INTRODUCTION TO DYNAMIC FINANCIAL ANALYSIS

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Abstract. In the last years we have witnessed growing interest in a discipline called Dynamic Financial Analysis (DFA). This phrase seems to be much more in use in the US and Canada than in Continental Europe. Moreover it is applied almost exclusively to nonlife insurance whereas a quite similar concept in life insurance is still called Asset Liability Management. We neither want to explain the difference between these two concepts nor do we want to introduce highly sophisticated modules of a DFA. There are some DFA models in place, mostly in the US. Our goal is to present an introduction into this field by giving an outline of common characteristics of different DFA models and by setting up a model framework for different modules of a DFA. We show how these modules can be constructed and related into an efficient risk management platform. An explicit DFA example is presented.

1. What is DFA

1.1. **Idea.** Nonlife insurance companies in the US, Canada and also in Europe have experienced certain developments during the past years. The most important ones have been: pricing cycles accompanied by volatile insurance profits, increasing catastrophe losses, and well performing capital markets giving rise to higher realized capital gains. These developments bore chances and risks for the two main objectives of an insurance company: solvency and profitability. The key management challenge will be the creation of shareholder value. Related to this objective is the decision on the amount of capital the company needs to run its business, and on the cost of this capital. In order to cope with these items one should be able to identify the sources of variability and quantify their levels and interrelations.

In order to analyze the financial effects of different entrepreneurial strategies for nonlife insurance companies over a given time horizon, one finds two primary techniques in use today. The first one, the so-called scenario testing, projects results under specific deterministic scenarios in the future. The disadvantage of this approach is the fact that only a few arbitrary scenarios are tested in order to decide how good a strategy is. The other technique is stochastic simulation, better known as *Dynamic Financial Analysis* (DFA). Here many different scenarios are generated stochastically with the aim of giving information about the distribution of some important variables, like surplus, written premiums or loss ratio.

1.2. **Fixing the Time Period.** On the one hand we would like to model over as long a time period as possible in order to see the long-term effects of a chosen strategy. In

Date: December 15, 1999.

Key words and phrases. Dynamic Financial Analysis, entrepreneurial strategy, efficient frontier concept, solvency testing, stochastic scenario, assets, liabilities, interest rate, losses, reinsurance, underwriting cycles, payment patterns.

The article is partially based on a diploma thesis written in cooperation with Zurich Financial Services. Further research of the first author was supported by Credit Suisse, Swiss Re and UBS through RiskLab.

particular, effects of decisions concerning long-tail business only appear after some years and can hardly be recognized in the first few years. Therefore to judge how good a strategy is we should take into consideration these long-term effects. On the other hand simulated values get more and more unreliable the longer this time period is. When we simulate, we start with given numbers and will most likely experience deviations from reality, which add up over time. For these reasons a compromise must be made in order to fix the length of the simulated time period.

1.3. **Which Risks Should be Modelled?** A DFA model is based on a stochastic model for the main factors of the whole complex. Hence we first have to identify the variables that determine the stochasticity on both the asset side and the liability side of the balance sheet. A good model should simulate stochastically the asset elements as well as the liability elements and interrelate both sides to reconcile with the intuition of an experienced actuary. This differs from traditional ALM-approaches (ALM=Asset-Liability Management) in life insurance. There for a long time liabilities have been assumed to be fixed, because they do not vary much from one year to another by reason of the long term structure of life insurance contracts. In nonlife insurance we cannot say in advance how liabilities will develop. We do neither know the time of a claim occurrence nor do we know its final size. The latter is different from life insurance where the claim size is for most traditional products known at the outset of a contract. In nonlife insurance the nature of liabilities appears to be more stochastic than in life assurance. In order to cope with this behaviour, we have to emphasize the stochastic simulation of liabilities.

It is important to note that it is neither possible nor appropriate to model all sources of risk and try to represent reality in detail. It can be dangerous to place confidence in a detailed, but perhaps inappropriate model. It is often better to use a simple model that captures only the key features. Smaller models tend to be more in line with intuition, and they make it easier to assess the influence of individual variables.

1.4. **Aim of DFA, Applications of DFA Models.** DFA gives the opportunity to compare the effects of different entrepreneurial strategies before applying them to reality. It does not necessarily give an optimal solution but instead leaves the decision of selecting a strategy to management. In other words DFA serves as a decision tool that requires a good understanding of the nonlife insurance business and some analytical/actuarial skills to be successfully implemented.

Before using a DFA model, management has to choose a financial or economic measure in order to assess a particular strategy which will be analyzed. The most common concept is the *efficient frontier concept* which is used in modern portfolio theory going back to Markowitz; see Markowitz [19]. When a company has chosen a measure for performance (e.g. expected surplus) and a measure for risk (e.g. expected policyholder deficit $EPD(X) = -\mathbb{E}_{\mathbb{P}}[X|X<0]$, see Lowe and Stanard [17], or worst conditional mean $WCM_\alpha(X) = -\inf\{\mathbb{E}_{\mathbb{P}}[\frac{X}{r}|A] \,|\, \mathbb{P}[A] > \alpha\}$ as a coherent risk measure, see Artzner, Delbaen, Eber and Heath [2] and [3]; r stands for the return on a reference instrument), it can compare different strategies by plotting the measured risk and the measured performance, as shown in Figure 1.1.

