Journal Of Financial And Strategic Decisions
Volume 10 Number 2 Summer 1997

AN EMPIRICAL INVESTIGATION OF PUT OPTION PRICING: A SPECIFICATION TEST OF AT-THE-MONEY OPTION IMPLIED VOLATILITY

Hongshik Kim*, Jong Chul Rhim** and Mohammed F. Khayum***

Abstract

We statistically test the robustness of implied volatility estimates across option pricing models for at-the-money put options. The results of the specification tests show that the implied volatility estimate recovered from the Black-Scholes European option pricing model is nearly indistinguishable from the implied volatility estimate obtained from the MacMillan/Barone-Adesi and Whaley's American put pricing model. We also investigate whether the use of Black-Scholes implied volatility estimates in American put pricing model significantly affect the prediction of American put option prices. It is shown that as long as the possibility of early exercise is carefully controlled for in the calculation of implied volatilities, predictions of American put prices are not significantly affected when the Black-Scholes implied volatility estimates are used in a specific American put option pricing model.

INTRODUCTION

Previous research has emphasized that at-the-money options are more likely to be efficient in estimating implied volatilities than away-from-the-money options [11]. Since the price of at-the-money options is more sensitive to the volatility of underlying stocks it is argued that it should provide more information about the true stock return volatility than the price of options away-from-the-money. Beckers [2] examined various weighting schemes used to calculate implied volatilities and found that the best estimates are obtained by using only at-the-money options. MacBeth and Merville [8] derived implied volatilities from the Black-Scholes European call option pricing model [3] and found that implied volatilities for out-of-the-money call options are less than implied volatilities obtained from at-the-money call options. Their results also showed that implied volatilities for in-the-money call options are greater than those for at-the-money call options. MacBeth and Merville assumed that at-the-money options are correctly priced by the Black-Scholes model and concluded that in-the-money call options are underpriced and out-of-the-money call options are overpriced. However, their conclusions are contingent upon the validity of implied volatilities recovered from the Black-Scholes European option pricing model. Due to the Black-Scholes model's restrictive assumptions, these estimates of implied volatilities are subject to biases resulting from various sources such as: (1) the stochastic nature of stock return volatilities, (2) misspecification of the terminal stock price distribution, and (3) the presence of early exercise possibilities.

Based on the observed linear relationship between at-the-money option prices and stock return volatilities,[1] it has been shown that most of the problems mentioned above can be avoided or minimized if at-the-money options are used in estimating implied volatilities. Corrado and Miller [4] extended Feinstein's [6,7] argument that the Black-Scholes option pricing model can recover virtually unbiased stock return volatility estimates when volatility behaves stochastically. They show that other biases, in estimating implied volatilities, resulting from misspecification of stock price dynamics can also be minimized if implied volatilities are obtained from at-the-

*Daewoo Research Institute
**University of Southern Indiana
***University of Southern Indiana

money options. They further show that the linear relationship between at-the-money option prices and stock return volatilities is well preserved for American options suggesting that implied volatility estimates are nearly indistinguishable across option pricing models for at-the-money options.

The purpose of this paper is to empirically test the robustness of implied volatility estimates obtained from at-the-money options using two different option pricing models. One set of volatility estimates is recovered from the Black-Scholes option pricing model and the other set of estimates is obtained from an American put valuation model developed by MacMillan [9] and Barone-Adesi and Whaley [1]. To minimize biases induced from not accounting for early exercise possibilities when recovering implied volatilities from the Black-Scholes European option pricing model, European option prices implied from observed American put prices are obtained using the put-call parity theorem [10]. We then test whether implied volatilities recovered from the Black-Scholes European model are significantly different from those derived from the MacMillan/Barone-Adesi and Whaley's American put pricing model. Results show that these two estimates of implied volatilities are not significantly different from each other, suggesting that Corrado and Miller's argument that implied volatility estimates are nearly indistinguishable across option pricing models for at-the-money options is correct. To further investigate whether the use of different implied volatility estimates affects pricing of American put options, theoretical prices based on the two different estimates of volatilities are compared against observed market prices. It is shown that theoretical prices based on the Black-Scholes implied volatilities fall outside observed dealers bid-ask spread boundaries slightly more than do theoretical prices based on the American model implied volatilities. However, statistical tests show that theoretical option prices based on the two different volatility estimates are not statistically different from each other. The rest of the paper outlines the estimation methodology, provides a description of the data, discusses the empirical results, and provides some concluding remarks.

