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Valuation of Exchangeable Convertible Bonds

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Abstract

This paper provides a structural valuation model for exchangeable convertible bonds, since such bonds are widespread by now. The model is solved through the Hopscotch finite difference method. As the issuer owns the underlying shares, exchangeable convertibles may be called and the exchange option may be exercised even as the issuer experiences financial distress. The value of exchangeable convertibles always decreases in the volatility of the issuer's assets (unlike the value of ordinary convertibles) and decreases in the correlation between the underlying shares and the issuer's assets. The analysis confirms that the dominant motive for issuing exchangeable convertibles is likely to be to dispose of the underlying shares.

Keywords: *bond valuation, structural model, default risk, exchangeable convertible, Hopscotch finite difference method.*

JEL classification: G13; G33.

1 INTRODUCTION

Exchangeable convertible bonds differ from "ordinary" convertible bonds in that they are issued by a company (issuer) and can be exchanged for the shares of another company (entity). Instead "ordinary" convertible bonds can be exchanged for shares of the issuer. So those who invest in exchangeable convertibles bear the credit risk of the issuer and the equity risk of the entity, whereas those who invest in ordinary convertibles bear the credit risk and the equity risk of the issuer. Typically an exchangeable convertible is a bond that pays periodic coupons and is callable at preset prices. The bond can be exchanged for a set number of the entity's shares at bond maturity or when the bond is called or before maturity, if bondholders wish to do so.

In 2001 the global convertible bond market was worth 460 billion dollars and a significant fraction of such market was made up of exchangeable convertible bonds. By now the vast majority of new convertible issues in the UK are exchangeable (FSA CP 149). Between 1998 and 2001 about 112 billion Euro worth of exchangeable convertibles were issued in Europe alone and in October 2001 the Association of Convertible Bonds Management (hereafter ACBM) reported that about one third of the European convertible bond market was made up of exchangeable convertibles. In the US fourteen per cent of convertibles are exchangeable as reported by Grimwood and Hodges (2002).

This paper attempts to fill a gap in the literature. In fact past literature has devoted much attention to the theoretical valuation of ordinary convertibles through a firm value approach as in Ingersoll (1977), Brennan and Schwartz (1977 and 1980), Nyborg (1996), Anderson, Pan and Sundaresan (1997 and 2000), or through an equity value approach as in McConnel J. and Schwartz E. (1986), Loshak (1996), Davis and Lischka (1999) or Tsiveriotis and Fernandes (1998). But such literature did not concentrate on the valuation of exchangeable convertibles, probably because only in recent years the latter have become widespread. Similarly the literature has devoted much attention to the possible motivations for issuing ordinary convertible, but different may be the motivations for the specific use of exchangeable convertibles. Hence this paper shows how the valuation and use of exchangeable convertibles differ from the valuation and use of ordinary convertibles. In particular the valuation of exchangeable convertibles should reflect the observation that "the issuer normally has a long position in the underlying shares and is disposing of a substantial shareholding" (Financial Services Authority CP 149).

The model for valuing exchangeable convertibles is solved numerically through the Hopscotch finite difference method and the main conclusions of the present analysis highlight the specificity of "exchangeable convertibles" as follows. Firstly, even when the issuer approaches financial distress, the "exchange option" is often valuable and worth exercising. Distress may not prevent the issuer from calling the bond and force exercise of the exchange option. These features markedly differentiate exchangeable from ordinary convertibles. The conversion option of ordinary convertibles is often "out-of-the-money" or lost when the issuer is insolvent, whereas "the rights accruing on the exchange property after a default on the issuer" (ACBM 2001) are material in the valuation of exchangeable convertibles. In particular, since the issuer normally owns the shares underlying the exchange option, the issuer's default may not compromise investors' right to exchange the bond, especially if before or after default the underlying shares are pledged to investors as suggested by the ACBM[1].

Secondly, if the exchange option can be exercised at any time, early exercise allows exchangeable investors to obtain the shares before default very much in the spirit of Stulz and Johnson (1987), who showed that early exercise of an American vulnerable call option may be optimal even in the absence of dividends. So pledge of the underlying shares and unrestricted early exercise make exchangeable convertibles quite insensitive to the issuer's credit risk.

Thirdly, higher volatility of the issuer's assets decreases the value of an exchangeable convertible, whereas it often increases the value of an ordinary convertible by boosting the value of the ordinary conversion option. Moreover,

[1] The Association of Convertible Bonds Management suggested that "the exchange property should be pledged to the exchangeable investor upon default by the issuer"(2001).

the value of an exchangeable convertible generally decreases as the correlation between the underlying shares and the issuer's assets rises.

The present analysis confirms that the dominant motive for issuing exchangeable convertibles is likely to be the disposal of the underlying shares. Furthermore, if the call price is high the shares are more likely to be disposed of as investors exchange their bonds for shares while the issuer experiences financial distress. In such case exchange of the bonds for shares can avert the issuer's bankruptcy and significantly reduce expected bankruptcy costs.

The paper is organised as follows. The structural valuation model for exchangeable convertibles is next presented under the assumption that the issuer owns the shares. Then the exercise of the exchange option as the issuer experiences distress is analysed in detail. Then the motives for issuing exchangeable convertibles and the case in which the issuer does not own the underlying shares are examined. The conclusions follow.

2 THE VALUATION MODEL

This section presents a structural valuation model for exchangeable convertible bonds. The issuer's default risk is explicitly modelled. The usual assumptions underlying structural models of credit risk are made also here, in particular dynamic market completeness. The following are the main assumptions.

$E(V, S, t)$ denotes the value of an exchangeable, which depends on time t, on the value of the issuer's assets V and on the total value of the shares S for which the bond can be exchanged. S and V follow the risk neutral processes

$$dS = Srdt + S\sigma_s dw_s \tag{1}$$

$$dV = V\left(r - b\right)dt + V\sigma_v dw_v \tag{2}$$

where r is the default free short interest rate which is assumed constant over time, σ_s is the entity's shares volatility, b is the issuer's assets payout rate, σ_v is the issuer's assets volatility, dw_s and dw_v are the differentials of the Wiener processes respectively driving S and V, such that $dw_v \cdot dw_s = \rho dt$ in the mean square sense.

As stated above, the analysis of this paper hinges on the observation that the issuer normally owns and keeps the shares worth S. So the issuer's total assets are here assumed to be the sum of the shares underlying the exchange option worth S plus the remaining assets of the issuer worth V. This assumption will heavily affect the valuation of exchangeable convertibles.

The issuer defaults when it lacks the liquidity to honour coupon payment obligations in the spirit of Kim-Ramaswamy-Sundaresan (1993). Since S is assumed to pay no dividends and coupons payments are approximated as a continuous stream, default is triggered as soon as V drops to the barrier $V_d = \frac{c \cdot F + c_o \cdot F_o}{b}\left(1 - \tau\right)$, where τ is the corporate tax rate, c and F are respectively the coupon rate and face value of the exchangeable bond, c_o and F_o are respectively the coupon rate and face value of other debt outstanding in the issuer's capital structure. The default barrier is constant over time, even at $t = T$, the maturity date of the exchangeable. This assumption entails little loss in generality, simplifies the analysis and is suitable when F is small in comparison to F_o or when b is low. Default is followed by bankruptcy and the recovery value of the bond deprived of the exchange option is assumed to be $R(V_d, S_{t_d}) = \min(\max(V_d(1-a) + S_{t_d} - F_o, 0), F)$, where t_d is the date when default occurs, S_{t_d} is the shares value at default and a is the fraction of assets value V that is lost to bankruptcy costs after default.

