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Managing Stochastic Volatility Risks of Interest Rate Options: Key Rate Vega

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Abstract: This paper empirically tested the one factor and two factor arbitrage-free interest rate models for swaptions in three currencies, U. S. dollar, Euro and Japanese yen. The models are shown to be robust in explaining the swaption valuation. Further, the implied volatility functions of the models are estimated. They are shown to be dynamic, exhibiting a three factor movement in all three currencies. The results show that the vega risks of interest rate contingent claims should be managed and that key rate vega measure may be needed to control the stochastic volatility risks.

Keywords: multi-factor arbitrage-free interest rate models, binomial lattice, interest rate derivatives, implied volatility function, vega risk, swaption, stochastic volatilities, volatility surface, key rate duration, key rate vega

1 Introduction

Arbitrage-free interest rate models such as Ho-Lee (1986, 2005), Heath, Jarrow and Morton (1992) HJM, Brace, Gatarek and Musiela (1997) BGM have broad applications in securities valuation. In particular, they are used extensively to value interest rate contingent claims such as derivatives and balance sheet items with embedded interest rate options. The models are also used to determine the key rate durations or PV 01 to specify the dynamic hedging strategies in managing interest rate risks by measuring the interest rate derivatives sensitivities to the key rates along the yield curve.

However, interest rate derivatives value can be significantly affected by the changes in

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the volatility, or that the vega measure is not negligible. Heidari and Wu (2003) uses the principal component analysis and shows that the volatility surface of swaptions has three orthogonal movements, independent of the principal movements of the yield curve. Collin-Dufresne and Goldstein (2002) uses interest rate straddles to provide empirical evidence of "unspanned stochastic volatility" showing that interest rate derivatives cannot be dynamically hedged or replicated by bonds (with no embedded options), because of the significant presence of the volatility risk. Other empirical studies of interest rate models have shown that the implied volatilities are stochastic (Amin and Morton (1994), De Jong, Driessen, and Pelsser (2001)). Further Amin and Ng (1997) has shown that the implied volatilities have informational content in predicting the future interest rate volatilities. Therefore, vega measure should be used to manage the volatility risk in hedging, risk reporting, integrating market risks on the trading floor or on the enterprise level.

To date, there are significant challenges to determine the vega buckets for interest rate derivatives. The first challenge is the computational intensity required in determining the measurement. Pietersz and Pelsser (2003) uses the BGM model to determine the vega buckets confining to swaptions on the anti-diagonal buckets (where the sum of expiry and tenor is 31 years) of the swaption volatility surface and that process requires 1 to 5.8 million scenario paths. However, Heidari and Wu shows that the volatility risk is not confined to the anti-diagonal buckets of the volatility surface. The entire surface generates the independent movements. Therefore, to extend the key rate duration measures to the volatility surface poses a practical problem.

Another challenge is the use of interest rate models. To determine the vega measure, we require the interest rate model to be (1) accurate in pricing the swaptions, (2) stable in the estimated parameters without overspecification, and (3) computationally efficient. Despite the prevalent use of arbitrage-free interest rate models, thus far, there is a lack of empirical evidence of an interest rate model that has the above three attributes.

Most empirical studies to date are limited to test the models using short dated interest rate derivatives or over a short sample period (Amin and Morton (1994), Mathis and Bierwag (1999), Gupta and Subrahmanyam (2005), Flesaker (1993)). And these tests assume non-stochastic term structure of volatilities in these tests. As a result, scant empirical studies have provided insights into the current practical use of the arbitrage-free interest rate models in valuing the long dated swaptions and measuring the vega, though some recent papers begin to deal with these issues.

Han (2005) and Jarrow, Li and Zhao (2004) extend the string model and the HJM model respectively to incorporate the unspanned stochastic volatility. These approaches require four stochastic factors for the yield curve movements and three stochastic factors for the volatility risks, and over ten parameters have to be estimated. Extensive computation may be required to compute the sensitivities of interest rate derivatives. These papers have not addressed these issues.

The purpose of this paper is to fill this void by providing a solution to manage the volatility risk. We do so by using the generalized Ho-Lee model (2005). We first show that the model has high explanatory power for the observed swaption prices across 1- 10 year expiry and 1-20 year tenor (the entire volatility surface), using a one factor and two factor model with four and five parameters, respectively. Then we show that the volatility surface movements can be represented by the movements of two volatility curves ("implied volatility functions") analogous to the yield curve in the volatility space. We can reduce the movement of the volatility surface to the movements of two curves. As a result, we can determine the vega at the key points on these curves, the "key rate vega". Finally, the recombining lattice framework of the generalized Ho-Lee model provides a computation efficient model to value interest rate derivatives. As a result, this paper, combining the use of key rate duration, provides a practical solution to manage the risk of interest rate derivatives.

This paper contributes to the extensive literature in interest rate modeling in several ways. First, this paper extends the empirical investigation. We examine the one factor and two factor generalized Ho-Lee models on the a broad range of swaptions, which are central to the derivatives market, over a five year period across three major currencies, U.S. dollar (USD), Euro (EUR), and Japanese yen (JPY). This extension provides the important validation of the model for a key segment of the derivatives market, over a longer sample period, in the major currencies. Second, we empirically specify the stochastic movements of the implied volatility functions to specify the principal movements of the volatility surface. As a result, this paper provides the empirical implications of the use of arbitrage-free interest rate models in the capital markets both in valuation and in managing the yield curve risks.

Our main results based on calibrating 91 at-the-money swaptions of the volatility surface show that (1) the time series of residual mean square errors of both the one factor and two factor are generally less than two Black volatility points, within typical bid-ask spreads for at-the-money swaptions in USD and EUR, and generally less than three Black volatility points in JPY; (2) the two factor model provides a slightly higher explanatory power, and both the one and two factor models have the least valuation errors in the USD market; (3) the implied volatility function has three orthogonal movements for each currency, together explaining over 95% of the implied volatility function movements; (4) the model parameters are stable, providing an effective key rate vega measure for risk management.

Our results can also provide some insights into the previous empirical literature. De Jong, Driessen, and Pelsser (2001) investigates the BGM model by studying the swaption returns. Our result shows that such tests have to control the vega risk. Our approach is similar to that of Amin and Morton in calibrating the implied volatility function. We differ in our sample. We analyze the swaptions and not the short dated Euro futures options, as in their test. Longstaff, Santa-Clara and Schwartz (2001) calibrates a four factor string model to 34 at-the-money swaptions, as opposed to 91 swaptions in this paper, with their errors ranging between 2% to 16%. Han (2005) extends the string model to incorporate an additional three stochastic volatility factors with lower errors. The performance of these models in fitting the swaption prices can be explained by the alternative approaches in modeling that we take in this paper.

The generalized Ho-Lee model differs from these models by calibrating (1) the correlations of the key rates, implying the yield curve movements from the swaption prices, and (2) the interest rate stochastic distribution (switching between two processes used in this paper), inferring a mix of "lognormal" and "normal" distributions also from the market prices. This approach is entirely consistent with the basic premise of the arbitrage-free rate movement models. This way, the model incorporates the continually changing views of the market in the yield curve movements, in the rates correlation and the distribution. By way of contrast, both the string model and the market model use historical correlations and pre-specified interest rate distributions. Such an approach would not incorporate the conditional expectation of the yield curve movements, for example, the anticipation of the change of yield curve steepening and the behavior of the tails of the distribution, to the valuation model.

The paper proceeds as follows. Section 2 describes the generalized Ho-Lee model, which provides the theoretical framework to specify the implied volatility function. Section 3 describes the empirical estimation method and the data. Section 4 presents the empirical results of the estimated errors and the specification of the implied volatility function movements for the one factor model of the three currencies. Section 5 compares the results of the two factor models to those of the one factor models. Section 6 introduces the key rate vega measure. Section 7 contains the conclusions.

2 Theoretical Model

Our approach is consistent with current practice of valuing interest rate contingent claims and the management of interest rate risks. Brace Gatarek and Musiela (1997) and Jamshidian (1997) proposed the market models, which are arbitrage-free models that fit not only the spot yield curve but also the caps/floors (LIBOR model) or a sample of swaptions (Swaption Model). In essence, these models fit the model volatilities to the benchmark options over the entire range of tenors instead of calibrating the implied volatility function to a sample of options as in Amin and Morton. The growing interests in such market models in capital markets underscore two important aspects of the arbitrage-free models assumed in practice.

First, the market benchmark options, for example, the caps/floors and at-the-money swaptions ("volatility surface"), are liquid. To many practitioners, the volatility surface is as important as the term structure of interest rates in valuing the interest rate derivatives. The entire market volatility surface, and not a volatility number, is often used to value derivatives, and therefore the arbitrage-free interest rate model must necessarily be consistent with these benchmark options. Second, the volatility function of the arbitrage-free model is not perceived to be constant. It is implicitly assumed that "prices in stochastic volatility models are of similar form to those in a constant volatility model, with volatility terms in the latter replaced by their conditional expected levels in the stochastic volatility environment. [Hull and White (1987)]" Thus, using a constant volatility model with market-implied volatility parameters achieves nearly the same effect.

2.1 Implied Volatility Function

Arbitrage-free interest rate models can be uniquely specified by the term structure of volatilities, a function of time and states. The parameters of the function can be implied from the observed prices of the traded options. Such a function is called the "implied volatility function", which can be interpreted as the market perceived interest rate uncertainties into the future.

Specifically, interest rate models may be specified as follows:

$$dr(t) = \alpha(t, r)dt + \sigma(t, r(t))dW(t) \tag{1}$$

where r is the instantaneous interest rate, $\alpha(t, r)$ is the drift term that fits the model to the observed spot curve, $\sigma(t, r(t))$ is the implied volatility function and dW is the standard Brownian motion.

Some examples of the implied volatility function are:

- Absolute (Ho-Lee): $\sigma(\cdot) = \sigma_0$
- Square Root (Courtedon) : $\sigma(\cdot) = \sigma_0 r(t)^{1/2}$
- Proportional (Cox, Ingersoll, Ross): $\sigma(\cdot) = \sigma_0 r(t)$
- Linear Absolute: $\sigma(\cdot) = \sigma_0 + \sigma_1(t)$
- Exponential (Vasicek): $\sigma(\cdot) = \sigma_0 \exp(-c(t))$

The generalized Ho-Lee model that we study empirically in this paper has the implied volatility function is given by

$$\sigma(a, b, c, d, R) = ((a + b(t)) \exp(-c(t)) + d) \min[r(t), R] \tag{2}$$

where R is called the threshold rate, enabling the model to switch from a normal model to a lognormal model when interest rates are low. This switch of regime would determine a lower bound for and disallow explosive rise of interest rates. If R is an arbitrarily large constant, then the model is a lognormal model. Conversely, if R is an arbitrarily small constant, then the model is a normal model. Figure 1 below depicts the behavior of the one factor generalized Ho-Lee model. The lattice shows that the interest rates rise linearly on the top boundary (a normal model) and the rates fall proportionally on the bottom boundary (a lognormal model). For the empirical test, we will fix the threshold rate. For clarity of the exposition, and without loss of generality, we will refer to the function below as the "implied volatility function" for the generalized Ho-Lee model. The level of the threshold rate R only affects the distribution of interest rates and not the specific shape of the implied volatility function. For this reason, keeping R constant does not affect the main conclusions of the paper.

$$\sigma(a, b, c, d) = (a + b(t)) \exp(-c(t)) + d \tag{3}$$

The parsimonious specification of the implied volatility function, using only four para-