ESTIMATION METHODOLOGY AND THE DATA

Implied Volatility Estimates

Implied volatilities are estimated by using Whaley's [12] non-linear regression procedure which allows option prices to provide an implicit weighting scheme that yields an estimate of the standard deviation with minimum prediction error. Let $P_j(\sigma)$ denote the theoretical price of a put option given an estimate, σ, of the stock return volatility. The observed market price of put option is denoted by P_j. The prediction error, ε_j, is defined as follows:

Equation 1

$$\varepsilon_j = P_j - P_j(\sigma)$$

The estimate of σ is then determined by minimizing the sum of squared errors,

Equation 2

$$\sigma_{Whaley} = \sigma^* : \underset{(\sigma^*)}{Min} \sum_{j=1}^{N} (P_j - P_j(\sigma^*))^2$$

where N is the number of at-the-money option prices in each day and σ^* is the estimated parameter. A numerical search routine is designed to find the optimal σ^*. An initialization value is set at $\sigma_0 = 0.30$, and then the equation 2 is solved iteratively using a Taylor expansion of Pj around the initialization value, σ_0. Ignoring the higher-order terms, we get:

Equation 3

$$P_j - P_j(\sigma_0) = (\sigma_1 - \sigma_0)\frac{\partial P_j}{\partial \sigma}\Big|\, \sigma_0 + v_j$$

Applying the ordinary least squares regression technique to equation 3 until $|(\sigma_1 - \sigma_0)/\sigma_0| < 0.0001$ yields an estimate of the optimal σ^*.

Using the above mentioned procedure, two types of implied volatility estimates are derived in this paper. One implied volatility estimate is recovered from a specific American option pricing model using the observed market price of American put options. Thus, the estimate of σ_{MBAW} is determined by minimizing the sum of squared errors,

Equation 4

$$\sigma_{MBAW} = \sigma^* : \underset{(\sigma^*)}{Min} \sum_{j=1}^{N} (P_j^{AM} - P_j^{MBAW} (\sigma^*))^2$$

where P_j^{AM} is the observed market price of American put option and P_j^{MBAW} is the theoretical price generated by the MacMillan/Barone-Adesi and Whaley American put pricing model.

The other implied volatility estimate is derived from the Black-Scholes option pricing model using the market prices of European put option implied from the put-call parity relationship. That is, the estimate of σ_{BS} is determined by minimizing the sum of squared errors:

Equation 5

$$\sigma_{BS} = \sigma^* : \underset{(\sigma^*)}{Min} \sum_{j=1}^{N} (P_j^{EU} - P_j^{BS} (\sigma^*))^2$$

where P_j^{EU} is the market price of European put option and P_j^{BS} is the theoretical Black-Scholes put price.

Derivation of Theoretical Prices

Implied volatility estimates obtained using the estimation technique described above are used to generate theoretical prices for an American put option based on the quadratic approximation approach developed by MacMillan [9] and Barone-Adesi and Whaley [1]. The rationale behind this approach is that, given that both American and European option prices satisfy the well-known Black-Scholes partial differential equation, it must be true that the difference in prices between an American option and an otherwise identical European option (i.e., the early exercise premium) must also satisfy the same partial differential equation. Using a quadratic approximation technique, a solution for an American put valuation formula is derived as follows:

Equation 6

$P^{MBAW}(\sigma^*) = P^{BS}(\sigma^*) + A_1(S/S^*)^{q1}$ where $S > S^*$, and
$P^{MBAW}(\sigma^*) = X - S$ where $S < S^*$,

where:

$P^{MBAW}(\sigma^*)$	denotes the MacMillan/Barone-Adesi and Whaley American put price, given the volatility estimate of σ^*
$P^{BS}(\sigma^*)$	denotes the Black-Scholes European put price, given the volatility estimate of σ^*
A_1	$= -(S^*/q_1)\{1 - N[-d_1(S^*)]\}$
q_1	$= (1/2)\{-(M-1) - [(M-1)^2 + 4M/K]^{1/2}\}$
M	$= 2r/\sigma$
K	$= 1 - e^{-r(T-t)}$

S^* is a critical stock price which is obtained by solving the following equation:

Equation 7

$$X - S^* = P^{BS}(S^*) - \{1 - N[-d_1(S^*)]\}S^*/q_1$$

To investigate whether the use of different implied volatility estimates affects pricing of American put options, equation 6 is used to yield two different theoretical prices, given σ_{MBAW} and σ_{BS}.

The Data

This study uses quotation data on the most heavily traded equity options on the Chicago Board of Trade Options Exchange (*CBOE*) from November 5 to November 30, 1990.[2,3] Real time price quotation data for IBM stock options are selected from the Berkeley Options Data Base.[4] An added feature is the use of a *resorted format* data base in this paper.[5]

The market price of European put options, which is not directly observable from the market, is obtained from using the put-call parity relationship. Assuming that options markets are efficient in the sense that European put-call parity holds, and that investors are rational in the sense that holders of American call options do not prematurely exercise their call options when no dividends are to be paid until option maturity, the European put-call parity equation is reconstructed by replacing a European call with an American call:

Equation 8

$$C^A - P^E = S - Xe^{-rT}$$

where:

C^A is the value of an American call with a striking price of X and a maturity of T, and
P^E is the value of a European put with a striking price of X and maturity of T.

By rearranging, we obtain the market value of a European put option:

Equation 9

$$P^E = C^A - S + Xe^{-rT}$$

To impute the market value of European put options, all put-call pairs that meet the following requirements are selected.

(1) Both put and call options in a put-call pair are options on the same underlying stock, with the same strike price and the same maturity.
(2) The length of time between put and call quotes for a put-call pair must be less than 2 minutes.
(3) Put and call prices are at least $1.00.[6]
(4) Put and call option prices within a put-call pair must satisfy the American put-call parity boundary condition.[7]

To isolate options on non-dividend paying stock from options on dividend paying stock, only options in a period where no dividends are paid before option maturity are selected.[8] The price of options used in this study are averages of bid and ask quotes. For these time intervals during which the underlying stock price remains unchanged, the highest and lowest option prices are averaged,[9] and all put-call pairs where stock prices are different, and put prices are unique, are included. After the screening process, 7,795 usable put-call pairs (daily average of 433 pairs) are identified. The average of bid and ask yield quotations on Treasury-bills that mature closest to option expiration is used to estimate a risk free rate of interest. Daily data on annualized T-bill rates are obtained from the *Wall Street Journal*.

HYPOTHESIS TESTING AND RESULTS

Each day, implied volatilities for January contract and December contracts are estimated separately using equation 4 and equation 5. Table 1 presents the results of the Black-Scholes implied volatility estimates and the *MBAW* implied volatility estimates for IBM stocks during November, 1990. It is shown that on average the *MBAW* implied volatilities are generally greater than the Black-Scholes implied volatilities during the first half of the month while the Black-Scholes implied volatilities are slightly greater than the *MBAW* implied volatilities during the rest of the month.

Test of Difference Between Alternative Implied Volatility Estimates

To investigate whether the two implied volatility estimates are statistically different from each other, we test the following null hypotheses that the mean value of the *MBAW* implied volatilities are equal to the mean value of the Black-Scholes implied volatilities:

H_0: $\sigma_{MBAW} = \sigma_{BS}$

Rejection of the null would imply that the *MBAW* implied volatilities are statistically different from the Black-Scholes implied volatilities. The results reported in Table 2 show that although the mean value of the Black-Scholes implied volatilities is slightly higher than the mean value of the *MBAW* implied volatilities, the two implied volatility estimates are not significantly different from each other. This finding lends support to Corrado and Miller's [4] argument that implied volatility estimates are nearly indistinguishable across option pricing models for at-the-money options. It also suggests that option prices generated from the Black-Scholes European option pricing formula and the *MBAW* American option pricing model may provide the same information about future stock return volatility.