A representative callable exchangeable convertible is now valued under the assumption that the underlying shares S pay no dividends. The valuation model applies to the "American" as well as to the "European" type exchangeable convertible, since early exercise will not be optimal before maturity. The convertible is callable for price P, where for simplicity P is assumed to be constant over time. Standard valuation arguments imply that the value of such an exchangeable $E(V,S,t)$ should satisfy the following partial differential equation:

$$\frac{\partial E(V,S,t)}{\partial t} + \frac{\partial^2 E(V,S,t)}{\partial V^2}\sigma_v^2 V^2 + \frac{\partial^2 E(V,S,t)}{\partial V \partial S}\rho\sigma_v V \sigma_s S + \frac{\partial^2 E(V,S,t)}{\partial S}\sigma_s^2 S^2 + \tag{3}$$

$$+\frac{\partial E(V,S,t)}{\partial V}(r-b)V + \frac{\partial E(V,S,t)}{\partial S} rS - rE(V,S,t) + c = 0$$

subject to

$$E\left(V\rightarrow\infty,S,t\right)=\frac{c}{r}\left(1-e^{-r(T-t)}\right)+Fe^{-r(T-t)}+O\left(S,t\right) \tag{4}$$

$$E\left(V_d,S,t_d\right)=\max\left(R\left(V_d,S_{t_d}\right),S_{t_d}\right) \tag{5}$$

$$E\left(V,S=P,t\right)=P \tag{6}$$

$$E\left(V,S\rightarrow 0,t\right)\rightarrow D\left(V,t\right) \tag{7}$$

$$E\left(V,S,t=T\right)=\max\left(S,F\right). \tag{8}$$

These boundary and terminal conditions are now commented in turn. Condition 4 states that the risk of the issuer's default vanishes as the assets V become very valuable. The expression $\frac{c}{r}\cdot\left(1-e^{-r(T-t)}\right)+F\cdot e^{-r(T-t)}$ is the default free value of the cash flows promised by the bond in the absence of the exchange option. $O\left(S,t\right)$ is the default free value of the exchange option, which is like callable call option with maturity T, which is called as soon as $S=P$ for a call price equal to $S-\frac{c}{r}\cdot\left(1-e^{-r(T-t)}\right)-F\cdot e^{-r(T-t)}$ and which pays the terminal payoff $\max\left(S-F,0\right)$ at time T. The exchange option $O\left(S,t\right)$ is independent of V as $V\rightarrow\infty$ because default risk vanishes and because V does not affect the call decision when the call price is $P\geq\frac{c}{r}\cdot\left(1-e^{-r(T-t)}\right)+F\cdot e^{-r(T-t)}$ as here assumed. In this case the exchangeable convertible is rationally called just when $S=P$. But if $P<\frac{c}{r}\cdot\left(1-e^{-r(T-t)}\right)+F\cdot e^{-r(T-t)}$ the exchangeable convertible may be called also when V rises sufficiently.

Condition 5 states that as $V=V_d$ the issuer defaults and exchangeable investors can still exercise the exchange option. At default the shares underlying the exchangeable are worth S_{t_d} and are owned by the issuer. So at default the issuer's total assets net of bankruptcy costs are worth $V_d(1-a)+S_{t_d}$ and the shares are still available for delivery if the exchange option is exercised at the time of default t_d. This is the case when for example the shares are pledged so that exchangeable investors are guaranteed to receive the shares whenever they exercise the exchange option. If the shares are pledged, it is possible that the exchange option be exercised also after default, but to confine the exercise to no later than the time of default seems a realistic approximation and implies the payoff $\max\left(S_{t_d}-R\left(V_d,S_{t_d}\right),0\right)$. $R\left(V_d,S_{t_d}\right)$ is again the recovery value of the exchangeable convertible in bankruptcy if the exchange option is not exercised. If the exchange option is lost upon default, condition 5 simply becomes $E\left(V_d,S,t_d\right)=R\left(V_d,S_{t_d}\right)$.

To gain insight, we can rewrite 5 as $E\left(V_d,S,t_d\right)=\max\left(\min\left(\max\left(V_d\left(1-a\right)+S_{t_d}-F_o,0\right),F\right),S_{t_d}\right)$, which implies that at default:

- if $S_{t_d}>F$, the exchange option is exercised and $E\left(V_d,S,t_d\right)=S_{t_d}$;

- if $F_o>V_d\left(1-a\right)$, the exchange option is exercised even if $S_{t_d}\leq F$ and again $E\left(V_d,S,t_d\right)=S_{t_d}$;

- if $F_o\leq V_d\left(1-a\right)$ and if $S_{t_d}\leq F$, the exchange option is not exercised and investors receive the minimum between F and the bankruptcy proceeds of $V_d+S_{t_d}$ net of bankruptcy costs and of the payment of the face value of senior creditors F_o, so that $E\left(V_d,S,t_d\right)=F-\max\left(F-V_d\left(1-a\right)+F_o-S_{t_d},0\right)$. When $V_d\left(1-a\right)\geq F_o$, exchangeable holders may receive the full value of the underlying shares even if they do not exercise the exchange option. When $F_o=0$, the exchangeable is the only outstanding debt and exchangeable holders will never exercise the exchange option if $S_{t_d} < F$.

Condition 6 states that, as the entity's share value rises to the call price P, the exchangeable is called and exercise of the exchange option is forced in the spirit of Ingersoll (1977). As mentioned above, forced exchange requires that $P \geq \frac{c}{r} \cdot \left(1 - e^{-r(T-t)}\right) + F \cdot e^{-r(T-t)}$.

If the exchangeable is were not callable, the third condition would be $E\left(V, S \rightarrow \infty, t\right) \rightarrow S$. This is now explained. In the absence of the call provision, the probability of the issuer's default would not vanish as $S \rightarrow \infty$, because the default barrier V_d is a fraction of the face value of outstanding $(F + F_o)$. This can be the case because default is triggered by lack of liquidity and/or because, as $S \rightarrow \infty$, $E\left(V, S \rightarrow \infty, t\right) \rightarrow \infty$ so that the default is now triggered when V drops to a fraction of F_o. So, even as $S \rightarrow \infty$ the probability of default is positive. But, if the issuer owns the shares, even upon default can investors exercise the exchange option and get $S_{t_d} \rightarrow \infty$ instead of $R\left(V_d, S_{t_d}\right)$.

Condition 7 states that, as the shares value vanishes, the conversion option becomes worthless and $E\left(V, S, t\right)$ approaches $D\left(V, t\right)$, i.e. the value of the bond deprived of the exchange option and whose collateral is only V.

Condition 8 states that, if the issuer neither has called the exchangeable nor has previously defaulted, at maturity T convertible holders can exchange the face value F of the bond for shares worth S.

The conditions just exposed imply that the value of the exchangeable convertible can also be viewed as the sum of a "straight" debt plus an exchange option as shown in Appendix A.

If, as is here assumed, the exchange option can be exercised at the issuer's default and the shares pay no dividends, early exercise of the "American" callable exchangeable convertible is not optimal if the bond is not called or if it does not experience default and the present valuation model applies also to the "American" case.

2.1 Comparative statics and the effect of correlation

Comparative statics using the above model reveal the characteristics of exchangeable convertibles. Equation 3 has been solved through the "line" Hopscotch method proposed by Gourlay and McKee (1977). Appendix B describes the details of the application of the "line" Hopscotch method to solve equation 3 subject to its boundary and terminal conditions.

The base case scenario assumes $V_d = 1$, $a = 0.2$, $\sigma_v = 0.2$, $b = 0.05$, $r = 0.04$, $F = 1$, $c = 0.03$, $T - t = 5$, $S = 1$, $\sigma_s = 0.3$, $F_o = 1$, $\rho = 0$, $P = 1.5$, $c_o = 0.047$, $\tau = 0.35$ and the corresponding results are portrayed in Figure 1. Figure 1 shows that the value of the exchangeable convertible rises in the shares value S, but when the shares value reaches the call price level $P = 1.5$ the bond is called by the issuer. As the value of the issuer's assets V drops to $V_d = 1$, default takes place and investors decide to exercise the exchange option to receive the shares value S_{t_d}. In the base case investors would get even less if they did not exercise the exchange option, because $F_o > V_d(1 - a)$. It follows that the value of the defaulted exchangeable convertible rises in the shares value and the value of the non-defaulted exchangeable convertible also rises in the value of the issuer's assets V, because default risk decreases.

The value of the exchangeable convertible decreases in assets volatility σ_v especially when the shares value is low. This is highlighted by the more pronounced concavity of the function $E(V,S,t)$ with respect to V when S is low. The reason is that as assets volatility increases, the probability of default increases and the payoff to the defaulted exchangeable increases in the shares value. So, as the shares value lowers, the loss given default on the exchangeable convertible is higher and the bond becomes more sensitive to changes in the probability of default and hence to changes in assets volatility σ_v. It follows that *higher volatility of the issuer's assets always decreases the value of the exchangeable convertible*, as described in Figure 2. This is a peculiarity of exchangeable convertibles as opposed to ordinary convertibles. In fact the value of the latter often increases in the issuer's assets volatility, since assets volatility increases the value of the ordinary conversion option.