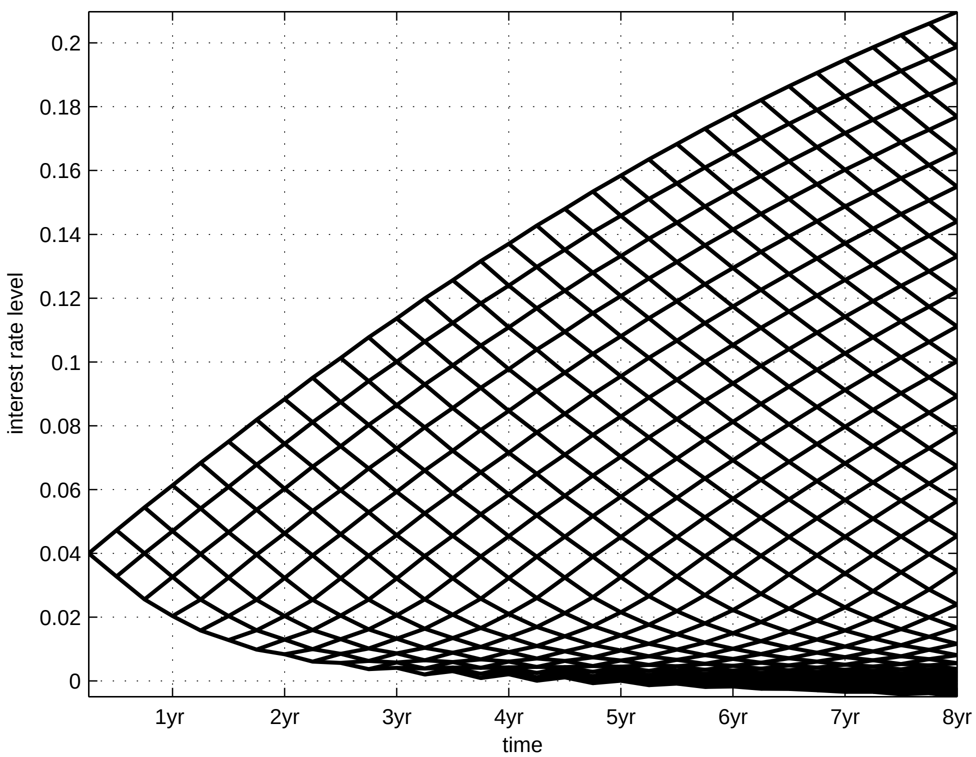

Fig. 1 The one factor generalized Ho-Lee model lattice. The lattice is constructed for quarterly step sizes with volatility parameters (a, b, c, d) equal to $(0.389, 0.042, 0.126, 0.096)$ respectively.

meters, avoids over specification of the model. In principle, any curve fitting methods using multiple parameters can be used to perfectly fit the implied volatility function to observed prices of the benchmark securities. See Lee and Choi (2005), which is akin to that employed by the market models. However, the purpose of this paper is not to show that the interest rate model can fit the spot curve and the volatility curve perfectly. The purpose of this paper is to show that the model with few parameters can explain many observed swaption prices over an extensive sample period. And for this reason, we are testing the functional form of equation (3) empirically.

The parameters (a, b, c, d) can be interpreted as follows. When interest rates are below the threshold rate, $(a - d)$ and d are the instantaneous and long term short rate volatilities, respectively. The constant c is the exponential decay rate which is directly related to the extent of the mean reversion process, and b determines the size of the hump of the volatility curve.

2.2 The Model

Intuitively, the model can be described as follows. The recursive construction of the recombining lattice is similar to that of the Black Derman and Toy (1990) BDT model. At each time step, ensuring the lattice to recombine, equation (2) is used to determine the local volatility at each node point, instead of requiring a lognormal distribution. Then an algebraic relationship is determine to fit the observed spot yield curve instead of using a Newton Ralphson's approach to calibrate the lattice. The lattice is then recursively constructed over n steps.

In the discrete time model, as usual, we assume the usual perfect capital market conditions and that everyone trades at the discrete time, $0 \leq i \leq n$. At each node, there are only two possible outcomes in the next period. The building blocks of the binomial model are the binomal volatilities δ_i^n, for $0 \leq i \leq n$. δ_i^n is the proportional decrease in the one period bond value P_i^n from state i to $i+1$ at time n at the end of a binomial period. Without loss of generality, we assume that the bond price decreases, and the bond yield increases, with state i, and hence $\delta_i^n < 1$. When $\delta_i^n = 1$, by definition, there is no risk at the binomial node with respect to the upstate and downstate outcomes. More generally, let $\delta_i^n(T)$ denote the binomial volatility of a T term bond. Since cash, bond with zero maturity, has no risk, by convention, we have

$$\delta_i^n(0) = 1 \tag{4}$$

The implied volatility function equation (3) in the binomial lattice framework is re-written as

$$\sigma(n) = (a + b \cdot n\Delta t)\exp(-c \cdot n\Delta t) + d \tag{5}$$

where Δt is the interval of one period, for $0 \leq i \leq n$. For example, if one period (the step size of the lattice) is one month, then Δt is 1/12.

The binomal volatilities δ_i^n are defined by the volatility function $\sigma(n)$ in the equation (5), as follows[‡]:

$$\delta_i^n = \exp(-2\sigma(n)\min[-\log P_i^n, R\Delta t]\Delta t^{1/2}) \tag{6}$$

[‡] Since this is a discrete time model, the interest rates can still become negative as a result of the discrete time approximation, even for some small volatilities when the rates are low. Equation (6) cannot ensure that δ_i^n are always bounded by one. For implementation, we use $\delta_i^n = \exp(-2\sigma(n)\max[\min[-\log P_i^n, R\Delta t]\Delta t^{1/2}, \epsilon])$ for some small ϵ , say, 0.0001 or 1 basis point, effectively

R is the threshold rate, as explained in the previous section. Equation (6) translates the volatility measure as the standard deviation of the proportional change in rates to the proportional change in prices.

By the construction of the arbitrage-free rate model, the binomial volatilities have to satisfy the recursive equation

$$\delta_i^n(T) = \delta_i^n \delta_i^{n+1}(T+1) \left[\frac{1+\delta_{i+1}^{n+1}(T-1)}{1+\delta_i^{n+1}(T-1)}\right] \tag{7}$$

The binomial volatilities in equation (7) specifies the one period bond pricing model at node (n, i):

$$P_i^n = \frac{P(n+1)}{P(n)} \prod_{k=1}^{n} \frac{(1+\delta_0^{k-1}(n-k))}{(1+\delta_0^{k-1}(n-k+1))} \prod_{j=0}^{i-1} \delta_j^{n-1} \tag{8}$$

This system of recursive equations (4)–(8) defines the binomial model of the generalized one factor Ho-Lee model.

The two factor Ho-Lee model can be specified analogously, below

$$\begin{aligned} P_{i,j}^n(T) = {} & \frac{P(n+1)}{P(n)} \times \prod_{k=1}^{n} \frac{(1+\delta_{0,1}^{k-1}(n-k))}{(1+\delta_{0,1}^{k-1}(n-k+1))} \\ & \times \frac{(1+\delta_{0,2}^{k-1}(n-k))}{(1+\delta_{0,2}^{k-1}(n-k+1))} \times \prod_{k=0}^{i-1} \delta_{k,1}^{n-1}(T) \times \prod_{k=0}^{j-1} \delta_{k,2}^{n-1}(T) \end{aligned} \tag{9}$$

where

$$\begin{aligned} \delta_{i,1}^n(T) &= \delta_{i,1}^n \delta_{i,1}^{n+1}(T+1) \left[\frac{1+\delta_{i+1,1}^{n+1}(T-1)}{1+\delta_{i,1}^{n+1}(T-1)}\right] \\ \delta_{i,1}^n(T) &= \delta_{i,2}^n \delta_{i,2}^{n+1}(T+1) \left[\frac{1+\delta_{i+1,2}^{n+1}(T-1)}{1+\delta_{i,2}^{n+1}(T-1)}\right] \end{aligned} \tag{10}$$

and by extending the specification of δ_i^n to the two factor model, we have,

$$\delta_{i,1}^n = \exp(-2\sigma_1(n) \min[-\log P_{i,1}^n, R\Delta t] \Delta t^{1/2}) \tag{11}$$

Similarly, we can define $\delta_{i,2}^n$ for the other factor, and we have

$$\delta_{i,2}^n = \exp(-2\sigma_2(n) \min[-\log P_{i,2}^n, R\Delta t] \Delta t^{1/2}) \tag{12}$$

The two factor generalized Ho-Lee model specifies the dynamics of the yield curve as two orthogonal movements. Figure 2 below depicts such a binomial interest rate lattice where one movement is a parallel movement and the other as a steepening movement. The steepening movement results in a stronger mean reversion behavior of the short term rate, as it is apparent at the elevation side when compared to the back side of the lattice.

switching the model to a normal model with an arbitrarily low volatility and determines a lower bound of the negative rates. See Ho and Lee (2005) for further extension of the model in controlling the extent of exhibiting negative rates of the model.

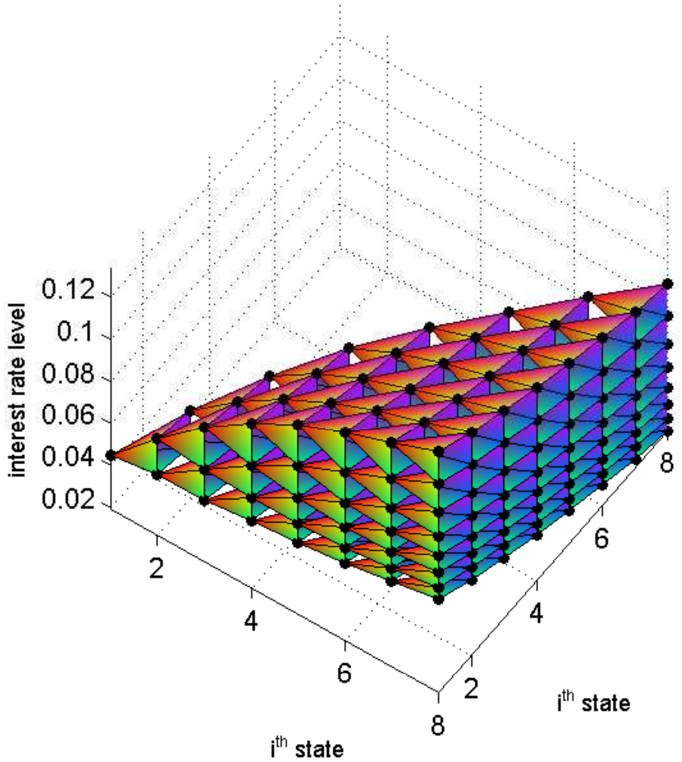

Fig. 2 Graphical representations of the two-dimensional recombining generalized Ho-Lee interest rate lattice. The lattice is constructed for annual step sizes with the first term structure of volatility parameters (a, b, c, d) equal to $(0.152, 0.077, 0.164, 0.125)$ and a flat second volatility of 0.083.

2.3 The Empirical Model

The empirical model is based on the one factor generalized Ho-Lee model (equations (4)–(8)) and the two factor model (equations (9)–(12)). The models are tested by the sample observations for each observation date (τ'), given the information set $\theta(\tau')$. Hence the implied volatility function of the one factor model (equation (5)) for the empirical model is re-expressed as:

$$\sigma(n|\theta(\tau')) = (a(\tau') + b(\tau') \cdot n\Delta t) \exp(-c(\tau') \cdot n\Delta t) + d(\tau') \tag{13}$$

That is, empirically, we assume that the volatility function is updated by the conditional expected levels at each observation date.