1.5. **Link Between DFA and Solvency Testing.** A better known concept than DFA is *solvency testing*, which deals with one central question: Does the company have enough capital compared to the level of risk it is exposed to, i.e. does the company have enough

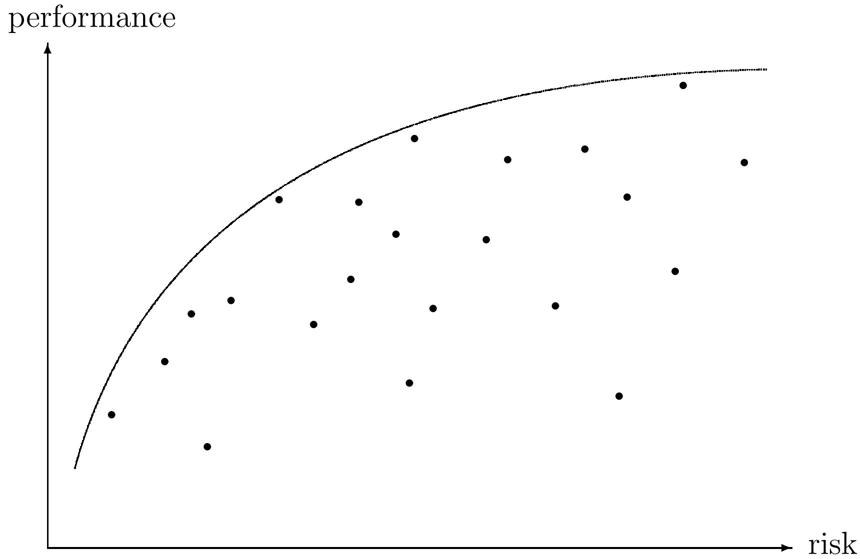

FIGURE 1.1. Efficient frontier.

capital to keep the probability of loosing $\alpha \cdot 100\%$ of its capital below a given level for the risks taken?

DFA gives us an estimate for the distribution of surplus, and therefore the probability of loosing $\alpha \cdot 100\%$ can be estimated. This means that DFA can serve as a solvency testing tool as well.

More information about solvency testing can be found in Schnieper [22] and [23].

1.6. **Main Structure of a DFA Model.** Most DFA models consist of three major parts. The *stochastic scenario generator* is the producer of stochasticity. Here random variables are generated. One scenario consists of a simulation of all implemented random variables. The second source of data is the company specific *input*, that includes historical data (e.g. mean severity of losses for every line of business, for every accident year), assumptions for model parameters (e.g. long term mean rate in a Cox, Ingersoll, Ross-interest rate model), and strategic assumptions (e.g. investment strategy). The last part, the *output* provided by the DFA model, can then be analyzed by management in order to improve the strategy. This is shown graphically in Figure 1.2.

2. Stochastically Modelled Variables

A very important step in the process of building an appropriate model is to find out which variables are the most important ones, and what are the sources of stochastic behaviour. After having identified such a variable, there still remains the problem of modelling or quantifying this factor. Moreover one has to trade-off improvement of accuracy versus increase in complexity, i.e. decrease in transparency.

There are different risk categories, and each of them we model with the help of a so-called generator. A possible and reasonable choice for the variables that a nonlife insurance company should model stochastically could be the following: On the asset side we need an *interest rate generator* in order to estimate interest rate risk. This is probably the most important asset risk since nonlife insurance companies are strongly exposed to it due to generally large investments in fixed income asset classes. Interest rates are strongly correlated with inflation, which itself influences the changes in claim size and

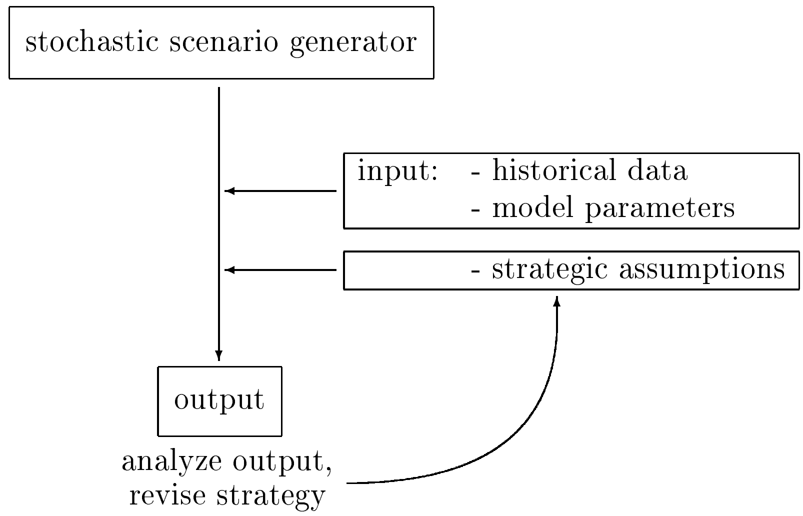

FIGURE 1.2. Main structure of a DFA model.

