Options Markets

Option Valuation

Option Valuation

- In this lecture we talk about how to value different options.
- We will value options by assuming that no arbitrage opportunities exist.
- Remember put-call parity.
- To value put options, we just have to value call options and apply PC parity.

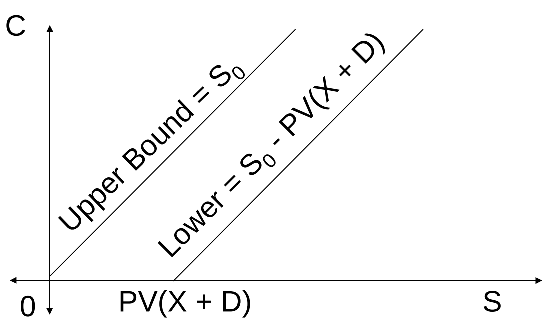
Option Valuation

- What determines option values?
  - » stock price, S (C goes up with S)
  - » strike price, X (C goes down with X)
  - » volatility, σ (C goes up with σ)
  - » time to maturity, T (C goes up with T)
  - » interest rate, r_f (C goes up with r_f)
  - » dividends, D (C goes down with D)
- Why do these relations hold?

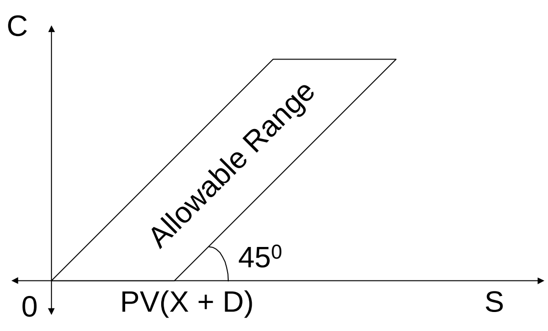
Option Valuation

- Restrictions on option values:
  - » C > 0 - why?
  - » If a call expires in the money, you can replicate its payoff by buying 1 share of stock and borrowing PV(X + D).
  - » The payoff to this would be (S_T - X).
  - » This strategy has downside risk.
  - » Thus, C > S_0 - PV(X) - PV(D).
  - » C < S_0 - why?

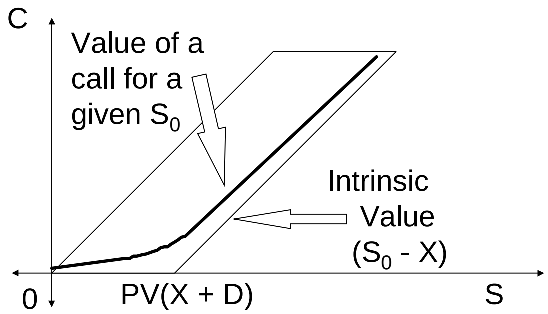
Option Valuation



Option Valuation



Option Valuation



Option Valuation

- When does a call holder want to exercise?
- First consider a call on a stock with D = 0.
- There are 2 ways to reverse a call position:
  - » exercise early
  - » sell the option
- Since C > [S_0 -PV(X)] > (S_0 - X), it is <u>never</u> optimal to exercise an American call early.

Option Valuation

- If D = 0 and you want to reverse your position, <u>sell</u> your call but don't exercise.
- European C = American C when D = 0.
- What if the stock pays a dividend?
- What if it pays a liquidating dividend?
- If D is high enough, it can be optimal to exercise your call option early.

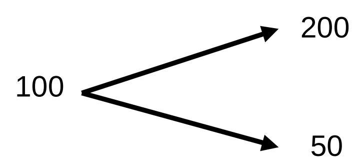
Option Valuation

- When does a put holder want to exercise?
- Unlike calls, it can be optimal to exercise American puts early when D = 0.
- What if, for example, S = $.10, X = $100, and T = 1 year?
- The time-value of the strike price can be greater than the potential gain from waiting.
- What if the stock is going to pay dividends?

Option Valuation

- Most option pricing is done with a binomial "tree" model.
- In a binomial tree, we assume that prices either go up or go down.
- For example, suppose we knew that the price of $100 stock would either go to $50 or $200 by year-end.

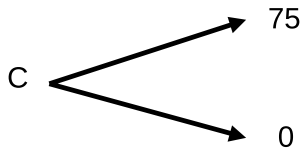
Option Valuation



This is our "binomial tree"

Option Valuation

- A call option with strike price $125 would have a payoff structure like:



Option Valuation

- We can replicate the call option with a position in the stock and in 8% T-bills.
- Suppose, for example, that you buy 1/2 share of the stock and borrow $23.15
- Your payoff at year-end will depend on S_T.
- If S_T = 50 then it is = 25 - 1.08*23.15 = 0.
- If S_T = 200 then it is = 100 - 25 = 75.
- This position costs you 50 - 23.15 = 26.85.

