THE EFFECT OF UNSTABLE BASIS RISK ON HEDGING EFFECTIVENESS FOR T-BOND FUTURES

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INTRODUCTION

The hedging effectiveness literature has concentrated on examining the size of the hedge ratios and associated hedging effectiveness values for ex-post data. Such measures implicitly assume that a static hedge ratio model is appropriate. Daigler and Smyser (1987) and Lasser (1987) show that the hedge ratios for T-bond and T-bill futures are not stable over time. The resultant question is whether this instability in hedge ratios has any effect on hedging effectiveness.

This paper develops two formulations of the effect of unstable hedge ratios on hedging effectiveness and then examines this effect by means of T-bond hedge ratios. The importance of determining the effect of unstable hedge ratios on hedging effectiveness is straightforward: using the previous period's unstable hedge ratio as an estimate of the current period's hedge ratio causes an upward bias in the hedging effectiveness measure that implies the effectiveness is greater than will actually occur. Large biases will create unexpected and undesired results for the unwary hedger. This paper shows that the degree of bias in the ex-post estimate of hedging effectiveness by employing a previous period's hedge ratio is related to the size of the change in the hedge ratio and the average change in the basis.

THE MODELS FOR HEDGING EFFECTIVENESS BIAS

Two models are developed to show the effect of an unstable hedge ratio on hedging effectiveness. The first model assumes that one wishes to hedge against all price changes except changes due to convergence. This simplification provides a straightforward result that is easy to calculate. The second model is based on the desire to hedge against <u>all</u> price changes. This model is more complicated in form but theoretically will be more accurate, especially for markets with trend changes, a large convergence factor, or for cross-hedging situations which have deviations between the behavior of the futures and cash markets.

A Simplified Model

The typical ex-post variance minimizing hedge ratio for time period t+1 is designated as b^*_{t+1} and is defined as:

$$b^*_{t+1} = \sigma_{SF}/\sigma_F^2 \quad (1)$$

Where:

σ_{SF} = the covariance between the spot (S) and futures (F) price changes during time period t+1

σ_F^2 = the variance of the futures price changes during time period t+1

The basis at a specific time k within the time interval t+1, as defined in terms of the ex-post minimum variance hedge ratio, is:

$$H^*_{t+1}(k) = \text{Basis} = S_{t+1}(k) - b^*_{t+1}\, F_{t+1}(k)$$

(2)

Where:

$H^*_{t+1}(k)$ = the basis at time k within time interval t+1, as determined by using the ex-post hedge ratio b^*_{t+1}

$S_{t+1}(k)$ = spot price at time k within interval t+1

$F_{t+1}(k)$ = futures price at time k within interval t+1

Similarly, we define the change in the basis from time k to time k+1 within time period t+1 as:

$$\Delta H^*_{t+1}(k,k+1) = \Delta S_{t+1}(k,k+1) - b^*_{t+1}\, \Delta F_{t+1}(k,k+1)$$

(3)

If one wishes to hedge against all price changes other than those due to convergence or to the average change in the basis over the period, then the variability of the basis change during time period t+1 can be determined by:

$$\mathrm{var}(\Delta H^*_{t+1}) = \sigma_S^2 + {b^*_{t+1}}^2\, \sigma_F^2 - 2\, b^*_{t+1}\, \sigma_{SF}$$

(4)

Where: σ_S^2 = the variance of spot price changes during period t+1

When an unstable minimum variance hedge ratio exists between time period "t" and time period "t+1" then b^*_{t+1} can be defined in terms of b^*_t and the change in the hedge ratio from "t" to "t+1":

$$b^*_{t+1} = b^*_t + \Delta b_t$$

(5)

