Volatility Surfaces

Calibration and maintenance of volatility surfaces for margin purposes

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Introduction

This document provides a brief description of the method developed by NOMX in order to account for volatility smiles in the margin calculations. The use of volatility surfaces will enhance the margin prices and hence also improve the margin calculations.

The method can be summarized in three steps.

1. NOMX will calibrate volatility surfaces to the option prices in the order book. The calibrations will be performed with the method of least squares and both call and put options will be used as input data. The calibrations will be performed once a week (at minimum).
2. NOMX will keep the shape of the volatility surfaces (approximately) constant in between the calibrations. The surfaces will, however, be moved upwards and downwards depending on the change of implied volatility of the option prices in the order book.
3. The volatility surfaces represent the option mid volatilities. In the margin calculations it is, however, the bid and ask volatilities that are used. NOMX will create the bid and ask volatilities from the option mid volatilities. In this process the spread between the bid and ask volatilities will be increased for option series that lack market prices, in order to account for the poor liquidity of these positions.

Basic processes

Calibration routine

The below process describes the calibration routine.

Option prices
- Option prices in the order book provide implied mid volatilitites.

Calibration
- Volatility surfaces are calibrated to the implied mid volatilitites.
- Two volatility surfaces are calibrated per underlying instrument, one for call and one for put options.

Bid/Ask volatilities
- A spread is applied to extract the bid and ask volatilities from the volatility surfaces. The spread will be larger for option series with poor liquidity.

Surface maintenance

The below process describes the surface maintenance.

Option prices
- Option prices in the order book provide implied mid volatilitites.

Surface shifting
- Option series with order book prices get their implied mid volatility updated.
- Option series without order book prices will get their mid volatility adjusted in order to keep the shape of the volatility surfaces (approximatively) constant.

Bid/Ask volatilities
- A spread is applied to extract the bid and ask volatilities from the volatility surfaces. The spread will be larger for option series with poor liquidity.

Calibration routine

Price types

An option series may have one of the below price types.

Market price

The definition of a market price differs depending on the price source.

Last paid

A last paid is a market price if the below criteria are fulfilled.

1. The last paid is not too old.
2. The implied volatility of the last paid is not too low.
3. The implied volatility of the last paid is not too high.

Bid/Ask pair

A bid/ask pair is a market price if the below criteria are fulfilled.

1. The bid/ask pair is not too old.
2. The implied bid volatility is not too low.
3. The implied ask volatility is not too high.
4. Either A or B is fulfilled.
    A. The bid/ask pair fulfills the market maker requirements.
    B. The spread between the implied bid and the implied ask volatility is not too wide.

Parity price

An option series is said to have a parity price if the below criteria are fulfilled.

1. The option series does not have a market price according to the above definition.
2. The option series with the same underlying instrument, strike price and expiration month but with the opposite option type has a market price.

No price

An option series that does not have a market price or a parity price is said to have no price.

Example: OMXS30

The below example explains the different price types. It further explains how the mid volatilities are calculated from the option prices in the order book.

Price types

Table 1 shows different price types for option series in OMXS30 with expiration in October 2009. The price data is taken from May 2009.

Table 1

Call						**Put**			**OMXS30, Oct 2009**	
Price type	Imp. Bid	Bid	Ask	Imp. Ask	Strike	Imp. Bid	Bid	Ask	Imp. Ask	Price type
None					380					None
Parity					400	47,1%	2,1	3,3	51,1%	Market
Parity					420	46,9%	3,1	4,5	50,5%	Market
…	…	…	…	…	…	…	…	…	…	…
Market	35,3%	91,5	96,5	37,1%	700	37,4%	54,0	58,5	38,4%	Market
Market	35,2%	86,0	90,3	36,6%	710	36,0%	57,8	62,3	37,8%	Market
Market	34,8%	80,3	84,5	36,3%	720	35,8%	62,3	66,8	37,6%	Market
…	…	…	…	…	…	…	…	…	…	…
Market	30,1%	25,3	29,3	32,0%	850					Parity
Market	29,6%	22,3	26,3	31,6%	860					Parity
None					870					None

Mid volatility calculations

Option series with market prices

The mid volatility for an option series with market price is calculated as the average of the implied bid and ask volatilities.

The mid volatility for OMXS309V400, in Table 1, is for example $\frac{47{,}1\%+51{,}1\%}{2} = 49{,}1\%$.

Option series with parity prices

The mid volatility for an option series with parity price is given as the mid volatility of the option series with opposite option type adjusted with the average difference of the call and put mid volatilities of the option series that has market prices in both the call and the put options.

