IMPLEMENTING SENSITIVITY CALCULATIONS FOR LONG INTEREST RATES FUTURE

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Abstract

In this thesis, Long interest rates Futures are studied, in particular its fundamental contract characteristics such as underlying theoretical bond, its related bond basket and conversion factors, Cheapest-To-Deliver (CTD) and basic market operations.

As these financial contracts have come to play an important role in the modern development of financial markets a close monitoring of its interest sensitivity is of critical importance. Moreover, as speed in treating a deal is also a success factor, more and more deals are treated using front-to-back platforms, such as the one proposed by Murex. The main objective of this thesis was therefore to understand how the interest rate sensitivities of Long interest rates Futures can be calculated, monitored and implemented within the Murex platform.

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1 INTRODUCTION

It is common to accept that the first futures appeared in 1865 on the CBOT trading mainly agricultural commodities and afterwards extending to oil, silver, natural gas, cattle, cotton, etc. Only during the 1970's was the future contracts extended to currency spot exchanges on the CME, without any big success, and later (1977 for the 30y Treasury bond futures contract – 1982 for the 10y Treasury notes – 88 for 5y and 90 for the 2y) to Interest Rates (IR) where these contracts grew to be a very popular and powerful financial contract.
According to the Bank for International Settlements' (BIS) statistics, around 4.08 billions of future contracts where exchanged during 2006, of which 2.62 billions where interest rates futures (1.33 billions were traded in North America) representing a notional nominal of 1 169 320.7 billions of USD.

This thesis is based on a document written during my internship at the Murex Headquarter in Paris, where I was asked to document and research the interest rate long future.
It was firstly thought as an introduction to basic features and functioning of the long interest rate futures, therefore, it first focuses on a general overview (basic notions, definitions, etc.) of the contracts and then on a more mathematical description. Options on long futures are not included in this document.
The term structure of the interest rates is assumed to be stochastic and in particular the Ho-Lee model is employed for the term structure modelling. Further the construction of a zero rate curve is studied as well as the relation between the long future prices and the cheapest-to-deliver prices.
Finally we propose an approach to the calculation of the long future sensitivity to rate changes.

In Chapter 2, a general description of interest rate long future is given, focusing on its purpose, its actors and key concepts as conversion factor, bond basket and cheapest-to-deliver.
In Chapter 3, basic pricing and sensitivity calculation are given.
In Chapter 4, the main results are presented, with the construction of a zero rate curve, pricing of bonds with that curve, comparison between identified cheapest-to-deliver and long future price evolution as well as the calculation for the interest rate sensitivity of the long future.
Chapter 5 is devoted to the different appendixes.

1 LONG FUTURE – A GENERAL OVERVIEW

2.1 Definition and description of financial long future contracts

As for the short future, the interest rate long future is a highly standardised, quoted forward contract but with a *fictive bond* as underlying. Standardisation includes:

- Quantity of underlying (i.e. how many contracts)
- Delivery details and delay penalties
- Currency
- Maturity (usually March, June, September or December)
- Daily max and min in case of “limit up or down”
- Quotation dates, hours…

Information on the basket bonds and a technical description of the long future are usually available on the corresponding trading market’s homepage. The technical description usually comprises the nominal of the contract, its theoretical yield (6% for the US treasury futures, British long gilt and the Euro-Bund Futures), characteristics of the bonds contained in the basket, the tick size (the value of a tick movement), delivery months, the unit of trading (and its quotation form), quotations hours, last trading day, last delivery date, settlement rules, payment, applying rules in case of delivery failure or default payment, etc. See Appendix 5.1 for an example of the 30 year US Treasury –bond future specification as presented on the CBOT brochure.

2.2 Actors & markets

The main actors on the future markets are usually placed in one of two groups:

- Hedgers seeking security instruments for their asset portfolio against an adverse yield shift
- Speculators, who seek to make a profit by predicting/anticipating market moves.

A third party, arbitrageurs, does exist and seeks to make a rapid gain between any miss pricing of an index and an underlying asset.

Since its introduction in the late 70’s on the CBOT, financial long futures have had a growing importance in the derivatives market, as can be seen from the following tables:

Notional Principal in billions USD	Amounts outstanding				Turnover	
Futures	Dec 2003	Dec 2004	Dec 2005	Ma 2007	2005	2006
Total :	**13752.9**	**18903.7**	**21609.9**	**30352.9**	**1005743.6**	**1262955.5**
IR	13123.7	18164.9	20708.8	28737.5	939590.2	1169320.7
Currency	79.9	103.5	107.6	423.9	11126.2	15275.3
Equity Index	549.3	635.2	793.5	1191.6	55027.3	78359.4
North America :	7 700,00	10465.9	12326.8	16522.7	564237.1	713937.6
IR	7384.6	10043.6	11855.2	15680.3	529120.9	667 386,00
Currency	64.9	91.5	90.7	268.6	10258.4	13684.8
Equity Index	250.4	330.7	380.8	573.8	24857.9	32866.8
Europe :	4363.2	5972.3	6275.4	9304.7	380 538,00	455733.1
IR	4200.2	5756.1	6050.5	8878.4	362066.3	427994.5
Currency	0.3	0.3	2.4	3.5	37	47
Equity Index	162.7	215.9	222.5	422.9	18434.7	27691.6

Number of contracts in millions	Contracts outstanding				Turnover	
Futures	Dec 2004	Dec 2005	Dec 2007	Ma 2007	2005	2006
Total :	**68.1**	**107.4**	**143.7**	**156.7**	**3 172,00**	**4085.9**
IR	48.8	54.2	83.7	91.2	2110.4	2621.2
Currency	2	4.1	4.2	5.8	143,00	231.1
Equity Index	17.4	49	55.8	59.7	918.7	1233.6
North America :	47.2	77.4	109.1	114.8	1487.5	1 951,00
IR	34.2	34.4	59.9	64.1	991,00	1 327,00
Currency	0.9	1	1.5	1.3	87.8	120.1
Equity Index	12	42	47.6	49.4	408.7	503.9
Europe :	11.8	16.4	18.5	22.8	1200.8	1468.5
IR	7.7	9.5	11,00	12.6	878.3	988,00
Currency	0.2	2.5	1.9	3.5	18.2	49
Equity Index	3.9	4.4	5.5	6.7	304.3	431.5

Source: BIS Quarterly Review June 2007

Similar products were soon introduced in other markets and are today traded on the CBOT (Chicago Board Of Trade), the CME (Chicago Mercantile Exchange), the Euronext-LIFFE, the SFE (Sydney Future Exchange) for example.

2.3 Purpose

The main purpose of the Long Future is to lock in an interest rate for a certain period of time, thus giving a hedge against the exposure of a certain portfolio to long term interest rates changes. For example, if you expect to receive a certain cash amount that you want to invest in interest rate products (US Treasury notes for example), taking a long position on a suitable future (10y-, 5y-, 2y-US Treasury notes futures for example) can hedge you from a possible yield drop (which would push up the Treasury notes' price). If the yields drop, the rise of the wanted financial products will essentially be compensated by the position in the futures products (if the hedge had been properly constructed). Conversely, if you are holding a portfolio of assets which are sensible to interest rates changes (i.e. a rise in yields might erode your assets value), a proper short position in long futures can guarantee you to offset any eventual losses on your portfolio.

Here follows some basic trading strategies:

Strategy Name	Against	User	Description
Hedge	Yield drop/rise	Hedger	By properly determining their Portfolio Hedge Ratio, traders buy (/sell) future contracts to compensate for the drop (/rise) in market yields
Bullish	Yield drop	Speculator	Traders expecting falling market yields enter long future position covering corresponding section of yield curve. A profit is made if the prices(/market yields) of the futures rises(/drops) on the corresponding maturities.
Bearish	Yield rise	Speculator	Traders expecting rising market yields short future (long) positions covering corresponding section of yield curve. A profit is made if the prices(/yields) of the futures drops(/rises).
Time- or Intra commodity- spread	Change in financing cost / misspricing of two contracts	Speculator / arbitrageur	-Simultaneous purchase (/sale) of a future contract and sale (/purchase) of similar contract but with different maturity . -Traders are either expecting a change of the financing costs for the different maturities or a mispricing of one or two of the contracts that will be leveled out by the market. -Entering, simultenously, on the two positions reduces the total market risk - even if expectations are not met a loss in one position is mainly offset by opposite position
Inter-commodity spread	Diverging yield development	Speculator	-Traders enter in a long and short positions in contracts with different underlying instruments but similar maturity. -Traders are either expecting a "steepening" of the yield curve or a "flattening", i.e that the yield differential will increase/decrease between short term, medium term or long term segmants
Cash&Carry arbitrage	Mispricing of future contracts/bonds	Arbitrageur	By holding a bond and going short on a future or conversely (called reverse cash&carry arbitrage), an arbitrageur hopes to make a profit from any mispricing of the two contracts and/or a positive carry

Source: CBOT- Eurex- and Liffe- publications

The strength of the long future market lies in its deep liquidity, high standardisation and high number of actors and products, ensuring that a desired security is quickly available and traded, with additionally relatively low transaction costs. Furthermore, the daily MTM minimizes any eventual credit risk, making it more reliable than OTC deals.

It should be clarified that not all financial long futures were successful. According to a study by the Bank of England[1] , 26 different financial futures had been created by the CBOT between 1987 and 1996 but only 17 were still traded on that date. The LIFFE market had created 16 interest rate derivatives between 1982 and 1994 but only 11 were still traded in 1996. In the same study, several factors were identified to contribute to the increase in demand and volume for futures:

- The effectiveness of the hedge (i.e the stronger the relationship between returns in the futures market and those in the spot market)
- Large and growing spot markets
- The increase of the spot market volatility
- First-mover advantage (first to emit a certain type of contract) reinforce by the competition from contracts listed on other exchanges with overlapping trading hours.

[1] « Features of a successful contract financial futures on LIFFE", Bank of England Quarterly Bulletin May 1997

2.4 Bond basket, conversion factors and Cheapest-to-Deliver (CTD)

Bond basket:

Since the underlying bond of the future is fictive, the issuer (CME, CBOT, Euronext-Liffe, Eurex) defines, for every contract (1y, 2y, 5y, etc.), a so called *bonds' basket* consisting of several (2 to 30) state bonds (US T-notes and T-bonds in the case of the CBOT, Euro-Bund in the case of Eurex) with fixed rates and maturities, from which the seller of the contract can choose to deliver from. For consistency reasons the maturities of the bonds included in each basket are close to the maturity of the underlying fictive bond.
The existence of a basket of securities effectively minimizes the problem of loss of liquidity of the Cheapest-To-Deliver (CTS – see below), especially if the cost of delivering the different securities is close.

Conversion Factor:

Even though the deliverable bonds included in the bond basket are issued from the same emitter, they are not homogeneous (different coupons and maturities). Therefore Conversion Factors (CF,) are needed to link the price of the notional (fictive) bond, used as underlying for the long future contract, to the prices of the different bonds in the basket. At the emission of the future contract, a specific CF is assigned to each cash instrument that meets the maturity specifications of the contract. This CF represents the price that would prevail in the cash market if 1 currency unit par of a certain basket-bond if it were trading at a yield equivalent to the contract's notional coupon (most commonly trade long futures use a 6% yield). A more practicable way to see the conversion factor is to say that, taking the 0.9387 conversion factor for the 4½ % 5y T-note of March 2012 deliverable in Jun 07 in the table in Appendix **5.4**, the cash 5y T-note is approximately 94% as valuable as a 6% 5y T-Note future of the same maturity.

When calculating the conversion factor, special attention should be taken to the considered contract's maturity date definition as well as some eventual rounding date rules. For example, the maturity date for the long futures traded on the CBOT is the first calendar day of the future delivery month. For Bond futures and 10y note futures, the underlying's remaining time to maturity is always rounded down to the nearest three month quarter whereas for 5y- and 2y Note futures underlying bonds' time to maturity are rounded down to the nearest one month period.
Most long futures trading markets publishes the conversion factors associated to each bonds constituting the bond basket (see Appendix **5.3**) and some markets, as the Eurex exchange for example, even proposes the formula giving the conversion factor (see Appendix **5.3**).

If we denote the quoted price of the long future of maturity T at time t as $F_t^{T\,i}$ or simply F_t and the conversion factor of the i:th bond of the bonds' basket as cf_i then the price paid (PP) at time T (maturity of the contract) is :

$$PP_i = cf_i \cdot F_T + ac_i(T)$$

Here $ac_i(T)$ denotes the accrued coupon for the i:th bond of the bond basket calculated at the maturity date T.

Cheapest-to-Deliver (CTD):

As for other futures contracts (on commodities for example), to avoid any obvious arbitrage situation, the future price converges to the spot (or cash market) price at maturity.

Because of the bond basket, the short seller of a future (i.e the person that contractually must deliver a cash security) has the possibility to choose (see also the Delivery Options section) which security he wishes to deliver. If not already owning one, the seller will go in the cash market and purchase a cash instrument that maximises the difference between what he pays for it and the invoice price he receives from the long buyer. That security is referred to as the **Cheapest-To-Deliver (CTD)**.