Where:

b^*_t = the minimum variance hedge ratio over the time period t

Δb_t = the change in the hedge ratio from time period t to time period t+1

Consequently, the change in the basis between time k and time k+1 within time interval t+1 can be redefined to consider the effect of employing the previous period's minimum variance hedge ratio b^*_t as an estimate of the true current period's minimum variance hedge ratio. Thus, if $b^*_t + \Delta b_t$ from (5) is substituted for b^*_{t+1} in (3) we have:

$$\Delta H^*_{t+1}(k,k+1) = \Delta S_{t+1}(k,k+1) - (b^*_t + \Delta b_t)\ \Delta F_{t+1}(k,k+1) \qquad (6)$$

The resultant equation for the variability in the basis change is:

$$\mathrm{var}(\Delta H^*_{t+1}) = \sigma_S^2 + (b^*_t + \Delta b_t)^2\ \sigma_F^2 - 2\ (b^*_t + \Delta b_t)\ \sigma_{SF} \qquad (7)$$

Likewise, if at the beginning of time period t+1 one uses the minimum variance hedge ratio b^*_t as the best estimate of b^*_{t+1}, then one may determine what the variability of the basis change would be during t+1 by using b^*_t:

$$\mathrm{var}(\Delta H^t_{t+1}) = \sigma_S^2 + b^{*\,2}_t\ \sigma_F^2 - 2\ b^*_t\ \sigma_{SF} \qquad (8)$$

Where:

$\mathrm{var}(\Delta H^t_{t+1})$ = the variance of the change in the basis during time period t+1 as determined by using the previous period's minimum variance hedge ratio b^*_t.

Subtracting (7) from (8) we can determine the additional basis risk from using b^*_t as an estimate of b^*_{t+1} when the minimum variance hedge ratio changes over time:

$$\mathrm{var}(\Delta H^t_{t+1}) - \mathrm{var}(\Delta H^*_{t+1}) = -\ \Delta b^{*\,2}_t\ \sigma_F^2 - 2\ b^*_t\ \Delta b_t\ \sigma_F^2 + 2\ \Delta b_t\ \sigma_{SF} \qquad (9)$$

$$= 2\ \Delta b_t\ (\sigma_{SF} - b^*_t\ \sigma_F^2) - \Delta b_t^2\ \sigma_F^2$$

$$= 2\ \Delta b_t\ \sigma_F^2\ (\sigma_{SF}/\sigma_F^2 - b^*_t\ \sigma_F^2/\sigma_F^2) - \Delta b_t\ \sigma_F^2$$

$$= 2\ \Delta b_t\ \sigma_F^2\ (b^*_{t+1} - b^*_t) - \Delta b_t^2\ \sigma_F^2$$

Since from (5):

$$\Delta b_t = b^*_{t+1} - b^*_t$$

we determine that:

$$\mathrm{var}(\Delta H^t_{t+1}) - \mathrm{var}(\Delta H^*_{t+1}) = \Delta b_t^2\ \sigma_F^2 > 0 \qquad (10)$$

Using $E^*_{t+1} = R_{t+1}^2$ as the typical measure of the minimum variance hedging effectiveness for period t+1, equation (11) states this definition in terms of the variability in the basis change by employing the minimum variance hedged position (ΔH^*_{t+1}) and the variability of the changes in the unhedged or cash (ΔS_{t+1}) position:

$$E^*_{t+1} = R_{t+1}^2 = 1 - \mathrm{var}(\Delta H^*_{t+1})/\mathrm{var}(\Delta S_{t+1}) \qquad (11)$$

Where:

E^*_{t+1} = the hedging effectiveness for period t+1 by using the minimum variance hedge ratio b^*_{t+1}

The upward bias in the t+1 minimum variance hedging effectiveness value when b^*_t is used as an estimate of b^*_{t+1} can be determined by using (10):

$$E^*_{t+1} - E^t_{t+1} = 1 - \mathrm{var}(\Delta H^*_{t+1})/\sigma_S^2 - [1 - \mathrm{var}(\Delta H^t_{t+1})/\sigma_S^2]$$

$$= \Delta b_t^2\ [\sigma_F^2/\sigma_S^2] \qquad (12)$$

Where:

E^*_{t+1} = the minimum variance hedging effectiveness measure when the ex-post hedge ratio b^*_{t+1} is employed during time period t+1

E^t_{t+1} = the hedging effectiveness when the ex-ante hedge ratio b^*_t from period t is employed during time period t+1

Equation (12) determines the upward bias inherent in E^*_{t+1} when the ex-post minimum variance hedge ratio b^*_{t+1} is employed to determine the hedging effectiveness and the hedge ratio is not stable over time. Equation (12) shows that this bias is related to the size of the change in the hedge ratio squared, Δb_t^2, and the volatility scale factor σ_F^2/σ_S^2.

Including the Average Change in the Basis in the Model

Another model of the effect of unstable hedge ratios on the ex-post hedging effectiveness can be determined by including the effect of the average change in the basis during time period t+1. Since the typical variance model employed in (12) above determines the variability around the mean of the distribution, any trend or convergence in the data that shows up as an average change in the basis will not be considered as variability by the model derived above. However, if we assume that the hedger wishes to minimize variability about a zero change in the basis, then the following model is appropriate to determine the extent of the bias in the hedging effectiveness measure.

Equations (1) through (3), (5), and (6) define basis and the change in the basis in terms of b^*_{t+1}, b^*_t, and the change in these hedge ratios from t to t+1, Δb_t. If we use the regression methodology to define the change in the cash price between intervals k and k+1 during period t+1 we have:

$$\Delta S_{t+1}(k,k+1) = a^*_{t+1} + b^*_{t+1}\ \Delta F_{t+1}(k,k+1) + e^*_{t+1}(k,k+1) \tag{13}$$

Where:

a^*_{t+1} = the y-intercept for the minimum variance hedge ratio regression equation during period t+1

e^*_{t+1} (k,k+1) = the error term for the minimum variance hedge ratio regression equation during period t+1, for the price change occurring during the time interval k to k+1

Then substituting into equation (3) we obtain:

$$\Delta H^*_{t+1}(k,k+1) = [a^*_{t+1} + b^*_{t+1}\ \Delta F_{t+1}(k,k+1) + e^*_{t+1}(k,k+1)] - b^*_{t+1}\ \Delta F_{t+1}(k,k+1)$$

$$= a^*_{t+1} + e^*_{t+1}(k,k+1) \tag{14}$$

Squaring each change in the basis and summing over all of the time intervals k in period t+1, one obtains the <u>total</u> variability in the basis during period t+1:

$$\sum_k (\Delta H^*_{t+1})^2 = \sum_k (a^*_{t+1} + e^*_{t+1})^2 \tag{15}$$

Alternatively, if one employs the previous period's minimum variance hedge ratio b^*_t during time period t+1 then the change in the basis for a given time interval is:

$$\Delta H^t_{t+1}(k,k+1) = \Delta S_{t+1} - b^*_t\ \Delta F_{t+1}$$

$$= [a^*_{t+1} + b^*_{t+1}\ \Delta F_{t+1}(k,k+1) + e^*_{t+1}(k,k+1)] - b^*_t$$

$\Delta F_{t+1}(k,k+1)$ (16)

Substituting from (5), $b^*_{t+1} = b^*_t + \Delta b_t$, squaring each basis change, and summing over k we obtain:

$$\sum_k (\Delta H^t_{t+1})^2 = \sum_k (a^*_{t+1} + e^*_{t+1} + \Delta b_t \ \Delta F_{t+1})^2 \tag{17}$$

The following formulas employ the squared variabilities being summed over the time intervals k during time period t+1 to define the hedging effectiveness measures:

$$E^*_{t+1} = R_{t+1}^2 = 1 - \sum_k (\Delta H^*_{t+1})^2 / \sum_k (\Delta S_{t+1})^2 \tag{18}$$

$$\text{and } E^t_{t+1} = 1 - \sum_k (\Delta H^t_{t+1})^2 / \sum_k (\Delta S_{t+1})^2$$

Note that the summation of the variability of ΔH is the <u>total</u> basis variability of the hedged position. This total basis variability depends on whether b^*_{t+1} or b^*_t is employed as the hedge ratio during period t+1 to determine E^*_{t+1} and E^t_{t+1}, respectively.

