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Implied Interest Rates*

With the introduction of put and call options a new risk-free asset has been created. The objective of this study is to estimate the rate implied in option prices and compare it to other riskless instruments. We have used transactions data on Chicago Board Options Exchange options taking into account the American feature of these options. We find that options markets provide rates that are competitive with other short-term rates. These rates are closer to the borrowing rate than to the lending rate.

I. Introduction

One of the most important assumptions in financial economics models is the existence of a risk-free rate, available for borrowing and lending. In any economy, however, the borrowing rate does not coincide with the lending rate even for the short run. The difference between the rates is considered a compensation for intermediation that is necessary in the imperfect real world. In a well-functioning financial market this difference would be small, and the smaller it is, the better off is the economy. The two short-term rates that best approximate riskless lending and borrowing rates are the Treasury-bill rate and the brokers' loan-call rate, respectively.

With the introduction of put and call options a new risk-free instrument has been created. Using options alone or options combined with the underlying asset[1] one could turn to these markets for his borrowing or lending needs, thereby

* We would like to thank Steve Freund for his valuable programming assistance. We would also like to thank Robert Reback for his helpful comments. This paper was presented at the Fourth Annual ASE Options Colloquium, New York, March 1984.

1. By buying (selling) the call spread and buying (selling) the put spread with the same striking prices one can lend (borrow) money essentially risklessly. This is also known as buying (selling) a "box." Alternatively, one can lend money by buying a stock, buying a put, and selling the call with the same strike creating a riskless position, known as a "conversion." The other side in the above transactions could be a borrower.

(*Journal of Business,* 1986, vol. 59, no. 3)

0021-9398/86/5903-0006$01.50

narrowing the gap between the borrowing and the lending rate. Assuming that the markets for stocks and options are efficient, we can test whether options markets provide rates that are competitive with the above short-term rates.

This new instrument should provide competitive rates when European options are involved, but for American options, where early exercise is possible, this may not be the case. Using American options for borrowing or lending exposes the borrower or the lender to a premature termination of the loan. It raises the cost of the loan to the borrower if rates have not changed or have increased. It may reduce the lender's income if rates have come down. Since the American feature of options cannot be derived analytically, we deal with it by isolating the cases in which early exercise is practically improbable.

Thus the objective of this study is to estimate the rate implied in options prices and compare it to other riskless rates. Also the value of the American feature can be estimated by comparing some of these implied rates to rates implied from other options or to the actual rates.

The estimates will be based on transactions data for options traded on the Chicago Board Options Exchange (CBOE). Since many observations over a relatively short time interval are available, the distribution properties of the estimates can be evaluated. The sample, containing various options series, will allow, though on a limited basis, us to study the effects of some of the parameters on the implied interest rate.

The results indicate that options markets provide a financial instrument that competes with existing riskless instruments. It seems, however, that the implied interest rate is closer to the effective borrowing rate than to the lending rate, suggesting that the introduction of options may have contributed to more competition and efficiency on the borrowing front.

In Section II we present the basic parity condition and the implied interest rate equation. Section III deals with the complication in the put-call relation that is due to the differences in values between American and European options and between dividend-protected and dividend-unprotected options. The effect of the options' terms on the implied interest rates will be specified. Section IV contains a description of the data employed in the study and of the estimation procedure. Section V provides the tests of the hypotheses. Section VI presents a summary and conclusions.

II. Put-Call Parity

Put-call parity (PCP) is a well-known relation that should exist, in a perfect capital market, between the prices of European call and put options with similar terms on the same underlying stock. This relation is described in a book by Castelli written in 1877. However, the analytical proof of the parity relation appeared only in 1969 in a paper by Stoll. He showed that with no transaction costs and taxation the following relation must hold:

$$C - P = V - Ke^{-r\tau}, \tag{1}$$

where r is the default-free interest rate, C and P are, respectively, the prices of the call and put options with striking price K and time to maturity τ. The value of the underlying stock is denoted by V.