Option Valuation

- Since this strategy costs you $26.85 today and its payoff is equal to the call's payoff, the call must cost $26.85.
- Why is this true?
- This is basically how "trees" work.

Option Valuation

- Binomial trees construct a riskless position out of stock, options and "cash."
- The payoff of the options plus cash is perfectly correlated with the stock price.
- In the example, holding 1/2 share and writing 1 call produces a risk-free position.
- The payoff to this position is always $25.
- It costs you $26.85 today to get $25 at T.

Option Valuation

- Suppose C = 30 (not 26.85), r_f = 8%.
- We can make an arbitrage chart:

Position	Initial CF	S = 50	S = 200
Write 2 C	60	0	-150
Buy 1 S	-100	50	200
Borrow 40	40	-43.20	-43.20
Total	0	6.80	6.80

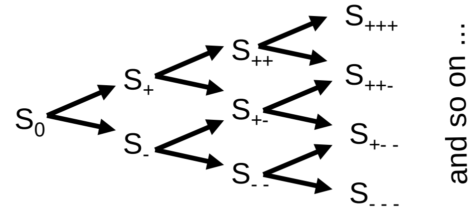
Option Valuation

- An important concept for option pricing is the "hedge ratio," or delta.
- The hedge ratio is defined as the number of shares of stock held for each call option written in the risk-free portfolio.
- The hedge ratio for this example is 1/2.
- We'll talk more about deltas later.

Option Valuation

- What if you assumed the price could only go up to 200 or down to 50 but you were wrong?
- In general, there are many more steps in a tree than just 2.
- By having more steps, we make the model much more realistic.

Option Valuation

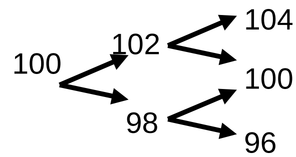


Option Valuation

- When we have a more complicated tree,we just solve the problem at each node of the tree starting at the end.
- We get a new hedge ratio at each node.
- The more nodes we add, the smaller the jump between each node needs to be.
- On Wall Street, analysts use computer programs with many nodes.

Option Valuation

- Numerical example:

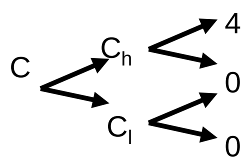


Stock Price Tree

Let's see how to value a call option with X = 100 using this tree and r_f = 2%

Option Valuation

- Numerical example:

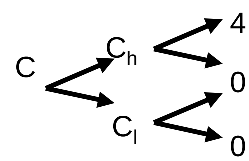


Option Price Tree

We need to solve for each of the C values in this diagram. C_l is particularly easy.

Option Valuation

- Numerical example:



Option Price Tree

To get C_h we need to calculate a hedge ratio. The ratio is:

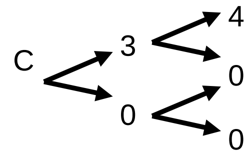
$$H = \frac{C+ - C-}{S+ - S-} = 1$$

Option Valuation

- Now we know that we could write 1 call and hold 1 stock and the payoff in the last node (assuming S = 102) is riskless.
- Since this is a half period, r_f = 1%
- So (S - C) = PV(100).
- 102 - C = 99.
- C = 102 - 99 = 3.
- Back to the tree diagram.

Option Valuation

- Numerical example:



Option Price Tree

To get C we need to find delta again.

$$H = \frac{C+ - C-}{S+ - S-} = .75$$

Option Valuation

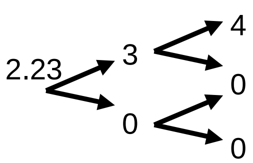
- Now we know that we could write 1 call and hold .75 stock and the payoff in the first node is riskless.
- Since this is a half period, r_f = 1%
- If we short 1 call and buy .75 shares:
  - » We get (.75)(98) = $73.50 if S goes down.
  - » We get (.75)(102) - 3 = $73.5 if S rises.

Option Valuation

- With .75 shares and 1 short call, we get $73.50 no matter what.
- So (.75)S - C = PV(73.5).
- 75 - C = 72.77.
- C = 75 - 72.77 = $2.23.
- Back to the tree diagram.

