Trade
- Theoretical Futures Price
  - Theoretical FP
    - = Price + Price_t(coupon/short rate differential)
      - asymmetrical lending and borrowing rates
      - cheapest to deliver changes
      - interim cash flows change

Interest Rate Swaps

- Swap Payments
  - Floating Rate Payments
    - Floating Rate Payment = Notional Amount*LIBOR*(Actual#Days/360)
  - Future Floating Rate Payments
  - Swap Rate and Swap Spread
  - Calculate PV of Floating Rate Pay. And PV Notional
    - Total $PV_{floating}$ = Total PV_{fixed}
    - Swap Rate and Swap Spread
  - Value a Swap

Options

- Components of the Option Price
  - Intrinsic Value
  - Time Value
- Factors that Influence Value of Option on Bond
  - Price of Underlying Security
  - Strike Price
  - Time to Expiration of Option
  - Exp. Interest Rate Volatility
  - Short-term Risk Free Rate
  - Coupon Payment

Option Pricing Models

- Black Scholes
  - risk free rate remains constant
  - volatility remains constant
  - log normal distribution for security prices
  - not good for fixed income securities
- Arbitrage-Free Binomial Model
  - estimate interest rate volatility and project rates forward
  - price bond at each node
  - value option at terminal nodes
  - work backward to determine present value
- Black Model
  - same as Black Scholes - was designed for European style options