In this example the option series with strike prices 700, 710 and 720 have market prices in both the call and the put options. These option series show a slightly higher mid volatility for the put options compared to the call options and the average of the difference in mid volatility equals $\frac{1}{3}(1{,}7\% + 1{,}0\% + 1{,}2\%) = 1{,}28\%$.

The mid volatility for OMXS309V850 is therefore given by $\frac{30{,}1\%+32{,}0\%}{2} + 1{,}28\% = 32{,}33\%$.

The mid volatility for OMXS309J400 is therefore given by 49,1% - 1,28% = 47,82%.

Option series with no prices

The option series with no prices have no order book mid volatility. They will get a mid volatility from the calibrated volatility surfaces.

Summary

Table 2 shows the order book mid volatilities for the option series listed in Table 1.

Table 2

Call						Put			OMXS30, Oct 2009	
Mid Vol	Imp. Bid	Bid	Ask	Imp. Ask	Strike	Imp. Bid	Bid	Ask	Imp. Ask	Mid Vol
					380					
47,82%					400	47,1%	2,1	3,3	51,1%	49,10%
47,42%					420	46,9%	3,1	4,5	50,5%	48,70%
…	…	…	…	…	…	…	…	…	…	…
36,20%	35,3%	91,5	96,5	37,1%	700	37,4%	54,0	58,5	38,4%	37,90%
35,90%	35,2%	86,0	90,3	36,6%	710	36,0%	57,8	62,3	37,8%	36,90%
35,55%	34,8%	80,3	84,5	36,3%	720	35,8%	62,3	66,8	37,6%	36,70%
…	…	…	…	…	…	…	…	…	…	…
31,05%	30,1%	25,3	29,3	32,0%	850					32,33%
30,60%	29,6%	22,3	26,3	31,6%	860					31,88%
					870					

Method of least squares

Concepts

The concept of volatility surfaces implies that the mid volatility, Z, can be seen as a function of time to expiration, x, and strike price, y, i.e. Z = Z(x,y). NOMX assumes that the volatility surfaces are given by a 3rd degree polynomial surface. This gives the below function for the mid volatility.

$$Z(x,y) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4y + c_5y^2 + c_6y^3 + c_7xy + c_8x^2y + c_9xy^2 \qquad (1)$$

The problem is to find the coefficients c_k. If they are found, then the implied volatility can be calculated for any given time to expiration and strike price.

The mid volatilities from the option series with market prices and parity prices provide several points in the 3 dimensional space [x y Z]. For example, the mid volatility of OMXS309V400 gives the data point [162 400 49,1%]. This data point can also be expressed using equation (1).

$$49{,}1\% = c_0 + c_1 \cdot 162 + c_2 \cdot 162^2 + c_3 \cdot 162^3 + c_4 \cdot 400 + c_5 \cdot 400^2 + c_6 \cdot 400^3 + c_7 \cdot 162 \cdot 400 + c_8 \cdot 162^2 \cdot 400 + c_9 \cdot 162 \cdot 400^2 .$$

There are 10 unknown coefficients, $c_{k,}$ for a 3rd degree polynomial surface. The minimum number of data points in order to calculate the coefficients are therefore 10. If there are market prices and parity prices corresponding to more than 10 data points these add up to an over determined system of linear equations. In this case there is no exact solution but there is a way to mathematically estimate the best approximation to the coefficients. This is called the method of least squares and it is described below.

Algorithm

The set of data points [x_k, y_k, Z_k] can be converted into a linear system of equations using equation (1). The linear system of equations can be expressed as a matrix multiplication **A**·**c** = **z** were **A** is a matrix containing the times to expiration, x_k, and the strike prices, y_k, **c** is a vector containing the unknown coefficients, c_k, and **z** is a vector containing the order book mid volatilities, Z_k.

$$A = \begin{bmatrix} 1 & x_1 & x_1^{\ 2} & x_1^{\ 3} & y_1 & y_1^{\ 2} & y_1^{\ 3} & x_1y_1 & x_1^{\ 2}y_1 & x_1y_1^{\ 2} \\ . & . & . & . & . & . & . & . & . & . \\ 1 & x_{end} & x_{end}^{\ \ 2} & x_{end}^{\ \ 3} & y_{end} & y_{end}^{\ \ 2} & y_{end}^{\ \ 3} & x_{end}y_{end} & x_{end}^{\ \ 2}y_{end} & x_{end}y_{end}^{\ \ 2} \end{bmatrix},$$

$$c = \begin{bmatrix} c_0 \\ . \\ c_9 \end{bmatrix}, \qquad z = \begin{bmatrix} Z_1 \\ . \\ Z_{end} \end{bmatrix}.$$

The method of least squares contains the following steps of linear algebra.