If the parity condition described in (1) is valid and the financial markets are efficient, the riskless interest rate for the options' maturity can be inferred from the prices C, P, V, and the striking price K by

$$r_{t,\tau} = \frac{1}{\tau} \ln \frac{K}{V - C + P}, \tag{2}$$

where r is the default-free nominal interest rate for the period left to maturity τ, and t is the current date.

There are five major reasons why equation (1) may not strictly hold for the data employed in this study. First, the options are American, while equation (1) holds for European options. Second, CBOE options are not dividend protected, and when the stock goes ex-dividend, the terms of the options are unaltered. Third, equation (1) assumes that all the proceeds from a short sale can be used by the seller, while most sellers can use only 80%–83% of the proceeds, depending on the seller and on the stock involved. Fourth, there may be a simultaneity problem in trading in the call, put, the underlying stock, and the riskless bonds. Fifth, because of the discreteness of price quotes on one hand and the bid-ask spread in prices on the other hand, nonsystematic deviations from parity can arise.[2] The latter two problems are handled by using transactions data to estimate the implied rate and then computing the average (or median) implied rate for a given week. The effect of deviations from parity that are due to lack of simultaneity, price discreteness, and bid-ask spread should be minimal. The effect of the first three problems on the parity condition, hence on implied interest rates, is discussed in the next section.

III. Parity Conditions and American Options

Equation (1) should hold in a frictionless market, where C and P are the prices of European dividend-protected options. Let us denote by sub-

2. Stoll (1969), Gould and Galai (1974), and Galai (1978) found violations of eq. (1) for the OTC options, which were European and dividend-protected options. Klemkosky and Resnick (1979, 1980) adjusted eq. (1) to account for the fact that CBOE options are American and dividend unprotected. Deviations from the revised conditions were still recorded, though most were not realizable profits that traders could have captured.

scripts a and e American and European options, respectively, and by subscripts p and u the protected and unprotected options, respectively. With this notation equation (1) can be rewritten as follows:

$$C_{ep} - P_{ep} = V - K_e^{-r\tau}. \tag{1'}$$

A. The Case of Dividend-protected American Options

Merton (1973*a*, 1973*b*) shows that equation (1) is invalid for a dividend-protected American put since it may be profitable to exercise it before maturity. Hence the value of the American put cannot be less than the value of the European put with identical terms. For the American call, however, he shows that it does not pay to exercise it before maturity. Thus

$$C_{ap} = C_{ep}, \quad P_{ap} \geq P_{ep}. \tag{3}$$

Therefore

$$V - K \leq C_{ap} - P'_{ap} \leq V - Ke^{-r\tau}. \tag{4}$$

Even in an efficient market, if the options are American, the implied interest rate will depict the upper boundary for $C_{ap} - P_{ap}$, and, in general, the implied interest rate will be downward biased.[3] The bias will be small for cases in which the put option has very low probability of early exercise. The probability of early exercise is a function of the time to maturity, τ, and the extent the option is away from the money, K/V. The shorter the time to maturity, τ, the smaller, in absolute values, will be the bias in the implied rate. The bias in the rate will also increase as a function of K/V. The bias will be smaller for out-of-the-money American puts compared with in-the-money puts.

Based on these considerations, for options with no ex-dividend days expected before their expiration, the following hypothesis can be tested: (*a*) implied interest rates will be below the observed Treasury-bill rates; and (*b*) implied interest rates will be lower for options when the put is deep in the money ($V \ll K$) than when the put is deep out of the money ($V \gg K$); and (*c*) compared to the observed Treasury bill rates, for example, the difference between the observed rates and the implied rates will diminish as we approach the expiration date. Alternatively, we can estimate the value of the American feature, the likelihood of early exercise, by the differences between the implied rates and the actual rates.

