The Term Structure of Interest Rates and Its Risks

Term Structure Theories
Yield Curve Risks
Valuing Embedded Options
The Binomial Model
Spreads including Option Adjusted Spread
Effective Duration and Convexity
Derivatives
Option Pricing Models

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The Yield Curve

- Shifts
  - parallel
  - nonparallel
  - twist in slope
  - flattening
  - steepening
  - butterfly shift

- Treasury Returns
  - level of rates
  - changes in slope
  - changes in curvature
Theoretical Spot Rate

- Select securities, then the methodology
  - On-the-run treasuries
    - bootstrapping
  - On-the-run treasuries and other off-the-run treasury issues
    - bootstrapping
  - Strips
    - yield = spot rate
  - All treasury coupon securities and bills
    - elaborate statistical techniques necessary
Term Structure Theories

- Pure Expectation Theory
  - Drawbacks
  - Interpretations
  - Forward Rates and Market Consensus
- Biased Expectations Theory
  - Liquidity Theory
  - Preferred habitat Theory
- Market Segmentation Theory
The Yield Curve
or Spot Yield Curve or Term Structure

• Spot yields vary (rise) with maturity
  • expectation that future short rates will rise
  • investors unwilling to invest long term unless rewarded by increased yield

• Pure expectation theory
  – forward rates of interest embodied in the term structure are unbiased estimates of expected future spot rates of interest
    • forward rates are biased (future spot rates) due to autocorrelation and by the amount of “term premium”
  – expected returns arising from different maturity strategies are equal
      • expected HPR are equal only for specific period, not all future holding periods
Pure Expectations Theory

• Investors are indifferent between each of the strategies below:
  • **Strategy I:** Buy 1 year bond today and roll over
  • **Strategy II:** Buy 2 year bond and hold to maturity
  • **Strategy III:** Buy 3 year bond and sell after year two.

• Ignores Uncertainty
Two More Theories

• **Long term premium** (Liquidity Preference): provides an upward tilt to long end to compensate for uncertainty of longer term investments.
  – borrowers borrow long, lenders lend short
  – uncertainty induces “term premium”

• **Segmentation modification**: supply and demand for bonds in various term spectrum segments dictates the current market rate.
  – legal and behavioral restrictions create “preferred” maturity demands

• *Compare and contrast three term structure theories.*
Measuring Yield Curve Risk

Rate Duration and Key Rate Duration

- **D(1):** \( (50/100) \times 2 + (0/100) \times 0 + (50/100) \times 0 = 1 \)
- **D(2):** \( (50/100) \times 0 + (0/100) \times 16 + (50/100) \times 0 = 0 \)
- **D(3):** \( (50/100) \times 0 + (0/100) \times 0 + (50/100) \times 30 = 15 \)
  - **Effective:** Portfolio III: \( (50/100) \times 2 + (0/100) \times 16 + (50/100) \times 30 = 16 \)
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Volatility

- Measuring Historical Yield Volatility
  - Determining Number of Observations
  - Annualizing Standard Deviation
  - Interpreting the Standard Deviation
- Historical versus Implied Volatility
- Forecasting Yield Volatility Alternatives
  - expected value equals zero
  - weighted observations
Valuing Bonds with Embedded Options

- What is an interest rate model?
  - one factor (binomial), two factor
  - I.E. MV bond $89. Model valuation $90 (based on 12% vol.). Model valuation $86 (based on 15% vol.)

- Option Free Bond Valuation
  - Benchmark Curve
  - Using Spot Curve or Forward Rates
The Binomial Model

- Binomial Interest Rate Tree
- Determining the Value of a Node
- Constructing the Binomial Interest Rate Tree
- Valuing an Option-Free Bond with the Tree
  - Value using binomial tree should be identical to bond value found when discounting at spot rates or 1 year forward rates.
Valuing the Callable Bond

- Determining the Call Option Value
  - value of call option = value of option free bond - value of callable bond

- Volatility and Arbitrage-Free Value
  - option value increases with volatility; option spread decreases with volatility.
  - value of callable bond decreases with volatility.