The upward bias in the minimum variance hedging effectiveness measure E^*_{t+1} when there exists an instability in the hedge ratio from periods t to t+1 is:

$$E^*_{t+1} - E^t_{t+1} = 1 - \Sigma(\Delta H^*_{t+1})^2 / \Sigma(\Delta S_{t+1})^2 - [1 - \Sigma(\Delta H^t_{t+1})^2 / \Sigma(\Delta S_{t+1})^2]$$

$$= [\Sigma(\Delta H^t_{t+1})^2 - \Sigma(\Delta H^*_{t+1})^2] / \Sigma(\Delta S_{t+1})^2 \tag{19}$$

Substituting equations (15) and (17) into (19), combining terms, rearranging, and noting that $\Sigma e = 0$:

$$E^{*}_{t+1} - E^{t}_{t+1} = \Sigma\ \Delta b_t^{2}\ \Delta F_{t+1}^{2} + \Sigma\ 2a^{*}_{t+1}\ \overline{\Delta b}_t\ \Delta F_{t+1} \tag{20}$$

Now, since:

$$\sigma_F^{2} = \Sigma \Delta F^{2}/N - \overline{\Delta F}^{2}$$

and thus

$$\Sigma \Delta F^{2} = N\ \sigma_F^{2} + N\ \overline{\Delta F}^{2} \tag{21}$$

Where:

σ_F^{2} = the variance of ΔF over time period t+1

$\overline{\Delta F}$ = the mean of ΔF over time period t+1

and similarly for $\Sigma \Delta S^{2}$, upon summing and substituting (21) into (20) we obtain:

$$E^{*}_{t+1} - E^{t}_{t+1} = [\Delta b_t^{2}\ \sigma_F^{2} + \Delta b_t^{2}\ \overline{\Delta F}^{2} + 2a^{*}_{t+1}\ \overline{\Delta b}_t\ \Delta F]/[\sigma_S^{2} + \overline{\Delta S}^{2}] \tag{22}$$

Where:

a^{*}_{t+1} = the average per period change in the basis during period t+1

<u>Interpreting the Models</u>

The models in the previous sections show that using the variance minimizing hedge ratio technique when hedge ratios are unstable over time results in an upward biased value for the hedging effectiveness measure. Conceptually, if b^{*}_{t+1} is the minimum variance hedge ratio during time t+1 using regression, then any other hedge ratio b_t that differs from b^{*}_{t+1} will have a larger sum of squared errors than b^{*}_{t+1} and thus possess a lower R^2 or E value.

Model (1) is based on the concept that one wishes to minimize the variance of the price changes around the average change in the basis. Hence, the assumption is made that a systematic change in the basis due to convergence or other external economic factors can not be hedged away. This results in the conclusion that the bias in the hedging effectiveness with an unstable hedge ratio is determined by (12):

$$E^*_{t+1} - E^t_{t+1} = \Delta b_t^2 \; [\sigma_F^2/\sigma_S^2] \tag{23}$$

Model (2) is based on the desire to minimize the variance of <u>all</u> price changes, i.e. to hedge against any change in the basis, including any systematic change in the basis. Equation (22) shows the bias in hedging effectiveness for model(2):

$$E^*_{t+1} - E^t_{t+1} = [\Delta b_t^2 \; \sigma_F^2 + \Delta b_t^2 \; \overline{\Delta F}^2 + 2a^* \; \Delta \overline{b_t} \; \Delta F]/[\sigma_S^2 + \overline{\Delta S}^2] \tag{24}$$

The implications of these models for the hedger of using minimum variance hedging effectiveness measures from period t+1 as an estimate of the actual effectiveness value for t+1 are obvious: if there is a large change in the hedge ratio or a large average change in the basis then the minimum variance effectiveness measure may contain a significant upward bias. Thus, unstable hedge ratios increase the basis risk of the hedge compared to the typical R^2 hedging effectiveness results.