B. The Case of the Dividend-unprotected American Options

If the stock pays dividends during the life of the options and the options are European but not dividend protected, then in the parity condition

3. The implied interest rate is obtained by equating $C_{ap} - P_{ap}$ to $V - K_e^{-r\tau}$. Since the difference between American call and put prices is less than for European options, the term $V - KE^{-r\tau}$ will be lower only if we substitute a lower interest rate.

(1) the price of the underlying stock, V, should be adjusted by subtracting from it the present value of the dividend stream. This adjustment assumes that future dividends and their timing are known in advance with certainty. For most established companies this is usually a good assumption.[4]

Since a dividend-unprotected call does not entitle its holder to the dividend distribution of the underlying stock, its value therefore cannot be more than that of the dividend-protected call with identical terms:

$$C_{ep} \geq C_{eu}, \quad C_{ap} \geq C_{au}. \tag{5}$$

The put holder benefits from unadjusting the terms of the put when the price of the stock is adjusted downward on the ex-dividend day. Hence,

$$P_{ep} \leq P_{eu}, \quad P_{ap} \leq P_{au}. \tag{6}$$

By combining (5) and (6) for American options it is easy to show that

$$C_{au} - P_{au} \leq C_{ap} - P_{ap}. \tag{7}$$

Therefore the upper boundary for the American dividend-protected options will apply also to the American unprotected options:

$$C_{au} - P_{au} \leq V - Ke^{-r\tau}. \tag{8}$$

The above condition is not necessarily the strictest. The parity condition for European unprotected options is given by

$$C_{eu} - P_{eu} = V - Ke^{-r\tau} - \sum De_{t_D}^{-r\tau} \tag{9}$$

where D_{t_D} is the dividend paid on day t_D.[5]

Since American options can be exercised on any day prior to maturity, specifically, just before the stock goes ex-dividend for the call and after an ex-dividend day for the put, therefore

$$C_{eu} \leq C_{au}, \quad P_{eu} \leq P_{au}, \tag{10}$$

and as a result

$$C_{au} - P_{au} \lesseqgtr C_{eu} - P_{eu}. \tag{11}$$

In general we cannot tell if the difference in American option prices is larger or smaller than the difference in the European option prices. If both the put and the call deviate by the same magnitude from their European counterparts, then condition (9) will also apply to $C_{au} - P_{au}$. Hence for dividend-unprotected American options the following hy-

4. Since General Motors grossly violated this assumption, we have not used it in this study.

5. For simplicity we ignore the difference between the ex-dividend day and the payment day. The difference is between 1 and 2 weeks and should have a negligible effect.

potheses could be tested: (*d*) if the dividend payment is small relative to V and $C_{au} - P_{au}$ is equated to $V - Ke^{-r\tau}$, the implied interest rate is expected to be downward biased; (*e*) as the price of the stock increases relative to the striking price, the probability of early exercise of the put diminishes while the probability of early exercise of the call increases. Therefore, with dividend adjustment of V, for in-the-money calls (i.e., $V > K$) an upward bias in the implied r is expected; and (*f*) for at-the-money options, with dividend adjustment, the direction of the bias in implied r is ambiguous, as indicated by (11). The bias will be small if the dividend is relatively small and the time period between the ex-dividend day and the option expiration day is long.

IV. Data and Estimation

A. Data on Stock Prices and Option Prices

This study uses transactions data on four stocks that had put and call options on the CBOE in the period June 1977–August 1978.[6] The four stocks are Avon Products (AVP), Eastman Kodak (EK), Honeywell (HON), and IBM, all having "thick" markets (for the stocks and the options). We use the "consolidated format" data base, which compresses into one record all information for each option over a time interval during which the underlying stock price remains unchanged. Though this format sacrifices some information, it makes the analysis manageable.[7] Each record provides the lowest and highest option prices observed during any time interval. The prices that are used are averages of the low and the high prices.[8]

The other data necessary for this study are the dividend record and estimates of the risk-free rate, which were obtained from *Moody's Dividend Record* and from the *Federal Reserve Bulletin,* respectively. Data were collected for the loan-call money rate and the 3- and 6-month Treasury-bill rates.

