Bonds: Additional concepts

NOTATION

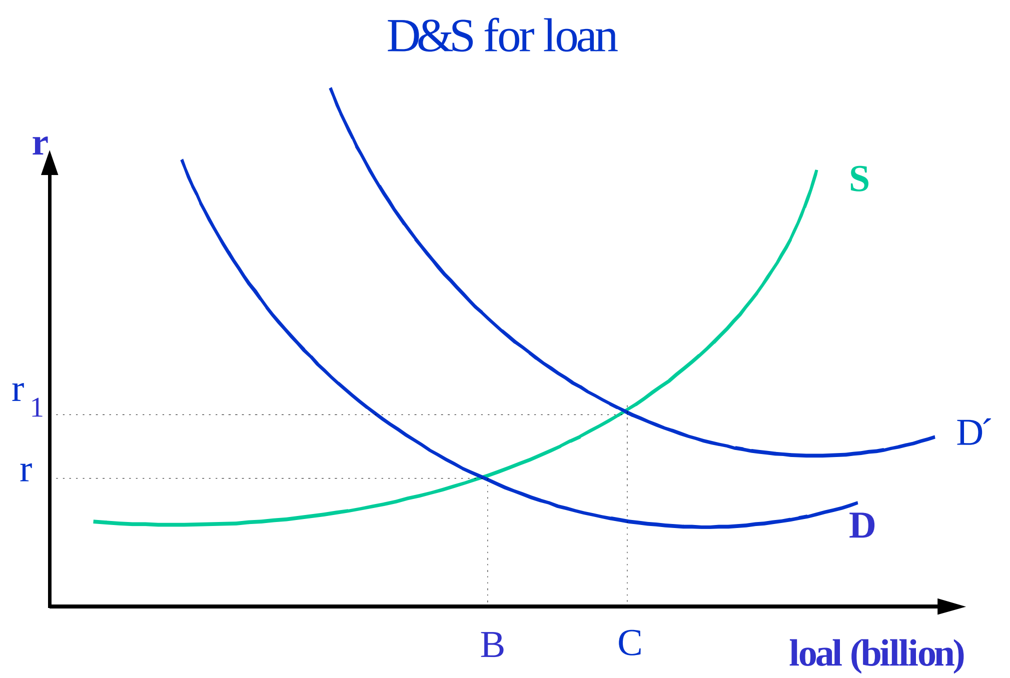
C = coupon payments
T = maturity period
F = Face Value (or Principal), (ofta SEK 1000)
γ = Yield to maturity, the return of a bond when it is kept to maturity

How the interest rate r or γ is decided?

The supply (S) and demand (D) for capital (loan) decide the value of r.

The D has a negative slope, because the cheaper the cost of capital is, the higher its demand, since many unprofitable projects turn into profitable now when IRR > r.

The S has a positive slope, because the higher the return of capital, the more we want to save.
(figure)



The PV of bonds

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{F}{(1+r)^T} \qquad (1)$$

The Market price of bonds

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+\gamma)^t} + \frac{F}{(1+\gamma)^T} \qquad (2)$$

At equilibrium, (1) = (2), i.e., $r = \gamma$.

Example 1: A company issues a 5-year bond with coupon, C = 50 and F = 1000. If the price att issue day is 852, what is its γ?

$$\sum_{t=1}^{5} \frac{50}{(1+\gamma)^t} + \frac{1000}{(1+\gamma)^5} = 852 \Rightarrow \gamma = 0{,}0878\text{, i.e. } \gamma = 8{,}78\ \%$$

Relationship between $\mathbf{P_B}$ and $\boldsymbol{\gamma}$

(a) **If $\boldsymbol{\gamma}$ increases, $\mathbf{P_B}$ falls.**

(If we derive P_B w.r.t. γ we obtain:

$$\frac{1}{(1+\gamma)}\left[\sum_{t=1}^{T} -(t)\frac{C_t}{(1+\gamma)^t} - (T)\frac{F}{(1+\gamma)^T}\right] \prec 0) \qquad (2a)$$

Since the second derivative is positive, the relationship between P_B and γ is negative, and convex.

From (2a) it is clear that a bond's interest rate sensitivity (volatility) depends *ceteris paribus* on (i) how often the coupons are paid, and (ii) remaining time to maturity.

(i) The more frequent the coupon payments, the lower the volatility.

That depends on the fact that when one receives C often, she can save them at higher interest rate if it increases. This is not possible when C do not occur often.

Example 2: Are these bonds equally volatile? Assume r = 10 %

	Price (t=0)	C_1	$C_2 + F$
A	1000	100	100 + 1000
B	1000	-	210 + 1000

Notice that $100/(1,1) + 1100/(1,1)^2 = 1210/(1,1)^2 = 1000$.

(1) If r = 20 %
$P_A = 100/(1,2) + 1100/(1,2)^2 = 847,22$
$P_B = 1210/(1,2)^2 = 840,28$

(2) If r = 5 %
$P_A = 100/(1,05) + 1100/(1,05)^2 = 1092,97$
$P_B = 1210/(1,05)^2 = 1097,51$

Volatility for A = (1092,97 - 847,22)/1000 = 24,575 %
Volatility for B = (1097,51 - 840,28)/1000 = 25,723 % (higher!)

(ii) Bonds with short maturity are less volatile

That depends on the fact that those who invest in longer bonds face higher risk if the interest rate increases.

Example 3: Are these 0-coupon bonds equally volatile?

r (%)	Price 1-year	Price change	Price 30-year	Price change
10	909,09	0	57,31	0
5	952,38	4,8 %	231,38	304 %
20	833,33	-8,3 %	4,21	-93 %

This principle can explain why the "short" interest rate is relatively risk-free.

Other relationships between $\mathbf{P_B}$ and $\mathbf{C_t}$, **F** or **T**

(b) **If $\mathbf{C_t}$ increases, $\mathbf{P_B}$ increases**.

(If we derive P_B w.r.t C_t we obtain:

$$\frac{1}{\gamma} - \frac{(1+\gamma)^{-T}}{\gamma} \succ 0, \text{ because } \sum_{t=1}^{T} \frac{C_t}{(1+\gamma)^t} = \frac{C_t}{\gamma} - \frac{C_t(1+\gamma)^{-T}}{\gamma})$$

(c) **If F increases, $\mathbf{P_B}$ increases**.

(If we derive P_B w.r.t F we obtain: $\frac{1}{(1+\gamma)^T} \succ 0$)

(d) **If T increases, $\mathbf{P_B}$ increases**, if and only if $C_t > \gamma F$

(If we derive P_B w.r.t T we obtain:

$$\frac{(C_t - \gamma F)(1+\gamma)^{-T} \ln(1+\gamma)}{\gamma} \succ 0)$$