Analogously, for the two factor model, we specify the implied volatility functions as:

$$\sigma_1(n|\theta(\tau')) = (a(\tau') + b(\tau') \cdot n\Delta t) \exp(-c(\tau') \cdot n\Delta t) + d(\tau') \tag{14}$$

and

$$\sigma_2(n|\theta(\tau')) = e(\tau') \tag{15}$$

That is, we assume that the second principal movement is a parallel movement.

3 Data and the Empirical Estimation Methodology

3.1 Swaps and Swaptions

The empirical tests are based on the swaption prices. To describe the sample data, we begin with an overview of the swaption markets to exposit the market conventions and terminologies used in this paper. The swaptions are based on the vanilla swaps, where two parties agree to exchange a stream of cash flows over some specific period of time, where the time to the termination date is the tenor. At the time the swap is initiated, the coupon rate on the fixed leg of the swap is specified. This rate is chosen to make the present value of the fixed leg equal to the present value of the floating leg. The fixed rate at which a new swap with tenor T can be executed is known as the swap rate and we denoted it by $S(0,0,T)$, where the first argument refers to time zero, the second argument denotes the start date which is time zero for a standard swap, and T is the termination date of the swap.

Once a swap is executed, the fixed payments of $S(0,0,T)/2$ are paid semi-annually at times $0.50, 1.00, 1.50, \cdots, T-0.50$ and T. Floating payments follow the convention of quarterly payments at times $0.25, 0.50, 0.75, \cdots, T-0.25$, and T and are equal to 0.25 times the three-month LIBOR rate at the beginning of the quarter. A floating rate note paying three-month LIBOR quarterly must worth par at each quarterly LIBOR reset date. Since the initial value of the swap is zero, the initial value of the fixed leg must also worth par. The swap rates are available from a variety of sources, such as Bloomberg Financial Services, for standard swap tenors such as $1, 2, 3, 4, 5, 7, 10, 12, 15, 20, 25$ and 30 years, in real time.

We use vanilla European swaptions in this paper. The holder of the swaption has an option to enter into a swap and receive (or pay) fixed payments. The holder of the option has a right at time τ to enter into a swap with a remaining term $T-\tau$, and receive (or pay) the fixed annuity of c. This option is called a τ into $T-\tau$ receivers (or payers) swaption, where τ is the time to expiration of the option and $T-\tau$ is the tenor of the underlying swap.

The convention in the swaptions market is to quote prices in terms of their implied volatilities relative to the Black (1976) model as applied to the forward swap rate. The Black model implies that the value of a τ by T European payers swaption at time zero is

$$V(0,\tau,T,c) = \frac{1}{2}A(0,\tau,T)\left[S(0,\tau,T)\Phi(d) - c\Phi(d-\sigma\sqrt{\tau})\right] \tag{16}$$

where

$$d = \frac{\ln(S(0,\tau,T)/c) + \sigma^2\tau/2}{\sigma\sqrt{\tau}} \tag{17}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function and σ is the volatility of the forward swap rate, and $A(0,\tau,T)$ is the present value of the annuity interest payments. In the special case where the swaption is at the money the above valuation formula reduces to

$$V(0,\tau,T,S(0,\tau,T)) = (D(0,\tau) - D(0,T)) \left[2\Phi(\sigma\sqrt{\tau}/2) - 1\right] \tag{18}$$

Since this receiver swaption is at-the-money forward, the value of the corresponding payers swaption is identical. Note that when an at-the-money swaption is quoted at an implied volatility σ, the actual price that is paid by the purchaser of the swaption is given by substituting σ into equation (18).

The sample period we have chosen is 7/21/2000-6/21/2005, based on monthly data. We have chosen this period because this period has experienced significant volatilities including the burst of the internet bubble, the September 11 tragedy that led to the dramatic fall in interest rates, particularly the short term rate leading to significant steepening of the yield curve, and the subsequent rise in interest rates. The significant yield curve movements over the period are depicted in Figure 3 for USD and in Appendix B for EUR and JPY.

The swaption prices and interest rates for this paper are obtained from Bloomberg Financial Services, which collects and aggregates market quotations from a number of brokers and dealers in the derivatives market. We use three major currencies: USD, EUR and JPY, and the swaption are at-the-money options with expiration 1, 2, 3, 4, 5, 7,10 years and tenor 1, 2, 3, 4, 5, 6, 7, 8, 9,10, 12, 15, 20 years. Therefore there are approximately 91 swaption observations for each month. There are 60 observation dates in three currencies, in total 16,380 swaption observations. A summary description of the data is provided in Appendix A. Figure 4 depicts the volatility surface of the USD market on four sample dates.

These figures plot the quoted volatilities of USD swaptions on four different dates of the sample. Each figure shows the quotes for the swaptions with tenors between one month and ten years on the underlying swaps with the times to expiration of the options between one and thirty years. The volatility surfaces show that the volatilities tend to decrease with the time to expiration of the options and the tenor. At time, the surface exhibits a hump for the short time to expiration and tenor. Such observations motivate the specification of the functional form of the implied volatility functions of equation (5).

3.2 Empirical Methodology

The empirical test seeks to estimate equations (13) (14) (15) using the Ho -Lee model (equations 4–12) to fit the observed swaption prices.

In our estimation, we fix the threshold rates R to be 3%, 5% and 7% for USD, EUR and JPY respectively. They are chosen to minimize the errors over the sample period. The threshold rates may also be used to calibrate the model for each sample date. But the additional complexity does not significantly affect the main results of this paper, as

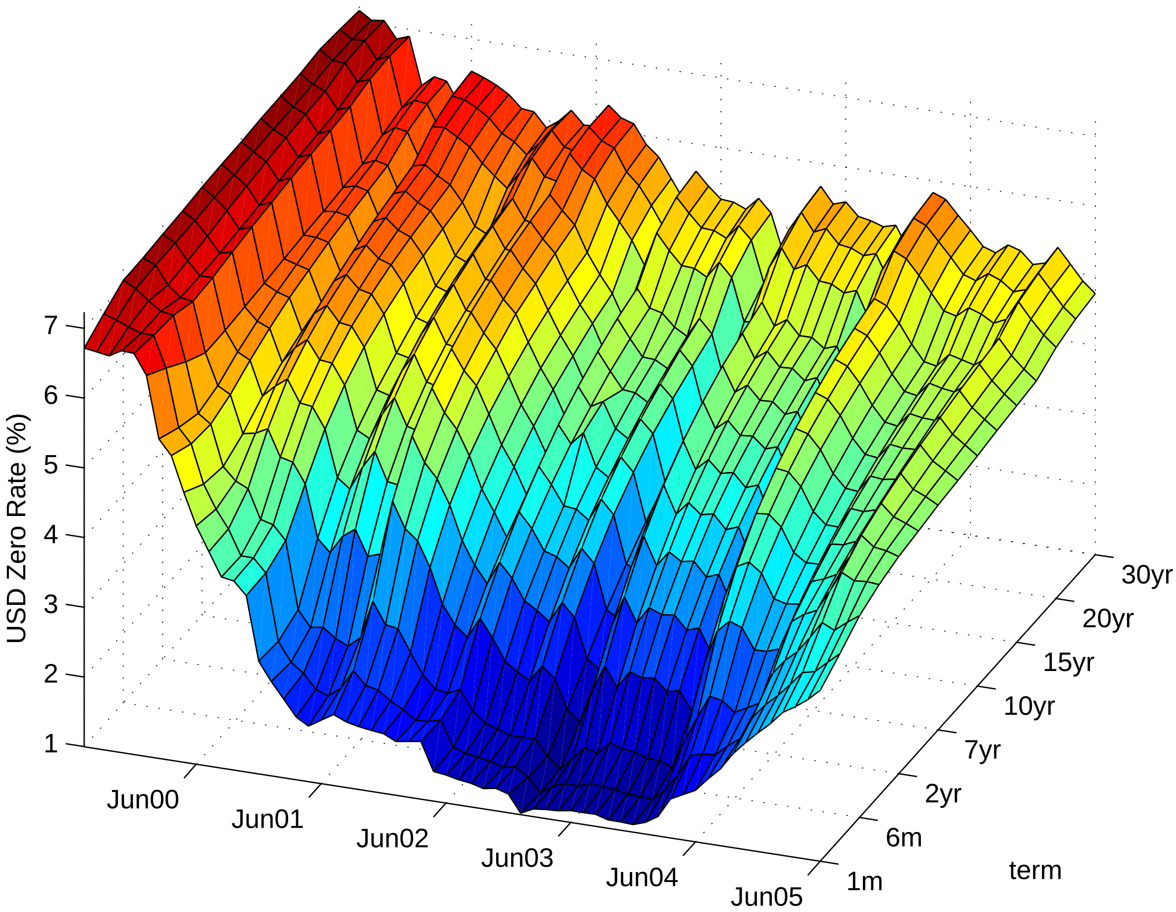

Fig. 3 Time Series of USD zero rates. The data set consists of monthly observations of USD zero curves with terms of one month to thirty years, for the period from June 2000 to June 2005. All data are obtained from Bloomberg Financial Services.

explained earlier in equation (3).

We minimize the sum of squared percentage price error of the swaptions by searching for the optimal parameters of the volatility function. Specifically, the function is:

$$F(a, b, c, d, e) = \sum_{i=1}^{n} \left(\frac{P_i^{\text{observed}} - P_i^{\text{model}}}{P_i^{\text{observed}}} \right) \tag{19}$$

where P is the price of swaption in dollars. The parameter (e) is not used for the one factor model.

The estimation procedure of the implied volatility function is similar to that used by Amin and Morton. For each date and each currency, we use the Levenberg-Marquardt algorithm, a non-linear estimation procedure, to minimize the objective function.

We use the percentage error instead of the volatility point error that Amin and Morton

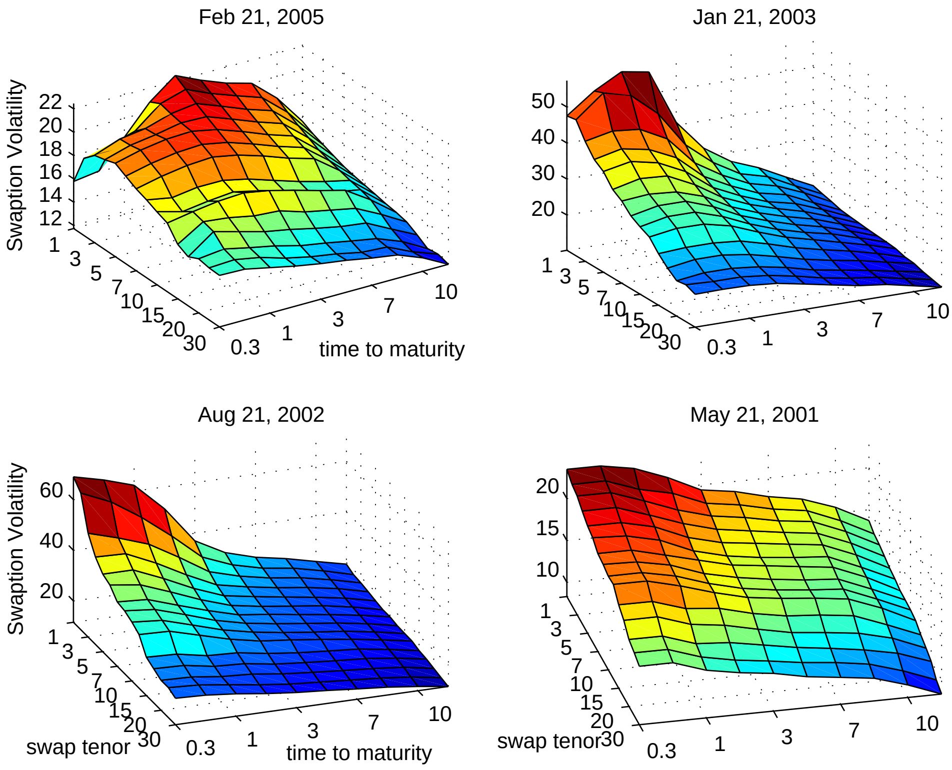

Fig. 4 Examples of USD Swaption Volatility Surfaces. Figures in the appendix show the corresponding swaptions surface for JPY and EUR.

uses because our measure enables us to appropriately compare across swaptions across the currencies. Since we do not fit the volatility function to the observed swaption prices, the goodness of the implied volatility function can be measured by the percentage errors of the Black volatilities converted from the swaption prices. Specifically, we define the error to be

$$\text{Error}_i = \left(\frac{\nu_i^{\text{observed}} - \nu_i^{\text{model}}}{\nu_i^{\text{observed}}} \right) \tag{20}$$

for each swaption, at each date, for each currency, ν is the Black volatility measured in percent.