Figure 3 shows that the value of the exchangeable convertible increases in shares volatility σ_s, as is suggested by the convexity of the function $E(V,S,t)$ with respect to S. Figure 1 shows that such convexity is more accentuated when the issuer's assets value is high and much less accentuated when the assets value is next to the default barrier V_d. The reason is that when V is high the default probability is negligible and a rise in shares volatility increases the value of the exchange option. But when V is low the probability of default is high and upon default the payoff from exercising the exchange option is linear rather than convex in the shares value S (in the base case).

Another peculiarity in the valuation of exchangeable convertibles is that a rise in the correlation between the value of the shares S and the value of the issuer's assets V decreases the value of the exchangeable: $E(V, S, t)$ decreases in ρ as portrayed both in Figures 4 and 5. This effect is most pronounced when the assets V approach the default barrier V_d. Figure 1 suggests the explanation for this since it shows that the value of the exchangeable convertible rises in the shares value, i.e. $\frac{\partial E(V,S,t)}{\partial S} > 0$, and that $\frac{\partial E(V,S,t)}{\partial S}$ decreases as V rises, i.e. $\frac{\partial^2 E(V,S,t)}{\partial V \partial S} < 0$. Equation 3 implies that, if $\frac{\partial^2 E(V,S,t)}{\partial V \partial S} < 0$, an increase in the variable coefficient term of the cross derivative $(\rho\sigma_v V \sigma_s S)$ reduces the value of the function $E(V, S, t)$.

Such result also has a more intuitive explanation: if ρ increases, high (low) equity values S are more likely to be associated with high (low) values of the issuer's assets V. Loosely speaking, this means that the "high S-high V" and "low S-low V" corner regions in Figure 1 become more likely while the "low S-high V" and "high S-low V" corner regions become less likely. But the the values of the exchangeable convertible in the "high S-high V" corner region are just slightly higher than in the "high S-low V" corner region, while the values of the exchangeable convertible in the "low S-low V" corner region are much lower than in the the "low S-high V" corner region. Hence as an increase in ρ makes the "high S-high V" and "low S-low V" corner regions more likely and the other two corner regions less likely, the value of the exchangeable convertible must decrease. The exchangeable is more sensitive to ρ when V is low and when equity value S is midway between P and 0. In fact, as S is further from P, the "call" probability decreases and the expected life of the exchangeable lengthens. The longer the expected life of the exchangeable, the more sensitive the exchangeable is to changes in ρ.

Figure 6 shows how reducing the call price P increases the value of the the issuer's call option and reduces the value of the exchangeable, even if a lower call price reduces the probability of default on the exchangeable convertible by increasing the "call" probability.

Increasing the default free interest rate r increases the risk neutral drifts of the V and S processes, so as to reduce the risk neutral probability of default and to increase the risk neutral probability that the issuer will exercise its call before maturity T. On the other hand higher r reduces the present value of the cash flows promised to exchangeable investors (payments of coupons and principal). The net effect is that the value of an exchangeable usually decrease in r, unless S and V are very low. When V and S are very low, an increase in r has the dominant effect of reducing the probability of default and of increasing the expected value of S_{t_d}, i.e. the expected payoff upon default. This fact is relevant for interest rate immunisation strategies involving exchangeable convertibles.

Increasing the assets payout rate b decreases the value of the exchangeable convertible by increasing the probability of default. Increasing proportional bankruptcy costs a and the nominal amount of the issuer's other debt may increase the loss given default and so decrease the value of the exchangeable.

The effect of maturity is mixed. Figure 7 shows that, ceteris paribus, shorter time to maturity increases (decreases) the value of the exchangeable convertible when S and V are low (high). If S and V are low, the exchange option is of little value and convertible value is below par, but as time to maturity decreases the convertible value increases because it is "pulled-to-par". If S is high, the convertible trades above par because the exchange option is valuable, but shorter maturity decreases the value of the convertible by reducing the value of the exchange option.

After exploring the comparative statics, it is useful to contrast exchangeable convertibles with ordinary convertibles and secured debt.

2.2 The difference with secured debt and with ordinary convertibles

Exchangeable convertibles share some commonalities with secured debt as well as with ordinary convertibles. Exchangeable convertibles are similar to secured debt as the issuer holds the underlying shares. Such shares constitute some sort of collateral, in particular when they are pledged either at bond issuance or as the issuer defaults. But shares often are a more risky than the collateral usually securing a firm's debt, such as real estates of sovereign bonds. Holders of secured debt can get hold of the collateral just in case of default, whereas holders of exchangeable convertibles can get hold of the shares also upon spontaneous exercise of the exchange option or upon forced exercise as the bonds are called. In bankruptcy holders of secured debt can be satisfied also through liquidation of the debtor's assets other than collateral, should the collateral not be valuable enough, whereas holders of exchangeable convertibles do not have any claim on the debtor's assets after they have decided to exercise the exchange option.

Exchangeable convertibles are also similar to ordinary convertibles. Both ordinary conversion options and exchange options are subject to equity risk. Equity risk is perfectly correlated with the default risk of the issuer of an ordinary convertible, but just partially correlated (if at all) to the default risk of the issuer of an exchangeable convertible. The exchange option may become virtually worthless even while the issuer remains perfectly solvent or the exchange option may become very valuable even while the issuer approaches bankruptcy. So when the convertible issuer experiences distress ordinary conversion options are usually "out-of-the-money" while exchange options can often be "in-the-money" , especially in case of low correlation between the shares value S and the value of the remaining assets of the issuer V. Moreover, unlike ordinary conversion options, exchange options may often be exercised even when the issuer approaches default, especially if investors expect to lose the exchange right during the reorganisation process that follows default. Next the effect of default risk on exchangeable convertibles and differences with ordinary convertibles are explored in further depth.

3 EXCHANGEABLE CONVERTIBLES AND DEFAULT RISK

This section focuses on how the value of an exchangeable convertible depends on default risk and on contractual provisions such as a pledge on the underlying shares, early exercise of the exchange option or call for early redemption.

3.1 Default, pledge on shares and early exercise of the exchange option

The exchange option may be lost upon default if the issuer does not own the underlying shares or if the issuer owns the shares but the shares are not pledged to exchangeable investors, so that in bankruptcy all creditors would have a claim on the shares. In this regard the ACBM suggested that "the exchange property should be pledged to the exchangeable investor upon default by the issuer" (2001). Such pledge would prevent the loss of the exchange option, because it guarantees exchangeable investors that they will be able to exercise their exchange option even at or after the issuer's default. If the issuer owns the shares and the shares are pledged for conversion, the issuer will honour the obligation to deliver the shares even if insolvent. In such case and if the shares pay no dividends, the exchange option would not be exercised before default, assuming default precedes maturity or early redemption.

The presence of a pledge on the shares increases the value of an exchangeable convertible especially when the exchange option is European or early exercise is restricted. Instead if the event of default is predictable as implied by the structural model presented above, if the exchange option is of American type and if it is "in-the-money", investors can exchange their bonds for shares before default, thus avoiding the risk of losing the exchange option upon default as the shares are not pledged. In this case, investors may prefer early exercise of the exchange option to a claim in bankruptcy, even in the absence of dividends paid by the shares. This result is similar to the one in Stulz and Johnson (1987): early exercise of a vulnerable call option eliminates the risk of losing the option upon the issuer's default.

So even if the shares are not pledged, the issuer's default risk may be mitigated by early exercise, if the exchange option is American. But such early exercise implies that the default free value of the exchangeable convertible is still higher than the value of a default prone exchangeable convertible, since the optimal exercise policy of the former implies no spontaneous early exercise before maturity. In fact, if the issuer has no default risk and if the shares pay no dividends, there is no incentive for investors to spontaneously exercise the exchange option before maturity, moreover by so doing investors would forego coupon payments. Finally, if the underlying shares pay dividends, the value of the exchangeable decreases and spontaneous early exercise of the exchange option may be optimal even for a default free exchangeable convertible.

3.2 Default and call of the exchangeable convertible

Whereas ordinary convertibles are called just when the issuer's equity value rises sufficiently, exchangeable convertibles may be called also when the issuer approaches distress. In fact, even if the issuer's assets value approaches the default barrier, the shares value may have risen sufficiently to enable the issuer to call the exchangeable and force exercise of the exchange option. Even if the issuer is on the brink of default, it can still honour its obligation of delivering the underlying shares in its possession.

As ordinary callable convertibles may be called and early conversion forced before adverse fortunes lead the debtor to default, the same could be said of exchangeable callable convertibles. Indeed for the latter this is a more likely prospect especially when the correlation ρ between the underlying equity and the issuer's assets is low. It follows that the call feature may significantly reduce the exposure of exchangeable convertibles to default risk.