If Pi is the "profit" made by the short seller of the contract, delivering a bond I, then the CTD is the bond i which maximizes:

$$P_i = cf_i \cdot F_T - B_i(T)$$

where $B_i(T)$ is the quoted market price of the bond i at time T (excluding the accrued coupon).

It is important to state here, that even though only around 1% of all interest rate long future contracts ends with a physical delivery, identifying the CTD is key to understanding how future prices may be expected to move.

The cost-and-carry relation can be of importance when determining the CTD. Carry is the relationship between the coupon income enjoyed by the owner of the security and the cost incurred to finance the purchase of that security until some future date when it is delivered into the futures market. A simplified example is to suppose an investor wishing to purchase a Treasury bond or note with a 10% coupon. He finances the purchase by going to the repo market (market for short term loans collateralized by U.S Treasury securities – one loans money at a certain “repo rate” giving a security as deposit), where, if the O/N repo rate is 6%, he will earn 4% on interest holding the security. This is called a positive carry and is characteristic of an upward sloping yield curve (where long term rates are higher than short term). On the contrary, if we had an inverted yield curve (ST rates higher than LT) an investor would spend more to finance the purchase than he earned in interest income. Therefore, to maintain a balance in the relationship between cash and future prices, the price of futures contracts should price below the adjusted (with the conversion factor) cash market price in case of positive carry, and conversely price above the adjusted price in case of negative carry. See also the section on Delivery options to see how these options can impact the price.

2.5 Clearing Service Provider – Credit risk

As opposed to forward contracts where the two parties deal directly with each other, with a subsequent credit default risk, one of the major benefits of the long future contracts is that buyers and sellers of contracts don't trade directly with each other but through so called Clearing Firms, acting as a broker between their customers and a Clearing Service Provider (CSP). This CSP serves as counterparty for both sellers and buyers, centralizing all the transactions ("mark to market" and delivery process). In the example of the CBOT, its CSP is the Clearing House of Chicago Mercantile Exchange Inc (a sub division of the CME exchange).

The CSP holds the clearing firms as accountable for every position it carries and thus financially responsible for their different customers' accounts. Conversely, the CSP is held accountable for the clearing members for the net settlement from all transactions on which it has been substituted, thus eliminating the credit default risk. To hedge itself against the credit default, the CSP usually requires from its customer certain guarantees in forms of security deposits, as well as fixing a maximal daily quote fluctuation ("limit up" if rise in prices or "limit down" if fall). Usually the maximum daily fluctuation allowed equals the deposits required by the CSP.

It is also the CSP who is responsible for matching the different short and long positions wishing to deliver/receive the underlying security. See appendix **5.5 Delivery Process** for more information.

2.6 Delivery options

By definition, a future contract holder is legally bonded to fulfil the contractual obligation to buy or sell the underlying Treasury securities. But since hedgers seek to lay off interest risk exposure rather to acquire the underlying contract most future contracts are liquidated before they enter their delivery cycle. The vast majority of these liquidations are either settle in cash or rolled, meaning that the offsetting trades in the expiring contracts (for example, selling a number of future contracts equal to the original short position) are combined with the establishment of corresponding new positions in the deferred contract month.

The cash settlements are so prevalent that less than 1% of all financial futures traded at the CBOT result in the actual delivery of the underlying security. Still, it is this very possibility of delivery that is the fundamental link between future contract prices and spot market prices.

As for commodities futures, financial long futures have several delivery options that are specified within the contracts thus affecting its price (any investor thinking having unveiled an arbitrage situation won't obtain any particular profit if he hasn't considered the delivery options in his price calculation):

- Quality option: the seller of a future contract can choose which underlying Treasury bond or note he wants to deliver (from a basket of bonds).

- Three Time option:

- Accrued Coupon option: gives the seller the opportunity to deliver any trade day of the delivery period (if the market rates differ from the bond rates the seller can either sell in the beginning of the period and invest the cash, or at the end and benefit of the bond's coupon)
- Wild Card option: For the T-bonds and T-notes futures that are quoted on the CBOT market, future trading closes at 14:00 whereas the spot market continues trading until 16:00. Thus traders with a short position, who, additionally, have the right to announce their intention to deliver until 20:00, can wait and see if the prices on the spot market declines (and thus buy the deliverable bond to a cheaper price).
- End-of-month option: for treasury bonds futures and 10Y Treasury note futures, last trading day incurs 7 business days before the last delivery day (which is the last business day of the delivery month), giving the short trader extra time to decide to deliver and which bond/note to deliver.

3 LONG FUTURE – PRICING AND SENSITIVITY CALCULATION

In order to give the price process of the CTD bond, we need first to set the frames of interest rate dynamics.

3.1 Term Structure Equation and Short Rate models

3.1.1 *Term Structure Equation*

In order to model interest rate dynamics, we rely on the following assumptions:

- Existence of a market for all zero coupon bonds (or T-bond), p(t T), for every T>0
- The existence of one exogenously given (local risk and default free) asset, the so called "money account" which dynamics are given by :

$$dB(t) = r(t) \cdot B(t)dt$$

Where r(t) is the short rate, with Q-dynamics (Q being a martingale measure) given by :

$$dr(t) = \mu(t, r(t))dt + \sigma(t, r(t))dW_t \tag{3.1}$$

If we denote, for every T, the price of a T-bond by p(t,T) = G(t,r(t),T), where G is a smooth function of three real variable (here r is the outcome of the stochastic short rate process). To avoid any arbitrage possibility, the model of the family zero coupon bonds must satisfy the ***term structure equation***, see Björk [1]***:***

$$\begin{cases} G_t^T + \mu G_t^T + \frac{1}{2}\sigma^2 G_{rr}^T - rG^T = 0 \\ G^T(T, r) = 1 \end{cases} \tag{3.2}$$

Here the superscript refers to the maturity T, since the term structure should hold for every maturity T. The subscript refers to the partial derivation taken over the different variables, i.e. $G_t^T = \frac{\partial G(t,r(t),T)}{\partial t}$.

A solution to the equation (3.2) is given by the risk neutral valuation:

$$G(t, r(t), T) = E_{t,r}^Q \left[e^{-\int_t^T r(s)ds} \right] \tag{3.3}$$

3.1.2 *Short Rate models*

Thus to price interest rates derivatives we need to specify the r-dynamics under the martingale measure Q. We present here some of the most popular models:

- Vasicek model, $dr(t) = (b - a \cdot r(t))dt + \sigma dW_t$, $a > 0$
- Dothan model, $dr(t) = r(t)dt + \sigma r(t)dW_t$
- Cox-Ingersoll-Ross model (CIR), $dr(t) = a\big(b - r(t)\big)dt + \sigma\sqrt{r(t)}dW_t$
- Black-Derman-Toy model, $dr(t) = \Theta(t)r(t)dt + \sigma(t)r(t)dW_t$
- Ho-Lee model, $dr(t) = \Theta(t)dt + \sigma dW_t$
- Hull-White(extended CIR), $dr(t) = \big(\Theta(t) - a(t)r(t)\big)dt + \sigma(t)\sqrt{r(t)}dW_t$

For more on short rate models see [9].

We will in this study consider the Ho-Lee model. Thomas S.Y. Ho and Sang Bin Lee proposed the first no-arbitrage model of the term structure in 1986. Instead of modelling the short-term interest rate, Ho-Lee developed a discrete time model of the evolution of the whole yield curve. The model was presented in the form of a binomial tree of bond prices with two parameters, namely the short-rate standard deviation and the market price of risk of the short-rate. The continuous-time limit of the model is given above.

The main advantage of this model is its relative simplicity and the fact that it can be calibrated so as to fit the current term structure. Its main disadvantages is that it does not incorporate a mean reversion, meaning that all shocks to the short rate are permanent and do not wear off with time. Secondly, we will show in the sections below that interest rates are normally distributed under the Ho-Lee model. Thus there will be a non-negative probability the rate becomes negative which is contra-intuitive.

3.2 <u>Affine Term Structure and Bond price with the Ho-Lee model</u>

3.2.1 Definition

If the solution to the term structure (3.2), could be rewritten on the form:

$$G(t, r; T) = e^{A(t,T) - B(t,T)r} \tag{3.4}$$

where A(t,T) and B(t,T) are deterministic function, then the model is said to have an affine term structure (ATS).

By substituting (3.4) into the term structure (3.2) we obtain:

$$A_t(t,T) - [1 - B_t(t,T)]r - \mu(t,r)B(t,T) + \frac{1}{2}\sigma^2(t,r)B^2(t,T) = 0 \qquad (3.5)$$

The initial condition becomes: $G^T(T,r) = \begin{cases} A(T,T) = 0 \\ B(T,T) = 0 \end{cases}$

By observing that if both μ and σ^2 are affine functions of r then (3.5) becomes separable and substituting we get:

$$\begin{cases} \mu(t,r) = \alpha(t)r + \beta(t) \\ \sigma^2 = \gamma(t)r + \delta(t) \end{cases} \qquad (3.6)$$

$$\begin{cases} B_t(t,T) + \alpha(t)B(t,T) - \frac{1}{2}\gamma(t)B^2(t,T) = -1 \\ B(T,T) = 0 \end{cases} \qquad (3.7)$$

$$\begin{cases} A_t(t,T) = \beta(t)B(t,T) - \frac{1}{2}\delta(t)B^2(t,T) \\ A(T,T) = 0 \end{cases} \qquad (3.8)$$

Thus by assuming the normal distribution of the rate, equation (3.3) boils down to determine the expectation value of a log-normal distributed stochastic variable.

As the model described above is arbitrage free (existence of a (local) martingale measure) but not complete (the martingale measure is not unique), the idea will be to go to the market and get information about the market price of risk and use that information to fit our term structure. This is called “inverting the yield curve”.
Thus we want to analytically compute the arbitrage free bond prices under our Ho-Lee model, and then fit them to the observed Zero coupon bond prices (also called the yield curve or rate curve), and thus obtain the parameter vector to “pin down” the martingale measure we are working with.

3.2.2 ATS applied to the Ho-Lee model

Using the Ho-Lee model described above :

$$dr(t) = \Theta(t)dt + \sigma dW_t$$

Equations (3.8) and (3.9) become:

$$(1) \quad \begin{cases} B_t(t,T) = -1 \\ B(T,T) = 0 \end{cases}$$

$$(2) \quad \begin{cases} A_t(t,T) = \Theta(t)B(t,T) - \dfrac{1}{2}\sigma^2 B^2(t,T) \\ A(T,T) = 0 \end{cases}$$

Solutions to these equations are given by :

$$\begin{cases} B(t,T) = T - t \\ A(t,T) = -\int_t^T \Theta(s)(T-s)ds + \frac{\sigma^2}{6}(T-t)^3 \end{cases} \tag{3.10}$$

If we now use the definition of the forward rates:

$$f(t,T) = -\frac{\partial}{\partial T} lnP(t,T) \tag{3.11}$$

Remembering equ.(3.4) :

$$P(t,T) = G(t,T) = e^{A(t,T)-B(t,T)r(t)}$$

We find that (3.11) becomes:

$$f(t,T) = -\frac{\partial}{\partial T} lnP(t,T) = -\frac{\partial}{\partial T}[A(t,T) - B(t,T)r(t)]$$

$$f(t,T) = -\frac{\partial}{\partial T}\left[-\int_t^T \Theta(s)(T-s)ds + \frac{\sigma^2}{6}(T-t)^3 - (T-t)r(t)\right]$$

$$f(t,T) = \left[\Theta(T)(T-t) - \frac{\sigma^2}{2}(T-t)^2 + r(t)\right]$$

And deriving once more by $\frac{\partial}{\partial T}$, we obtain :

$$\frac{\partial}{\partial T} f(t,T) = \Theta(T) - \sigma^2(T-t)$$

If we now observe in the market the initial term structure $\{p^*(0, T); T \geq 0\}$, we can determine analytically our parameter function $\Theta(T)$ as :

$$\Theta(T) = \frac{\partial}{\partial T} f^*(0,T) + \sigma^2 T \tag{3.12}$$

Using this result in expression (3.10) for A(t,T) we obtain :

$$A(t,T) = -\int_t^T \left(\frac{\partial}{\partial s} f^*(0,s) + \sigma^2 s\right)(T-s)ds + \frac{\sigma^2}{6}(T-t)^3$$

$$A(t,T) = -\int_t^T \frac{\partial}{\partial s} f^*(0,s)(T-s)ds - \int_t^T \sigma^2 s(T-s)ds + \frac{\sigma^2}{6}(T-t)^3$$

Integrating by parts the first integral gives us:

$$A(t,T) = -[f^*(0,s)(T-s)]_t^T + \int_t^T f^*(0,s)\cdot(-1)ds - \int_t^T \sigma^2 s(T-s)ds + \frac{\sigma^2}{6}(T-t)^3$$