claim frequency. A second necessary generator on the asset side simulates *stock returns*, depending on interest rates. On the liability side we have several risk categories. In this paper we consider four different ones: *non-catastrophe losses*, *catastrophe losses*, *underwriting cycles*, and *payment patterns*. Catastrophes are separated from non-catastrophe losses, and for each of these two groups the number and the severity of claims are modelled separately. Another approach would be to integrate the two by using a heavy-tailed distribution; see for instance Embrechts, Klüppelberg and Mikosch [10]. Underwriting cycles model the current market situation, like the competition among insurance companies, or a general recession. With payment patterns we model when and how losses are paid. We need these patterns to estimate loss reserves.

To model these risk categories there are many different possibilities. Here we present one possibility in order to give an idea how components can be modelled stochastically.

2.1. **Interest Rates.** We assume, as for example Daykin, Pentikäinen and Pesonen [9, p. 231], that inflation is correlated with interest rates. So we construct one single generator that simulates all the following: short term interest rate, long term interest rates, general inflation, and inflation by line of business.

The probably best known model for simulating investment return and inflation in the insurance world is the Wilkie model, see for example Wilkie [25], or Daykin, Pentikäinen and Pesonen [9, pp. 242-250].

2.1.1. *Short Term Interest Rate.* There are many different interest rate models used by financial economists. An overview of some common ones is given by Ahlgrim, D'Arcy and Gorvett [1], see also Musiela and Rutkowski [21, pp. 281-302].

To simulate the annualized short term interest rate for every year t we can use for example a discretization of the mean reversion model proposed by Cox, Ingersoll and Ross (CIR model).

The CIR model in continuous time is characterized by the equation

$$dr_t = a\,(b - r_t)\,dt + s\,\sqrt{r_t}\,dZ_t\,. \tag{2.1.1}$$

where

$$\begin{aligned} r_t &= \text{the instantaneous short term interest rate,}\\ b &= \text{the long term mean of the interest rate,}\\ a &= \text{a constant, that determines the speed of reversion}\\ &\quad\ \text{of the interest rate towards its long-run mean } b,\\ s &= \text{the volatility of the interest rate process,}\\ Z_t &= \text{a standard Brownian motion.} \end{aligned}$$

By discretizing this mean reversion model we get

$$r_t = r_{t-1} + a\,(b - r_{t-1}) + s\,\sqrt{r_{t-1}}\,Z_t\,, \tag{2.1.2}$$

where

$$\begin{aligned} r_t \ &= \ \text{the instantaneous short term interest rate}\\ &\quad\ \text{at the beginning of year } t,\\ Z_t &\sim \ \mathcal{N}(0,1),\ Z_1, Z_2, \ldots \ \text{i.i.d.},\\ &a,\ b,\ s \text{ as in (2.1.1).} \end{aligned}$$

Whereas the realizations of the CIR model in continuous time are almost surely positive, when discretizing one may lose this property. If r_t becomes negative we cannot calculate the right side of equation (2.1.2). So we change it to

$$r_t = r_{t-1} + a\,(b - r_{t-1}) + s\,\sqrt{{r_{t-1}}^{+}}\,Z_t\,, \tag{2.1.3}$$

or

$$r_t = r_{t-1} + a\,(b - r_{t-1}) + s\,\sqrt{|r_{t-1}|}\,Z_t\,. \tag{2.1.4}$$

In order to model how volatility depends on the current short term interest rate r_t we introduce a new variable g:

$$r_t = r_{t-1} + a\,(b - r_{t-1}) + s\,|r_{t-1}|^g\,Z_t\,. \tag{2.1.5}$$

Chan, Karolyi, Longstaff and Sanders [4] describe how suitable values for a, b, s, and g can be found based on historical data.

A disadvantage of (2.1.3), (2.1.4) and (2.1.5) is the fact that some of the simulated values for r_t may become negative. When we try to cope with this, an obvious possibility is to take the positive value of formula (2.1.5):

$$r_t = \big(r_{t-1} + a\,(b - r_{t-1}) + s\,r_{t-1}^g\,Z_t\big)^+\,. \tag{2.1.6}$$

There is also a disadvantage when we use this formula: b can no longer be interpreted as the long term mean of the interest rate, indeed $b < \lim_{t\to\infty} \mathbb{E}\,[r_t]$.

If we observe that the short term interest rates simulated by a *yearly* discretization are not good enough, we can change to smaller time intervals, and simulate for example monthly, weekly, or even daily values of r_t.