Test of Difference Between Theoretical Option Prices Based On σ_{MBAW} and σ_{BS}

Equation 6 is used to yield two different theoretical prices, given σ_{MBAW} and σ_{BS}, respectively. Table 3 presents the observed market price and the theoretical prices, given the two volatility estimates, for all options, for in-the-money options, for at-the-money options, and for out-of-the-money options. Both sets of theoretical prices overvalue in-the-money options and undervalue out-of-the-money options. The degree of mispricing is slightly greater for the theoretical prices based on Black-Scholes implied volatility than for those based on the *MBAW* implied volatility. To examine whether the two theoretical price predictions are indistinguishable from each other, we test the null hypothesis that the mean value of theoretical prices generated from using the black-Scholes implied volatility is equal to the mean value of theoretical prices based on the *MBAW* implied volatility. The results in Table 4 show that although the mean value ($2.85) of theoretical prices based on the Black-Scholes implied volatility is slightly greater than the mean value ($2.84) of theoretical prices based on the *MBAW* implied volatility, on average, both predictions are not statistically different from each other.

To further investigate whether the use of different implied volatility estimates affects pricing of American put options, theoretical prices based on the two different estimates of volatilities are compared against observed bid and ask quotes. Table 5 shows that the proportions of theoretical prices based on each of the two implied volatility estimates which falls outside dealers bid-ask spread boundaries are nearly indistinguishable (79.6% vs 79.8%) for in-the-money options. However, for out-of-the-money options the theoretical prices based on the American model fall outside the observed bid-ask spread boundaries slightly more (97.2%) than do theoretical prices based on the Black-Scholes implied volatility (94.6%). These results suggest that predictions of American put pricing are not significantly affected by the estimation of implied volatility whether the volatility estimate is recovered from the Black-Scholes European option pricing model or from a specific American put pricing model.

CONCLUSION

We statistically test the robustness of implied volatility estimates across option pricing models for at-the-money put options. The results of the specification tests show that the implied volatility estimates recovered from the Black-Scholes European option pricing model is nearly indistinguishable from the implied volatility estimates obtained from the MacMillan/Barone-Adesi and Whaley's American put pricing model. We also investigate whether the use of Black-Scholes implied volatility estimates in an American put pricing model significantly affect the prediction of American put option prices. It is shown that as long as the possibility of early exercise are carefully controlled for in the calculation of implied volatilities, predictions of American put prices are not significantly distorted when the Black-Scholes implied volatility estimates are used in the American put option pricing model used in this paper.

TABLE 1
Daily Implied Volatilities (σ) Of IBM Stocks During 11/05/90 - 11/30/90*

σ_{MBAW} is implied volatility recovered from *MBAW* American model using market prices of American put options.
σ_{BS} is implied volatility recovered from Black-Scholes European model using market prices of European put options.