Using now again equ (3.11), we find :

$$A(t,T) = f^*(0,t)(T-t) + \int_t^T \frac{\partial}{\partial s} lnP^*(0,s)ds - \int_t^T \sigma^2 s(T-s)ds + \frac{\sigma^2}{6}(T-t)^3$$

$$A(t,T) = f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \sigma^2\left[T\frac{s^2}{2} - \frac{s^3}{3}\right]_t^T + \frac{\sigma^2}{6}(T-t)^3$$

$$A(t,T) = f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \sigma^2\left(\frac{T^3}{2} - \frac{T^3}{3} - T\frac{t^2}{2} + \frac{t^3}{3}\right) + \frac{\sigma^2}{6}(T-t)^3$$

$$A(t,T) = f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \sigma^2\left(\frac{T^3}{2} - \frac{T^3}{3} - T\frac{t^2}{2} + \frac{t^3}{3}\right) + \frac{\sigma^2}{6}(T^3 - 3tT^2 + 3Tt^2 - t^3)$$

Collecting some terms we find,

$$A(t,T) = f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \frac{\sigma^2}{6}(-6Tt^2 + 3t^3 + 3tT^2)$$

Putting 3t in factor we get,

$$A(t,T) = f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \frac{\sigma^2}{2}t(t^2 - 2Tt + T^2)$$

Or,

$$A(t,T) = f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \frac{\sigma^2}{2}t(T-t)^2$$

Hence, plugging this result and the result for B(t,T) gives us :

$$p(t,T) = e^{A(t,T) - B(t,T)r(t)} = e^{f^*(0,t)(T-t) + lnP^*(0,T) - lnP^*(0,t) - \frac{\sigma^2}{2}t(T-t)^2 \; -(T-t)r(t)}$$

Which we easily rewrite as:

$$p(t,T) = \frac{P^*(0,T)}{P^*(0,t)} e^{f^*(0,t)(T-t) - \frac{\sigma^2}{2}t(T-t)^2 \; -(T-t)r(t)} \tag{3.13}$$

Here, $P^*(0,T)$ and $P^*(0,t)$ denote the observed bond prices at time t =0. The term $f^*(0,t)$ denotes the observed forward rate at time t =0 $(with\ f(0,t) = -\frac{\partial}{\partial t} lnP^*(0,t))$, and r(t) is the short rate modeled by the Ho-Lee model.

3.2.3 Pricing of a coupon bearing bond

When a bond has a series of coupon c_{t_i} that are paid at $t_i = 1, \dots, N$, where t_N is the maturity of the bond,and nominal N, then the price of this bond is :

$$V(c_{t_i}, t_i, t) = \sum_{i=1}^{N} c_{t_i} P(t, t_i) + N \cdot P(t, t_N) \tag{3.14}$$

Where $P(t,t_i)$ is the price at time t of a zero-coupon bond that pays 1 at time t_i.

3.3 Sensitivity of a bond – Duration

The duration of a bond is defined as:

$$D = \frac{\sum_{i=1}^{n} t_i c_i e^{-y t_i}}{V\left(c_{t_i}, t_i, t\right)} \quad (3.15)$$

Where y is the yield of the bond. The duration is a weighted average of the times when payments are made, with the weight provided by the cash flow at time t_i .

As explained in [2], it can be shown that the following relation holds:

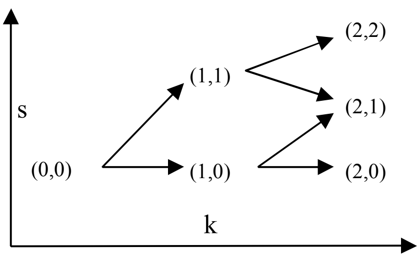
$$\frac{\Delta B}{B} \approx -D\Delta y \quad (3.16)$$

Thus, for small variation in the yield, Δy, i.e small changes in interest rates, the duration is a measure of the sensitivity of a bond to movements in interest rates.

3.4 The discrete Ho-Lee model

Besides being easily calibrated to market data, the Ho-Lee model is also well suited for building recombining binomial lattices. This is used to discretize the continuous model, as shown by [3] and studied in [5]. It shows that for sufficiently small intervals, the discrete model converges to the continuous.

A recombining lattice is set up for the time period needed. At each node a short rate is assigned, representing the one period interest rate for the next period. Under the risk neutral probability, the node transition probabilities are set to $\frac{1}{2}$. The nodes in the lattice are indexed by (k,s), where k is the time variable , k=0,..., T, for maturity T, and s represents the state, s= 0, ..., k, as shown by the figure below :



If we define $P_0(k,s)$ to be the ***elementary price***, the price at time t=0 of a contract paying one unit at time k and state s only, then the price of $P_0(k+1,s)$ is defined under risk-neutral valuation to be :

$$P_0(k+1,s) = \frac{1}{2}d(k,s-1)P_0(k,s-1) + \frac{1}{2}d(k,s)P_0(k,s)$$

Since only 2 other states [(k,s-1) and (k,s)] lead to (k+1,s) and where $d(k,s) = e^{-\Delta t \hat{r}(k,s)}$is the one-period discount factor .

For the nodes on the top and the bottom of the lattice the prices are given by:

$$P_0(k+1,k+1) = \frac{1}{2}d(k,k)P_0(k,k)$$

$$P_0(k+1,0) = \frac{1}{2}d(k,0)P_0(k,0)$$

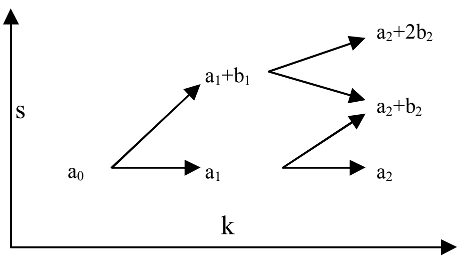
Summing all the elementary prices at time k, gives us the price of a zero-coupon with maturity k :

$$P(0,k) = \sum_{s=0}^{k} P_0(k,s)$$

Now in the Ho-Lee model, the short rate at node (k,s) is represented as :

$$\hat{r}(k) = a(k) + b(k) \cdot s$$

Where a(k) is a measure of aggregate drift from 0 to k and b(k) is the volatility parameter.



Now as shown in [5], by matching the implied zero rates by the tree, with the observed zero rates, one can adjust the parameter a(k) in such a way that the term structure is perfectly fitted. Furthermore, if $\hat{r}$ denote the discrete rate and r the continuous one, then, as for the elementary prices:

$\hat{r}(k+1) - \hat{r}(k) = a(k+1) - a(k) + [b(k+1) - b(k)]s$ with probability $\frac{1}{2}$ (under the risk free probability measure), and

$\hat{r}(k+1) - \hat{r}(k) = a(k+1) - a(k) + [b(k+1) - b(k)]s + b(k+1)$, with probability $\frac{1}{2}$.

Thus the conditional expectation and variation equal:

$$\mathbb{E}(\hat{r}(k+1) - \hat{r}(k)|\hat{r}(k)) = a(k+1) - a(k) + [b(k+1) - b(k)]s + \frac{b(k+1)}{2} \qquad (3.17)$$

And the variation is easily found from : Var(X) = E(X^2) – [E(X)]^2

$$Var(\hat{r}(k+1) - \hat{r}(k)|\hat{r}(k)) = \frac{[b(k+1)]^2}{4} \qquad (3.18)$$

The expectations and variations for the continuous model are known and equal to:

$$\mathbb{E}(r(t+\Delta t) - r(t)|\mathcal{F}_t) = \theta(t)\Delta t + o(\Delta t) \qquad (3.19)$$

$$Var(r(t+\Delta t) - r(t) \mid \mathcal{F}_t) = \sigma^2 \Delta t + o(\Delta t) \qquad (3.20)$$

Thus to have a convergence in the L^2-sense, that is $\mathbb{E}[\,|\,X_k - X\,|\,^2] \longrightarrow 0$, we choose a(k) and b(k) such that both expectations and variations are equal:

$$a(k+1) - a(k) + [b(k+1) - b(k)]s + \frac{b(k+1)}{2} = \theta(t)\Delta t \qquad (3.21)$$

$$\frac{b(k+1)}{2} = \sigma\sqrt{\Delta t} \qquad (3.22)$$

We note that as the Ho-Lee model is assumed to have constant volatility over a time step, b(k) is therefore independent of k and equal to $\sigma\sqrt{\Delta t}$.

In [5], numerical tests showed that 300 steps were sufficient to obtain convergence. We will therefore use the same number of time steps.

3.5 Pricing of Long Futures

We consider a probability space (Ω , **F**, P) on which is defined a Wiener process $(\mathrm{W}_t^{\mathrm{P}})$ and associated filtration, $\mathrm{F}_t = \sigma(\mathrm{W}_s^{\mathrm{P}}; s \leq t)$.

Futures' price process

Furthermore, let ***Y*** be a given contingent T-claim, and assume that market prices are obtained from the fixed risk neutral martingale measure Q. Then the futures price process, F_t^T , is given by:

$$F(t;T;\boldsymbol{Y}) = F_t^T = E_{t,X_t}^Q[\boldsymbol{Y}]$$

Here our T-claim is the fictive bond described in section **2.1**. We have seen that this fictive bond is replaced by the CTD-bond (see section **2.4**) at the maturity of the contract should the short position choose to deliver. Thus to avoid any arbitrage possibilities, the price process of the IR Long Future is that of its CTD-bond (times its conversion factor).

4. ANALYSIS

4.1 Zero Curve from money market data

4.1.1 Introduction

The construction of an “observed term structure”, or also called “zero-curve”, has no real theoretical basis but is more of a practical problem where different construction could be used depending on the final use of the curve.
Different construction options include :

- Choice of “pillars” types (i.e deposit rate, short futures, long futures, Interest Rates Swaps, bonds)
- Choice of the maturities of these pillars (1 month, 2 months, 1years, 5 years, etc.)
- Choice of interpolation methods (linear interpolation, spline, other)

While choosing the different instruments for the curve, one should give special attention to its liquidity, available maturities and correlation with fixed-income products.

In this study we choose to use:

- USD LIBOR deposit rates up to 1year for the short end of the curve
- USD LIBOR Swap rates, with maturities from 1 year up to 10 years, for the long end of the curve

Note that interest rates short futures were not used, as these require a convexity adjustment which is outside the scope of this study.

The swap market offers a variety of advantages. It has almost no government regulations, making it more comparable across different markets and provides a high degree of privacy. Furthermore it is an increasingly liquid market, with narrow bid-ask spreads and a wide spectrum of maturities. The supply of swaps is solely dependent on the number of counterparties wishing to transact at any given time. Moreover it is highly correlated with other fixed income products, rendering its derived term structure a fundamental pricing mechanism for these products and a relevant benchmark for measuring the relative value of different fixed-income products.

4.1.2 Deriving the swap curve

Please refer to appendix 5.6 for the data used for the swap curve construction.

The following equation is solved to compute the continuously compounded zero swap rate (for the short end of the curve) :

$$1 + r_d \frac{t_m}{t_y} = e^{z_c \frac{t_m}{t_y}}$$

$$\Leftrightarrow z_c = \frac{t_y}{t_m} \times ln\left[1 + \frac{r_d}{\frac{t_y}{t_m}}\right] \tag{4.1}$$

r_d is the the observed market rate (USD cash market is supposed to be quoted in ACT/360

t_m represent the remaining days to maturity for the observed market rate (USD swap rates are supposed to be quoted in 30/360)

t_y represent the number of days in a year as specified according to the day count convention used.

For the long end of the curve, the bootstrap method is used to derive zero-coupon interest rates from the swap par rates. The "swap rate" is defined at emission of the contract (t=0) as :

$$R = \frac{p(0,T_0) - p(0,T_n)}{\delta \sum_1^n p(0,T_i)}$$

Where $\delta = T_i - T_{i-1}$.

Now, the plain vanilla swaps are such a liquid instrument, that new swaps are emitted every day, and therefore the published swap rates represents the fixed rate for which the market is willing to swap for a floating rate.

By setting T_0 = 0, and substituting $p(0,T_n) = e^{-z_{T_n} T_n}$, with z_T is the zero rate.

Starting from the first swap rate, given all the continuously compounded zero rates for the coupon cash flows prior to maturity, the next zero rate is bootstrapped as follow :

$$z = -\frac{1}{T} ln[1 - R \cdot \sum_{i=1}^{T-1} e^{-z_i t_i}] \tag{4.2}$$

Here,

T, is the maturity date (equal to the different maturity pillars chosen)

z_i represent the calculated zero rate for the maturity i

t_i is the maturity

R is the quoted swap rate

For the maturities for which no data was available (for example the 1,5 years-, 2,5 years- , 3,5 years pillars) the zero rate for this maturity was set to the precedent available pillar (in the case of the above: 1 year, 2 year and 3 year pillar).

Furthermore, for the overlapping value at the maturity of 1 year, the average of the two calculated values was retained as final value for the curve construction.