4 One Factor Model Empirical Results: the Model Errors and the Implied Volatility Function Movements

4.1 Analysis of the Model Errors

The result shows that the average percentage absolute errors over the sample period are 2.55, 3.36 and 5.74 for the USD, EUR and JPY respectively. These estimation errors are within the bid-ask spreads in the market. To clarify the measure of errors, consider a numerical illustration. Suppose a swaption value is quoted as 30 (Black) volatility points, then 1% error as quoted in this paper is only 0.3 volatility point. This calibration is an optimal search over only four parameters to fit 91 swaptions in most of the dates.

The results also show that the model can fit the USD market better than the EUR market, which in turn is better than the JPY market. However, the coefficient of variations (standard deviation/mean) are similar in magnitude, with the JPY value being lowest. They are 0.34, 0.40 and 0.26 for USD, EUR and JPY respectively.

Next we analyze the errors in terms of the swaption tenor over the sample period. Table 1 below presents the average percentage absolute errors over the sample period grouped by the tenor. The results show that the percentage errors are largest for the one and the

	Swap Tenor (years)											
Currency	1	2	3	4	5	6	7	8	9	10	15	20
USD	3.66	2.64	2.21	1.81	1.85	1.94	1.73	2.37	2.69	2.73	2.67	3.12
EUR	6.50	4.39	2.94	2.29	2.49	2.41	2.33	2.33	2.29	2.57	2.25	3.46
JPY	7.94	6.19	4.54	3.92	4.73	4.56	5.20	4.85	4.73	4.77	6.00	8.46

Table 1 Average absolute swaption price percentage errors by swap tenor

20 year term for all three currencies.

Similarly, we consider the percentage errors as a function of the time to expiration of the swaptions in Table 2 below. The results show that the percentage errors appear highest

	Option Term (years)						
Currency	1	2	3	4	5	7	10
USD	4.30	2.66	2.12	2.03	2.09	1.92	2.03
EUR	3.77	3.48	2.75	2.35	2.33	2.49	3.98
JPY	8.43	5.23	4.37	5.24	4.83	5.19	5.15

Table 2 Average absolute swaption price percentage errors by option term

for the options expiring in one year. However, the errors remain reasonable given the market bid-ask spreads.

4.2 Analysis of the Errors over Time

Given the significant change in the interest rate levels over this sample period, one may expect that the model errors change over time. The results below depict the average of the swaption errors across the time to expiration and tenor for each date and currency. The plots of the percentage errors over the sample period for the three currencies are presented in the figure below. For the most part the absolute Black volatility points

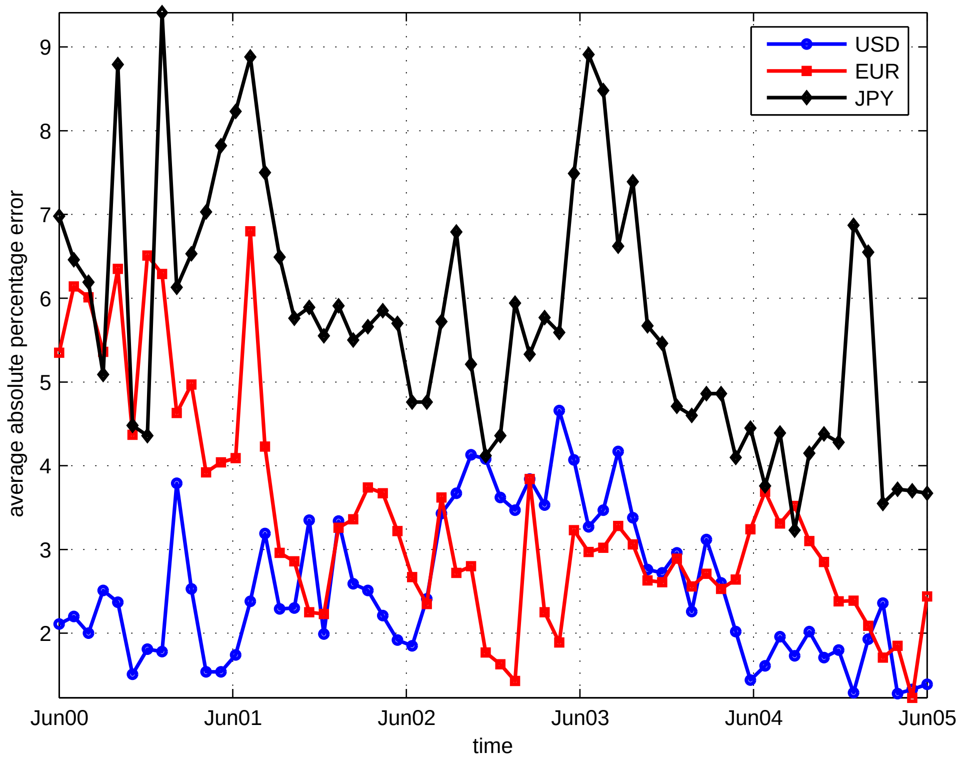

Fig. 5 Time series of average absolute swaptions price errors of the generalized one factor Ho-Lee model for USD, EUR and JPY. The absolute swaptions price errors are the difference between the Ho-Lee model swaption prices and the market prices expressed as a percentage of the market prices.

are below one volatility point for USD and EUR, and below 3 volatility points for JPY. Most trading desks tolerate a bid-ask spread of below 2 volatility points for USD and EUR. The volatility errors for the JPY are heavily influenced by the short term which has very high volatility. For example a 5% error on a 100% volatility would translate for 5 volatility points, whereas the same percentage error on 30% volatility would translate to 1.5 volatility points.

The results show that the errors have decreased in recent months. This observation may be explained by the relative calm of the markets in recent months. The level of errors tends to be correlated to the market volatility, and this may be explained by the positive correlation of the volatility of the market to the bid-ask spreads. In general, we find that the USD and EUR swaptions have lower errors than those of the JPY swaptions. This again may be explained by the relative illiquidity of the JPY market resulting in higher bid-ask spreads.

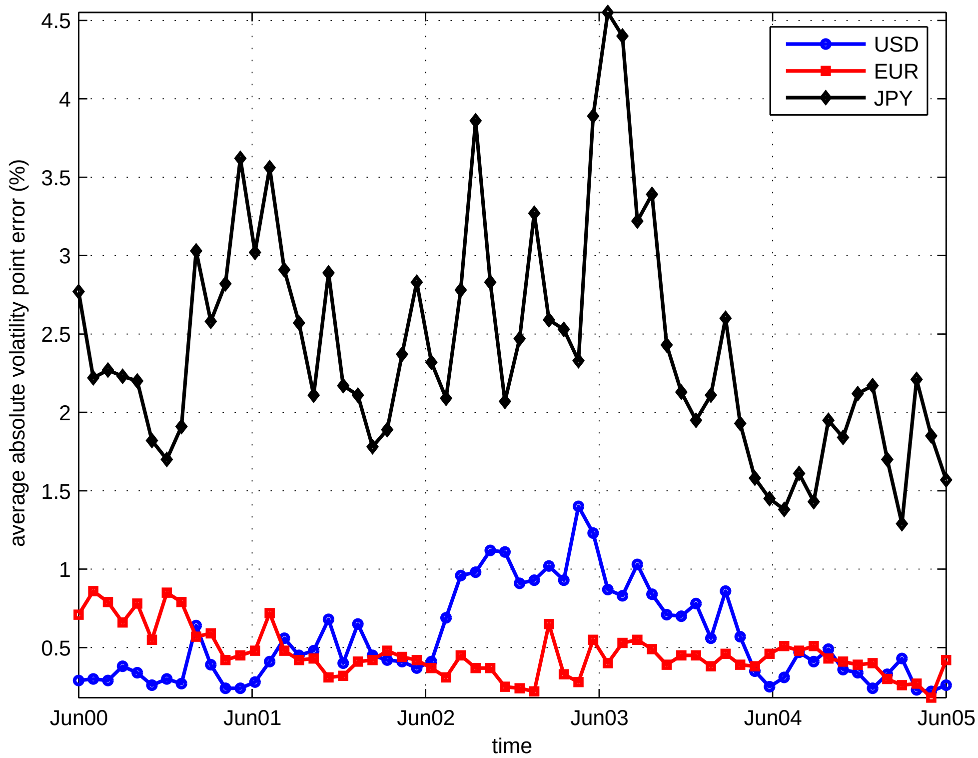

Fig. 6 Time series of absolute swaptions Black volatilities unit difference of the generalized one factor Ho-Lee model for USD, EUR and JPY. The absolute volatility unit difference the difference between the Ho-Lee model swaptions Black model implied volatilities and the quoted market volatilities expressed as a percentage.

4.3 Empirical Results on the Implied Volatility Function Movements

This section proceeds to analyze the estimated implied volatility function and its movements for each currency. The implied volatility function is estimated for each date and each currency.

Table 3 presents the mean and standard deviations of the estimated parameters of the implied volatility functions for the one factor model for each currency. The results show that on average over the sample period, the instantaneous short term volatility for the USD, EUR and JPY, as measured by $(a + d)$ are 48.5%, 26.3%, 95.2% respectively. And the "long term volatility" (d) are 9.6%, 10.7%, 22.8%, showing that the short term volatilities are higher than the long term volatilities. The decay rates (c) are 12.6%, 25.5% and 40.2% for the USD, EUR and JPY respectively. The implied volatility functions have a hump in all the currencies, as measured by the positive value of (b). All the parameters show significant coefficient of variations, and hence the implied volatility functions change in shape and level over time.