It is interesting that forced as well as non-forced exercise of the exchange option decreases leverage and the default barrier. For example, as default is liquidity triggered and as the shares pay no dividends, the default barrier drops from $V_d = \frac{c \cdot F + c_o \cdot F_o}{b}(1-\tau)$ before option exercise to $V_d = \frac{c_o \cdot F_o}{b}(1-\tau)$ after option exercise. So even if the issuer is about to default, forced or spontaneous exercise of the exchange option may make the issuer return to be fully solvent, reduce expected bankruptcy costs and increases total firm value.

4 MOTIVES FOR ISSUING EXCHANGEABLE CONVERTIBLES

The above analysis can suggest the possible motives underpinning the decision to issue exchangeable convertibles. Brennan and Schwartz (1988) suggested that ordinary convertibles may be issued when investors are particularly uncertain about the issuer's assets volatility, since ordinary convertibles are less sensitive to assets volatility than "straight" bonds are. But this motive seems less capable to explain the issuance of exchangeable convertibles, whose value always decreases in the issuer's assets volatility.

Alternatively it is held that the issuer of exchangeable convertibles intends to dispose of the underlying shares. Before maturity disposal of the shares can take place as investors exchange their bonds for the underlying shares, either when the bonds are called or when the issuer experiences distress.

A low (high) call price entails a high (low) probability that the issuer can force exercise of the exchange option and a reduced (increased) probability of exercise as the issuer experiences distress. When the issuer experiences distress, the exercise of an exchange option seems more likely than the exercise of the conversion option of an ordinary convertible, especially in case of low correlation between the value of the shares underlying the exchangeable and the value of issuer's other assets. "Exchange in distress", as seen above, can prevent or eliminate the insolvency of the debtor, thus reducing the probability of bankruptcy and hence expected bankruptcy costs.

To corroborate the thesis that the issuer's motive for issuing an exchangeable convertible is disposal of the underlying shares, it is now instructive to consider the alternative case in which the issuer does not own the underlying shares.

4.1 When the issuer does not own the shares

A key assumption in the preceding analysis has been that the issuer owns the underlying shares. Such assumption is now dropped. Before proceeding, it is necessary to introduce the issuer's equity value when, ceteris paribus, the exchangeable is absent from the issuer's capital structure. Such equity value can be assumed to be a time independent function of V as in Leland (1994) and is denoted as $E_q(V) = T(V) + V - B_k(V) - D(V, F_o)$, where $T(V)$ is the present value of the debt induced tax shield, $B_k(V)$ are expected bankruptcy costs, $D(V, F_o)$ is the value of debt (with face value F_o). The formula for $E_q(V)$ is reported in Appendix C. To simplify the exposition, we now consider the case of a European type non-callable exchangeable convertible $E'(V, S, t)$. Then, since assets worth V are now the only assets of the issuer, the liquidity default barrier for $t < T$ is still V_d, but at T default is triggered if $E_q(V) \leq \max(S, F)$, i.e. if equity value after honouring the commitment to pay $\max(S, F)$ at maturity T is not greater than the commitment itself. This condition implies that the issuer cannot liquidate its assets in order to be able to pay $\max(S, F)$ at T. $E'(V, S, t)$ still satisfies equation 3 and conditions 4 and 7, but conditions 5, 6 and 8 are now respectively substituted by

$$E'\left(V_d, S_{t_d}, t_d\right) = \min\left(R\left(V_d\right), \max\left(F, S_{t_d}\right)\right) \tag{9}$$

$$E'\left(V, S \to \infty, t\right) \to E\left(V, t\right) \tag{10}$$

$$\begin{aligned} &E'\left(V, S, t = T\right) = 1_{E_q(V) > \max(S,F)} \cdot \max\left(S, F\right) + \\ &+ 1_{E_q(V) \leq \max(S,F) = S} \cdot \max\left(F \cdot 1_{E_q(V) > F}, R\left(V\right)\right) + \\ &+ 1_{E_q(V) \leq \max(S,F) = F} \cdot \max\left(S \cdot 1_{E_q(V) > S}, R\left(V\right)\right), \end{aligned} \tag{11}$$

where $R\left(V\right) = \max\left(V\left(1-a\right) - F_o, 0\right)$, where 1_A is the indicator function of event A and where $E\left(V, t\right)$ satisfies

$$\frac{\partial E\left(V,t\right)}{\partial t} + \frac{\partial^2 E\left(V,t\right)}{\partial V^2}\sigma_v^2 V^2 + \frac{\partial E\left(V,t\right)}{\partial V}\left(r-b\right)V - rE\left(V,t\right) + c = 0$$

subject to

$$E\left(V \to \infty, t\right) \to \frac{c}{r}\left(1 - e^{-r(T-t)}\right) + Fe^{-r(T-t)} \tag{12}$$

$$E\left(V_d, t_d\right) = R\left(V_d\right) \tag{13}$$

$$E\left(V, t = T\right) = \max\left(F \cdot 1_{E_q(V) > F}, R\left(V\right)\right). \tag{14}$$

Condition 9 implies that the insolvent issuer may be unable to deliver the shares underlying the exchange option if the issuer does not own them and the exchange option is exercised. In such case the exchange option resembles the type of vulnerable options studied by Stulz and Johnson (1987). It may be optimal to exercise vulnerable options before maturity even if the underlying shares pay no dividends, when early exercise in unrestricted[2].

Condition 10 states that the bond value tends to $E(V,t)$ as $S \to \infty$. $E(V,t)$ is the value of a bond similar to $E(V,S,t)$ but that is deprived of the exchange option and that gives investors the option to receive the greater between the recovery value $R(V)$ and the face value F, when $E_q(V) > F$ at T. The reason is that as $S \to \infty$ and investors exercise the exchange option at T, the issuer will be insolvent and investors will have the right to receive the recovery value $R(V)$ at T. But investors will choose not to exercise the exchange option and to keep the issuer solvent if $E_q(V) > F$ and $F > R(V)$ at T.

The terminal condition 11 states the following. If the issuer is solvent at T, i.e. if $E_q(V) > \max(S,F)$, investors receive the payoff $\max(S,F)$. If the issuer is not solvent and the exchange option is "in-the-money", i.e. $E_q(V) \leq \max(S,F) = S$ investors exercise the exchange option and cause the issuer to default, thus getting the recovery value $R(V)$. But they will do so just if exercising pays them more than not exercising the exchange option, as by not exercising they can keep the issuer solvent and thus receive F. If the exchange option is "out–of-the-money" and the issuer is insolvent, i.e. $E_q(V) \leq \max(S,F) = F$, investors receive the recovery value $R(V)$, unless by exercising the exchange option they can keep the issuer solvent and so receive

[2] Moreover the value of vulnerable options can decrease with maturity, because the default probability increases with maturity.

$S > R(V)$, i.e. unless $F \geq E_q(V) > S > R(V)$. *So condition 11 implies that the exchange option may not be exercised even it is nominally "in-the-money" or that it may be exercised even if it is nominally "out-the-money", since the exercise decision can affect the state of solvency of the issuer when the issuer does not own the shares.*

Condition 11 highlights other differences between ordinary and exchangeable convertibles. Whenever $V > F$ at maturity T, ordinary convertibles will not experience default. Instead exchangeable convertibles may experience default at T even when $V > F$ if the issuer does not own the shares and if the exchange option is "in the money". Even if $V > F$ at maturity T, there is no guarantee that the issuer will able to honour its obligation to deliver the shares, which it would have to buy in the market as the exchange option is exercised. So, even if $V > F$ at maturity T, the issuer may default if $E_q(V) < S > F$.

We can conclude that, when the exchangeable convertible issuer does not own the underlying shares, the issuer's default probability increases, the expected payoffs to the exchange option at default and at maturity decrease and hence the value of the exchangeable convertible decreases. These considerations discourage the issuance of exchangeable convertibles when the issuer does not own the shares and support the widely held view that the issuer chooses to offer an exchangeable rather than an ordinary convertible precisely in order to dispose of the underlying shares in his possession.

5 CONCLUSIONS

This paper has for the first time studied the valuation and use of exchangeable convertible bonds through a structural credit risk model. The following distinctive characteristics of exchangeable convertibles have emerged.