4.1.3 Interpolation algorithm

There is no single correct way to link deposit, futures and swap interest rates (or any other product used in the to construct the complete swap term structure; however, several fundamental characteristics and conventions should be followed, to ensure yield curve validity. The derived yield curve should be consistent and smooth, and should closely track observed market data points. However, over-smoothing the yield curve might cause the elimination of valuable market pricing information. Thus when choosing an interpolation method the following should be taken into account :

- Are the forward rates positive (thus avoiding any arbitrage possibilities)
- Are the forward rates continuous? This is required for forward rates sensible interest rate instruments.
- When a change in the input does it only have an affect locally or is the entire curve affected?
- Stability of the forward rates to a change in the input of the curve.

A summary of existing methods is exposed in [7] and presented in more detail in [8].

We present in the following table the most prevalent algorithms:

Interpolation type	Forwards >0 ?	Forward smoothness	Method local	Forward stable
Linear on zero rates	no	not continuous	excellent	excellent
Linear on dscount factors	no	not continuous	excellent	excellent
Linear on the log of discount rates	yes	not continuous	excellent	excellent
Natural Cubic spline	no	smooth	poor	good
Monotone convex	yes	continuous	good	good

As the monotone convex method is quite complex to implement, we choose to use the Natural Cubic Spline and the linear interpolation on the log of discount factors to study the effect of interpolation on bond and future pricing.
The later one was used in this study, as this technique offers the advantage of fitting perfectly the observed data points, ensures a smooth curve that additionally avoids wide oscillations at end points (as is the case of a n-1 polynomial, where n is the number of observed points).

4.1.4 Linear interpolation on the log of discount factors

This method corresponds to piecewise constant forward rates, which can be seen as follows:
For $t_i \leq t \leq t_{i+1}$, the continuously compounded risk-free rate for maturity t is :

$$z(t) = \frac{t-t_i}{t_{i+1}-t_i}\frac{t_{i+1}}{t}z(t_{i+1}) + \frac{t_{i+1}-t}{t_{i+1}-t_i}\frac{t_i}{t}z(t_i) \tag{4.3}$$

4.1.5 Constructing a cubic spline

To construct a set of cubic splines, let the function $R_i(t)$ denote the cubic polynomial associated with the t segment $[t_i; t_{i+1}]$:

$$R_i(t) = a_i(t-t_i)^3 + b_i(t-t_i)^2 + c_i(t-t_i) + r_i \tag{4.4}$$

where n is the number of market observations,
r_i represents market observation (knot point) i,
t_i represents time to maturity of market observation i.

The $3 \cdot (n-1)$ unknown coefficients are obtained by imposing the following conditions:

- $\forall i \in [0; n-1]$, the piecewise polynomials must pass through the data value, leading to the (n-1) conditions :

  $$R_i(t_{i+1}) = r_{i+1} \Leftrightarrow a_i(t_{i+1}-t_i)^3 + b_i(t_{i+1}-t_i)^2 + c_i(t_{i+1}-t_i) = r_{i+1} - r_i \tag{4.5}$$

- $\forall i \in [1; n-1]$, the first and second derivatives are set to be equal around interior points to ensure a smooth continuation. This gives rise to 2(n-2) conditions:

$$\begin{cases} R'_{i-1}(t_i) = R'_i(t_i) \\ R''_{i-1}(t_i) = R''_i(t_i) \end{cases} \Leftrightarrow \begin{cases} 3a_{i-1}(t_i - t_{i-1})^2 + 2b_{i-1}(t_i - t_{i-1}) + c_{i-1} - c_i = 0 \\ 6a_{i-1}(t_i - t_{i-1}) + 2b_{i-1} - 2b_i = 0 \end{cases} \quad (4.6)$$

- The final 2 conditions are to set the second derivative to zero at the end of the curve:

$$\begin{cases} R''_0(t_0) = 0 \\ R''_{n-1}(t_n) = 0 \end{cases} \Leftrightarrow \begin{cases} b_0 = 0 \\ 6a_{n-1}(t_n - t_{n-1}) + 2b_{n-1} = 0 \end{cases} \quad (4.7)$$

Conditions (4.4), (4.5) and (4.6) generate our linear system, which is solved using classical matrix calculus.

A more detailed view on numerical implementation of cubic splines is given in [7] *More Yield Curve modeling at the Bank of Canada*.

4.1.6 Ho-Lee model parameterization

We recall from the Ho-Lee model (see §3.2), that the Ho-Lee parameter was determined by:

$$\Theta(T) = \frac{\partial}{\partial T} f^*(0,T) + \sigma^2 T \quad (3.12)$$

With $f(0,t) = -\frac{\partial}{\partial t} lnP^*(0,t)$, which we determined in §4.1.4 and §4.1.5, we are now left to determine the volatility **σ** in our Ho-Lee model.
This could be done by:

- Make an assumption concerning the volatility: assume for example that the volatility is 0.01 per year (meaning that the short rate is likely to fluctuate about 1 percentage point per year)
- Use historical volatility. Taking for a similar time length as the period covered by our study (i.e. September 07th 2010 to the last trading day of the December 2010 3y T-Notes' future, December 31st 2010) we determine the various standard deviation, using $\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, of the various prices of the different instruments composing our zero rate curve. In our case we found that approximately$\sigma = 0{,}001$.
- As proposed by [5], it is possible to refine the volatility calculated above by calculating a theoretical price of an option on the future, using the Ho-Lee tree, and comparing it to the market value of the option, and iterate towards a better estimate of the value for σ^2.v

For the purpose of this study we choose a fix value for the volatility equal to 0.001.

4.2 Bond pricing with the zero curve

Now that a zero curve has been obtained, we can now attempt to price any bond. The total price of a coupon-bearing bond is the sum of the discounted coupon payments and the discounted notional. The price is given by (3.14):

$$V(c_{t_i}, t_i, t) = \sum_{i=1}^{N} c_{t_i} P(t, t_i) + N \cdot P(t, t_N) \tag{3.14}$$

4.3 CTD determination

4.3.1 Using the Ho-Lee tree

Now, as shown in [5] we also make use of the Ho-Lee lattice constructed in §3.4, to compute the prices of various bonds included in the 3y T-Note December 2010 future's bond basket (see Appendix 5.7 for the prices of the various bonds included). As explained in §2.4 the delivered bond will be the one which maximizes the following position:

$$P_i = cf_i \cdot F_T - B_i(T)$$

Thus by computing the price evolution between "today", date set to September 07th 2010, and the last trading day of the future contract, that is according to Appendix 5.1, the last business day of the contract expiry month, or December 31st 2010, we can identify the CTD bond.
With the same notations as in §4.2, the price of a bond at time t is:

$$V(c_{t_i}, t_i, t) = \sum_{i=1}^{N} c_{t_i} P(t, t_i) + N \cdot P(t, t_N) \tag{3.14}$$

Introducing the following notations:

- $H\big((M,s), t\big)$ as the discount factor at node (M,s) of a bond that pays one unit at time t
- $K\big(\vec{c}^{\,j}, \vec{t}_c^{\,j}, (M,s)\big)$ as the price at node (M,s) of a bond j with coupons $\vec{c}^{\,j}$ paid at times $\vec{t}_c^{\,j}$

The tree having M steps (corresponding to the number of time steps) and s states (s=1,... M).
Then, (3.14) transforms into:

$$K\big(\vec{c}^{\,j}, \vec{t}_c^{\,j}, (M,s)\big) = \sum_{i=1}^{N} c_{t_i}^{j} H\left((M,s), t_i^{j}\right) + H\left((M,s), t_N^{j}\right) \tag{4.8}$$

Using (3.15) we can calculate all the bond prices at time for the last trading date (at the end nodes of the tree), using the various parameters defined in §4.1.6.

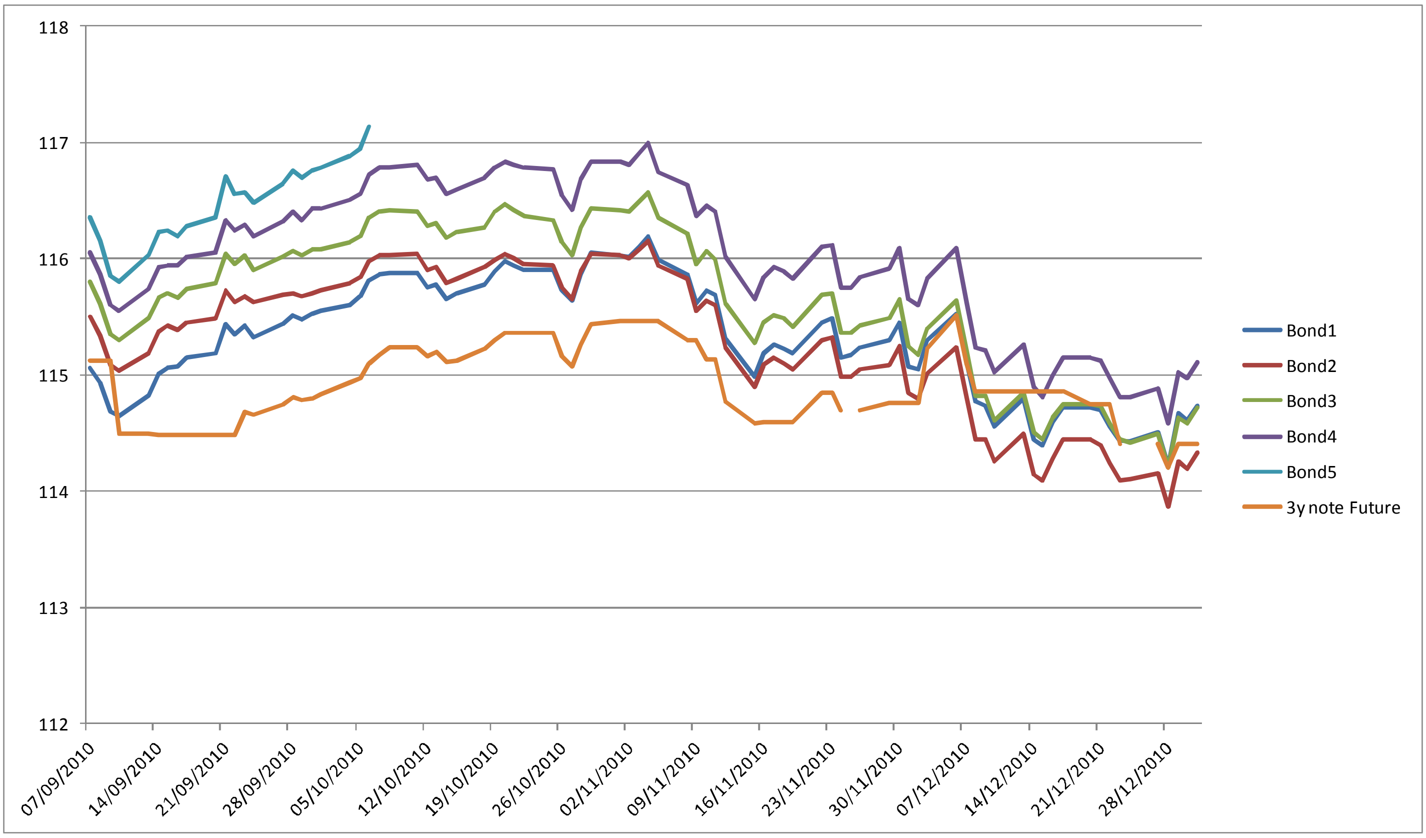
Now, using the same technique as in §3.4, we can go backwards in the tree saying that at some node (M-1, s) then, one time step later, we are either at node (M , s) with probability ½ or (M, s+1) with the same probability. Thus the price of a bond j at node (M-1 , s) is :

$$K\left(\vec{c}^{j}, \vec{t}_{c}^{j}, (M-1, s)\right) = \frac{1}{2} d_{M-1,s}\left[K\left(\vec{c}^{j}, \vec{t}_{c}^{j}, (M, s)\right) + K\left(\vec{c}^{j}, \vec{t}_{c}^{j}, (M, s+1)\right)\right] \quad (4.9)$$

Where $d_{M-1,s}$ is the discount factor at the state s for the time period M-1 to M, or, using the notation of §3.4, $d_{M-1,s} = e^{-\hat{r}(M-1)\times\frac{T}{M}}$, $\frac{T}{M}$ being the time interval of our model.