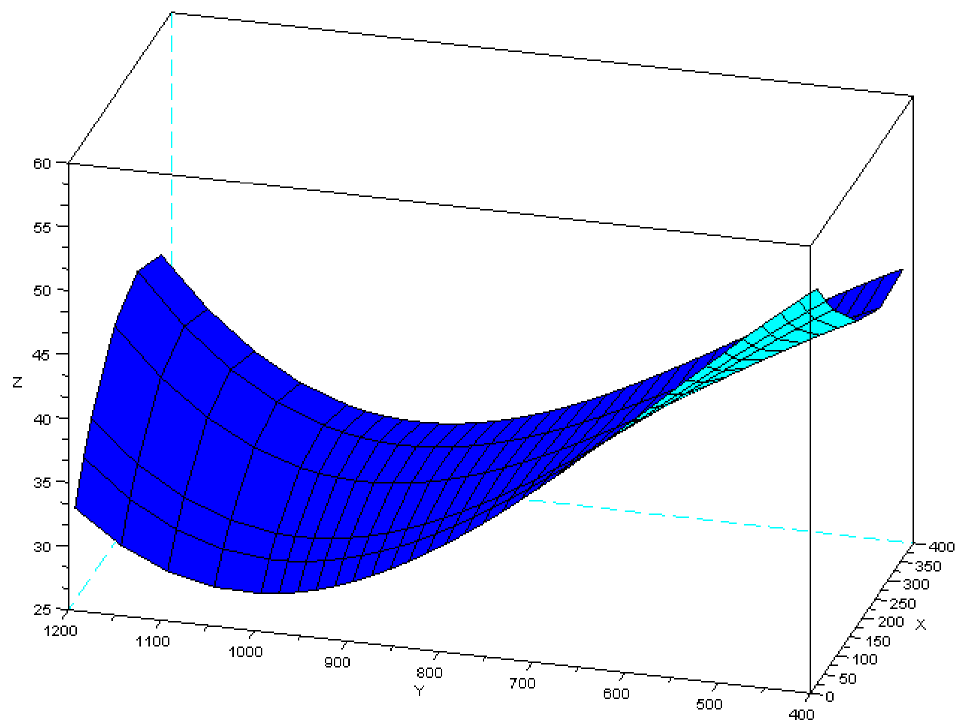
1. $\mathbf{A} \cdot \mathbf{c} = \mathbf{z}$
2. $\mathbf{A}^T\mathbf{A} \cdot \mathbf{c} = \mathbf{A}^T \cdot \mathbf{z}$
3. $\mathbf{c} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T \cdot \mathbf{z}$

This algorithm provides the set of coefficients, c_k, for the surface Z(x,y) that minimizes the sum of the squares of the distances from $Z(x_k, y_k)$ to the corresponding data point $[x_k, y_k, Z_k]$.

Example: OMXS30

Figure 1 shows the calibrated volatility surface for put options in OMXS30. The input data is taken from May 2009.

Figure 1



Adjusting the mid volatilities

Option series with market prices

The option series with market prices will continue to use their order book mid volatility.

Other option series

The option series with parity prices and the option series with no prices will use a mid volatility taken from the calibrated volatility surfaces.

Example: OMXS30

Table 3 shows the mid volatilities for the option series listed in Table 1. The mid volatilities that have been taken from the calibrated volatility surfaces are colored grey in Table 3.

Table 3

Call						**Put**			**OMXS30, Oct 2009**	
Mid Vol	Imp. Bid	Bid	Ask	Imp. Ask	Strike	Imp. Bid	Bid	Ask	Imp. Ask	Mid Vol
48,95%					380					50,10%
47,90%					400	47,1%	2,1	3,3	51,1%	49,10%
47,30%					420	46,9%	3,1	4,5	50,5%	48,70%
…	…	…	…	…	…	…	…	…	…	…
36,20%	35,3%	91,5	96,5	37,1%	700	37,4%	54,0	58,5	38,4%	37,90%
35,90%	35,2%	86,0	90,3	36,6%	710	36,0%	57,8	62,3	37,8%	36,90%
35,55%	34,8%	80,3	84,5	36,3%	720	35,8%	62,3	66,8	37,6%	36,70%
…	…	…	…	…	…	…	…	…	…	…
31,05%	30,1%	25,3	29,3	32,0%	850					32,50%
30,60%	29,6%	22,3	26,3	31,6%	860					31,80%
29,70%					870					30,70%

Boundaries

Concepts

The accuracy of the calibration is dependent on the length of the extrapolation. This implies that the result will be best for option series that lie close to option series with market prices or parity prices. NOMX will apply boundaries to the volatility surfaces in order to ensure that the extrapolation is not performed too far.