To get reasonable estimates of the implied interest rate we restricted our sample to call and put prices that were at least \$1.00 and to expiration dates that were at least 50 days away. In doing so we may have lost a few observations, but we also eliminated many more observations

6. The data were taken from the Berkeley Options Data Base, which consists of all transactions on options traded on the CBOE for the 2-year period August 1976–August 1978. The structure and content of the data base is discussed in detail in Bhattacharya and Rubinstein (1978).

7. The logic for such a consolidation is that in an efficient market there is no reason for the option price to change if none of the underlying factors have changed. In the short interval between two price changes no other parameter is expected to change, and the only reason for changes in option prices are temporary random demand/supply imbalances.

8. Every stock price change for which both put and call prices were reported was selected for the sample.

that would provide unreasonable estimates caused by the discreteness of price change and, mainly, by "thinness" in these options. Since short-term options are not interest sensitive, the information loss due to the omission of very short term options is minimal.[9] In general, long maturity options have a more pronounced interest rate effect and should yield better estimates of the interest rate, but these options also contain more dividends before expiration and introduce more noise in our estimates.

B. *Estimation Procedure*

To account for discrete dividend payments we use an approximation suggested by Black (1975) and employed by several researchers. We subtract the present value of all realized dividends paid before the option's maturity from the stock price and use this adjusted price in the put-call parity.[10] This approach assumes that dividend payments are known with certainty, a reasonable assumption for most large companies.

Using the European put-call parity, we estimated the implied interest rates by

$$\hat{r}_{t,\tau} = \frac{1}{\tau} \ln \frac{K}{P + V - C}, \tag{12}$$

where $\hat{r}_{t,\tau}$ is the estimate of the interest rate at time t with τ months to maturity, and V is adjusted for dividends.

For every month in our sample and for every stock we used all the put-call pairs in the first whole week of trading (5 trading days) to generate the distribution of implied interest rates. All pairs with a given stock price interval were employed in our sample.

If there are no dividend payments left to maturity, if the probability of early exercise of the put is zero, and if all parameters are measured simultaneously and without error, we should not expect any distribution for the implied rate. Even if the first two conditions hold, which is true in many cases, the error measurement problem and the simultaneity problem always hold at least to some extent. Thus we shall always observe a distribution even if put-call parity strictly holds.[11]

To test the hypothesis that the rate implied by option prices is equal to some known rate in the economy we must make assumptions about the distribution of the error in

9. For very short term options we may obtain negative or zero implied rates due to violations of the boundaries.

10. For simplicity we have discounted the dividends at the Treasury-bill rate, an appropriate approximation under the null hypothesis of an implied risk-free rate.

11. The approach that looks at the distribution of the implied parameters rather than at single observations is introduced and described in Brenner and Galai (1981).

$$\hat{r}_{t,T} = r_{t,T} + \tilde{e}_{t,T}, \tag{13}$$

where ˆ denotes an estimate and ˜ denotes a random variable.

If we assume that the errors are independent and identically distributed, then a standard t-test can be used to compare the mean implied rate to the observed interest rate. Since some of our observations, over time and across exercise prices and stocks, are dependent, the standard errors are downward biased, and the t-values are upward biased. The results of these tests, therefore, should be interpreted with caution. Another statistic that is presented is the median. In some cases, because of problems mentioned above, few extreme observations may have a strong effect on the mean, and the median may be a more proper statistic to look at.[12]

V. Tests and Results

The put-call relation described in Section II is based on the assumption of frictionless markets. In such markets there is no difference between the borrowing rate and the lending rate. In reality, however, if an investor lends money by buying a pure discount government short-term bond, his holding period interest rate is the Treasury-bill rate. If, on the other hand, the investor wants to borrow money against a riskless future income, he would pay a rate higher than the Treasury-bill rate. Stock or option dealers pay the loan-call rate when they borrow.[13] A lender that uses the options market will require at least the Treasury-bill rate, while a borrower will pay at most the loan-call rate. Thus we would expect the implied interest rate to lie in between these two rates. This, however, may not be the case, first, because of the American feature of options and, second, because of the unequal treatment of long and short positions in the stock. Even when early exercise is unimportant, the restriction on short-position funds will bias the implied rate.