4.3.2 Using the zero-rate curve

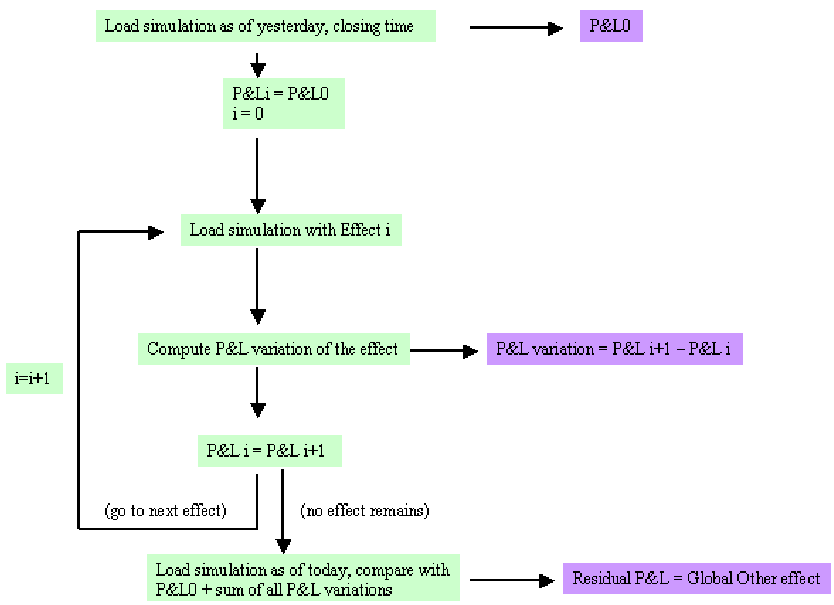
Using the fact that futures prices converge to the spot price at maturity, as shown by the following graph of the price evolutions of the different bonds of the bonds' basket and the December 2010 3y Note future:



One can determine the CTD today, by using the market expectations (embedded within the price of the long future as well as the different market quotes for the instruments used to construct the zero curve). Thus with the market data of the start date of September 07th 2010:

4.4 Sensitivity calculation at Murex

In order to understand the daily P&L changes of their portfolios the traders and P&L control users are using different methodologies. One of them is based on the "Greeks" (partial derivatives). This methodology, called P&L Predict (at Murex), is applying the market data changes on the computed Greeks in order compute the expected P&L change. This P&L change will not match exactly the one monitored by the traders but should give a correct figure – its accuracy depending on the cross effects and the convexity of the different effects.
This methodology is summarized in the following diagram:

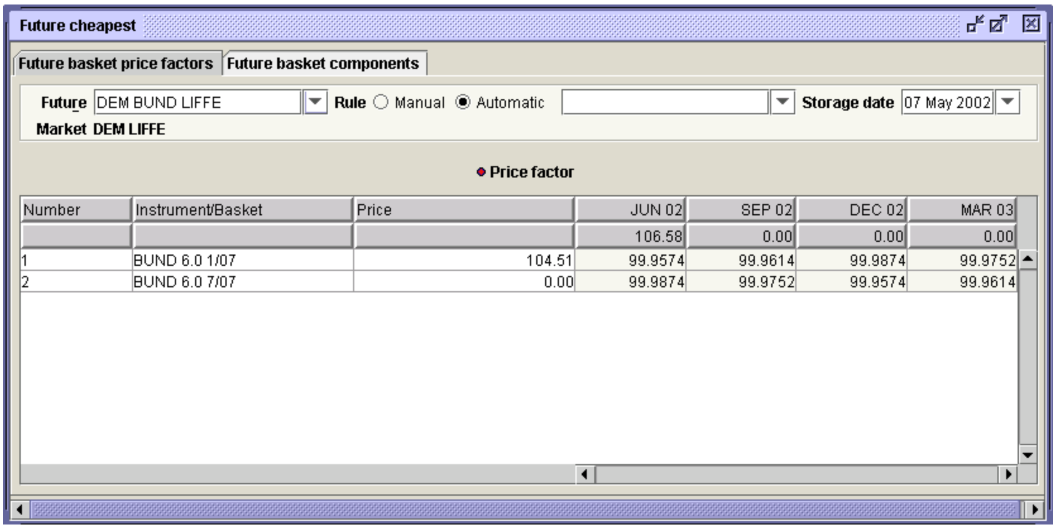


We look closer to the calculation of the "delta"-sensitivity of the long future:
As specified in the chapter §3.4, to avoid any arbitrage possibilities, the price process of the IR Long Future is that of its CTD-bond (times its conversion factor).
Therefore, the long future sensitivity is calculated as the forward sensitivity of the forward cheapest-to-deliver (CTD) e.g. the CTD bond priced on the zero curve with a settlement date equal to the future maturity adjusted by the conversion factor (CF).

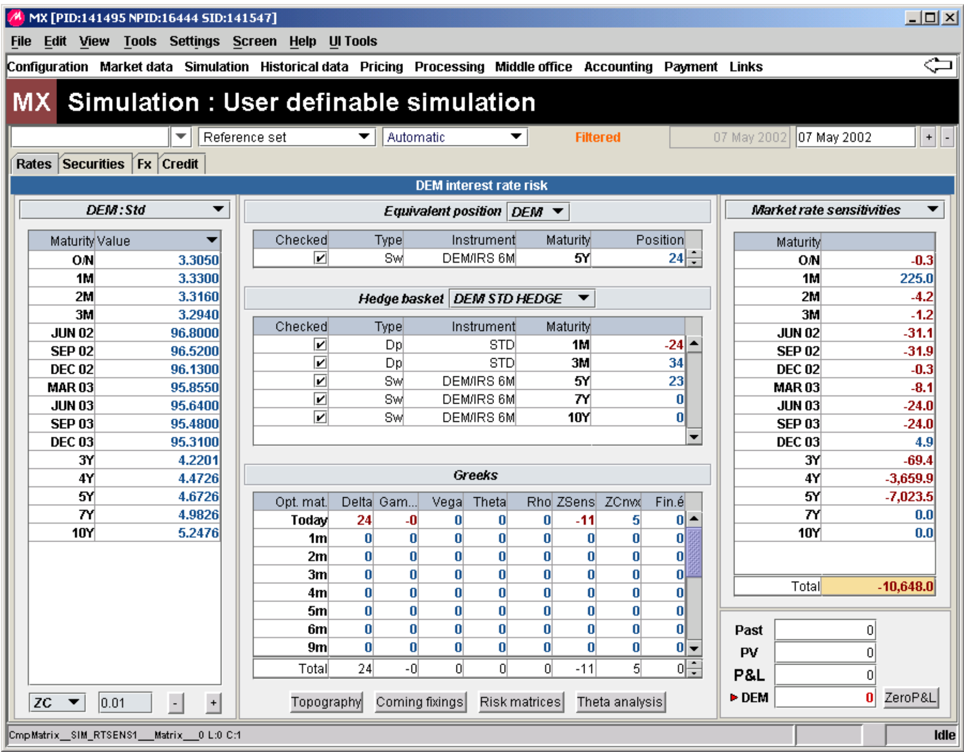
Now we look at an example implemented using the Murex-program (for this example we used Liffe's June 2002 Bund Future, implementation was performed with data as of May 07th 2002), the aim being to reproduce the sensitivities calculated:

The price of the future on May 07th 2002 was: 106.58

The CTD was determined to be the bond BUND 6.0 01/07, which "clean price" (i.e. without accrued coupons) on May 07th 2002 was 104.51 and the conversion factor (CF) for the June 02 future was 99.9574. This is summarized in the following screen:



The Zero-sensitivities for a portfolio consisting of the future given by Murex are:



The screen on the right gives the sensitivity for the various pillars. The total sensitivity was determined to -10648.01605663744 DEM.

According to Liffe delivery processes, the future maturity is defined as June 6th. The underlying CTD will be delivered 3 open days later: June 10th.

Using the fact that the Long Future will have the same price process as its underlying (see §3.5), then it will have its sensitivity. Thus the following relationship must hold:

$$\frac{\partial P_F}{\partial Z_j} = \frac{\partial P_B}{\partial Z_j} * \frac{N_F}{N_B} * \frac{1}{CF} * \frac{1}{\Phi(T_0, V_i, Z+S)} \qquad (4.10)$$

Where

$\frac{\partial P_F}{\partial Z_j}$ is the sensitivity of the future price with respect to the jth zero-rate

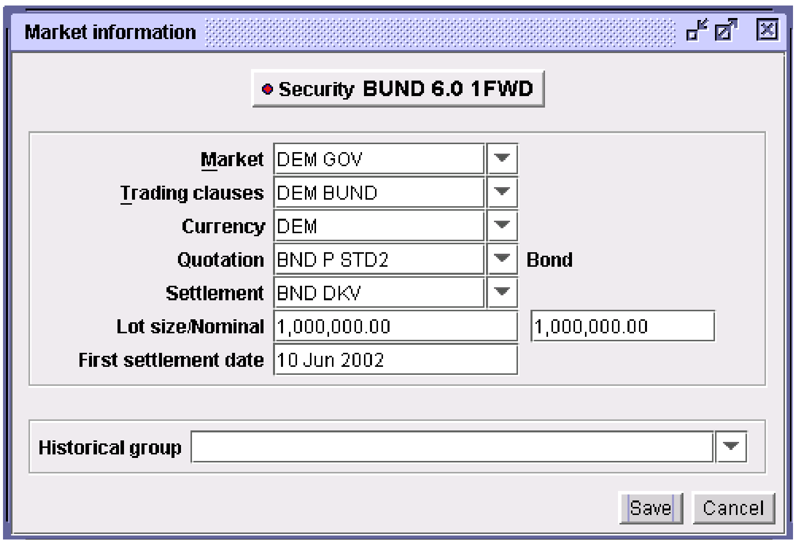
$\frac{\partial P_F}{\partial Z_j}$ is the sensitivity of the bond price with respect to the jth zero-rate

N_F is the future nominal

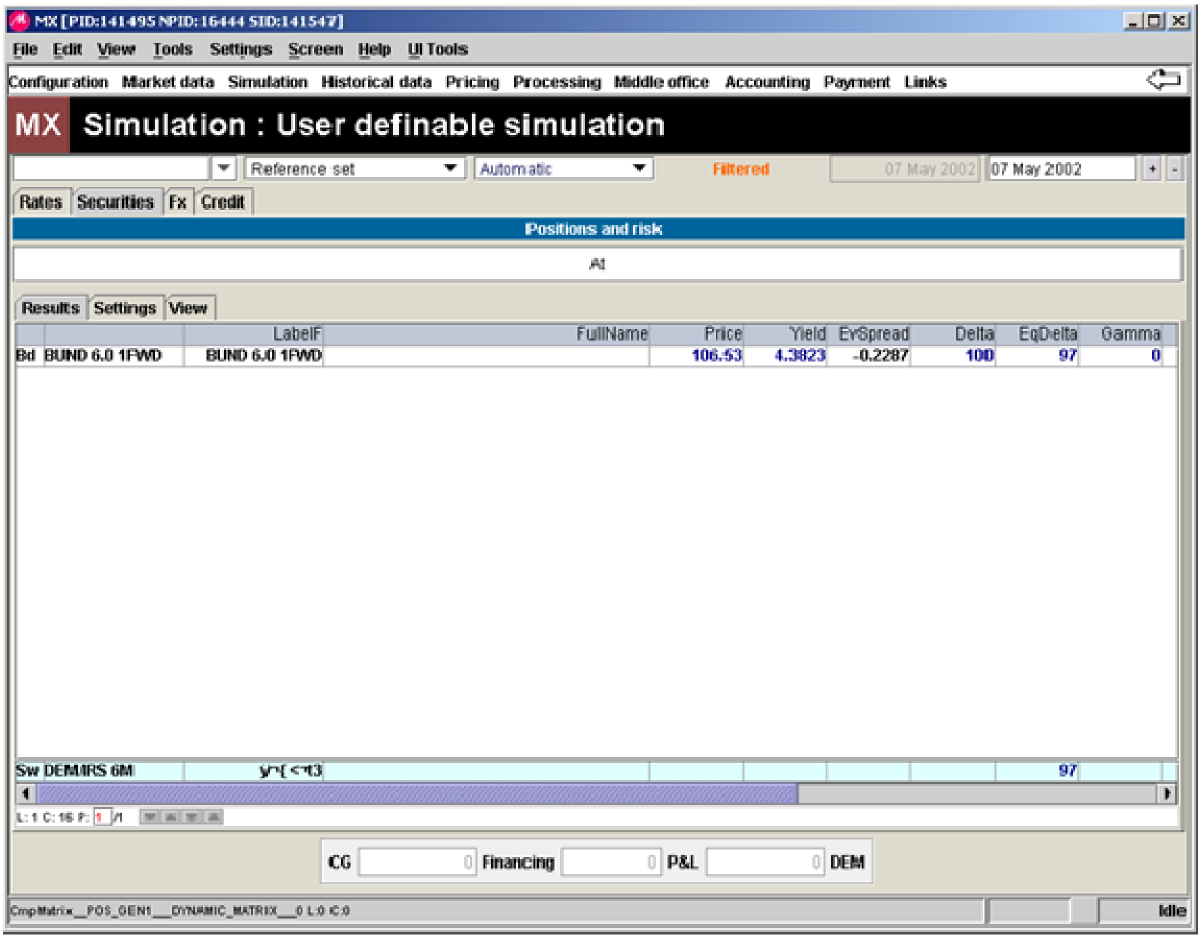
N_B is the bond nominal.

$\Phi(T_i, T_j, Z+S)$ the discount factor calculated between T_i and T_j on the zero curve Z where the zero-rates are shifted by the spread S.