Since the minimum variance $E^*_{t+1} = R_{t+1}^2$ values have been employed in most of the previous research to determine hedging effectiveness, and since unstable hedge ratios affect the more realistic E^t_{t+1} values, the empirical implications of the above result need to be examined. Specifically, to what extent do unstable hedge ratios affect the hedging effectiveness of the model? The next section explores this question.

DATA AND RESULTS

Data

This paper examines the effect of unstable hedge ratios on hedging effectiveness empirically by employing two series of T-Bonds, namely the Bellweather T-bond series and the 9% 2001 T-Bonds.[1] These cash bond data are employed in the analysis with the T-Bond futures contracts for the period 1/81 through 12/85. The bias in hedging effectiveness is determined by using quarterly periods which consist of weekly futures and cash price changes. Prices from the last trade of the week, typically Friday, are used to generate the weekly price changes. This data provides twenty quarters of data to generate the results in this paper.[2]

Results

Models (1) and (2) as represented by equations (23) and (24) respectively, are employed to determine the extent of the bias in the hedging effectiveness measures given unstable hedge ratios for T-bond futures hedges. Tables I and II present the results of these models.

TABLES I AND II ABOUT HERE

These tables show that the hedge ratios are unstable for these two series, with the 9% bond having a greater degree of instability. The Bellweather bond series has a significant degree of liquidity, which reduces the timing problems between the cash and futures end of week prices; this may help to explain the larger R^2 values for this series.

The biases shown by the two models typically are small, but several periods show biases for the R^2 values of over five percent. There are four periods for model (1) for the 9% bond that have biases of at least five percent, with the largest bias being over 11%. Model (2) has three periods with biases above four and one-half percent.

The Bellweather bond has four periods for model (1) that have biases of at least five percent, with the largest bias two biases being 12.7 and 13.0 percent. Model (2) has two periods with biases over five percent.

IMPLICATIONS AND CONCLUSIONS

This paper derives two models which determine the extent of the bias in the R^2 values when hedge ratios are unstable over time and the previous period's minimum variance hedge ratio is employed as the estimate of the current period's hedge ratio. Empirical results showing the size of this bias is then determined for T-bond futures hedges.

The importance and implications to the hedger of unstable hedge ratios and the resultant effect on hedging effectiveness is obvious, namely: the use of past data to forecast future hedge ratios and hedging effectiveness must be undertaken with greater care. Previous research implicitly assumed that the hedger possessed ex-post data to determine the hedging effectiveness, and hence whether a hedge should be employed and the resultant consequences of the proposed hedge position. Since the use of the previous period's hedge ratio as an estimate of this period's true hedge ratio when $b^*_t \neq b^*_{t+1}$ increases the basis variability of the hedge compared to the minimum variance results (i.e. the true hedging effectiveness is less than R^{*2}_{t+1}) the hedger may need to reevaluate the firm's analysis procedure for hedging.

FOOTNOTES

[1] The Bellweather bond series is the most recently issued bond series by the Treasury. This series has a significant degree of liquidity due to the large amount of trading by dealers. Moreover, these bonds are hedged in large quantities by the dealers. The Bellweather bond was chosen for its liquidity and near constant maturity. The 9% bond was chosen since its liquidity was marginal and it possessed a changing maturity, but had a constant coupon. The data was obtained from the Chicago Board of Trade statistical annuals, supplemented by The Wall Street Journal to check for errors.

[2] The quarterly periods for the two futures maturities end on the same calendar day, with the nearby contract ending at least four, but no more than nine, trading days prior to the futures expiration. Only the nearby results are presented here for space considerations; the first deferred results are almost identical to the nearby results. When the Bellweather bond series changes bonds during the quarter then the bond being removed from the series is sold on the nearest Friday and the new Bellweather bond is purchased on that day; consequently, the price changes employed are always between the same bond issue.