If the rate implied by option prices is higher than the Treasury-bill rate, then lenders will buy the stock, buy the put, and sell the call, raising the price of the put relative to the call till the implied rate equals the Treasury-bill rate. If, on the other hand, the implied rate is below the Treasury-bill rate, it may just stay there since no borrowers are coming in. A borrower that raises funds by shorting the stock, buying a call, and selling a put can use approximately 80% of the balance.[14] The borrower in this market will thus require a higher price for the put

12. See, e.g., Brenner and Galai 1984.

13. The borrowing rate is somewhat higher for individual customers. Also the loan-call rate is for short-term lending and borrowing and not for a holding period of a few months.

14. If the stock that is being shorted is difficult to borrow, as in a pending merger case, there will be an additional cost to the borrower, and the effective rate will be even higher.

TABLE 1 Actual 3- and 6-Month Interest Rates in the Period June 1977–August 1978

	Treasury Bill		Loan Call	
Date	3	6	Low	High
June 1977	5.03	5.23	6.00	6.00
July 1977	5.12	5.29	5.75	6.25
August 1977	5.35	5.47	6.00	6.25
September 1977	5.56	5.86	6.25	6.50
October 1977	6.09	6.33	6.62	7.00
November 1977	6.20	6.49	7.00	7.25
December 1977	6.04	6.37	7.00	7.25
January 1978	6.20	6.45	6.87	7.25
February 1978	6.42	6.70	7.12	7.50
March 1978	6.29	6.64	7.12	7.50
April 1978	6.37	6.73	7.12	7.50
May 1978	6.38	6.90	7.50	7.75
June 1978	6.62	7.14	7.75	8.00
July 1978	6.80	7.43	8.50	8.75
August 1978	6.70	7.24	8.50	8.75

relative to the call that he needs to buy. He has no incentive to use this borrowing vehicle unless options prices are such that the implied rate is lower than 80% of the loan-call rate. Thus, the implied rate could be as low as 80% of the loan-call rate.

Before presenting the results of the hypotheses outlined in Section II we provide in table 1 the evolution of the 3- and 6-month Treasury-bill rate and the loan-call rate from June 1977 through August 1978. In this period the rates have increased steadily but at a rather slow pace. The Treasury-bill rate is about 1% lower than the loan-call money rate. In the next few tables the rates implied from option prices are compared to these rates.

If the possibility of early exercise of a call or a put is negligible, then the implied rates should be the same across stocks and across striking prices and, as discussed above, could be below the Treasury-bill rate.

The first test uses all data, across stocks, for options with approximately 3- and 6-month maturity dates. The results of the analysis of variance (ANOVA) are presented in table 2. The mean values for in-the-money, at-the-money, and out-of-the-money options with 3 and 6 months appear in part A along with the Treasury-bill rate. Part B provides the test statistics for the ANOVA. It turns out that there is no striking price effect and no interaction effect but that, as expected, there is a time to maturity effect. There is only a 2.9% probability that the latter happened by chance. For all striking prices the mean implied rates were higher for the longer maturity than for the shorter one, as was the case for the actual interest rates. Though the probability of early exercise of a put is largest for in-the-money puts with short matu-

TABLE 2 Mean Implied Interest Rates and an Analysis of Variance Test

A. Means

	$K/V < 1$	$K/V = 1$	$K/V > 1$	Total	$r(TB)$	$r'(LC)$
3 month	4.13	4.73	4.39	4.53	6.13	5.61
6 month	5.09	5.23	5.15	5.17	6.75	...
Total	4.76	4.98	4.85	4.89	...	...