Unlike the conversion option of ordinary convertibles, the "exchange option" is often valuable and worth exercising even when the issuer experiences financial distress. Since the issuer normally owns the shares underlying the exchange option, the issuer's default may not compromise investors' exchange right, especially if the shares are pledged as suggested by the ACBM (2001). The exchange option is particularly valuable if exchangeable investors are paid little in bankruptcy, either because of high bankruptcy costs or because the exchangeable is a subordinated bond. If the exchange option can be exercised at any time, early exercise allows investors to obtain the underlying shares before default and so makes the value of an exchangeable relatively insensitive to default risk even if the underlying shares are not pledged and even if the exchange option were lost upon default. Overall, pledge of shares and unrestricted early exercise are shown to make the exchangeable quite insensitive to the issuer's default risk.

Unlike ordinary convertibles, exchangeable convertibles may be called also when the issuer approaches distress. Forced as well as voluntary exercise of the exchange option can stave off bankruptcy.

Unlike the value of ordinary convertibles, the value of exchangeable convertibles always decreases as the volatility of the issuer's assets increases and also as the correlation between the underlying shares and the issuer's assets rises.

When the issuer does not own the shares underlying the exchange option, investors may not exercise the option even if the option is nominally "in-the-money" or may exercise the option even if the option is nominally "out-the-money", since the exercise decision can affect the state of solvency of the issuer.

Finally, the present analysis suggests that the motives for issuing exchangeable convertibles may differ from the motives for issuing ordinary convertibles. The analysis confirms that the main motive for the firm to issue exchangeable convertibles is likely to be to dispose of the underlying shares. When the call price of the exchangeable is high, disposal of the shares is likely to take place as and when the issuer approached distress and so it can reduce expected bankruptcy costs and increase total firm value.

A The exchangeable convertible as a "straight" debt plus a callable exchange option

The value of exchangeable convertibles can be decomposed as the sum of "straight" debt plus a callable exchange option, such that:

$E(V,S,t) = D(V,S,t) + O(V,S,t)$, where $D(V,S,t)$ is the value of "straight" debt and $O(V,S,t)$ is the value of the callable exchange option. Under the same assumptions as in section 2 above, $D(V,S,t)$ will satisfy equation 3, but subject to:

$$D(V \to \infty, S, t) = \frac{c}{r}\left(1 - e^{-r(T-t)}\right) + Fe^{-r(T-t)} \tag{15}$$

$$D(V_d, S, t_d) = R(V_d, S_{t_d}) \tag{16}$$

$$D(V, S \to \infty, t) = D(V, t, R(V_d, S_{t_d}) = F) \tag{17}$$

$$D(V, S \to 0, t) = D(V, t) \tag{18}$$

$$D(V, t) = F. \tag{19}$$

where $D(V, t)$ is the value of "straight" debt when the issuer does not own S and where $D(V, t, R(V_d, S_{t_d}) = F)$ is the value of "straight" debt when its recovery value in bankruptcy is $R(V_d, S_{t_d}) = F$. Also $O(V, S, t)$ will satisfy equation 3, but subject to:

$$O(V \to \infty, S, t) = O(S, t) \tag{20}$$

$$O(V_d, S, t_d) = \max(S - R(V_d, S_{t_d}), 0) \tag{21}$$

$$O(V, S = P, t) = \max(P - D(V, S, t), 0) \tag{22}$$

$$O(V, S \to 0, t) \to 0 \tag{23}$$

$$O(V, t) = \max(S - F, 0). \tag{24}$$

where $O(S, t)$ is a default free callable exchange option as described in section 2. The callable exchange option is referred to throughout the text simply as "exchange option".

B The line Hopscotch method

Partial differential equation 3 was solved using and adapting the "line" Hopscotch method of Gourlay and McKee (1977) as follows. Assume that i, j and k are integer numbers. Let $E_{i,j}^{k}$ denote the approximation of the exact solution to the PDE [i.e.$E\left(V,S,t\right)$] when $V=i\cdot dV$, $S=j\cdot dS$, $t=T-kdt$. In other words

$$E_{i,j}^{k}\approxeq E\left(idV,jdS,T-kdt\right) \tag{25}$$

An "interior" grid point $\left(idV,jdS,T-kdt\right)$ of the feasible region is a point such that $i_{\min}<i<I$, $0<j<J$, $0<k<K$. Here $i_{\min}$ is such that $dV\cdot i_{\min}=V_{d}$, I is the grid's upper limit to i, J is the grid's upper limit to j and K is such that $t_{0}=T-Kdt.$, where t_{0} is the present time. Then equation 3 can be approximated as follows:

- first at those "interior" grid points for which $k+j$ is even by the explicit scheme

$$\frac{E_{i,j}^{k+1}-E_{i,j}^{k}}{dt}=\frac{1}{2}\left(jdS\sigma_s\right)^2\frac{E_{i,j+1}^{k}-2E_{i,j}^{k}+E_{i,j-1}^{k}}{\left(dS\right)^2}+\frac{1}{2}\left(idV\sigma_v\right)^2\frac{E_{i+1,j}^{k}-2E_{i,j}^{k}+E_{i-1,j}^{k}}{\left(dV\right)^2}+ \tag{26}$$

$$+\left(jdS\sigma_s\right)\left(idV\sigma_v\right)\rho\frac{E_{i+1,j+1}^{k}-E_{i-1,j+1}^{k}-E_{i+1,j-1}^{k}+E_{i-1,j-1}^{k}}{4dVdS}+ \tag{27}$$

$$+\left(r-b\right)idV\frac{E_{i+1,j}^{k}-E_{i-1,j}^{k}}{2dV}+rjdS\frac{E_{i,j+1}^{k}-E_{i,j-1}^{k}}{2dS}-rE_{i,j}^{k}; \tag{28}$$

- then at those "interior" grid points for which $k+j$ is odd by the implicit scheme

$$\frac{E_{i,j}^{k+1}-E_{i,j}^{k}}{dt}=\frac{1}{2}\left(jdS\sigma_s\right)^2\frac{E_{i,j+1}^{k+1}-2E_{i,j}^{k+1}+E_{i,j-1}^{k+1}}{\left(dS\right)^2}+\frac{1}{2}\left(idV\sigma_v\right)^2\frac{E_{i+1,j}^{k+1}-2E_{i,j}^{k+1}+E_{i-1,j}^{k+1}}{\left(dV\right)^2}+ \tag{29}$$

$$\begin{aligned}&+\left(jdS\sigma_s\right)\left(idV\sigma_v\right)\rho\frac{E_{i+1,j+1}^{k+1}-E_{i-1,j+1}^{k+1}-E_{i+1,j-1}^{k+1}+E_{i-1,j-1}^{k+1}}{4dVdS}+\\&+\left(r-b\right)idV\frac{E_{i+1,j}^{k+1}-E_{i-1,j}^{k+1}}{2dV}+rjdS\frac{E_{i,j+1}^{k+1}-E_{i,j-1}^{k+1}}{2dS}-rE_{i,j}^{k+1}.\end{aligned} \tag{30}$$

The implicit scheme can be quickly solved through a successive over-relaxation (SOR) algorithm. The boundary conditions to this Hopscotch scheme can be conveniently approximated as follows:

$E^{k+1}_{I,j} = 2E^{k+1}_{I-1,j} - E^{k+1}_{I-2,j}$;

$E^{k}_{i_{\min},j} = R_{i_{\min},j}$;

$E^{k+1}_{i,J} = 2E^{k}_{i,J-1} - E^{k}_{i,J-2}$;

$E^{k+1}_{i,0} = 2E^{k}_{i,1} - E^{k}_{i,2}$.

$R_{i_{\min},j}$ approximates the bond recovery value, $R\left(V_d, S_{t_d}\right)$, so that

$$R_{i_{\min},j} = \min\left(\max\left(dV \cdot i_{\min}\left(1-a\right) + j \cdot dS - F_o, 0\right), F\right) \approxeq R\left(V_d, S_{t_d}\right).$$

To ensure accuracy and stability, the simulation results displayed in the Figures were derived using the following grid parameters: $dV = \frac{5 \cdot V_d}{I}$, $dS = \frac{2}{J}$, $I = 50$, $J = 50$, $i_{\min} = \frac{V_d}{dV}$, $dt = \frac{1}{300}$, $Kdt = T - t_0$.