Outer series

NOMX will apply a flat volatility structure for option series that lie outside of the boundaries of the volatility surfaces.

Surface maintenance

NOMX will calibrate the volatility surfaces once a week (at minimum). This section describes the method that will be used to maintain the volatility surfaces in between the calibrations.

Option series with market prices

The option series with market prices will continue to use their order book mid volatility and hence their mid volatility will be updated as the implied volatilities in the order book changes.

Option series with parity prices

The mid volatility for the option series with parity prices will be adjusted in order to keep yesterdays difference in mid volatility between the option series with parity price and the corresponding option series with market price. Hence, their mid volatility will be updated as the implied volatilities of the option series with market prices are changed.

Option series with no prices

The option series with no prices have no order book mid volatility. Their mid volatilities will be adjusted in order to keep the difference in mid volatility between the closest option series with market price or parity price.

Example: OMXS30

Suppose the volatility surfaces were calibrated on day T and the result were as given in Table 3. Further suppose that the order book mid volatilities changes for the option series with market prices. This would give the following result on day T+1.

Table 4

Call				**Put**		**OMXS30, Oct 2009**
Price Type	Change	Mid Vol	Strike	Mid Vol	Change	Price Type
None	2,00%	50,95%	380	52,10%	2,00%	None
Parity	2,00%	49,90%	400	51,10%	2,00%	Market
Parity	1,00%	48,30%	420	49,70%	1,00%	Market
…	…	…	…	…	…	…
Market	1,70%	37,90%	700	39,20%	1,30%	Market
Market	1,50%	37,40%	710	38,90%	2,00%	Market
Market	0,8%	36,35%	720	37,70%	1,00%	Market
…	…	…	…	…	…	…
Market	2,00%	33,05%	850	34,50%	2,00%	Parity
Market	1,00%	31,60%	860	32,80%	1,00%	Parity
None	1,00%	30,70%	870	31,70%	1,00%	None

Bid/Ask spreads

Concepts

The above presented procedures give the option mid volatilities. In the margin calculations it is, however, the bid and ask volatilities that are used. NOMX will create the bid and ask volatilities from the option mid volatilities. The bid volatility will be the mid volatility minus a spread and the ask volatility will be the mid volatility plus a spread. The size of the spread depends on the liquidity of the option series.

Inner series

Option series with market prices

All option series with market prices will use a spread that is equal to half the difference between their implied bid and ask volatilities. This means that option series with market prices will use their implied bid and ask volatilities in the margin calculations.

Other option series

All option series with parity prices and with no prices will use a spread that is equal to half the difference between the implied bid and ask volatilities of the closest option series with market price. This spread will further be increased linearly depending on the distance between the series itself and the closest option series with market price. NOMX will, however, decide a maximum volatility spread parameter and the spread can never be larger than this value.

Outer series

All option series that lie outside the boundaries to the volatility surfaces will have a volatility spread that is equal to the maximum volatility spread parameter.

Example: OMXS30

Table 5 shows the bid and ask volatilities for the option series listed in Table 1. The option series that lack market prices are colored grey.

Table 5

Call						Put			OMXS30, Oct 2009	
Spread	Imp. Bid	Bid	Ask	Imp. Ask	Strike	Imp. Bid	Bid	Ask	Imp. Ask	Spread
4,2%	46,85%			51,05%	380	48,1%			52,2%	4,1%
4,1%	45,85%			49,95%	400	47,1%	2,1	3,3	51,1%	4%
4%	45,3%			49,3%	420	46,9%	3,1	4,5	50,5%	3,6%
…	…	…	…	…	…	…	…	…	…	…
1,8%	35,3%	91,5	96,5	37,1%	700	37,4%	54,0	58,5	38,4%	1,0%
1,4%%	35,2%	86,0	90,3	36,6%	710	36,0%	57,8	62,3	37,8%	1,8%
1,5%%	34,8%	80,3	84,5	36,3%	720	35,8%	62,3	66,8	37,6%	1,8%
…	…	…	…	…	…	…	…	…	…	…
1,9%	30,1%	25,3	29,3	32,0%	850	31,35%			33,65%	2,3%
2,0%	29,6%	22,3	26,3	31,6%	860	30,60%			33,00%	2,4%
2,1%	28,65%			30,75%	870	29,45%			31,95%	2,5%