2.1.2. *Long Term Interest Rates.* We can also use the CIR model (2.1.1) to calculate the long term interest rates $R_{t,T}$ with time to maturity T (in years), starting at time t. Lamberton and Lapeyre [16, pp. 129-133] prove the following result:

$$\mathbb{E}\,[e^{-\int_0^T r_{t+s}\,ds}] = e^{\ln A_T - r_t\,B_T}, \tag{2.1.7}$$

where

$$A_T = \Big(\frac{2G\, e^{(a+G)\,T/2}}{(a+G)\,(e^{GT}-1)+2G}\Big)^{2ab/s^2},$$

$$B_T = \frac{2(e^{GT}-1)}{(a+G)\,(e^{GT}-1)+2G},$$

$$G = \sqrt{a^2+2s^2}\,.$$

Because $R_{t,T}$ is the yield of a zero coupon with term to maturity T we can write the discounting factor $\mathbb{E}\,[e^{-\int_0^T r_{t+s}\,ds}]$ as $e^{-T\,R_{t,T}}$. With (2.1.7) we get

$$R_{t,T} = \frac{r_t\,B_T - \ln\,A_T}{T}\,. \tag{2.1.8}$$

2.1.3. *General Inflation.* Following our introductory remark to Section 2.1 we simulate the general inflation i_t – that we need for modelling loss payments – by using the short term interest rate r_t. We can do this by using a linear regression model:

$$i_t = a^I + b^I\,r_t + \sigma^I\,\epsilon_t^I, \tag{2.1.9}$$

where

$\epsilon_t^I \sim \mathcal{N}(0,1),\ \epsilon_1^I, \epsilon_2^I, \ldots$ i.i.d.,
a^I, b^I, σ^I: parameters that can be estimated by regression, based on historical data.

2.1.4. *Change by Line of Business.* The impact of inflation is not the same for different lines of business. Daykin, Pentikäinen and Pesonen [9, p. 215], and Walling, Hettinger, Emma and Ackerman [24] explain more in detail why it makes sense to model the changes caused by inflation for each line of business separately.

To model the change in loss frequency δ_t^F (i.e. # losses/# written exposure units), the change in loss severity δ_t^X, and the resulting change in prices δ_t^P, we can use the following formulas:

$$\delta_t^F = \max\,(a^F + b^F\,i_t + \sigma^F\,\epsilon_t^F, -1), \tag{2.1.10}$$

$$\delta_t^X = \max\,(a^X + b^X\,i_t + \sigma^X\,\epsilon_t^X, -1), \tag{2.1.11}$$

$$\delta_t^P = (1+\delta_t^F)\,(1+\delta_t^X) - 1, \tag{2.1.12}$$

where

$\epsilon_t^F \sim \mathcal{N}(0,1),\ \epsilon_1^F, \epsilon_2^F, \ldots$ i.i.d.,
$\epsilon_t^X \sim \mathcal{N}(0,1),\ \epsilon_1^X, \epsilon_2^X, \ldots$ i.i.d., $\epsilon_{t_1}^F, \epsilon_{t_2}^X$ independent $\forall\, t_1, t_2$,
a^F, b^F, σ^F, a^X, b^X, σ^X: parameters that can be estimated by regression, based on historical data.

The technical restriction of setting δ_t^F and δ_t^X to at least -1 is necessary in order to assure that we do not get any negative simulated values for loss numbers and loss severities.

The reason why we model also the changes in loss frequency dependent on general inflation is the empirical observation that in certain situations (e.g. when the inflation is high) insurants report more losses in certain lines of business.

The corresponding cumulative changes $\delta_t^{F,c}$ and $\delta_t^{X,c}$ can be calculated by

$$\delta_t^{F,c} = \prod_{s=t_0+1}^{t} (1 + \delta_s^F), \tag{2.1.13}$$

$$\delta_t^{X,c} = \prod_{s=t_0+1}^{t} (1 + \delta_s^X), \tag{2.1.14}$$

where

$$t_0 + 1 = \text{first year we model.}$$

2.2. **Stock Returns.** In order to model assets suitably, it is necessary to simulate stock returns.