Trade Date	Option Maturity	Number of Obs.	σ_{MBAW}	σ_{BS}
November 5, 1990	Dec. '90	224	0.2612	0.2566
	Jan. '91	98	0.2670	0.2718
November 6, 1990	Dec. '90	246	0.2357	0.2290
	Jan. '91	141	0.2404	0.2302
November 7, 1990	Dec. '90	221	0.2487	0.2435
	Jan. '91	139	0.2545	0.2473
November 8, 1990	Dec. '90	304	0.2545	0.2570
	Jan. '91	117	0.2604	0.2596
November 9, 1990	Dec. '90	245	0.2343	0.2337
	Jan. '91	162	0.2395	0.2360
November 12, 1990	Dec. '90	272	0.2281	0.2173
	Jan. '91	173	0.2290	0.2199
November 13, 1990	Dec. '90	174	0.2268	0.2218
	Jan. '91	178	0.2295	0.2220
November 14, 1990	Dec. '90	208	0.2209	0.2216
	Jan. '91	217	0.2260	0.2223
November 15, 1990	Dec. '90	151	0.2249	0.2276
	Jan. '91	129	0.2281	0.2246
November 16, 1990	Dec. '90	281	0.2084	0.2115
	Jan. '91	133	0.2171	0.2124
November 19, 1990	Dec. '90	300	0.2091	0.2110
	Jan. '91	157	0.2196	0.2176
November 20, 1990	Dec. '90	312	0.1965	0.2010
	Jan. '91	161	0.2121	0.2112
November 21, 1990	Dec. '90	252	0.1923	0.2013
	Jan. '91	174	0.2062	0.2110
November 26, 1990	Dec. '90	288	0.2116	0.2224
	Jan. '91	221	0.2238	0.2306
November 27, 1990	Dec. '90	297	0.1987	0.2060
	Jan. '91	168	0.2157	0.2213
November 28, 1990	Dec. '90	315	0.2015	0.2085
	Jan. '91	160	0.2206	0.2262
November 29, 1990	Dec. '90	356	0.2290	0.2401
	Jan. '91	200	0.2412	0.2504
November 30, 1990	Dec. '90	374	0.2081	0.2160
	Jan. '91	247	0.2241	0.2299

*November 23 data were excluded from the sample due to extremely thin trading during that day which was Friday after the Thanksgiving holiday.

TABLE 2
Results Of T-Tests That Mean Values Of σ_{MBAW} and σ_{BS} Are Equal

σ_{MBAW} is implied volatility recovered from *MBAW* American model using market prices of American put options.
σ_{BS} is implied volatility recovered from Black-Scholes European model using market prices of European put options.

T-Test	σ Type	N	Mean	Std. Dev.	t-value	Prob.>|t|
H0: $\sigma_{MBAW} = \sigma_{BS}$	σ_{MBAW}	36	0.2262	0.0188	-0.1678	0.8672
	σ_{BS}	36	0.2269	0.0171		

TABLE 3
Mean Values Of *MBAW* Theoretical Prices Of IBM Put Options: 11/05/90 - 11/30/90

$P(\sigma_{MBAW})$ is a theoretical price generated from *MBAW* American model given σ_{MBAW}
$P(\sigma_{BS})$ is a theoretical price generated from *MBAW* American model given σ_{BS}
P_{obs} is an observed market price of American put option

Moneyness	*N*	P_{obs}($)	$P(\sigma_{MBAW})$($)	$P(\sigma_{BS})$($)
All Options	7795	2.9610	2.8374	2.8518
In-the-Money Options	1351	5.1238	5.3427	5.3716
At-the-Money Options	2864	3.2307	3.2306	3.2617
Out-of-the-Money Options	3580	1.9289	1.5774	1.5729

σ_{MBAW} is implied volatility recovered from *MBAW* American model using market prices of American put options.
σ_{BS} is implied volatility recovered from Black-Scholes European model using market prices of European put options.

TABLE 4
Results Of T-Tests That Mean Values Of Theoretical Prices Are Equal

$P(\sigma_{MBAW})$ is a theoretical price generated from *MBAW* American model given σ_{MBAW}
$P(\sigma_{BS})$ is a theoretical price generated from *MBAW* American model given σ_{BS}

T-Test	Model Price	N	Mean($)	Std. Dev.	t-value	Prob.>|t|
H0: $P(\sigma_{MBAW}) = P(\sigma_{BS})$	$P(\sigma_{MBAW})$	7795	2.8374	1.6545	-0.5452	0.5856
	$P(\sigma_{BS})$	7795	2.8518	1.6511		

TABLE 5
Proportions Of *MBAW* Theoretical Prices Outside Bid-Ask Dealer Spread Boundaries

$P(\sigma_{MBAW})$ is a theoretical price generated from *MBAW* American model given σ_{MBAW}
$P(\sigma_{BS})$ is a theoretical price generated from *MBAW* American model given σ_{BS}
P_{obs}^{bid} is an observed bid price of American put options
P_{obs}^{ask} is an observed ask price of American put options