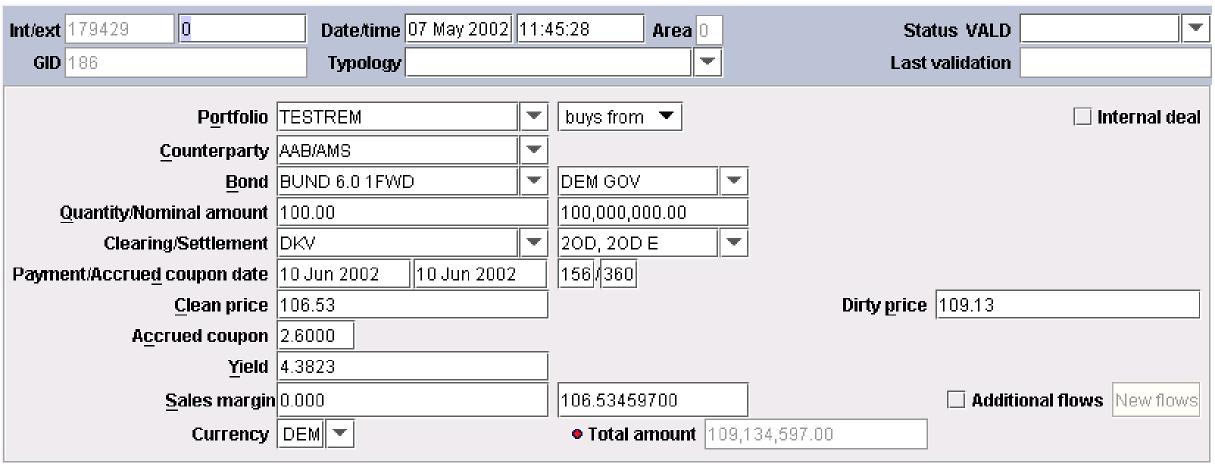
Using Murex internal Bond-pricing it is possible to recreate a bond with the same characteristics as the underlying fictive bond, i.e a bond with maturity in 8.5 years, first settlement date on June 10th 2002, and a bond price equal to today's future price, in this example 106.58, times the conversion factor obtained from the Liffe's homepage, CF = 0.999574) :



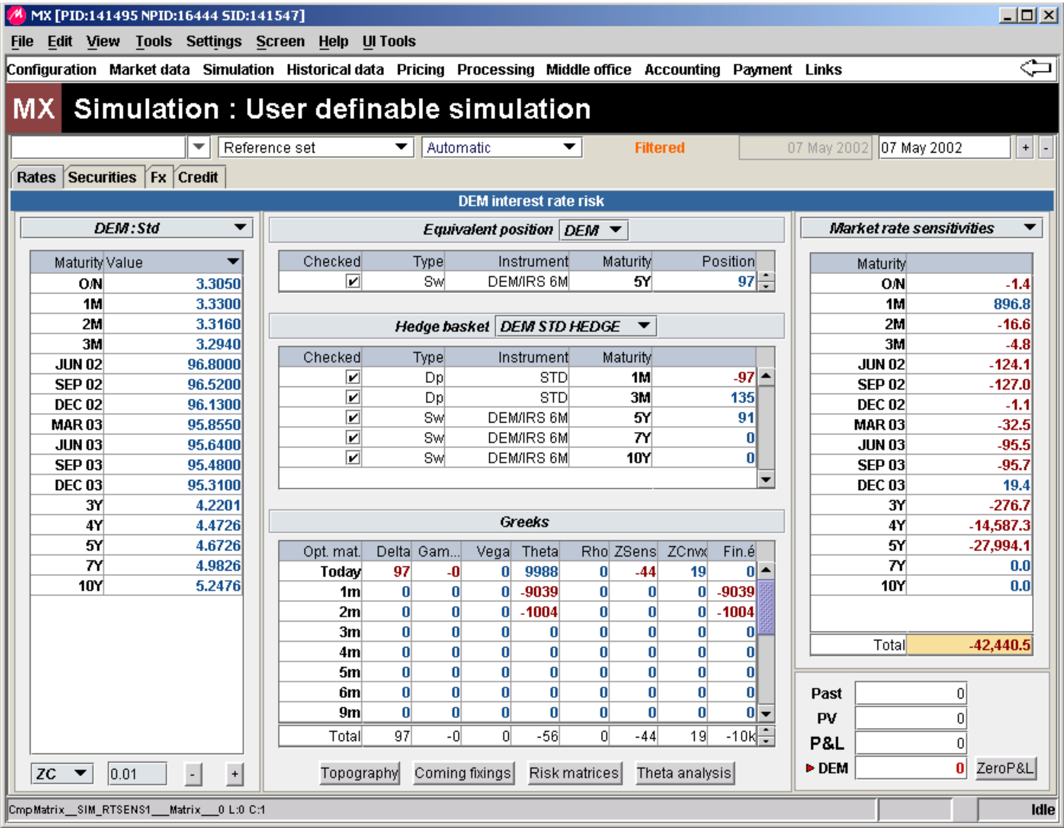
The spread resulting from the pricing on the defined yield curve is automatically displayed in the following tab:



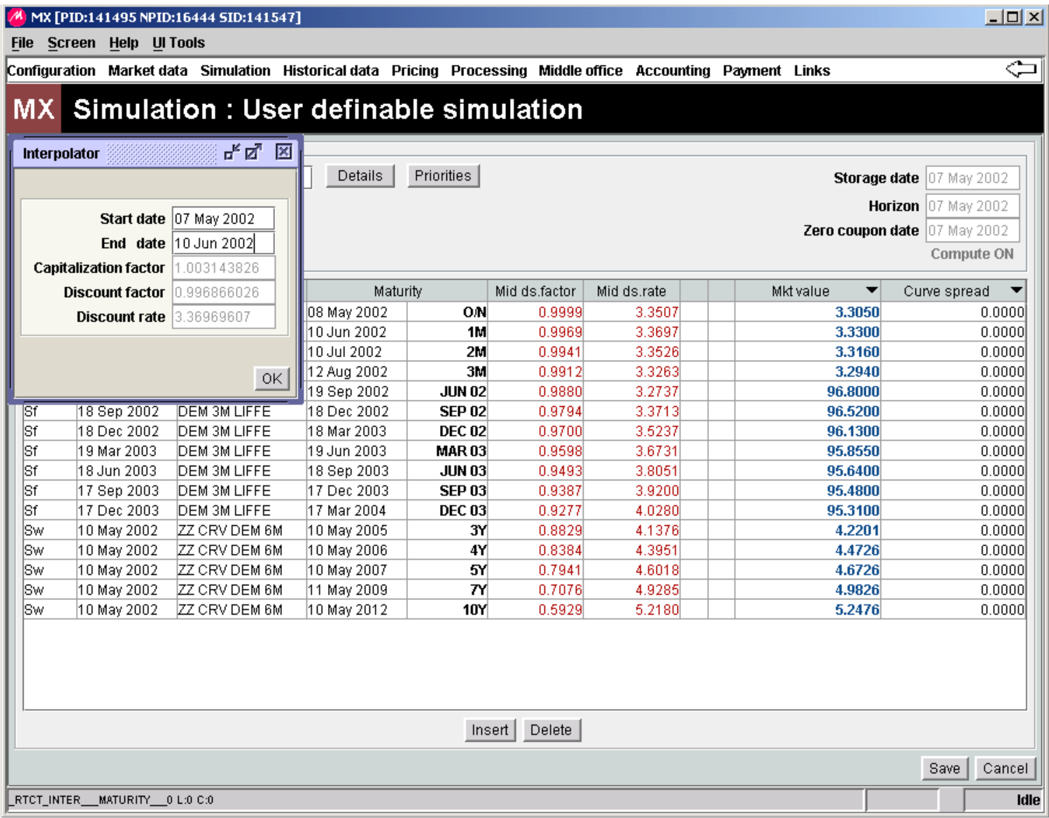
By taking a position on that peculiar bond (with price equal to the theoretical bond price), the resulting Zero Coupon sensitivity allows to deduce the long future one.



And the zero sensitivities of the new portfolio consisting of one theoretical fictive bond are given by:



Using the internal discount factor calculator we find that the capitalization factor (equal to 1.003143826) :



And applying the formula (4.10), we find that the global future sensitivity is equal to:

-42440.4939731713 * 250000/1000000 * 1/(0.99.9574) * 1.003143826 = -10648.0159301805 DEM.

The discrepancy with the actual result comes from the use of a truncated capitalisation factor provided by the interpolator.

5 CONCLUSION

After having presented and studied the different characteristics of the Long interest rates Futures, which are fundamental in understanding the dynamics of the contract, we turned to establish a theoretical financial environnement in which sensitivities could be calculated. We found that the CTD bond could be determined using a Ho-Lee model and a binomial tree and in turn used to establish the Long Futures interest rate sensitivities. We then showed how this could be implemented using a “front-to-back” financial platform as the one proposed by Murex.

6. APPENDIXES

6.1 30y US Treasury-bond and 3y US T-note future specification

30-Year U.S. Treasury Bond Futures

Contract Size
One U.S. Treasury bond having a face value at maturity of $100,000 or multiple thereof.

Deliverable Grades
U.S. Treasury bonds that, if callable, are not callable for at least 15 years from the first day of the delivery month or, if not callable, have a maturity of at least 15 years from the first day of the delivery month. The invoice price equals the futures settlement price times a conversion factor plus accrued interest. The conversion factor is the price of the delivered bond ($1 par value) to yield 6 percent.

Tick Size
Minimum price fluctuations shall be in multiples of one thirty-second (1/32) point per 100 points ($31.25 per contract) except for intermonth spreads, where minimum price fluctuations shall be in multiples of one-fourth of one-thirty-second point per 100 points ($7.8125 per contract). Par shall be on the basis of 100 points. Contracts shall not be made on any other price basis.

Price Quote
Points ($1,000) and thirty-seconds of a point.
For example, 80-16 equals 80 16/32.

Contract Months
Mar, Jun, Sep, Dec

Last Trading Day
Seventh business day preceding the last business day of the delivery month. Trading in expiring contracts closes at noon, Chicago time, on the last trading day.

Last Delivery Day
Last business day of the delivery month.

Delivery Method
Federal Reserve book-entry wire-transfer system.

Trading Hours
Open Auction: 7:20 am - 2:00 pm, Chicago time, Monday - Friday
Electronic: 6:00 pm - 4:00 pm, Chicago time, Sunday - Friday

Ticker Symbols
Open Auction: US
Electronic: ZB

Daily Price Limit
None

3-Year U.S. Treasury Note Futures

Contract Size
One (1) U.S. Treasury note having a face value at maturity of $200,000 or multiples thereof.

Deliverable Grades
U.S. Treasury notes that have an original maturity of not more than 5 years and 3 months and a remaining maturity of not less than 2 years and 9 months from the first day of the delivery month but not more than 3 years from the last day of the delivery month. The invoice price equals the futures settlement price times a conversion factor plus accrued interest. The conversion factor is the price of the delivered note ($1 par value) to yield 6 percent.

Tick Size
One-quarter of one thirty-second of one point ($15.625)..

Price Quote
Points ($2,000) and quarter-32nds of a point ($15.625). For example, 91-16 equals 91-16/32, 91-162 equals 91-16.25/32, 91-165 equals 91-16.5/32, and 91-167 equals 91-16.75/32..

Contract Months
Mar, Jun, Sep, Dec

Last Trading Day
The last business day of the contract expiry month. Trading in expiring contracts closes at 12:01 p.m., Central Time (CT), on the last trading day..

Last Delivery Day
Third business day following the last trading day

Delivery Method
Federal Reserve book-entry wire-transfer system.