C Equity value when the issuer does not own the shares

Following Leland (1994), if the issuer does not own the shares underlying the exchangeable convertible, if outstanding debt with face value F_o is a perpetuity, if default is liquidity triggered, if the exchangeable convertible is absent or has been removed from the issuer's capital structure, the issuer's equity value is:

$E_q\left(V\right) = T\left(V\right) + V - B_k\left(V\right) - D\left(V, F_o\right)$,

where

$T\left(V\right) = \tau \frac{c_o \cdot F_o}{r}\left(1 - \left(\frac{V}{V_d}\right)^q\right)$,

$$B_k(V) = aV_d\left(\frac{V}{V_d}\right)^q,$$

$$D(V,F_o) = \frac{c_o \cdot F_o}{r} + \left(-\frac{c_o \cdot F_o}{r} + \min\left(F_o, V_d\left(1-a\right)\right)\right)\left(\frac{V}{V_d}\right)^q,$$

$$q = \frac{-\left(r-b-\frac{1}{2}s_v^2\right)-\sqrt{\left(r-b-\frac{1}{2}s_v^2\right)^2+2rs_v^2}}{s_v^2},$$

$$V_d = \frac{c_o \cdot F_o}{b}\left(1-\tau\right).$$

References

[1] ACBM (Association of Convertible Bond Management), Oct 2001, ”Recommendation for the standardisation of convertible issuance”.

[2] Amman M., Kind A., Wilde C., 2003, "Are convertible bonds underpriced? An analysis of the French market", Journal of Banking and Finance 27, 635-653.

[3] Anderson R., Pan Y. and Sundaresan S., 1997, Corporate Bond Yield Spreads and the Term Structure, Columbia PaineWebber Working Paper Series in Money, Economics and Finance: PW/97/15 December 1997, 45.

[4] Anderson R., Pan Y. and Sundaresan S., 2000, Corporate Bond Yield Spreads and the Term Structure, Finance vol. 21, n.2, 15-37.

[5] Black F. and Scholes M., 1973, ”The pricing of options and corporate liabilities”, Journal of political economy 637-659.

[6] Brennan M. and Schwartz E., 1977, ”Convertible bonds: valuation and optimal strategies for call and conversion”, Journal of finance 32, n.5, 1699-1715”.

[7] Brennan M. and Schwartz E., 1980, "Analysing convertible bonds", Journal of financial and quantitative analysis 15, n.4, 907-929.

[8] Brennan M. and Schwartz E., 1988, "The case for convertibles", Journal of applied corporate finance n.1, 55-64.

[9] Davis and Lischka, 1999, "Convertible bonds with market risk and credit risk", Working paper Tokyo Mitsubishi plc.

[10] Essig S., 1991, "Convertible securities and capital structure determinants", PHD dissertation (Grad school of Business, Univ. of Chicago).

[11] Fan H. and Sundaresan S., 2000, "Debt valuation, renegotiation and optimal dividend policy", Review of financial studies 13, n.4, 1057-1099.

[12] Flannery M., 2002, "No pain no gain? Effecting market discipline via "reverse convertible debentures"", Working paper University University of Florida.

[13] FSA Consultation Paper 149, July 2002, "Market abuse: pre-hedging convertible and exchangeable bond issues".

[14] Grimwood R. and Hodges S., 2002, "The valuation of convertible bonds: a study of alternative pricing models", 2002, Warwick Business School Working paper, FORC Pre-Print 02/121.

[15] Gourlay AR. and McKee S., 1977, "The construction of Hopscotch methods for parabolic and elliptic equations in two space dimansions and mixed derivative", Journal of computational and applied mathematics 3, 201-206.

[16] Ingersoll J.E., 1977, "A contingent claims valuation of convertible securities", Journal of financial economics 4, n.2, 289-322.

[17] Johnson H. and Stulz R., 1987, The pricing of options under default risk, Journal of Finance 42, 267-280.

[18] Kim J., Ramaswamy K. and Sundaresan S., 1993, "Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model", Financial Management 117-131.

[19] Kurpiel A. and Roncalli T., 1999, "Hopscotch methods for two state financial models", Working paper of City University Business School.

[20] Leland H., 1994a, "Corporate debt value, bond covenants and optimal capital structure", Journal of finance 49, n.4, 1213-1252.

[21] Lewis, Rogalski and Seward, 1998, "Understanding the design of convertible debt" Journal of Applied Corporate Finance 11, 45-53.

[22] Loshak, 1996, "The valuation of defaultable convertible bonds under stochastic interest rate", Krannert graduate school of management, Purdue University.

[23] McConnel J and Schwartz E., 1986, "LYON taming", Journal of Finance 42, n.3, 561-576.

[24] Nyborg K.G., 1996, "The use and pricing of convertible bonds", Applied mathematical finance 3, 167-190.

[25] Safaty N. (2001) "The \$460bn global convertible bond market", Fixed income market review.

[26] Stump P.M., Ziets K., Marshella T., Rowan M. "Critical issues in evaluating the creditworthiness of convertible debt securities", 2001, "Moody's Investors Service" special comment.

[27] Tsiveriotis and Fernandes, 1998, "Valuing convertible bonds with credit risk", Journal of Fixed Income 8, 95-102.

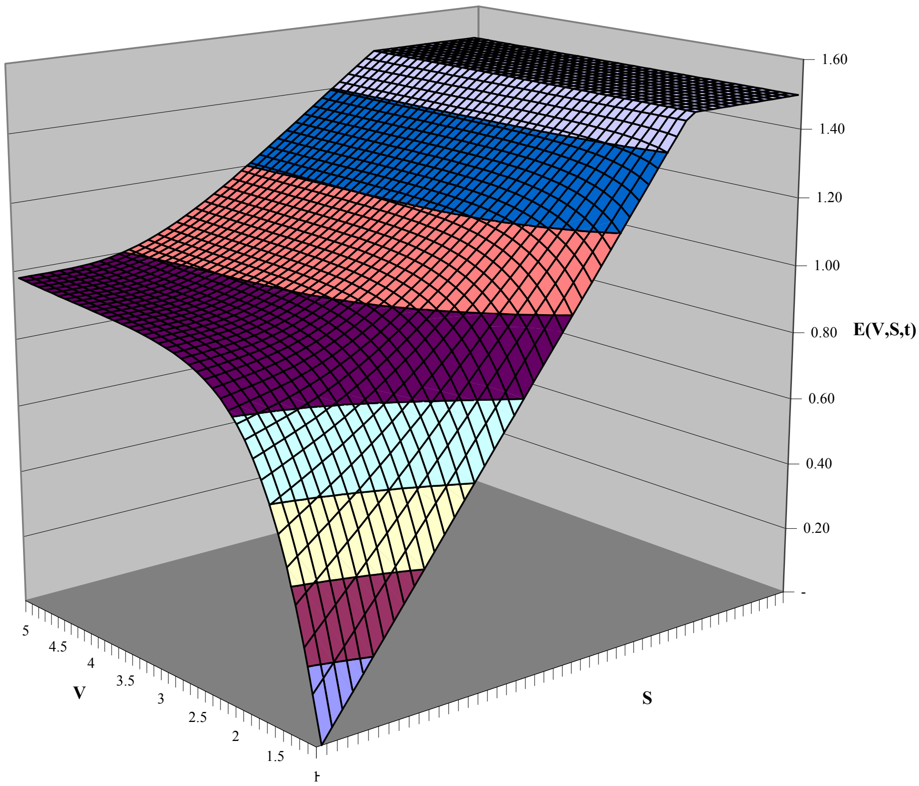

Figure 1: Value of a callable exchangeable convertible in the base case.