			Proportion of $P(\sigma_{MBAW})$ Outside Bid-Ask Boundary			Proportion of $P(\sigma_{BS})$ Outside Bid-Ask Boundary		
Moneyness	P_{obs}^{bid}	P_{obs}^{ask}	**N**	**%**	**Ave. Dev.($)**	**N**	**%**	**Ave. Dev.($)**
All Options	2.8972	3.0247	6056	77.7	0.1041	6365	81.7	0.0781
In-the-Money Options	5.0189	5.2288	1076	79.6	0.1625	1078	79.8	0.2021
At-the-Money Options	3.1625	3.2990	1502	52.4	0.0000	1899	66.3	0.0000
Out-of-the-Money Options	1.8843	1.9735	3478	97.2	0.3177	3388	94.6	0.3327

σ_{MBAW} is implied volatility recovered from *MBAW* American model using market prices of American put options.
σ_{BS} is implied volatility recovered from Black-Scholes European model using market prices of European put options.

ENDNOTES

1. For a graphical illustration, see pp. 278-280 of Cox and Rubinstein [5].
2. Among the 30 most actively traded equity options during this period, 11 to 15 options were IBM options. Daily average trading volume for IBM options exceeds 10,000 contracts.
3. November 23 data are excluded from the sample due to extremely thin trading on that day which was the Friday immediately following the Thanksgiving holiday.
4. The Berkeley Options Data Base is derived from the Market Data Report of the *CBOE*. The data base consists of records of bid-ask quotes and transaction data, time-stamped to the nearest second.
5. The resorted data are sorted into files, one for each trading day. Within each file, the records are sorted by ticker symbol and by chronological order.
6. Thinness in these options may result in unreasonable estimates due to the discreetness of the price change.
7. By filtering a sample based on this criterion, we avoid a joint test of market efficiency and model accuracy.
8. Since IBM stock went ex-dividend on November 5, 1990 and February 5, 1991, December and January option contracts traded during November 5 to November 30, 1990 are selected.
9. In an efficient market, no option prices are to be changed during a short time interval where underlying stock prices remain unchanged.

REFERENCES

[1] Barone-Adesi, G. and R.E. Whaley, "Efficient Analytic Approximation of American Option Values," *Journal of Finance* 42, June 1987, pp. 301-320.

[2] Beckers, S., "Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Volatility," *Journal Banking and Finance* 5, 1981, pp. 363-381.

[3] Black, F. and M.S. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, May/June 1973, pp. 637-659.

[4] Corrado, C. and T. Miller, "Efficient Volatility Estimates from Option-Implied Volatility," Working Paper, University of Missouri-Columbia, 1994.

[5] Cox, J.C. and M. Rubinstein, Options Market. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1985.

[6] Feinstein, S., "The Black-Scholes Formula is Nearly Linear in s for At-the-Money Options; Therefore Implied Volatilities From At-the-Money Options are Virtually Unbiased," Working Paper, Federal Reserve Bank of Atlanta and Boston University, 1989.

[7] Feinstein, S., "The Hull and White Implied Volatility," Working Paper, Boston University, 1992.

[8] MacBeth, J.D. and L.J. Merville, "An Empirical Examination of the Black-Scholes Call Option Pricing Model," *Journal of Finance* 34, 1979, pp. 1173-1186.

[9] MacMillan, L.W., "An Analytic Approximation for the American Put Option," *Advances in Futures and Options Research* 1, 1986, pp. 141-183.

[10] Stoll, H.R., "The Relationship Between Put and Call Option Prices," *Journal of Finance* 24, 1969, pp. 801-824.

[11] Stoll, H.R. and R.E. Whaley, *Futures and Options: Theory and Applications*, Cincinnati, OH: South-Western Publishing Co, 1993.

[12] Whaley, R.E., "Valuation of American Call Options on Dividend-Paying Stocks: Empirical Tests," *Journal of Financial Economics* 10, 1982, pp. 29-58.