To simulate the stock return r_t^S we make use of the Sharpe-Lintner CAPM pricing equation – for details on the Capital Asset Pricing Model (CAPM) see for example Ingersoll [14]. We need a model for the expected return $\mathbb{E}\left[r_t^M\right]$ on the market portfolio. For simplicity reasons we use a linear model:

$$\mathbb{E}\left[r_t^M | R_{t,1}\right] = a^M + b^M \left(e^{R_{t,1}} - 1\right), \tag{2.2.1}$$

where

$$\begin{aligned} e^{R_{t,1}} - 1 &= \text{risk-free return, see (2.1.8),} \\ a^M, b^M &= \text{parameters that can be estimated by} \\ & \text{regression, based on historical data.} \end{aligned}$$

Now we can use the CAPM formula to get the expected return on a stock:

$$\mathbb{E}\left[r_t^S | R_{t,1}\right] = \left(e^{R_{t,1}} - 1\right) + \beta_t \left(\mathbb{E}\left[r_t^M | R_{t,1}\right] - \left(e^{R_{t,1}} - 1\right)\right), \tag{2.2.2}$$

where

$$\begin{aligned} e^{R_{t,1}} - 1 &= \text{risk-free return,} \\ r_t^M &= \text{return on the market portfolio,} \\ \beta_t &= \beta\text{-coefficient of this asset} \\ &= \frac{Cov\left(r_t^S, r_t^M\right)}{Var\left(r_t^M\right)}. \end{aligned}$$

Usually the price of a stock is modelled with a geometric Brownian Motion. Therefore we can assume a lognormal distribution for $1 + r_t^S$:

$$1 + r_t^S \sim \text{lognormal}\,(\mu, \sigma^2), \quad r_1^S, r_2^S, \ldots \text{ independent,} \tag{2.2.3}$$

with μ chosen to yield

$$m_t = e^{\mu + \frac{\sigma^2}{2}},$$

where

$$\begin{aligned} m_t &= 1 + \mathbb{E}\left[r_t^S | R_{t,1}\right], \text{ see (2.2.2),} \\ \sigma^2 &= \text{estimated variance of logarithmic historical values.} \end{aligned}$$

Of course, in the above, more refined econometric models can be used as there are APT, stochastic volatility, GARCH models.

2.3. **Non-Catastrophe Losses.** Non-catastrophe losses of various lines of business develop quite differently. Therefore we simulate loss numbers and loss severities for every line of business separately. For the sake of better legibility, in this section we drop the index that represents the line of business.

Statistical considerations also show that losses depend on the age of insurance contracts. The *aging phenomenon* describes the fact that the loss ratio – i.e. the ratio of losses divided by earned premiums – decreases when the age of policy increases. For this reason it might prove useful to divide insurance business into three classes, as proposed by D'Arcy, Gorvett, Herbers, Hettinger, Lehmann and Miller [7]:

- new business (superscript 0),
- renewal business – first annual (superscript 1), and
- renewal business – second annual and subsequent (superscript 2).

More information about the aging phenomenon can be found in D'Arcy and Doherty [5] and [6], Feldblum [13], and in Woll [26].

To simulate loss numbers N_t^j and loss severities X_t^j for period t and renewal category j we can utilize the mean values $\mu^{F,j}$, $\mu^{X,j}$ and the standard deviations $\sigma^{F,j}$, $\sigma^{X,j}$ of (discounted) historical data for loss frequencies and loss severities, and some other factors that affect losses. Because loss frequencies are more stable than loss numbers, we propose to use estimations for loss frequencies instead of estimating loss numbers directly.

For numbers of losses we can use for example the negative binomial, Poisson, or binomial distribution function with mean $m_t^{N,j}$ and variance $v_t^{N,j}$. When we decide for the negative binomial distribution function, we can simulate as follows:

$$\begin{aligned} &N_t^j \sim \mathrm{NB}\,(a,p),\ j = 0,1,2\,, \\ &N_1^j, N_2^j, \ldots \text{ independent}, \end{aligned} \tag{2.3.1}$$

with a and p chosen to yield

$$\begin{aligned} m_t^{N,j} &= \frac{a\,(1-p)}{p}, \\ v_t^{N,j} &= \frac{a\,(1-p)}{p^2}, \end{aligned} \tag{2.3.2}$$

where

$$\begin{aligned} m_t^{N,j} &= w_t^j\,\mu^{F,j}\,\delta_t^{F,c}\,, \\ v_t^{N,j} &= (w_t^j\,\sigma^{F,j}\,\delta_t^{F,c})^2, \\ w_t^j &= \text{written exposure units, modelled in (3.0.3)}, \\ \mu^{F,j} &= \text{estimated frequency, based on historical data}, \\ \sigma^{F,j} &= \text{estimated standard deviation in frequency,} \\ & \text{ based on historical data}, \\ \delta_t^{F,c} &= \text{cumulative change in loss number, see (2.1.13)}. \end{aligned}$$

For negative binomial distributed variables N we have over-dispersion: $Var(N) \geq \mathbb{E}[N]$. Therefore this distribution yields a reasonable model only if $v_t^{N,j} \geq m_t^{N,j}$.

When we try to solve (2.3.2), we have the problem that the variable a must be an integer. So we cannot just use

$$\begin{aligned} a &= \frac{(m_t^{N,j})^2}{v_t^{N,j} - m_t^{N,j}}, \\ p &= \frac{m_t^{N,j}}{v_t^{N,j}}. \end{aligned} \tag{2.3.3}$$

One possibility is to round a mathematically, and then use the first equation of (2.3.2) to get a value for p:

$$\begin{aligned} a &= \max(\lfloor \frac{(m_t^{N,j})^2}{v_t^{N,j} - m_t^{N,j}} + 1/2 \rfloor, 1), \\ p &= \frac{a}{m_t^{N,j} + a}. \end{aligned} \tag{2.3.4}$$

The notation $\lfloor \ldots \rfloor$ means that the integer part of this expression is taken.