B. Analysis of Variance

Source of Variance	Degrees of Freedom	Sum of Squares	Mean Square	F	$s(F)$
Exercise prices	2	2.866	1.433	.420	.658
Time of expiration	1	16.518	16.518	4.845	.029
Interaction	2	1.315	.658	.193	.825
Explained	5	19.149	3.830	1.123	.351
Residuals (error)	144	490.964	3.409	...	...

NOTE.—T is time to maturity; V is the average price in the first week of the month; K is the striking price; $r(TB)$ is the Treasury-bill rate; and $r'(LC)$ is 80% of the loan-call rate (low).

rity implying relatively low rates, these rates cannot be distinguished from rates implied by out-of-the-money puts.[15] The mean implied rate is significantly lower than the Treasury-bill rate. Since this low rate is obtained by aggregation across stocks and by ignoring the early exercise feature of calls and puts, we continue our tests using subsets of the above data.

To test whether different stocks differ in the rate they imply, at-the-money options for all 15 months were used. Table 3 provides the means and standard deviations of implied rates and the Treasury-bill rate for the corresponding maturities. All mean implied rates are lower than the Treasury-bill rate, but only in four out of eight cases is this statistically significant. The longer maturity rates deviate less and have lower variability. This is expected since the probability of early exercise is smaller for longer maturities. If we compare the implied rates to 80% of the loan-call rate, then, except for AVP, all the deviations are not statistically significant.

Since there seem to be differences among stocks and such differences may be expected because of dividend payments, we turn to some more specific tests. We first carry out a detailed analysis of IBM, the most active stock in the sample with the largest number of transactions in options, and to reduce the effect of early exercise we used out-of-the-money puts. Table 4 presents the implied rates from IBM stock and options. Options with about 3 and 6 months to expiration that are out of

15. Another way to test for the effect of striking prices and interaction is to apply the ANOVA to the differences between the implied rate and the observed rate (e.g., Treasury-bill rate). The results were similar and showed no effect.

TABLE 3 Summary Statistics of the Implied Interest Rates from at-the-Money Options, Using Four Stocks and Two Expiration Dates

	Stock					
Treasury Bill	AVP	EK	HON	IBM	$r(TB)$	$r'(LC)$
3 month	2.97	5.68	4.99	4.95	6.13	5.61
	(2.30)	(2.04)	(2.30)	(1.16)	(.57)	(.83)
6 month	3.95	6.24	5.58	5.47	6.47	...
	(1.20)	(1.24)	(1.42)	(.84)	(.69)	...

NOTE.—Standard deviations are in parentheses.

the money for the put are employed here. The 6-month implied rates are usually higher than the 3-month rates, and both are lower than their respective Treasury-bill rates. Again the implied rates in most cases are very close to 80% of the loan-call rate, the rate obtained from a "reversal" position.

The hypotheses outlined in Section II*A* are tested in table 5. In our sample only HON and IBM had periods close to 3 months with no dividend payments before the options expired. In 11 out of 15 cases the average implied rate was significantly smaller than the Treasury-bill rate. When we take 80% of the loan-call rate, we still find the implied rates to be lower, but the difference is much smaller. Also if we use the median rather than the mean as our estimate we find the differences to be even smaller. The median in many instances is a more appropriate estimate because of the peculiarities of the data for some stocks and options.[16] The early-exercise effect should be stronger the lower is V relative to K, and indeed the mean implied rates get smaller for larger K, which is consistent with the hypothesis in Section II*B*.[17]

In table 6 we present the results of tests that deal with the dividend-unprotected options. The mean, standard deviation, and median implied rates for three stocks (AVP, EK, and IBM) with known dividends are presented. These rates were estimated when there were about 3 months left to expiration and about 1 month before the next ex-dividend date.