- Option Adjusted Spread (OAS)
  - constant spread that when added to rates (1yr) will make value (arbitrage-tree) equal to market price.
  - Volatility dependent
  - OAS attempts to remove from the nominal spread the amount that is due to the option risk
    - a spread
    - adjusts cash flows for option when computing spread to benchmark
Valuing the Callable Bond

- Callable Bond = Non-callable bond - call option.
  - the greater the value of the call feature, the lower the value of callable bond relative to that of non-callable bonds.
  - the level and volatility of interest rates are key factors in giving value to call feature.
  - value of option greater the greater the expected volatility

- Generally, the greater the variance of expected future interest rates, the higher value placed on call options; hence, higher spreads.
Facts About Spreads and OAS

- Static spread considers only one interest rate path.
- OAS contains MODEL risk. Must estimate call rule and volatility. Different analysts will arrive at different OASs.
- OAS uses averaging. OAS is not the spread the investor will earn over tsy.
- OAS should be used in conjunction with other measurements.
Valuing and Analyzing a Callable Bond

- **Modified Duration**
  - Macaulay Duration/[1+ytm/2]
  - \( \frac{dP}{P} = -(MD) \cdot (dR) \)
  - only good for "small" changes in rates

- **Convexity**
  - \( \frac{1}{P} \) (second derivative price/yield)
  - \( \frac{dP}{P} = -(MD)(dR) + \frac{1}{2}(Cvx)(dR)^2 \)
  - be able to compute approximate price change for interest rate changes.
  - Suppose \( MD = 3.95 \) and convexity = 19.45, compute price change if rates fell 1%. 
Effective Convexity and Duration

**Effective Duration** (vs Modified Duration)
- effective used to evaluate bond’s with embedded options
- approximation of modified duration
- \( \frac{V_- - V_+}{2V_0 (\delta r)} \)
- valid bond pricing model
- high interest rate levels, two measurements similar

**Effective Convexity** (vs Modified Convexity)
- \( \frac{(V_+ + V_- - 2V_0)}{2V_0 (\delta y)^2} \)
- considers cash flow stream changes - i.e. price compression
- lower than standard convexity for callable bonds
Factors that Affect Duration

- Lower coupon => greater duration
- Longer maturity => greater duration
- Duration is less than the time to maturity for coupon bonds
- The Macaulay duration of a zero coupon bond is its time to maturity
- When yields are high, durations are lower than when the general level of yields is low.
Factors that Affect Convexity

- Maturity: the longer the maturity, the greater the convexity.
- Coupon: the lower the coupon rate the greater the convexity. Zeros have greater convexity than coupon bonds of the same maturity.
- Coupon: the lower the coupon rate the smaller the convexity. Zeros have smaller convexity than coupon bonds of same duration.
- As yield increases, convexity decreases.
Valuing a Other Bonds

- A Putable Bond
- A Step-Up Callable Note
  - when more than one embedded option, determination made at each node
- A Capped Floater
  - coupon rate is set at begin period and paid at end period; affects cash flows at nodes
A Convertible Bond

- Basic Features
- Minimum Value
- Market Conversion Price
- Current Income vs Common Stock
- Downside Risk
- Investment Characteristics
Valuation of Interest Rate Derivative Instruments

- Interest Rate Futures Contracts
  - The Cash and Carry Trade
  - The Reverse Cash and Carry Trade

- Theoretical Futures Price
  - Theoretical FP
    - \[ \text{Price} + \text{Price}_t \text{(coupon/short rate differential)} \]
      - asymmetrical lending and borrowing rates
      - cheapest to deliver changes
      - interim cash flows change
Interest Rate Swaps

- **Swap Payments**
  - **Floating Rate Payments**
    - Floating Rate Payment = Notional Amount \times Libor \times (Actual\#Days/360)
  - **Future Floating Rate Payments**
  - **Swap Rate and Swap Spread**
  - **Calculate PV of Floating Rate Pay. And PV Notional**
    - Total \( PV_{floating} = Total \ PV_{fixed} \)
  - **Swap Rate and Swap Spread**

- **Value a Swap**
Options

 Components of the Option Price
  - Intrinsic Value
  - Time Value

 Factors that Influence Value of Option on Bond
  - Price of Underlying Security
  - Strike Price
  - Time to Expiration of Option
  - Exp. Interest Rate Volatility
  - Short-term Risk Free Rate
  - Coupon Payment
Option Pricing Models

- **Black Scholes**
  - risk free rate remains constant
  - volatility remains constant
  - log normal distribution for security prices
  - not good for fixed income securities

- **Arbitrage-Free Binomial Model**
  - estimate interest rate volatility and project rates forward
  - price bond at each node
  - value option at terminal nodes
  - work backward to determine present value

- **Black Model**
  - same as Black Scholes - was designed for European style options