The average implied volatility function over the sample period is depicted in Figure 7 below. The result shows that the implied volatility functions decline exponentially, with

Currency		Estimated Vol. Fcn. Parameters			
		a	b	c	d
USD	Average	0.389	0.042	0.126	0.096
	Std. dev.	0.188	0.031	0.038	0.056
EUR	Average	0.156	0.008	0.255	0.107
	Std. dev.	0.087	0.016	0.080	0.024
JPY	Average	0.724	0.307	0.402	0.228
	Std. dev.	0.554	0.402	0.133	0.044

Table 3 Implied volatility functions of the three currencies

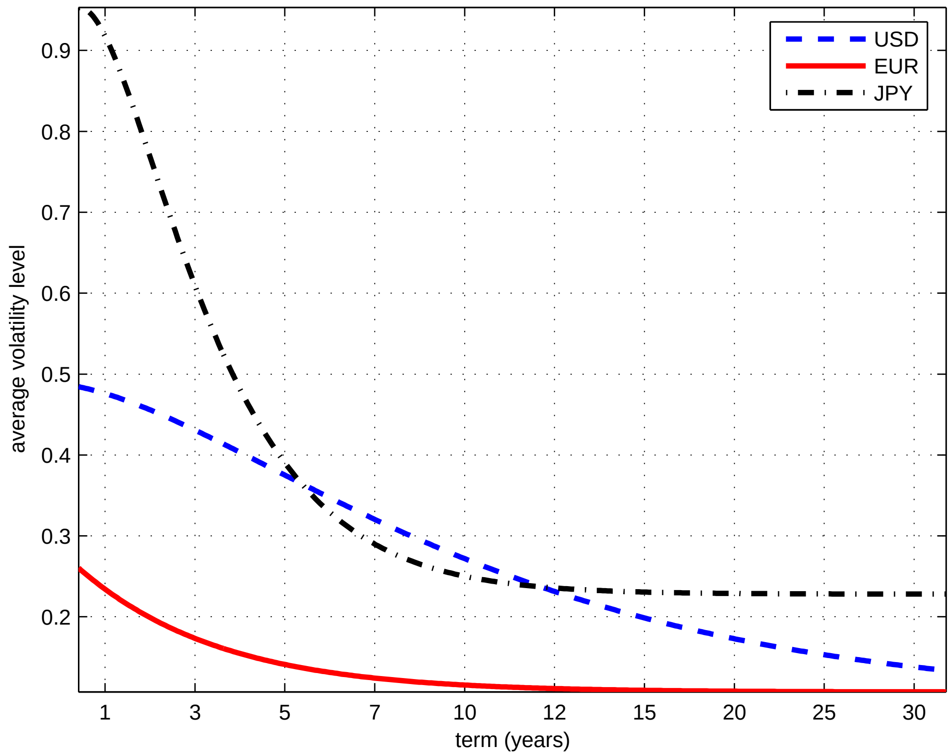

Fig. 7 The average implied volatility functions of the three currencies.

the JPY function shows a hump in the one year range.

Given the estimated parameters of the implied volatility function for each date, we can determine the estimated volatility function for each date over the sample period. Figure 8 below depicts the dynamic nature of the implied volatility function for the USD. The results show that the implied volatility function is stochastic and at the same time the function does not fluctuate wildly suggesting that the model is reasonable in its specification. Given the dynamic movements of the implied volatility function, we can

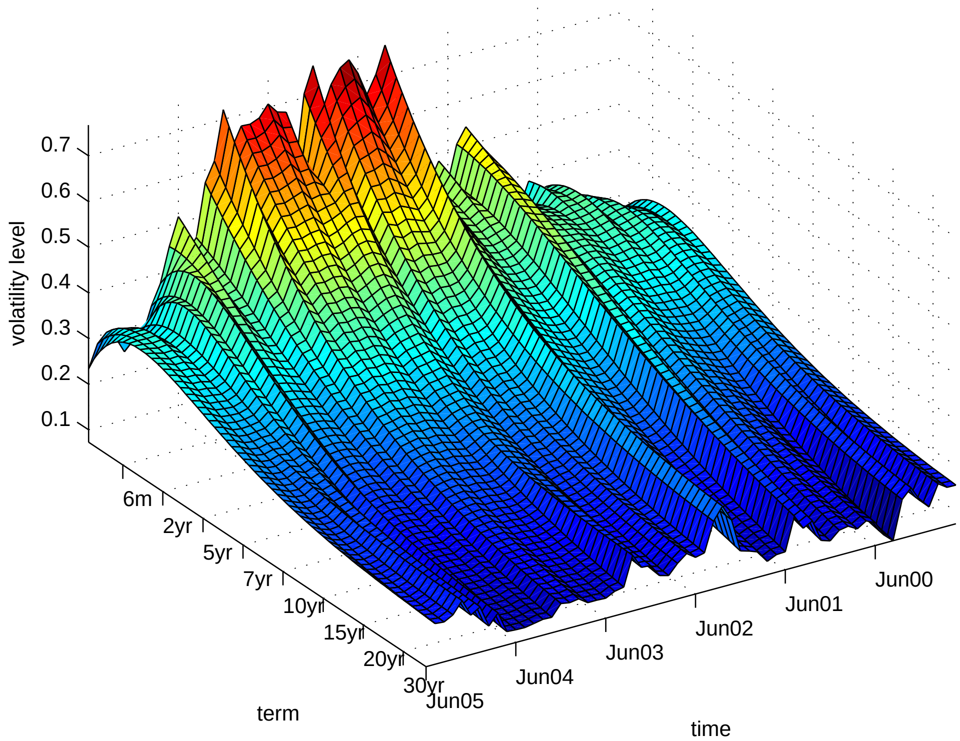

Fig. 8 Movements of the implied volatility function. The implied volatility function changes not only in the level but also the shape, with the short term volatility rises significantly in year 2001.

further specify the movements in terms of their principal components. The proportions of the percentage errors explained by the principal components are presented Table 4 below. The results show that the first three principal components explain 98.12%, 98.13%,

	Principal Components			
Currency	1st	2nd	3rd	Sum
USD	68.53%	24.55%	5.04%	98.12%
EUR	63.37%	26.34%	8.41%	98.13%
JPY	62.89%	21.79%	12.92%	97.61%

Table 4 The principal component of the implied volatility functions

97.61% of the movements in USD, EUR, and JPY respectively. The third component is quite significant for the JPY, while much less important for the USD.

These principal movements are depicted by the factor loadings, and these results are depicted in Figure 9a, 9b, 9c below. The results show that the first principal component for USD and EUR is “level”, representing the change of the level of the volatility across the term spectrum. There is a slightly higher volatility for the short (less than 3 years)

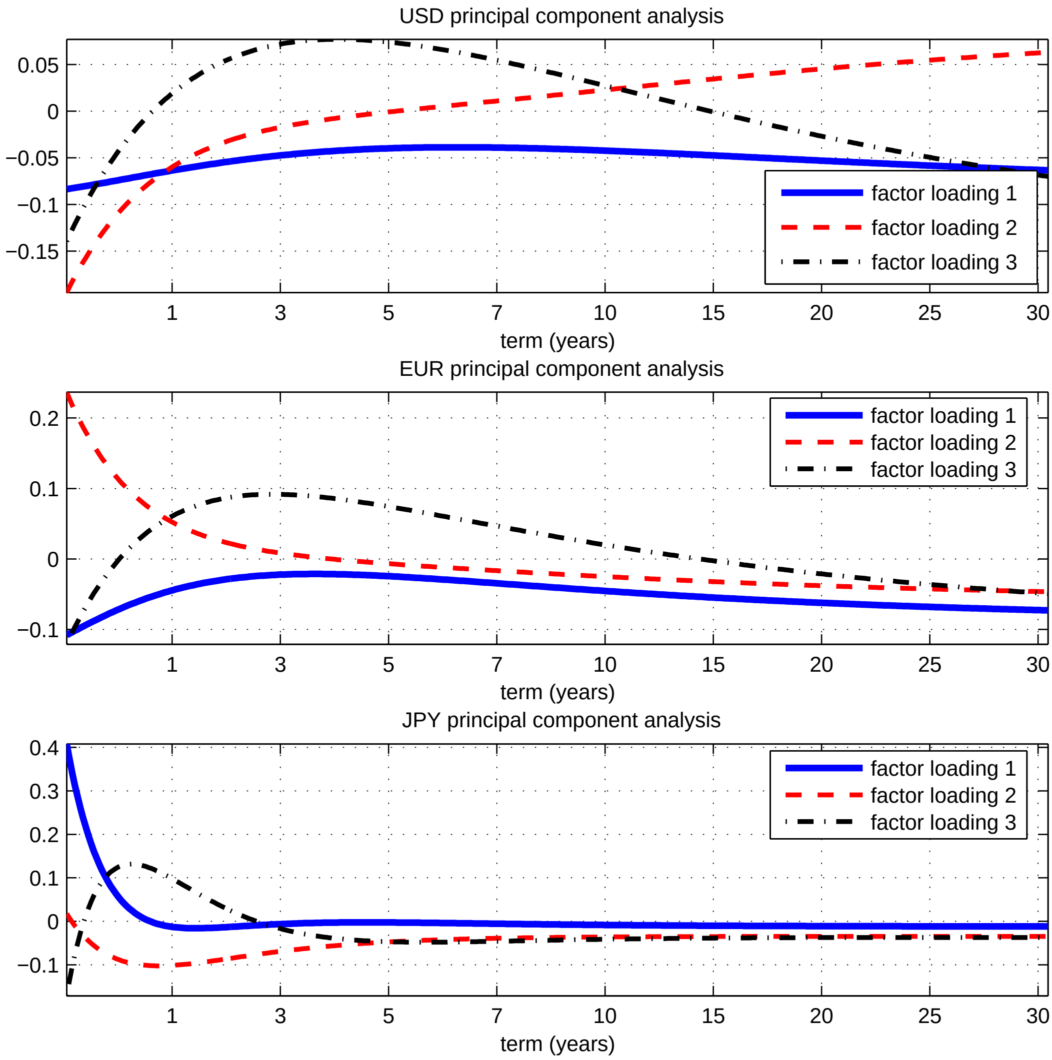

Fig. 9 Factor loading of the principal components of the implied volatility functions for the three currencies.

and the long ends (exceeding 15 years.) However, such is not the case with JPY. The first principal component for JPY is the short term volatility.

The second principal component represents the short term and "long term" volatilities for USD and EUR, while it is the "level" movement for JPY. We should note that the "long term" volatilities are implied mainly from the long dated option delivering a swap with long tenor. The 30 year volatility is estimated from the volatility of a 10 year expiration option on a 20 year swap. This combining of the two terms leads to a particular volatility movement.