Except for EK with $K = 50$, for all the other cases the mean-implied rate is significantly[18] lower than the Treasury-bill rate for different

16. Because of simultaneity problems, e.g., we may obtain some extreme implied rates, usually negative rates, that affect the average significantly. For IBM for at- or out-of-the-money calls, this problem is not a very serious one.

17. Since we exclude the period that is close to maturity and options with low prices, we could not test for the effect of time on the difference between the rates.

18. It should be noted that there is a little problem with the normality of the distribution of implied rates, as is evident from the values of the studentized range (SR). Since in our sample the observations are somewhat dependent, the estimated standard error will be downward biased, the test statistics will be upward biased, and the difference in rates may not be as significant as it looks.

TABLE 4 **Implied Interest Rates for Options on IBM Stock with 3 and 6 Months to Expiration, Using out-of-the-Money Put Options**

Date	r_3	$s(r_3)$	N	$r_3(TB)$	$r'(LC)$	$s(r_6)$	N	$r_6(TB)$	r_6
July 1977	4.64	.50	159	5.12	4.6	.33	91	5.29	4.87
October 1977	5.41	.40	170	6.09	5.3	.68	103	6.33	5.85
November 1977	5.79	.56	199	6.20	5.6	.28	191	6.49	6.20
January 1978	6.07	.37	91	6.20	5.5	.24	95	6.45	6.46
February 1978	. . .	. . .	. . .	. . .	. . .	.30	101	6.70	5.95
July 1978	5.77	.68	190	6.80	6.8	.37	111	7.43	6.45
August 1978	7.45	.87	352	6.70	6.8	.57	93	7.24	7.19

NOTE.—r_3 is the mean 3-month implied rate; $s(r_3)$ is the standard deviation of r_3; r_6 is the mean 6-month implied rate; and $s(r_6)$ is the standard deviation of r_6.

TABLE 5 Implied Interest Rates for Stocks with No Dividend Payments before Options Expiration Date

Date	Stock	V	K	Mean	SD	Median	SR	N	$r_3(TB)$	$r'(LC)$
June 1977	HON	51	50	1.41	1.80	1.57	4.37	112	5.03	4.8
November 1977	IBM	257	240	5.79	.56	5.73	4.57	199	6.20	5.6
			260	4.72	.96	4.85	4.78	345		
			280	2.13	.46	2.16	5.74	43		
December 1977	HON	46	45	8.66	1.77	8.88	5.49	110	6.04	5.6
February 1977	IBM	258	260	4.99	.75	5.03	5.45	251	6.42	5.7
			280	2.44	1.33	2.42	14.27	199		
March 1978	HON	44	45	4.33	1.59	4.19	5.29	69	6.29	5.7
May 1978	IBM	262	240	3.79	.81	3.86	5.11	313	6.38	6.0
			260	3.51	.92	3.72	5.69	394		
			280	3.45	.76	3.51	5.29	240		
June 1978	HON	58	60	6.82	1.85	7.25	5.84	112	6.62	6.2
August 1978	IBM	289	260	7.45	.87	7.45	9.82	352	6.70	6.8
			280	6.54	1.19	6.63	12.56	510		
			300	6.18	1.09	6.27	10.57	369		

NOTE.—SD is the standard deviation; SR is the studentized range.

stocks and different K's. In the case of dividends the bias in the implied rate may go either way since there may be an incentive to exercise the call earlier. If the probability of early exercise of the call is dominant, relative to early exercise of the put, then we should find the implied rates, in these cases, to be biased upward. This would be the case for deep-in-the-money calls, but these are traded infrequently. Thus we

TABLE 6 Implied Interest Rates for Stocks with Dividend Payments at Least 1 Month Away and 3 Months to Expiration