Trading Hours
Open Auction: 7:20 am - 2:00 pm, Chicago time, Monday - Friday
Electronic: 5:30 pm - 4:00 pm, Chicago time, Sunday - Friday

Ticker Symbols
Open Outcry: 3YR CME Globex: Z3N

Source: http://www.cmegroup.com/trading/interest-rates/us-treasury/

6.2 Other Futures contracts specifications

Type of contract	Nominal	Notional Yield	Underlying	Tick Size	Delivery Months
CBOT - 30y Treasury Bond Futures	Face value 100,000 USD	6% Invoice price = futures settlement price x CF + Accrued interests	US Treasury bond : (if callable) not callable for the next 15years from 1st delivery day (if not callable) at least 15years to maturity from 1st delivery day	1/32 of 100 points (31,25 per basis point) - except for intermonths spreads Price quote: 80-16 = (80+16/32)x1000 USD	Mar, Jun, Sep, Dec
CBOT - 10y Treasury Note Future	Face value 100,000 USD	6% Invoice price = futures settlement price x CF + Accrued interests	US Treasury notes with maturity from (min) 6,5y up to 10y from 1st delivery day	1/64 of 100 points (15,625 per basis point) - except for intermonths spreads Price quote: 80-165 = (80+16,5/32)x1000 USD	Mar, Jun, Sep, Dec
CBOT - 5y Treasury Note Future	Face value 100,000 USD	6% Invoice price = futures settlement price x CF + Accrued interests	US Treasury notes with maturity of (min) 4y2m up to 5y3m from 1st delivery date. 5y T-notes delivered after last trading date are not elligible	1/64 of 100 points (15,625 per basis point) - except for intermonths spreads Price quote: 80-165 = (80+16,5/32)x1000 USD	Mar, Jun, Sep, Dec
CBOT - 2y Treasury Note Future	Face value 200,000 USD	6% Invoice price = futures settlement price x CF + Accrued interests	US Treasury notes with original maturity no more 5y3m from 1st day of delivery month and remaining maturity of min 1y9m on 1st delivery date and max 2y from last delivery month date	1/128 of 100 points (15,625 per basis point) - except for intermonths spreads Price quote: 80-162 = (80+16,25/32)x1000	Mar, Jun, Sep, Dec
LIFFE - Long Gilt	Face value 100,00 GBP	6% Invoice price = futures settlement price x CF + Accrued interests		10£ per bp Quotation per 100£ nominal	Mar, Jun, Sep, Dec
Eurex - Euro Bund	Face value 100,000 EUR	6% Invoice price = futures settlement price x CF + Accrued interests	Bonds issued by Federal Republic of Germany with maturity between 8,5y to 10,5y	10EUR per bp	Mar, Jun, Sep, Dec

Specific delivery dates:

Type of contract	Last Intention day	Last Notice day	Last Trading day	Last Delivery day	Trading hours
CBOT - 30y Treasury Bond Futures	Second BD before last BD of contract's delivery month	Next-to-last BD of contract's delivery month	7 BD before last BD of delivery month - 12.00 Chicago Time	Last BD of delivery month	Open auction: Mon-Fri, 7.20 - 14.00, Chicago Time Electronic: Sun-Fri, 18.00-16.00, Chicago Time
CBOT - 10y Treasury Note Future	Second BD before last BD of contract's delivery month	Next-to-last BD of contract's delivery month	7 BD before last BD of delivery month - 12.00 Chicago Time	Last BD of delivery month	Open auction: Mon-Fri, 7.20 - 14.00, Chicago Time Electronic: Sun-Fri, 18.00-16.00, Chicago Time
CBOT - 5y Treasury Note Future	1st BD following end of delivery month	2nd BD following end of delivery month	Last BD of the delivery month	3rd BD following last Trading day (=end of delivery month)	Open auction: Mon-Fri, 7.20 - 14.00, Chicago Time Electronic: Sun-Fri, 18.00-16.00, Chicago Time
CBOT - 2y Treasury Note Future	1st BD following end of delivery month	2nd BD following end of delivery month	Last BD of the delivery month	3rd BD following last Trading day (=end of delivery month)	Open auction: Mon-Fri, 7.20 - 14.00, Chicago Time Electronic: Sun-Fri, 18.01-16.00, Chicago Time
LIFFE - Long Gilt	BD following last Trading day	BD following last Trading day	2 BD prior to last BD of delivery month at 11.00	Last BD of delivery month	08.00-18.00 London Time
Eurex - Euro Bund	Last Trading day	Last Trading 20.00 CET	2BD prior to Last Delivery day -12.30 CET	10th calendar day (or 1BD follow) of delivery month	08.00-19.00 CET

6.3 Conversion Factor calculation for the Eurex stock exchange :

Source : www.eurexchange.com

EUR-Denominated bonds (Lin Act/Act) :

$$CF = \frac{1}{1.06^{f}} \cdot \left[\frac{c}{100} \cdot \frac{\delta_i}{act_2} + \frac{c}{6} \cdot \left(1.06 - \frac{1}{1.06^{n}} \right) + \frac{1}{1.06^{n}} \right] - \frac{c}{100} \cdot \left(\frac{\delta_i}{act_2} - \frac{\delta_e}{act_1} \right)$$

f $\quad 1 + \frac{\delta_e}{act_1}$

δ_e NCD1y - DD

act_1 $\quad \begin{cases} NCD - NCD1y & ,\delta_e < 0 \\ NCD1y - NCD2y & \delta_e \geq 0, \end{cases}$

δ_i NCD1y - LCD

act_2 $\quad \begin{cases} NCD - NCD1y & ,\delta_i < 0 \\ NCD1y - NCD2y & \delta_i \geq 0, \end{cases}$

c coupon

n integer years from the NCD until the maturity date of the bond

DD Delivery Date

NCD Next Coupon Date

NCD1y 1 year before the NCD

NCD2y 2 years before the NCD

LCD Last Coupon Date before the delivery date

CHF-Denominated bonds:

$$CF = \frac{1}{1.06^{f}} \cdot \left[\frac{c}{6} \cdot \left(1.06 - \frac{1}{1.06^{n}} \right) + \frac{1}{1.06^{n}} \right] + \frac{c \cdot (1 - f)}{100}$$

n Number of integer years until maturity of the bond

f Number of full months until the next coupon date, divided by 12

(except for f = 0, where f = 1 and n = n-1)

c Coupon

6.4 Conversion Factor publication at the CBOT

Data published by the CBOT and available on its homepage.

3-YEAR U.S. TREASURY NOTE FUTURES CONTRACT

This table contains conversion factors for all short-term U.S. Treasury notes eligible for delivery as of September 7, 2010.
(The next auction is tentatively scheduled for October 12, 2010.)
Conversion factors in this document are based on a 6 percent notional coupon.

		Coupon	Issue Date	Maturity Date	Cusip Number	Issuance (Billions)	déc. 2010	mars. 2011
1,)		1 1/8	06/15/10	06/15/13	912828NH9	$36,0	-----	-----
2,)		3 3/8	06/30/08	06/30/13	912828JD3	$20,0	-----	-----
3,)		1	07/15/10	07/15/13	912828NN6	$35,0	-----	-----
4,)		3 3/8	07/31/08	07/31/13	912828JG6	$21,0	-----	-----
5,)		3/4	08/16/10	08/15/13	912828NU0	$34,0	-----	-----
6,)		3 1/8	09/02/08	08/31/13	912828JK7	$22,0	-----	-----
7,)	Bond1	3/4	09/15/10	09/15/13	912828NY2	$33,0	0,8687	-----
8,)	Bond2	3 1/8	09/30/08	09/30/13	912828JM3	$24,0	0,9280	-----
9,)	Bond3	2 3/4	10/31/08	10/31/13	912828JQ4	$24,0	0,9164	-----
10,)	Bond4	2	12/01/08	11/30/13	912828JT8	$26,0	0,8944	-----
11,)	Bond5	1 1/2	12/31/08	12/31/13	912828JW1	$28,0	0,8781	0,8874
12,)		1 3/4	02/02/09	01/31/14	912828JZ4	$30,0	-----	0,8907
13,)		1 7/8	03/02/09	02/28/14	912828KF6	$32,0	-----	0,8911
14,)		1 3/4	03/31/09	03/31/14	912828KJ8	$34,0	-----	0,8849
15,)		1 7/8	04/30/09	04/30/14	912828KN9	$35,0	-----	-----
16,)		2 1/4	06/01/09	05/31/14	912828KV1	$35,0	-----	-----

6.5 Delivery process

As mentioned in the section 2.6 Delivery Options, only around 1% of the futures contracts ends with the physical delivery of a treasury security. Other contracts are either closed out (selling of the contract or buying of an opposite contract) or rolled forward (closing of actual contract and establishing a similar position in a deferred contract). Since there is exactly the same number of short contracts as there are long, all remaining “open positions” have an opposite matching contract. The procedure of rolling or closing being quite simple (buying or selling in the usual market), we will only discuss, in this section the physical delivery.

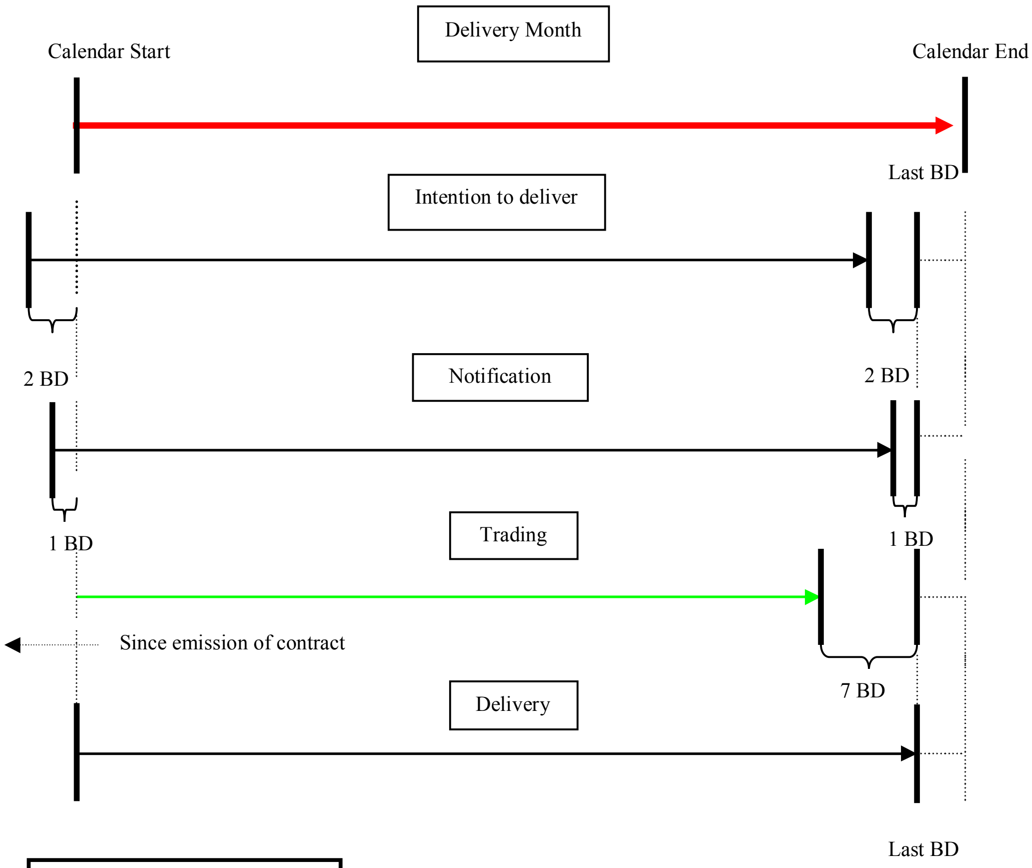
As opposed to short futures, long financial futures are not settled within a day of trading and the delivery process can span several days up to a month (see appendix A for last delivery date for example). In the following table, an overview of the delivery timetable for CBOT contracts is presented:

	Short	Clearing Service Provider	Long
First Position Day (Two business days prior to the named delivery month.)			By 8:00 pm, two days prior to the first day allowed for deliveries in expiring futures, clearing firms report to the Clearing Services Provider all open long positions, by origin (i.e., house or customer) and trade date.
Day 1: **Intention day**	By 8:00 pm, the short clearing firm notifies the Clearing Services Provider that it intends to make delivery on an expiring contract. Once the Clearing Services Provider has matched the short clearing firm with the long clearing firm(s), this declaration cannot be reversed.	By 10:00 pm, the Clearing Services Provider matches the delivering short's clearing firm to the clearing firm (or firms) with long positions having the oldest trade date(s). Each party – long and short – is informed of the opposite party's intention to make or take delivery.	By 8:00 pm all clearing firms report to the Clearing Services Provider all open long positions in expiring futures contracts, by origin (i.e., house or customer) and trade date.
Day 2: **Notice Day**	By 2:00 pm (3:00 pm on Last Notice Day), using calculations based on the expiring contract's Intention Day settlement price, the short clearing firm invoices the long clearing firm through the Clearing Services Provider.		By 4:00 pm, the long clearing firm provides the short clearing firm with the name and location of its bank.
Day 3: **Delivery Day**	Short and long clearing firms have until 9:30 am to resolve invoice differences. By 10:00am, the short clearing firm deposits Treasury securities for delivery into its bank account, and it instructs its bank to transfer the securities, via Fed wire, to the long clearing firm's account versus payment no later than 1:00 pm.		By 7:30 am the long clearing firm makes funds available and notifies its bank to remit the funds and accept Treasury securities. By 1:00 pm, the long clearing firm's bank has accepted the Treasury securities and, at the same time, has remitted the invoice amount via Fed wire to the short clearing firm's account.