5 A Comparison of the One Factor and Two Factor Models

Thus far, we have presented the errors of the one factor model. We now compare these results with the two factor model. Table 5 shows that the two factor model provides lower absolute percentage errors. This is particularly the case for the USD swaptions where the use of the two factor model leads to a 16.44% reduction in error. This reduction is particular significant considering that the two factor model and the one factor model for the case of USD use the same number of parameters in the calibration. In all the cases, the standard deviations of the errors are quite small. Next we analyze the variation of

	Currency		
	USD	EUR	JPY
average % error	2.19	3.04	5.43
std. dev.	0.70	1.25	1.63
error reduction	16.44%	9.50%	5.40%

Table 5 Two factor model percentage average errors

the absolute errors over time. Figure 9 shows that the two factor model for USD does well particularly over the 2002–2004 period. During this period the interest rate risks were higher. By way of contrast, the interest rate volatility is low in recent months, and the difference between the two models is relatively small. However, the improvements using a two factor model is relatively small for EUR and JPY. The results confirm that the use of one factor model can be quite robust for valuation. The percentage errors for EUR and JPY over the sample period are provided in Appendix B. In comparing the two factor model and the one factor model for the USD, we find that the model is over specified, with parameter d providing little explanatory power. For this reason, we restrict the parameter d to be zero. Table 6 provides the estimates of the parameters of the implied volatility functions. In comparing the estimates of the parameters of the

		Volatility 1				Volatility 2
Currency		a	b	c	d	e
USD	Average	0.401	0.050	0.128	0.046	0.099
	Std. dev.	0.211	0.027	0.034	0.062	0.049
EUR	Average	0.225	0.008	0.189	0.066	0.112
	Std. dev.	0.096	0.011	0.039	0.048	0.014
JPY	Average	0.739	0.399	0.430	0.159	0.122
	Std. dev.	0.516	0.314	0.088	0.065	0.044

Table 6 The estimated parameters of the implied volatility functions

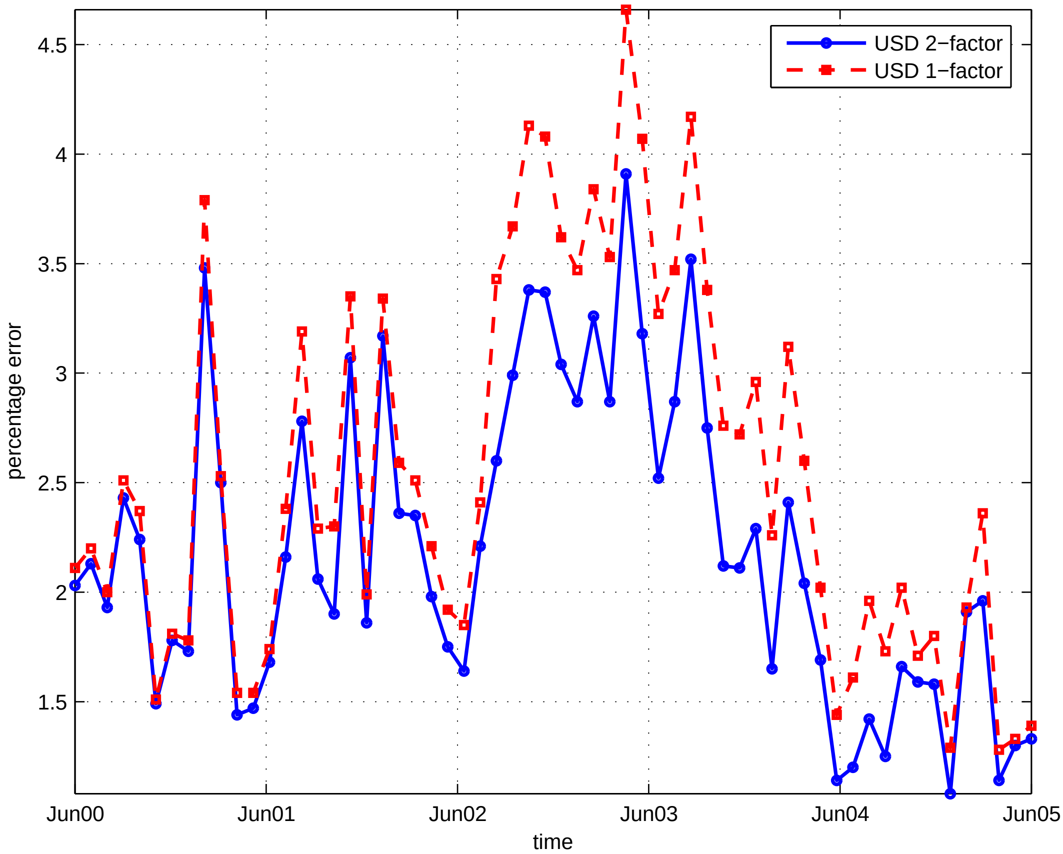

Fig. 10 Comparison of the percentage absolute errors of the one factor and two factor models.

implied volatility function for the two factor and the one factor model, we see that the implied volatility functions do not change significantly for USD and JPY. For the EUR, the parameters a and b have changed but the qualitative behavior of the function remains unchanged.

Following the analysis on the one factor model, we now proceed to analyze the dynamics of the implied volatility function using the principal components of the movements. Consider the results in Table 7. By introducing a second stochastic factor, the second principal component becomes more significant in all the currencies, providing explanatory power of more than 26% in all cases. Meanwhile, the first principal component remains significantly dominant, exceeding 50% for all the currencies. The dynamic movements of the implied volatility functions of the two factor model for USD are depicted below. The results show that the volatility functions are quite dynamic, exhibiting higher volatility in the short term.

This result is confirmed by estimating the factor loading of the principal components. Figure 10 shows that the variation in the short term is captured by the first principal

	Principal Components			
Currency	1st	2nd	3rd	Sum
USD	59.35%	32.56%	5.94%	97.86%
EUR	62.19%	26.7%	9.67%	98.56%
JPY	50.15%	27.55%	14.05%	91.75%

Table 7 Explanatory power of the principal components

components. We have described the results for the USD swaptions so far. However, these observations also apply to the EUR and JPY swaptions, whose results are provided in Appendix C.

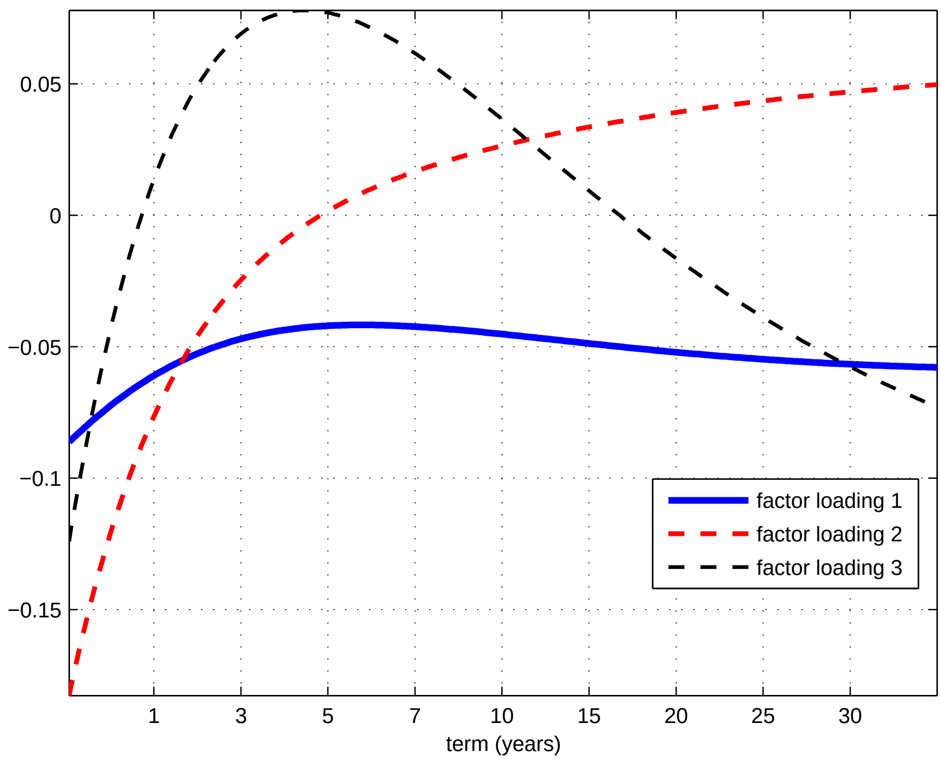

Fig. 11 Factor loading of the principal components of the USD two factor model.

6 Implications and Key Rate Vega

The empirical results have important implications in the valuation of interest rate contingent claims. The results show that for the major currencies, interest rate contingent claims can be valued relative to the observed spot curve and the swaptions. The implied volatility function can be specified quite simply by four parameters in the one factor model or five parameters of the two factor model.

The results also have important implications to hedging interest rate derivatives. Derman and Taleb (2005) have shown that delta hedging is often not effective in hedging equity options using the underlying stocks because of the vega effect. Our paper suggests that the volatility surface is also stochastic, consistent with previous empirical studies, and that the duration hedging of some interest rate derivatives also may not be effective because of the vega effect in the interest rates. This paper suggests that both swaps and swaptions should be used in hedging interest rate derivatives. Furthermore, in hedging the volatility risk, we cannot use one vega measure. For example, we cannot use a short dated option to hedge the volatility risk of a long dated option. Instead, we need to measure the value sensitivity of an option to the change in the implied volatility function. The vega buckets can be defined along the implied volatility functions.

The construction of these changes is analogous to the construction of the changes on the yield curve to determine the key rate durations. These value sensitivities are called "key rate vegas". The result shows that in hedging an interest rate option, we should match the option to a portfolio of swaps and swaptions, such that the sets of both key rate durations and key rate vegas are matched. In managing the volatility risk using swaptions, the effectiveness of the hedge should improve. Our result suggests that three key rate vegas would be effective.

We here describe the key rate vega for the one factor model. Key rate vegas for the two factor model can be defined analogously. The shift of the implied volatility function is depicted in Figure 12. The first key rate shift is a shift at the nearby term and linearly decline to the key rate. The second key rate shift is the shift of the key rate, with a linearly decline in the shift in either directions of the key rate. The third key are shift is the shift of the long volatility point and the shift linearly decline to the key rate on the left hand side and stays constant on the right hand side. The shifts are then added to the initial implied volatility function as depicted in Figure 12. Then a key rate vega is defined as the proportional change of the security value per unit change in the implied volatility function at a key term. Specifically, we define KRV(i) the i^{th} key rate vega to be

$$KRV(i) = \frac{\Delta V}{V} / \Delta\sigma(i) \tag{21}$$

where V is the value of the interest rate derivative and $\Delta\sigma(i)$ is a small shift of the implied volatility function at the i^{th} term.

Intuitively, equation (21) suggests that the shift $\Delta\sigma(i)$ is a small increase in the volatility of forward short rate at the i^{th} term. For example, if the second key rate vega is specified as the 7^{th} year shift in the volatility (as depicted in figure 12), then the shift increases the 7^{th} year short rate forward volatility. To the extent that we may consider the 7^{th} year forward short rate as a "key rate", then the proportional change in the interest rate derivatives to this increase in volatility is the "key rate vega".

Given this definition, we conclude with a few observations. As noted before, the implied volatility function is related to the Black volatilities. To the extent that trading floor vega buckets are measured in terms of Black volatilities, key rate vega can be converted to vega buckets, just as key rate durations can be converted to PV 01. Also note that the sum of the three key rate vegas equals to a unit parallel shift of the implied volatility function and key rate vegas can identify the difference of the vega risk between a short dated option and a long dated option. Specifically, a short dated option would have a high short term key rate vega and a low long term key rate vega. Conversely, the long dated options would have a significant long term key rate vega. The key rate vegas should enable the managers to control the vega risks of interest rate derivatives.

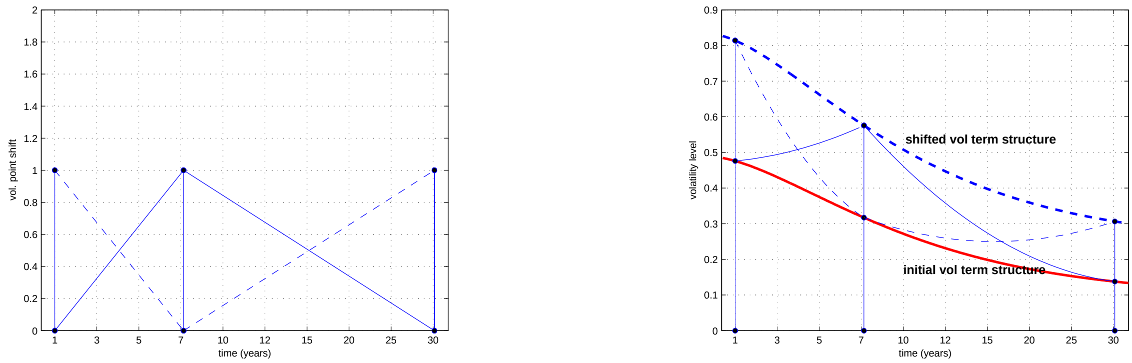

Fig. 12 Key rate vega. The left graph shows the shift of key rate vega and the right graph shows the shifted implied volatility function using key rate vega.