Option Valuation

- Numerical example:



Option Price Tree

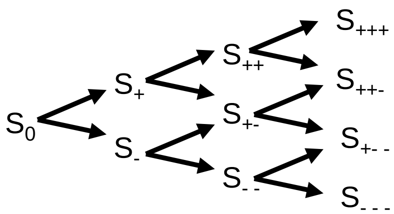
This is the completed tree.

Option Valuation

- You do not need to know how to construct trees exactly like this.
- If you ever use a tree algorithm, you should know what it is doing.
- The tree model uses *dynamic hedging.*
- When we allow for infinitely many nodes, this model converges to the Black-Scholes option pricing model.

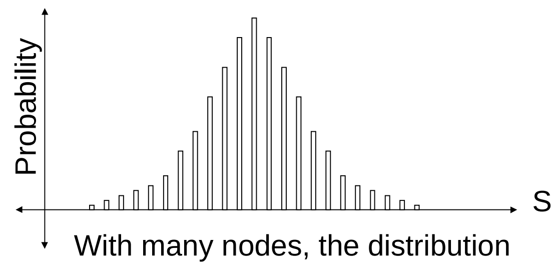
Option Valuation

Remember this tree?



How many paths are there to each node?

Option Valuation



With many nodes, the distribution of prices becomes *lognormal*.

Option Valuation

- The Black-Scholes formula:

$$C_0 = S_0\, N(d_1) - X\, e^{-rT} N(d_2)$$

$$d_1 = \frac{ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\, T^{(1/2)}}$$

$$d_2 = d_1 - \sigma\, T^{(1/2)}$$

Option Valuation

- Components of the formula:
  - » C_0 = call price, S_0 = stock price, X = strike.
  - » *N*(d*) = cumulative normal distribution.
  - » *e* = 2.71828..., *ln* = natural logarithm.
  - » r = risk-free rate, T = maturity.
  - » σ = standard deviation of log returns.
- You can calculate this with a calculator as long as you know the normal distribution.

Option Valuation

- Using C, we get put prices with PC parity.
- The BS model makes many assumptions.
  - » The stock pays no dividends.
  - » r and σ are constant over the life of the call.
  - » Stock prices are continuous - no jumps.
  - » Constant dynamic hedging is possible.
- More complicated models drop these.
- Binomial trees usually work.

Option Valuation

- We can observe all the components of the BS formula except for σ.
- Often we will calcuate the value of σ implied by one option value and apply it to valuing other options.
- Volatilities calculated this way are called *implied* volatilities.

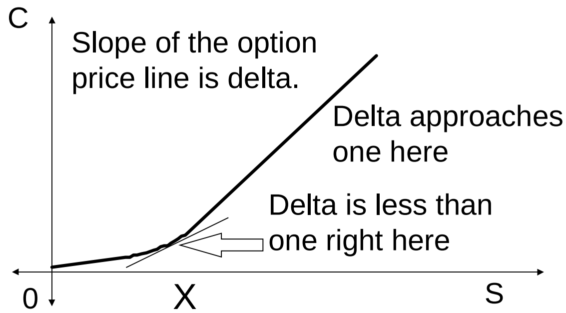
Option Valuation

- One way to adjust BS for dividends and possible early exercise is to calculate BS values assuming early exercise and exercise at maturity.
- After calculating both types of call values, we pick the maximum value.
- Its probably better to use a tree model for possible early exercise.

Option Valuation

- We can get the hedge ratio, delta, quite easily from the BS formula.
- With BS, delta = $N(d_1)$.
- We can interpret delta as $\Delta C/\Delta S$.
- This means that delta is just the slope of the graph that we saw before.

Option Valuation



Option Valuation

- The delta of a call is the number of shares we need to be hedged with 1 call written.
- The delta of a call is positive.
- The delta of a put is always negative.
- The delta of a put is $\Delta P/\Delta S = N(d_1) - 1$.
- The put's delta is the *percentage* of our **stock** holdings we need to put in T-bills to create a synthetic protective put position.

Option Valuation

Summary:

- Option values depend on lots of variables.
- We can place bounds on option values.
- Early exercise is never optimal for a call that does not pay dividends.
- If a call pays large dividends, early exercise may be optimal.
- Puts with D = 0 may require early exercise.

Option Valuation

Summary:

- Binomial tree models are used to value call options.
- With lots of nodes, a binomial tree becomes the Black-Scholes model.
- The delta of a call is $\Delta C/\Delta S = N(d_1)$.
- The delta of a put is $\Delta P/\Delta S = N(d_1) - 1$.