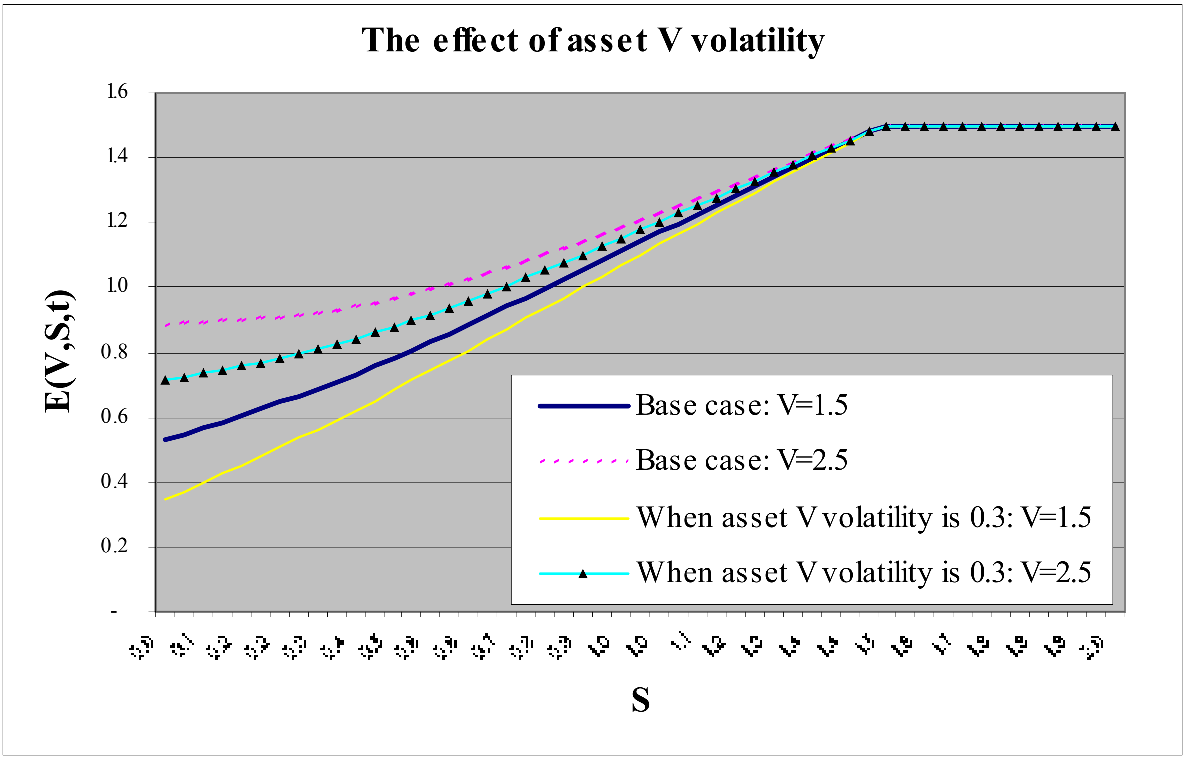

Figure 2: Base case scenario as asset V volatility is changed.

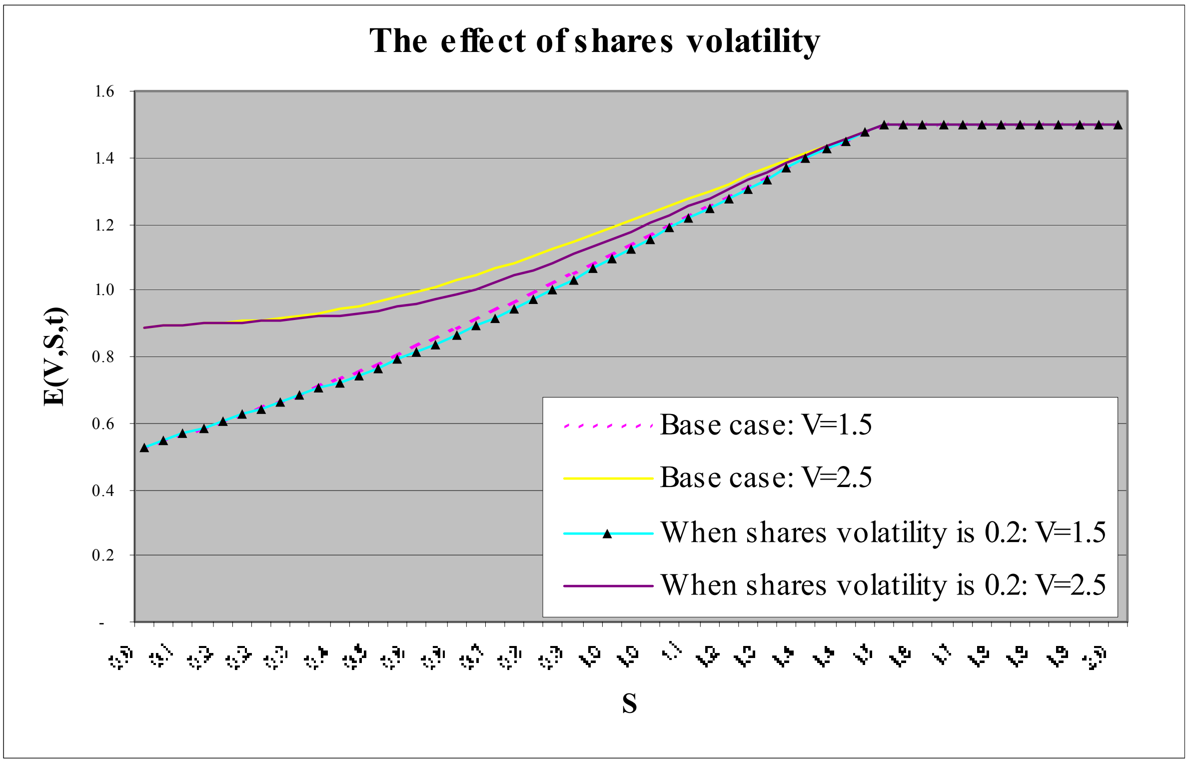

Figure 3: Base case scenario as the shares volatility is changed.

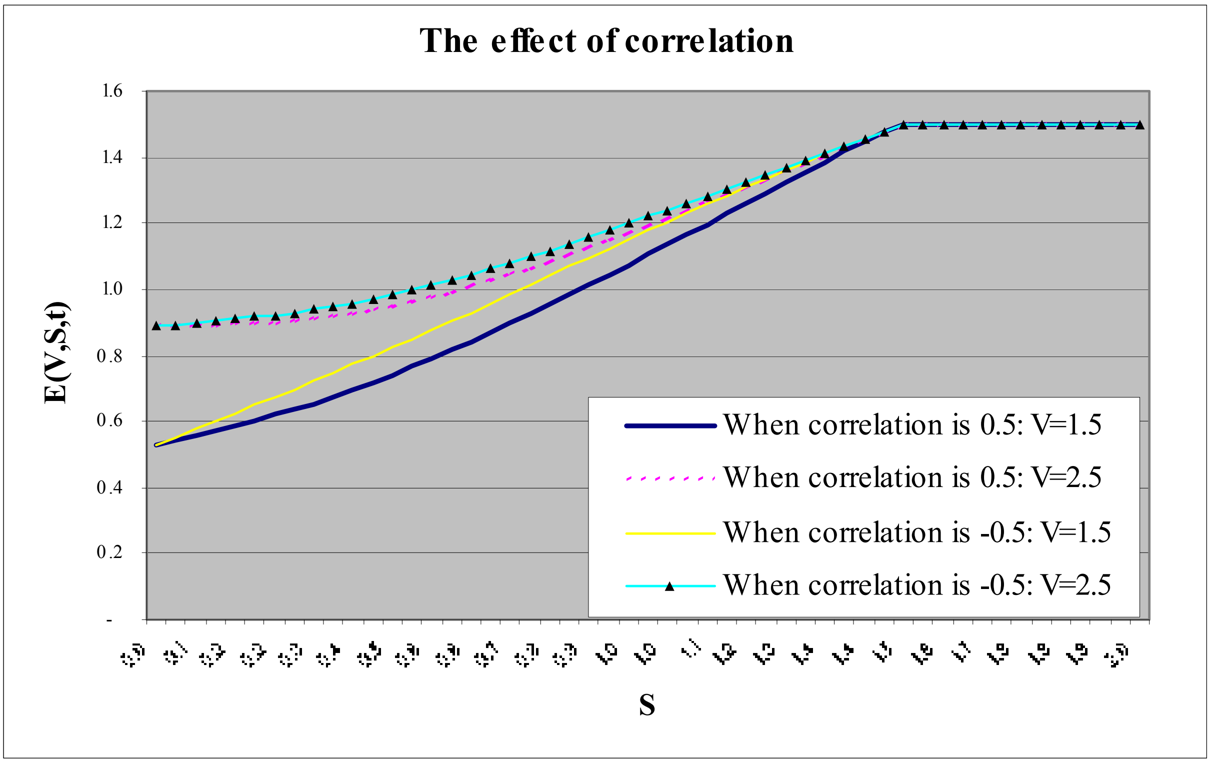

Figure 4: Base case scenario as correlation is changed.