7 Conclusions

This paper uses monthly data of swaptions in three major currencies to study the robustness of the generalized Ho Lee models, their implied volatility functions and movements. The empirical results show that the implied volatility functions are stochastic and they can be used to define key rate vega to manage the volatility risk of interest rate derivatives.

Specifically, we show that the implied volatility function exhibits movements with three significant components. This result shows that the use of durations to implement dynamic hedging of derivatives or the use of short term options to hedge the vega of the long dated options may not be effective. A more effective hedging approach would employ also the swaptions that would match all the principal movements of the implied volatility function, as well as the yield curve movements.

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Appendix A

In the table below, we present the summary statistics (mean, minimum, maximum and standard deviation) for at-the-money European swaption volatilities used for our empirical study. The data consists of 60 monthly observations from July 21, 2000 to June 21, 2005 of mid-market implied Black model volatilities.

Option Term	Swap Tenor	USD mean	USD min	USD max	USD std	EUR mean	EUR min	EUR max	EUR std	JPY mean	JPY min	JPY max	JPY std
1	1	33.4	13	56.2	13.2	22.6	12.9	33.3	5.6	90.5	52.5	115	15.1
3	1	24.7	14.8	35.1	5.6	17.7	12.8	22.9	2.7	62.1	44.9	80.5	8.2
5	1	21	14.6	27.1	3.1	15	12.1	18.3	1.6	44.9	28.5	64	7.7
7	1	15.7	12.5	20	1.6	12.1	10.1	15	1	27.5	19.8	40.8	4.8
1	3	28.3	13.4	45.9	8.8	19.2	12.2	27.9	3.9	64.5	43.3	93	10
3	3	22.6	14.2	31.1	4.4	15.5	11.4	19.7	2.2	47.4	29	66.5	8
5	3	19.6	13.9	25	2.7	13.4	10.8	16.9	1.5	35.3	23	55	7
7	3	14.7	11.3	18.6	1.5	11	8.9	14	1	24.4	17.8	36.5	4.3
1	5	25.5	13.2	38.6	6.9	16.8	11	24.2	3.2	53.6	33.8	72.5	8.8
3	5	21.2	13.4	28.1	3.6	13.9	10.4	18.1	1.8	38.8	24.5	56.5	7.3
5	5	18.5	13.4	23.5	2.3	12.3	9.9	16.1	1.3	30.2	21.5	47	6
7	5	13.9	10.5	17.6	1.4	10.3	7.5	13.9	1.1	22.3	17	33.8	3.9
1	7	23.6	13.2	33.8	5.5	15.1	10.6	21	2.5	45.3	28.5	64	8.4
3	7	20	13.5	26.6	3.1	13	9.8	17.2	1.6	33.2	22.4	50	6.3
5	7	17.6	12.7	22.4	2.1	11.8	9.5	15.6	1.3	27.4	20	42.3	5.1
7	7	13.3	10.2	17.1	1.3	10	7.9	13.8	1.2	21.3	16.3	31.5	3.5
1	10	21.6	13.2	29.4	4.2	13.6	10	18.2	2	35.6	24.4	59.5	7.3
3	10	18.8	13.2	24.6	2.5	12.2	9.1	16.4	1.5	28.9	21.4	45.3	5.3
5	10	16.6	12.5	21	1.7	11.3	9	15.2	1.2	25.2	19.7	39	4.4
7	10	12.7	9.9	16.4	1.3	9.8	7.6	13.7	1.2	20.7	15.8	30.3	3.3
1	15	19.4	15.6	25.2	2.5	12.7	9.5	16.9	1.6	29.7	21	44	6.1
3	15	17	14	22	1.7	11.6	8.7	15.4	1.2	25.9	20.3	38.1	4.3
5	15	15.1	13	19.1	1.2	10.8	8.2	14.6	1.2	23.8	18.4	35.7	3.9
7	15	11.7	9.5	15.2	1.1	9.5	7	13.1	1.2	20.6	15.4	31	3.1
1	20	17.9	14.5	23	2.1	11.9	9	15.7	1.4	27.4	18.5	43	5.8
3	20	15.8	13.3	20.2	1.5	11	8.1	14.9	1.2	24.4	19.3	36.5	4.1
5	20	14	12	17.6	1.2	10.3	7.5	14.2	1.2	22.7	17.2	33.3	3.6
7	20	11	8.7	14.2	1.1	9.2	6.5	12.5	1.2	20.3	15	29.5	3.1

Table 8 Swaption values measured in Black volatility points (%) change significantly over the sample period for all currencies

Appendix B

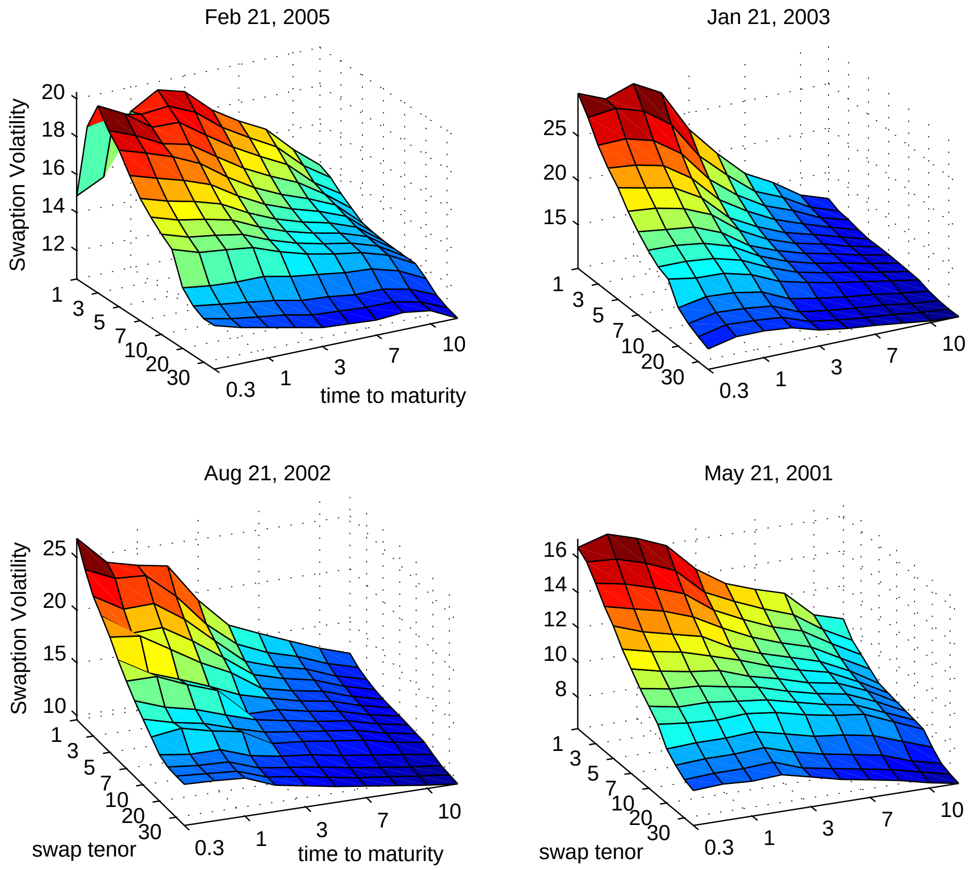

Fig. 13 Examples of EUR Swaption Volatility Surfaces. Each figures shows quotes for swations with maturities 0.5 and 10 years on underlying swaps with horizons at the maturity of the options between 1 and 20 years. All data is obtained from Bloomberg Financial Services.

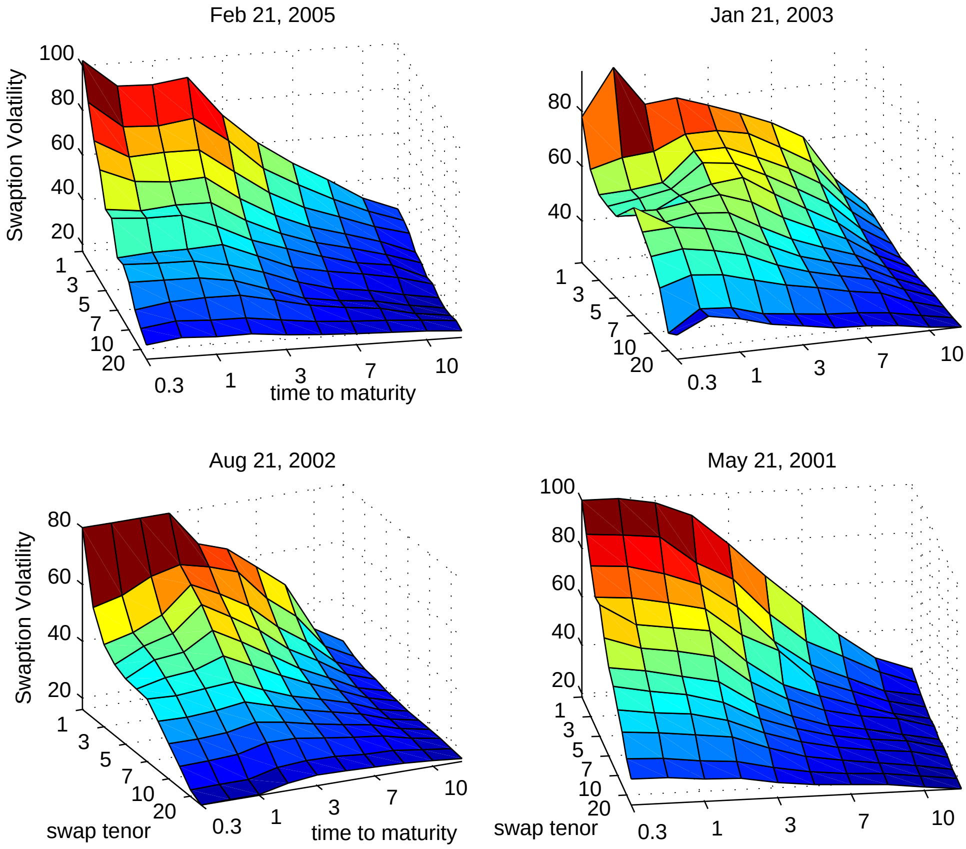

Fig. 14 Examples of JPY Swaption Volatility Surfaces. Each figures shows quotes for swations with maturities 0.5 and 10 years on underlying swaps with horizons at the maturity of the options between 1 and 20 years. All data is obtained from Bloomberg Financial Services.

