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The inescapable need for fractal tools in finance

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Summary. This short paper advances and defends a strong statement concerning financial modeling. It argues that, even when the present fractal models become superseded, fractal tools are bound to remain central to finance. The reasons are that the main feature of price records is roughness and that the proper language of the theory of roughness in nature and culture is fractal geometry.

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General questions that are often asked may take time to be answered. Everyone has wondered why – without ever trying explicitly – fractal geometry impinges deeply on so many issues in otherwise very different fields. How to explain that, as early as in the 1960s, my works in natural and social sciences (namely, turbulence, hydrology, and financial prices) could rely on closely analogous tools? Those questions can, at long last, be given the following short answer:

FRACTAL GEOMETRY MEASURES ROUGHNESS INTRINSICALLY.
HENCE IT MARKS THE BEGINNING OF A QUANTITATIVE THEORY
SPECIFIC TO ROUGHNESS IN ALL IT MANIFESTATIONS.

Roughness is ubiquitous in nature and culture (the latter term being used to denote all the works of Man, including financial markets). This is why fractality is also ubiquitous and why fractal geometry will never lack problems to deal with. An *Overview of fractals and multifractals* written in this spirit is featured in [3] in the form of a long Chapter H1.

Despite its ubiquity and antiquity, it is unquestionable and worth noting that the study of roughness has lagged very far behind the studies of other comparably ancient concepts: steepness (of a road or a trend), heaviness, pitch, color, hotness, and the like. The key drawback was that the intensity of roughness could not be measured. The first intrinsic quantitative measure of roughness had to wait for fractal geometry. Let me elaborate.

The steepness of a smooth incline came, of course, to be defined by the derivative of the height $h(x)$ along the incline. In theory, this definition implies that the increments' ratio dh/dx tends to a limit as $dx \to 0$. Custom has made this ratio be viewed as "normal". In practice, it suffices that dh/dx be nearly constant. But – almost by definition – rough profiles and surfaces are such that dh/dx varies all over without limit. By contrast, a basic feature of many models of price variation – the Bachelier model and my fractal/multifractal models – is that the derivative is not the proper tool. Instead, a limit exists for the highly "anomalous" ratio log (dh)/log (dx). Its existence is a manifestation of "scaling." According to the Bachelier model, this ratio has the limit $\alpha = 1/2$. Being the same at all instants in all financial data is a very important property. It is both a big asset – because of its simplicity, and a big flaw – because a limit equal to 1/2 is not available as parameter to be fitted to the data. To the contrary, the fractal/multifractal models allows $\alpha \neq 1/2$.

In inverse historical sequences and decreasing generality, I have originated and investigated three cases that are compared in [2]. The value of α may vary in some specific way from instant to instant; this characterizes multifractality. The value of α may be the same at all time instants but different from 1/2; this characterizes unifractality or the HHM (Hurst-Hölder-Mandelbrot) model. There is also a very important intermediate case I call "mesofractality" or the PLM (Pareto-Lévy-Mandelbrot) model. The initials HHM and PLM are motivated in the preceding article [2].

For the derivative, the intuitive concept of slope long predated mathematics. That is, a quantitative measure of the intuitive notion of steepness came early and the mathematics came late. The concept of α took the opposite path. I devised it for the sake of science by modifying a concept that Hölder introduced in 1870 as being purely mathematical and totally separate from intuition. A century later, my work gradually identified the Hölder exponent with an exponent due to Hurst, and proved it to be a key aspect of roughness. Examine, indeed, the various cartoons that illustrate the preceding paper [2]. From one to another, the intuitive, "eyeball," levels of roughness are immediately seen to be different.

To summarize, a key feature of fractal geometry is that it begins by measuring roughness by α and/or related fractal concepts. Moreover, the value and/or the distribution of α is directly observable. It is not an elusive concept that has to be unscrambled indirectly from many other observations. The predominant role played by this exponent is an aspect of parsimony.

For reasons that can perhaps be guessed, are developed in [4], but cannot be repeated here, the arguments I deployed in the 1960s against the Brownian amounted to the following assertions. Firstly, the value $\alpha = 1/2$ characterizes a special "mild" form of roughness. Secondly, the roughness found in financial data takes a "wild" form that excludes $\alpha = 1/2$.

While my substitutes for the Brownian have long been resisted, my objections were widely heard and innumerable alternative models reacted by introducing "fixes," each specifically designed to avoid one of the "anomalies" I pioneered, such as discontinuity, divergent moments, and divergent dependence. Examined in the light of the fractal/multifractal approach, the fixes have a common feature: they automatically reset the local roughness to 1/2.

One basic "fix" is variable – stochastic – volatility. It postulates that short enough records follow Brownian motion but that motion's variance changes, either continually every so often. My first criticism is that the burden of modeling is thereby not eliminated but solely pushed on to the process that rules the variation of volatility. If those variations are fast, the Brownian input is diluted to homeopathic irrelevance.

If the variance changes slowly, a second criticism kicks in: if volatility only measures the scale of Brownian motion, it does not affect α, hence every such "variable variance" model implicitly assumes $\alpha = 1/2$. To my knowledge, this conclusion has not been tested and the evidence I know suggests otherwise.

In any event, my multifractal model is definitely *not* a variable volatility model.

On the short run, fixes may be defended as the quickest response to urgent needs. On a longer run, they are unacceptable and they cannot be allowed to multiply forever. When the number of fixes in a recipes exceed a certain number, that recipe collapses under its own weight and the need arises for a new start. Thomas S. Kuhn famously described this process as "a change of paradigm." The paradigm that I introduced and favor as an alternative is fractal/multifractal.

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