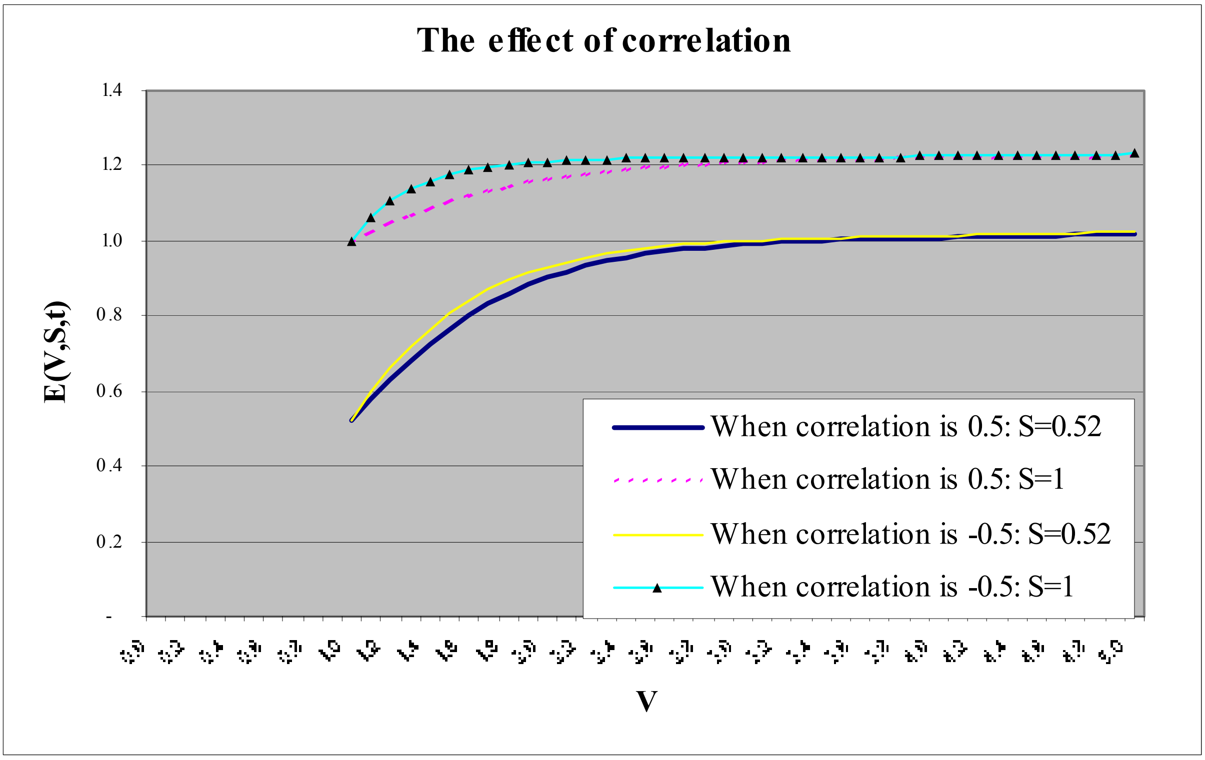

Figure 5: Base case scenario as correlation is changed.

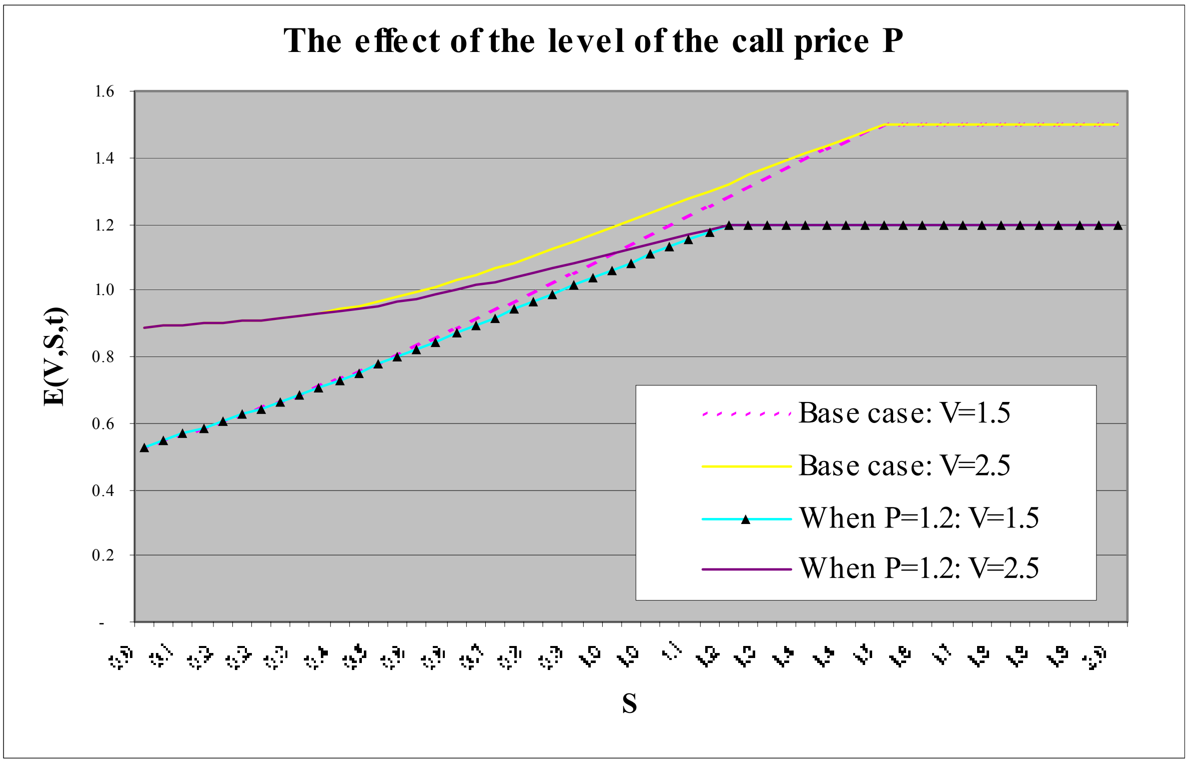

Figure 6: Base case scenario as the call price P is changed.

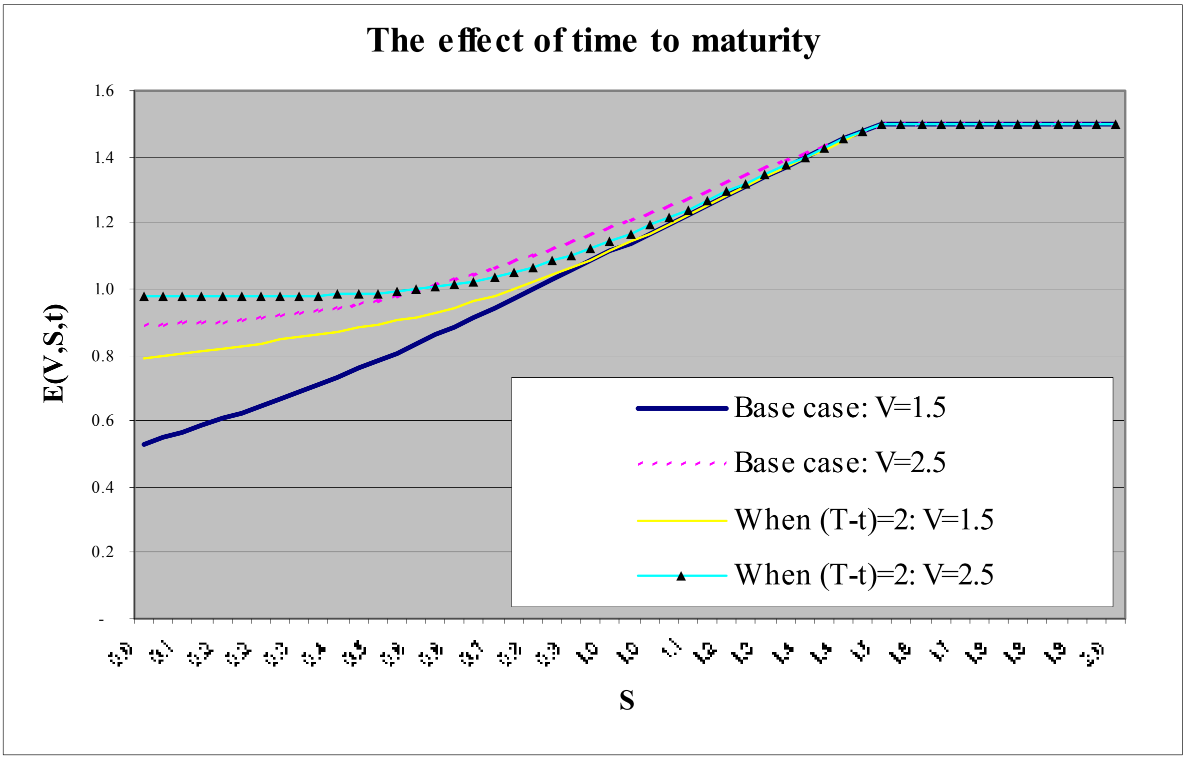

Figure 7: Base case scenario as time to maturity is changed.