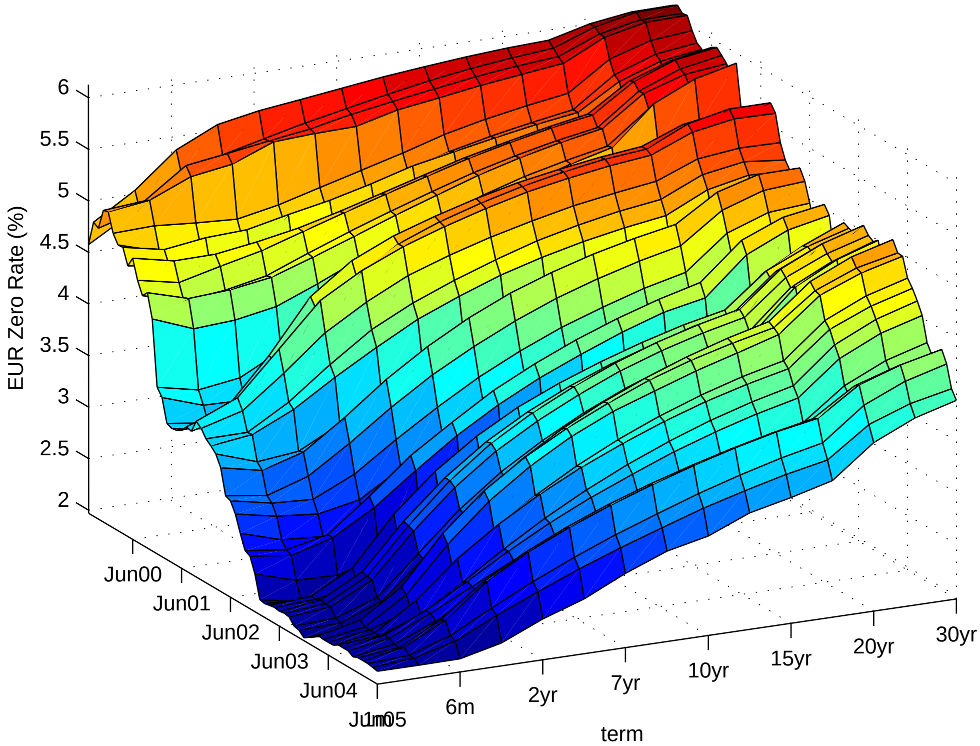

Fig. 15 Time series of EUR zero rates. The data consists of monthly observations of zero rates starting at 1 month to 30 years, for the period from July 2000 to June 2005. All data is obtained from Bloomberg Financial Services.

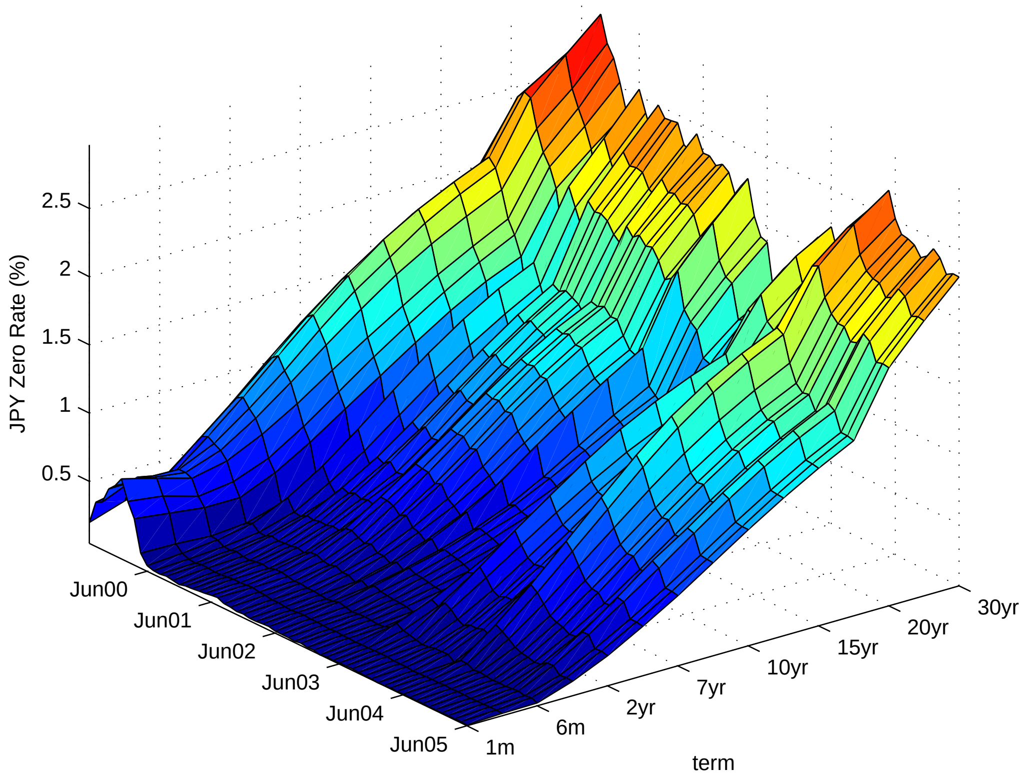

Fig. 16 Time series of JPY zero rates. The data consists of monthly observations of zero rates starting at 1 month to 30 years, for the period from July 2000 to June 2005. All data is obtained from Bloomberg Financial Services.

Appendix C

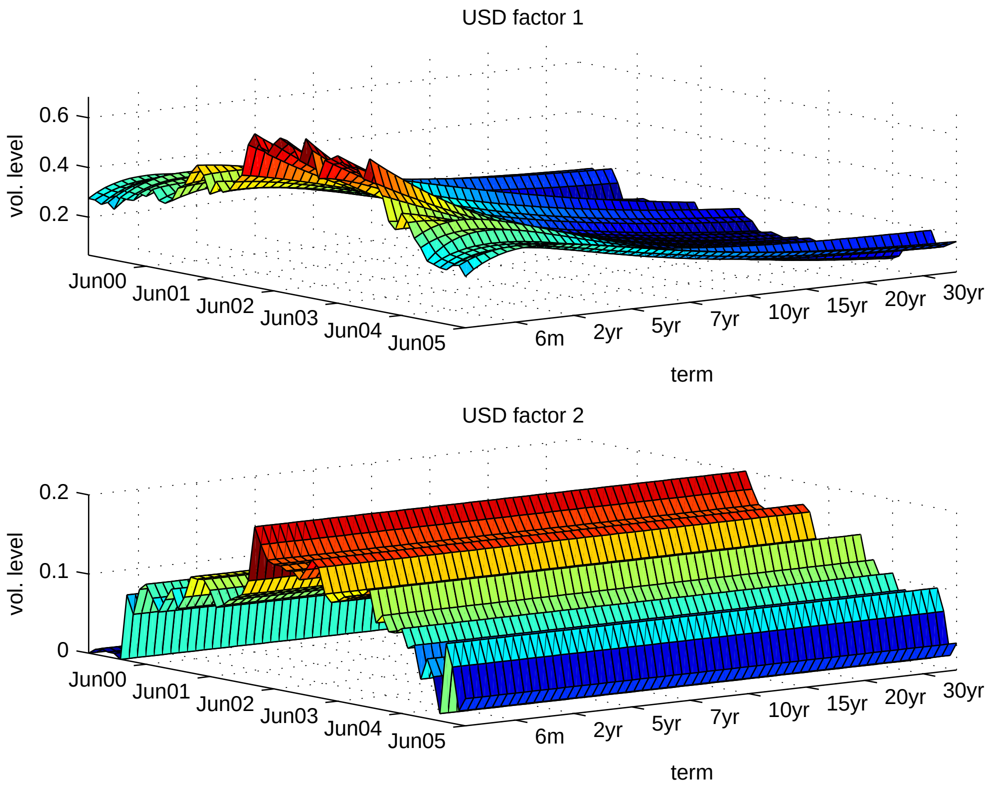

Fig. 17 Movements of the implied volatility functions of the two factor model in USD. The dynamic movements of the implied volatility functions show that the one-factor function exhibits downward sloping behavior when the market has high implied volatility. Otherwise, the function shows a slight hump in the short end of the interest rate spectrum.

The average shape of the implied volatility functions of the two factor models in USD, EUR and JPY are given in the figure below. JPY implied volatility function on average shows a significant hump in the short term.

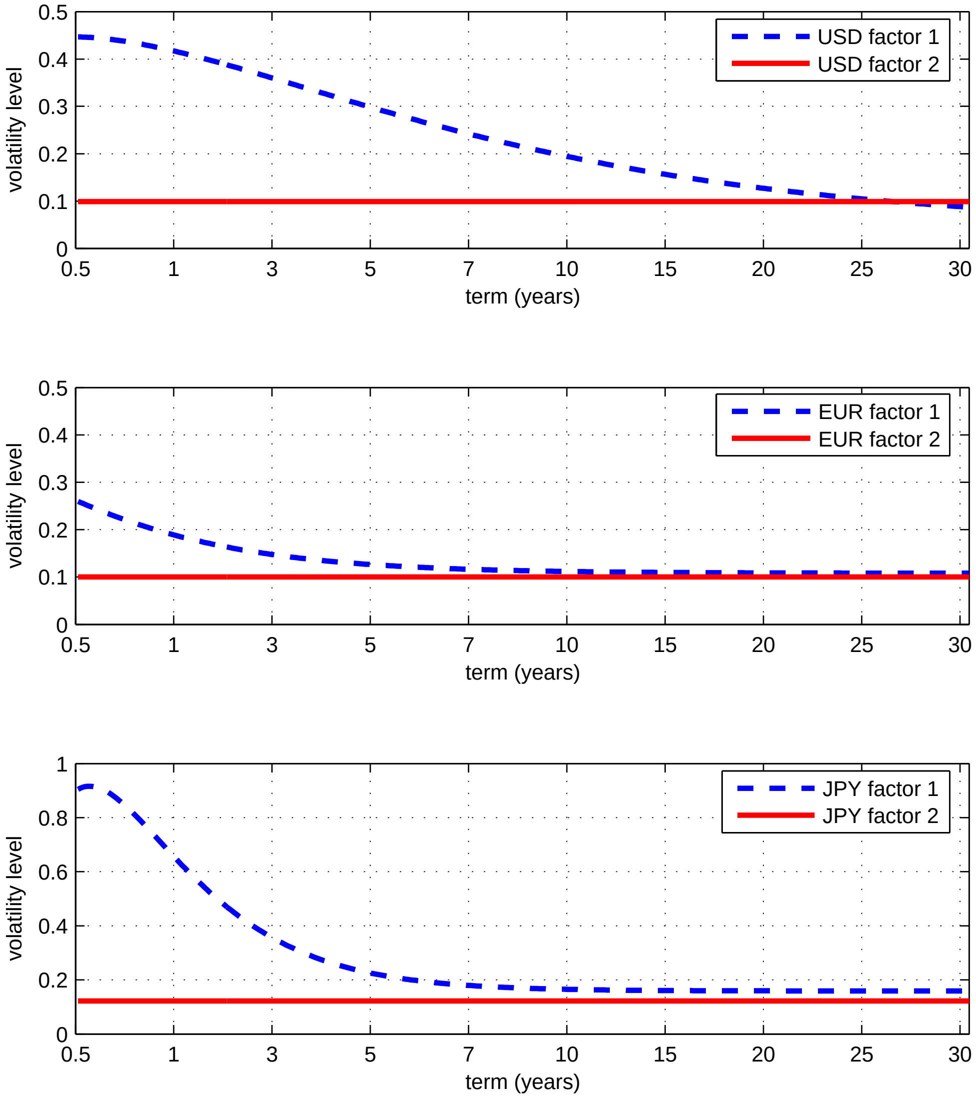

Fig. 18 The average implied volatility functions of the two factor model for all three currencies.