For mean loss severities one possibility is to use a GPD (generalized Pareto distribution) $G_{\xi,\beta}$. GPD's play an important role in the Extreme Value Theory, where $G_{\xi,\beta}$ appears as the limit distribution of scaled excesses over high thresholds, see for instance Embrechts, Klüppelberg and Mikosch [10, p. 165]. But also a Gamma distribution with mean $m_t^{X,j}$ and variance $v_t^{X,j}$ may be convenient. Because the density function of a Gamma distribution decreases exponentially, and since we model only non-catastrophe losses here, we can simulate mean loss severities by

$$\begin{aligned} &X_t^j \sim \text{Gamma}(\alpha, \theta), \; j = 0, 1, 2\,, \\ &X_1^j, X_2^j, \ldots \text{ independent}, \end{aligned} \tag{2.3.5}$$

with α and θ chosen to yield

$$\begin{aligned} m_t^{X,j} &= \alpha\,\theta\,, \\ v_t^{X,j} &= \alpha\,\theta^2, \end{aligned}$$

where

$$\begin{aligned} m_t^{X,j} &= \mu^{X,j}\,\delta_t^{X,c}\,, \\ v_t^{X,j} &= (\sigma^{X,j}\,\delta_t^{X,c})^2/\delta_t^{N,c}, \\ \mu^{X,j} &= \text{estimated mean severity, based on historical data}, \\ \sigma^{X,j} &= \text{estimated standard deviation, based on historical data}, \\ \delta_t^{X,c} &= \text{cumulative change in loss severity, see (2.1.14)}, \\ \delta_t^{N,c} &= \text{cumulative change in loss number, see (2.1.13)}. \end{aligned}$$

2.4. **Catastrophes.** In the simulation of loss numbers and loss severities we modelled only non-catastrophe losses. We simulate catastrophes separately, due to the quite different statistical behaviour of catastrophe and non-catastrophe losses. In general the volume of empirical data for non-catastrophe losses is much bigger than for catastrophe losses. By separating the two, we have more homogeneous data for non-catastrophe losses, which makes fitting the data by well known (right skew) distributions easier. In addition we experience a rapid development of a theory of distributions for extremal events, see Embrechts, Klüppelberg and Mikosch [10], and McNeil [20]. Therefore we consider the separate modelling for catastrophe and non-catastrophe losses as most appropriate.

For the number of catastrophes we can use for example the negative binomial, Poisson, or binomial distribution function with mean m^M and variance v^M. We assume that there is no trend in the number of catastrophes:

$$\begin{aligned} &M_t \sim \text{NB, Pois, Bin}, \ldots \text{ (mean } m^M, \text{variance } v^M), \\ &M_1, M_2, \ldots \text{ i.i.d.}, \end{aligned} \tag{2.4.1}$$

where

$$\begin{aligned} m^M &= \text{estimated number of catastrophes, based on historical data,} \\ v^M &= \text{estimated standard deviation, based on historical data.} \end{aligned}$$

Contrary to the modelling of the non-catastrophe losses, we simulate the *total* loss severity (i.e. not only the part the insurance company in consideration has to pay) for *every catastrophe* $i \in \{1, \ldots, M_t\}$ separately. Again, there are different distribution functions that proved to be adequate in the past:

$$\begin{aligned} &Y_{t,i} \sim \text{lognormal, Pareto, GPD}, \ldots \text{ (mean } m_t^Y, \text{variance } v_t^Y), \\ &Y_{t,1}, Y_{t,2}, \ldots \text{ i.i.d.}, \\ &Y_{t_1,i_1}, Y_{t_2,i_2} \text{ independent } \forall\, (t_1, i_1) \neq (t_2, i_2), \end{aligned} \tag{2.4.2}$$

where

$$\begin{aligned} m_t^Y &= \mu^Y\, \delta_t^{X,c}\,, \\ v_t^Y &= (\sigma^Y\, \delta_t^{X,c})^2, \\ \mu^Y &= \text{estimated loss severity, based on historical data,} \\ \sigma^Y &= \text{estimated standard deviation, based on historical data,} \\ \delta_t^{X,c} &= \text{cumulative change in loss severity, see (2.1.14).} \end{aligned}$$

Now we can divide up the total severity $Y_{t,i}$ among the various lines of business which are affected by the catastrophic event:

$$Y_{t,i}^k = a_{t,i}^k\, Y_{t,i}\,,\ k = 1, \ldots, l\,, \tag{2.4.3}$$

where

$l = \#$ lines of business,
$\forall\, i \in \{1, \ldots, M_t\}$: $(a_{t,i}^1, \ldots, a_{t,i}^l) \in \{x \in [0,1]^l, \|x\| = 1\} \subset \mathbb{R}^l$ is a random convex combination, whose distribution within the (l-1) dimensional tetraeder can be arbitrarily specified.