Date	Stock	V	K	Mean	SD	Median	SR	N	$r_3(TB)$	$r'(LC)$
July 1977	AVP	50	50	4.09	.84	4.02	4.12	45	5.12	4.6
	EK	59	60	4.51	.99	4.49	4.87	114		
	IBM	263	240	4.64	.50	4.58	6.53	159		
			260	3.42	.72	3.59	6.25	214		
October 1977	AVP	47	45	2.42	1.40	2.36	5.21	72	6.09	5.3
	EK	61	60	5.85	.90	5.79	5.03	148		
	IBM	259	240	5.41	.40	5.41	5.40	170		
			260	5.23	.39	5.26	5.90	225		
			280	3.53	.40	3.55	4.66	149		
January 1978	AVP	46	45	3.34	1.08	3.15	5.11	79	6.20	5.5
	EK	49	50	9.31	1.67	9.69	5.99	209		
	IBM	266	240	6.07	.37	6.10	5.43	91		
			260	5.54	.40	5.54	6.08	315		
			280	5.54	.79	5.47	10.45	222		
April 1978	AVP	47	45	2.00	1.42	1.83	5.73	63	6.37	5.7
	EK	43	45	4.32	1.01	4.41	5.28	121		
	IBM	238	240	5.83	.86	5.94	12.68	305		
			260	4.95	.52	4.87	5.16	212		
July 1978	AVP	54	50	3.19	1.44	3.19	4.54	49	6.80	6.8
			60	5.20	1.05	5.03	4.03	16		
	EK	53	50	5.02	1.03	5.23	4.77	115		
			60	5.09	.79	5.07	5.19	65		
	IBM	259	240	5.77	.68	5.84	4.76	190		
			260	6.19	.52	6.20	5.79	201		
			280	5.85	.54	5.91	7.41	158		

NOTE.—SD is the standard deviation; SR is the studentized range.

can only compare at-the-money options with in-the-money calls with enough observations. While, as just mentioned, we cannot in general tell which direction the bias will take, we should find that r should be less downward biased for in-the-money calls than for at-the-money calls. Again, IBM is the only stock for which the comparison can be done. It turns out that in 4 out of 5 months the in-the-money calls provided a higher implied rate than the rate implied from at-the-money calls.

Finally, we propose an estimate of the value of early exercise of various options. The estimate is based on implied rates from options with different exercise prices. The purest estimate is obtained in the period when no dividends are to be paid before maturity. The benchmark implied rate is obtained from deep-out-of-the-money puts, the least likely to be exercised (so are the corresponding calls). The rates implied from puts with other exercise prices are compared to the benchmark rate, and the difference measures the value of early exercise to the participants. Because of the limitations of our data, we again used IBM options. For at-the-money options the average difference is about 0.9% (compare the data in tables 3 and 4). For in-the-money puts (see table 5) the difference could be as high as 3.5%.[19]

VI. Summary and Conclusions

In a well-functioning capital market there is a narrow spread between the borrowing rate and the lending rate. For example, in 1977–78 the spread between the 3-month Treasury-bill rate and the broker loan-call rate was about 1%. The introduction of options in the mid-1970s has attracted many borrowers and lenders that found the new instrument to be a substitute to the existing short-term instruments, providing perhaps a narrower spread. This paper discusses the conditions under which this financial innovation is a substitute for other instruments and tests the extent to which the rate provided by this instrument competes with other short-term rates.

Using put-call parity, the implied interest rate is derived. Since short positions are not treated the same as long ones, the estimated implied rate could be less than the observed lending rate. The possibility of early exercise could also cause the implied rate to deviate from the observed rates. Still, we find that, in general, the implied rates track well the trends in market rates. Moreover, when we isolate the cases in which early exercise is unlikely, we find that the estimated implied rates are not significantly different from the effective borrowing rate,

19. Interest rate uncertainty coupled with early exercise may push the implied rates even lower.

suggesting that options markets do provide a financial instrument that competes with existing riskless instruments.

We also test several hypotheses regarding the effect of early exercise of calls and puts on the implied rates. We find that in cases in which early exercise is more likely the implied rates deviate significantly from the short-term rates, and under such conditions these options could not be substitutes for short-term borrowing or lending.

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