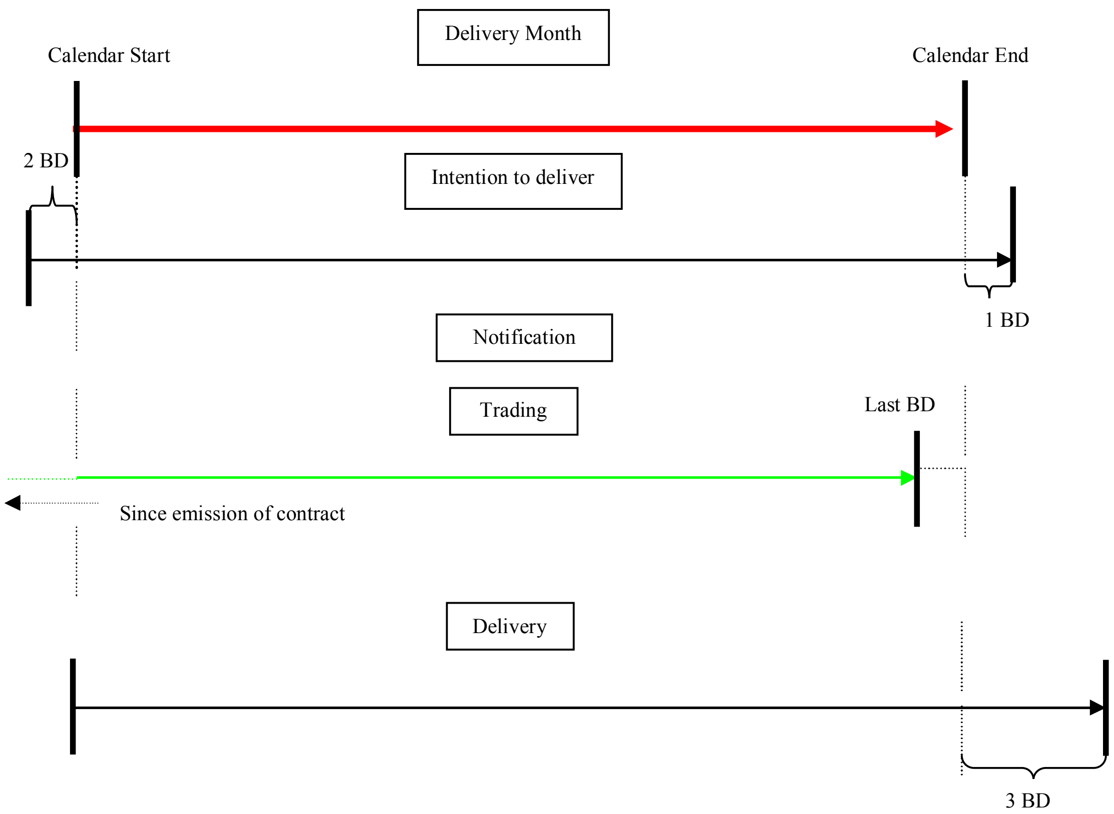
Source: CBOT

Note that the different days where the different positions and intentions are declared are not unique but also spans during the delivery month (i.e there is a first and last day of intention/notice/delivery/trading), as explained by the following graph :

30y T-bonds& 10y T-note futures



5y T-Notes& 2y T-note futures



Delivery matching :

Short Clearing Firms report no later than 8.00 pm the short positions that have declared intention to deliver:

F – Customer 900 contracts
G – Customer 100 contracts
H – House 1 000 contracts
Total contracts :2000 contracts

Long Clearing Firms by 8.00 pm EVERY open long positions by origin and vintage date

Clearing Service Provider « builds » the long position pool by selecting the long positions by vintage date

Vintage Date1	H – Customer	150 contracts	Total contracts : 290
	J – Customer	50 contracts	
	J – House	90 contracts	
Vintage Date2	H – Customer	10 contracts	Total contracts : 60
	L – Customer	50 contracts	
Vintage Date3	G – House	1 000 contracts	Total contracts : 9 000
	M – Customer	5 000 contracts	
	M – House	3 000 contracts	

The total nb of contracts exceeds short stack :

The CSP extracts a pro rata pool of long positions from last vintage date

Final Long position pool

H – Customer 150 contracts
J – Customer 50 contracts
J – House 90 contracts

H – Customer 10 contracts
L – Customer 50 contracts

G – House 183 contracts
M – Customer 917 contracts
M – House 550 contracts

Short positions declared on a specific intention day

F – Customer 900 contracts
G – Customer 100 contracts
H – House 1 000 contracts
Total contracts :2000 contracts

Clearing Service Provider randomly selects a short position to be matched

For example:

G-Customer 100

Pool of long positions as established by the Clearing Service Provider

H – Customer 150 contracts
J – Customer 50 contracts
J – House 90 contracts

H – Customer 10 contracts
L – Customer 50 contracts

G – House 183 contracts
M – Customer 917 contracts
M – House 550 contracts

Clearing Service Provider randomly selects a short position to be matched

For example:

J-House 90

Not Enought to cover the short position

CSP extracts a new RANDOM position

For example:

G-House 183

10 contracts from this position are needed, the rest are returned to the long position pool

Process is repeated until every short position is matched

6.6 Data for Zero rate curve construction

Date	US Libor O/N	US Libor 1W	US Libor 2W	US Libor 1M	US Libor 3M	US Libor 6M	US Libor 12M	SWAP LIBOR 1y	SWAP LIBOR 2y	SWAP LIBOR 3y	SWAP LIBOR 4y	SWAP LIBOR 5y	SWAP LIBOR 7y	SWAP LIBOR 10y
07/09/2010	0,22688	0,25181	0,25213	0,25766	0,29188	0,48875	0,82594	0,44	0,68	0,98	1,31	1,62	2,13	2,6

Source : Bloomberg & www.swap-rates.com

6.7 Data for Dec 2010 3y

Date	912828NY2 Govt CUSIP	912828JM3 Govt CUSIP	912828JQ4 Govt CUSIP	912828JT8 Govt CUSIP	912828JW1 Govt CUSIP	3YZ0 Comdty
Bond n°	Bond1	Bond2	Bond3	Bond4	Bond5	3y note Future
Conversion Factor	0,8687	0,928	0,9164	0,8944	0,8781	n/a
07/09/2010	115,060579	115,5037716	115,806416	116,051962	116,3556258	115,125
08/09/2010	114,9346725	115,3353987	115,618862	115,859794	116,1598907	115,125
09/09/2010	114,6828594	115,0828394	115,346055	115,597747	115,857391	115,125
10/09/2010	114,6468862	115,0323276	115,294904	115,545338	115,8040087	114,5
13/09/2010	114,8267526	115,1838631	115,482459	115,737506	116,035332	114,5
14/09/2010	115,0066191	115,3690733	115,670013	115,929674	116,2310671	114,4844
15/09/2010	115,060579	115,4195851	115,704114	115,947143	116,2488612	114,4844
16/09/2010	115,0785657	115,3859106	115,670013	115,947143	116,1954789	114,4844
17/09/2010	115,1505123	115,4532597	115,738215	116,017023	116,2844494	114,4844
20/09/2010	115,1864856	115,4869343	115,789366	116,051962	116,3556258	114,4844
21/09/2010	115,4382986	115,7226563	116,045122	116,331479	116,7115078	114,4844
22/09/2010	115,3483654	115,6216325	115,95987	116,24413	116,5513609	114,4844
23/09/2010	115,420312	115,6721444	116,028072	116,29654	116,569155	114,6797
24/09/2010	115,3303787	115,6216325	115,908719	116,191721	116,4801845	114,6641
27/09/2010	115,4382986	115,6889817	116,011021	116,314009	116,6403314	114,7422
28/09/2010	115,5102452	115,705819	116,062173	116,401358	116,7648901	114,8125
29/09/2010	115,4742719	115,6721444	116,028072	116,331479	116,6937137	114,7891
30/09/2010	115,5282318	115,705819	116,079223	116,436298	116,7648901	114,7969
01/10/2010	115,5462185	115,7226563	116,079223	116,436298	116,7826842	114,8281
04/10/2010	115,6001784	115,7900054	116,147425	116,506177	116,8894488	114,9297
05/10/2010	115,672125	115,8405172	116,198576	116,558587	116,9428311	114,9688
06/10/2010	115,8160182	115,9752155	116,35203	116,715815	117,1385662	115,1016
07/10/2010	115,8609416	116,0257543	116,403208	116,785666	n/a	115,1719
08/10/2010	115,8790146	116,0257543	116,420231	116,785666	n/a	115,2344
11/10/2010	115,8790146	116,0425647	116,403208	116,80322	n/a	115,2344
12/10/2010	115,7530793	115,9078664	116,283828	116,680903	n/a	115,1641
13/10/2010	115,7800161	115,9246767	116,300851	116,698345	n/a	115,1953
14/10/2010	115,6541959	115,7899784	116,18158	116,558587	n/a	115,1172
15/10/2010	115,7080695	115,8237069	116,232649	116,593582	n/a	115,125
18/10/2010	115,7800161	115,9246767	116,266805	116,698345	n/a	115,2266
19/10/2010	115,8879936	115,9920259	116,403208	116,785666	n/a	115,3047
20/10/2010	115,977898	116,0425647	116,47141	116,838104	n/a	115,3672
21/10/2010	115,9419823	116,008944	116,420231	116,80322	n/a	115,3672
22/10/2010	115,9059514	115,9584052	116,369053	116,785666	n/a	115,3672
25/10/2010	115,9059514	115,9415948	116,335007	116,768225	n/a	115,3594
26/10/2010	115,7261425	115,7563578	116,147425	116,541145	n/a	115,1641
27/10/2010	115,6361229	115,6552802	116,028045	116,418828	n/a	115,0781
28/10/2010	115,8700357	115,891056	116,266805	116,680903	n/a	115,2656
29/10/2010	116,0498446	116,0425647	116,437254	116,838104	n/a	115,4375
01/11/2010	116,0318867	116,0257543	116,420231	116,838104	n/a	115,4688
02/11/2010	116,0139289	116,008944	116,403208	116,80322	n/a	115,4688
03/11/2010	116,1038333	116,0762931	116,488433	116,907983	n/a	115,4688

Date	912828NY2 Govt CUSIP	912828JM3 Govt CUSIP	912828JQ4 Govt CUSIP	912828JT8 Govt CUSIP	912828JW1 Govt CUSIP
Bond n°	Bond1	Bond2	Bond3	Bond4	Bond5
Conversion Factor	0,8687	0,928	0,9164	0,8944	0,8781
04/11/2010	116,1937378	116,1604526	116,573658	116,995304	n/a
05/11/2010	115,9958559	115,9415948	116,35203	116,750783	n/a
08/11/2010	115,8700357	115,8237069	116,215626	116,628466	n/a
09/11/2010	115,6181651	115,5543103	115,959843	116,366391	n/a
10/11/2010	115,7261425	115,6384698	116,0622	116,453824	n/a
11/11/2010	115,6901117	115,6048491	115,993998	116,401386	n/a
12/11/2010	115,3124209	115,234375	115,618835	116,016995	n/a
15/11/2010	114,9886382	114,8976293	115,277826	115,650157	n/a
16/11/2010	115,1864856	115,0828664	115,448385	115,842352	n/a
17/11/2010	115,2584321	115,1502155	115,516587	115,929674	n/a
18/11/2010	115,2225164	115,0996767	115,482431	115,89479	n/a
19/11/2010	115,1864856	115,0491379	115,41423	115,824911	n/a
22/11/2010	115,456314	115,3017241	115,687036	116,104428	n/a
23/11/2010	115,4922298	115,3185345	115,704168	116,121869	n/a
24/11/2010	115,1505698	114,9817888	115,36316	115,755031	n/a
25/11/2010	115,1685277	114,9817888	115,36316	115,755031	n/a
26/11/2010	115,2404743	115,0491379	115,431362	115,842352	n/a
29/11/2010	115,294463	115,0828664	115,482431	115,912232	n/a
30/11/2010	115,456314	115,2511853	115,65299	116,086874	n/a
01/12/2010	115,0785657	114,8470905	115,24378	115,650157	n/a
02/12/2010	115,0425924	114,7966595	115,175578	115,597719	n/a
03/12/2010	115,294463	115,0155172	115,397206	115,824911	n/a
06/12/2010	115,5282606	115,234375	115,635967	116,086874	n/a
07/12/2010	115,114539	114,8134698	115,209625	115,632715	n/a
08/12/2010	114,7727984	114,4429957	114,817547	115,230881	n/a
09/12/2010	114,7368251	114,4429957	114,817547	115,213439	n/a
10/12/2010	114,5569587	114,2578664	114,612942	115,021243	n/a
13/12/2010	114,7907793	114,4935345	114,851593	115,265877	n/a
14/12/2010	114,4490388	114,1399784	114,510585	114,898927	n/a
15/12/2010	114,3950731	114,0894397	114,442383	114,811606	n/a
16/12/2010	114,592932	114,2746767	114,629965	115,003801	n/a
17/12/2010	114,7188327	114,4429957	114,749345	115,14356	n/a
20/12/2010	114,7188327	114,4429957	114,749345	115,14356	n/a
21/12/2010	114,7008518	114,3925647	114,732213	115,126118	n/a
22/12/2010	114,5569587	114,2409483	114,578787	114,968806	n/a
23/12/2010	114,4310464	114,0894397	114,442383	114,811606	n/a
24/12/2010	114,4310464	114,10625	114,42536	114,811606	n/a
27/12/2010	114,502993	114,1567888	114,493562	114,881485	n/a
28/12/2010	114,2152066	113,8705819	114,203732	114,584526	n/a
29/12/2010	114,6648786	114,2578664	114,629965	115,021243	n/a
30/12/2010	114,6109129	114,1905172	114,578787	114,968806	n/a
31/12/2010	114,7368251	114,3252155	114,71519	115,108564	n/a

Source : Bloomberg

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