By simulating the percentages $a_{t,i}^k$ stochastically, we model the diversification benefit that occurs when the company writes different lines of business.

Now, if we know the market shares of the insurance company in consideration by line of business, and if we know its reinsurance structure, we can calculate how catastrophes affect the liabilities.

The construction in this section creates dependence between total catastrophe losses in different lines of business, although all generated variables are *independent*.

2.5. **Underwriting Cycles.** After deregulation, even in Europe, for an insurance company the general market conditions are too important to be ignored. So we try to model the underwriting cycles, although they are quite complex. With these cycles we try to capture states like competition among insurance companies for certain lines of business, or general recession.

We can use a homogeneous Markov chain model (in discrete time), that resembles the one proposed by D'Arcy, Gorvett, Hettinger and Walling [8]: We classify each line of business for every year into one of the following states:

1 Weak competition,
2 Average competition,
3 Strong competition.

In state 1 (weak competition) the insurance company demands high premiums because it can increase its market share anyway. In state 3 (strong competition) the insurance company must demand low premiums in order to at least keep its current market share. High premiums are equivalent to high profit margin over pure premium, and low premiums equal low profit margin. Changing from one state to another causes significant changes in premiums.

The transition probabilities p_{ij}, $i, j \in \{1, 2, 3\}$ which denote the probability of changing from state i to state j from one year to the next are assumed to be equal for every year. This means that the Markov chain is homogeneous. The p_{ij}'s can be written in a matrix T:

$$T = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}.$$

There are many different possibilities to set these transition probabilities p_{ij}, $i, j \in \{1, 2, 3\}$. It could be useful to model the p_{ij}'s depending on the current market conditions of *all* lines of business. One possibility is the following one. When the company writes l lines of business, there are 3^l states of the world. Because business cycles of different lines of business are strongly correlated, only few of the 3^l states are attainable. So we have to model $L \ll 3^l$ states, where the transition probabilities p_{ij}, $i, j \in \{1, \ldots, L\}$ are still equal for every year. It is possible that some of them are zero, because there may exist some states that cannot be attained directly from certain other states. When L states are attainable, the matrix T has dimension $L \times L$:

$$T = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1L} \\ p_{21} & p_{22} & \cdots & p_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ p_{L1} & p_{L2} & \cdots & p_{LL} \end{pmatrix}.$$

In any of the above mentioned cases, in order to fix the transition probabilities p_{ij} we should first consider every state i separately, and assign estimated percentages to the variables $p_{i1}, \ldots, p_{iL}$ (such that $\sum_{j=1}^{L} p_{ij} = 1 \; \forall i$). Then, as a control, we should consider the stationary probability distribution π, which in general – i.e. if the Markov chain is irreducible and positive recurrent – our probability distribution converges to, no matter which starting state we choose. Because it is easier to estimate the stationary probability distribution π than to find suitable values for the p_{ij}'s we should use the fact that $\pi = \pi T$, to *check* whether the estimated values for the transition probabilities are realistic.

A central point in this consideration is to set the initial market conditions correctly in order to get a realistic simulation.

2.6. **Payment Patterns.** Until now we have modelled claim number and claim size, but we did not yet model *when* losses are paid. For that purpose we need payment patterns.

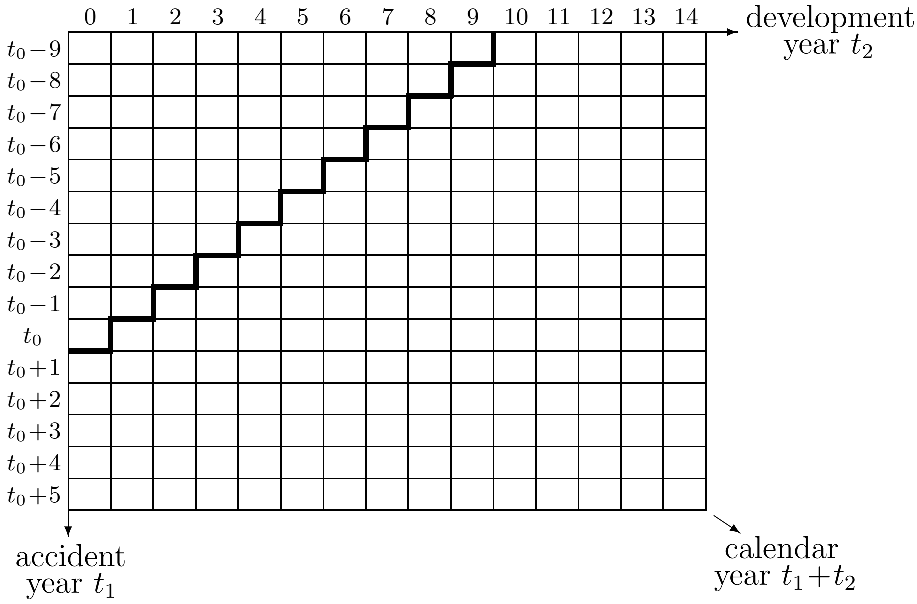

FIGURE 2.1. Paid losses in the triangle on the left side of the thick line are known, while the ones in the other part we have to estimate.