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TABLE I

HEDGING EFFECTIVENESS BIASES WITH UNSTABLE HEDGE RATIOS:

BELLWEATHER BOND

Year	Quarter	b^{*}_{t+1}	Δb_t	R^{*2}_{t+1}	Hedging Effectiveness Bias: Model (1)*	Hedging Effectiveness Bias: Model (2)**
1981	1	1.391		.990		
	2	1.508	.117	.990	.031	.007
	3	1.493	-.015	.988	.001	.002
	4	1.574	.081	.996	.016	.002
1982	1	1.490	-.084	.984	.016	-.002
	2	1.456	-.035	.994	.003	.005
	3	1.579	.124	.994	.038	.003
	4	1.319	-.261	.931	.127	.073
1983	1	1.184	-.135	.950	.027	.011
	2	1.132	-.052	.976	.003	.002
	3	1.175	.043	.970	.002	.000
	4	1.355	.181	.954	.060	.000
1984	1	1.395	.040	.990	.003	.000
	2	1.561	.165	.972	.069	.005
	3	1.285	-.276	.968	.130	.055
	4	1.288	.003	.964	.000	.000
1985	1	1.375	.087	.980	.014	.004
	2	1.246	-.129	.966	.027	.023
	3	1.203	-.043	.982	.003	.001
	4	1.111	-.092	.972	.011	.010

* Model (1): $E^{*2}_{t+1} - E^{2}_{t+1} = \Delta b_t^{2} \, [\sigma_F^{2}/\sigma_S^{2}]$

** Model (2): $E^{*2}_{t+1} - E^{2}_{t+1} = [\Delta b_t^{2} \, \sigma_F^{2} + \Delta b_t^{2} \, \Delta \bar{F}_{t+1}^{2} + 2a^{*}_{t+1}$

$$\Delta b_t \ \Delta F_{t+1}]/[\sigma_S^2 + \Delta S_{t+1}^2]$$

TABLE II

HEDGING EFFECTIVENESS BIASES WITH UNSTABLE HEDGE RATIOS:

9% BOND

Year	Quarter	b^*_{t+1}	Δb_t	R^{*2}_{t+1}	Hedging Effectiveness Bias: Model (1)*	Hedging Effectiveness Bias: Model (2)**
1981	1	.859		.931		
	2	.955	.096	.810	.010	.017
	3	.886	-.068	.734	.005	.001
	4	1.163	.276	.929	.111	.049
1982	1	.852	-.311	.745	.094	.099
	2	.859	.007	.835	.000	.000
	3	1.081	.222	.954	.060	.035
	4	.841	-.240	.733	.056	.070
1983	1	.842	.001	.974	.000	.000
	2	.736	-.105	.885	.007	.018
	3	.828	.091	.910	.006	.009
	4	.760	-.068	.899	.003	.006
1984	1	.796	.036	.884	.001	.001
	2	.847	.051	.856	.002	.001
	3	.862	.015	.931	.000	.001
	4	.930	.068	.927	.004	.004
1985	1	.881	-.048	.925	.002	.003
	2	.930	.048	.949	.002	.005
	3	.867	-.063	.933	.003	.005
	4	.626	-.240	.828	.027	.032

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* Model (1): $E^{*2}_{t+1} - E^2_{t+1} = \Delta b_t^2 \, [\sigma_F^2/\sigma_S^2]$

** Model (2): $E^{*2}_{t+1} - E^2_{t+1} = [\Delta b_t^2 \, \sigma_F^2 + \Delta b_t^2 \, \overline{\Delta F}_{t+1}^2 + 2a^*_{t+1} \, \Delta b_t \, \Delta F_{t+1}]/[\sigma_S^2 + \Delta S_{t+1}^2]$