For every accident year t_1 these patterns give us the information which part of the total loss is paid in which development year t_2. We model payment patterns separately for every line of business, because they develop quite differently. For the sake of better legibility, for the most part of this section we drop the index that represents the line of business.

We make the assumption that for every line of business there is an ultimate development year τ until when all claims will be paid. We know the claim payments Z_{t_1,t_2} for previous years $t_1 + t_2 \leq t_0$, as shown in Figure 2.1. The ultimate losses $Z_{t_1}^{ult} := \sum_{t=0}^{\tau} Z_{t_1,t}$ vary by accident year t_1. We need to estimate these variables $Z_{t_1}^{ult}$ in order to determine adequate loss reserves. We will do this by applying a chain-ladder type procedure (for the chain-ladder method, see Mack [18]), i.e. we apply ratios to cumulative payments per accident year. Therefore we need to define the following type of loss development factor

$$d_{t_1,t_2} := \frac{Z_{t_1,t_2}}{\sum_{t=0}^{t_2-1} Z_{t_1,t}}, \; t_2 \geq 1, \tag{2.6.1}$$

which describes how losses change from one development year to the next.

We distinguish two cases. For previous accident years ($t_1 \leq t_0$) we can simulate for each calendar year $t_1 + t_2 \geq t_0 + 1$ losses paid in this year. Since a lognormal distribution gives a good fit for historical incremental loss development factors, we can use the following model:

$$Z_{t_1,t_2} = d_{t_1,t_2} \cdot \sum_{t=0}^{t_2-1} Z_{t_1,t} \,, \tag{2.6.2}$$

where

$$\begin{aligned} d_{t_1,t_2} &\sim \text{lognormal}(\mu_{t_2}, \sigma_{t_2}^2), \\ \mu_{t_2} &= \text{estimated logarithmic loss development factor for} \\ &\quad\ \text{development year } t_2\text{, based on historical data,} \\ \sigma_{t_2} &= \text{estimated logarithmic standard deviation, based} \\ &\quad\ \text{on historical data.} \end{aligned}$$

As already mentioned before, τ stands for the ultimate development year. So we have a simulation for the ultimate loss for accident year $t_1 \leq t_0$:

$$Z_{t_1}^{ult} = \sum_{t=0}^{\tau} Z_{t_1,t}. \tag{2.6.3}$$

For future accident years ($t_1 \geq t_0 + 1$) we simulated the ultimate losses already in Sections 2.3 and 2.4:

$$Z_{t_1}^{ult}(k) = \sum_{j=0}^{2} N_{t_1}^{j}(k)\ X_{t_1}^{j}(k) + b_{t_1}(k) \sum_{i=1}^{M_{t_1}} Y_{t_1,i}^{k} - R_{t_1}(k)\,, \tag{2.6.4}$$

where

$$\begin{aligned} N_{t_1}^{j}(k) &= \text{number of non-catastrophe losses in accident year } t_1 \\ &\quad\ \text{for line of business } k \text{ and renewal class } j\text{, see (2.3.1),} \\ X_{t_1}^{j}(k) &= \text{severity of non-catastrophe losses in accident year } t_1 \\ &\quad\ \text{for line of business } k \text{ and renewal class } j\text{, see (2.3.5),} \\ b_{t_1}(k) &= \text{market share of the company in year } t_1 \text{ for line of business } k, \\ M_{t_1} &= \text{number of catastrophes in accident year } t_1\text{, see (2.4.1),} \\ Y_{t_1,i}^{k} &= \text{severity of catastrophe } i \text{ in line of business } k \text{ in} \\ &\quad\ \text{accident year } t_1\text{, see (2.4.3),} \\ R_{t_1}(k) &= \text{reinsurance recoverables; a function of the } Y_{t_1,i}^{k}\text{'s.} \end{aligned}$$

Now we have to split these ultimate losses between development years. We can simulate the incremental percentages A_{t_1,t_2} for example by using beta distribution functions with parameters based on payment patterns of previous calendar years:

$$A_{t_1,t_2} = B_{t_1,t_2} \left(1 - \sum_{t=0}^{t_2-1} A_{t_1,t}\right), \tag{2.6.5}$$

where

$$\begin{aligned} B_{t_1,t_2} &= \text{percentage of remaining payments for} \\ &\quad\ \text{accident year } t_1 \text{ in development year } t_2 \\ &\sim \text{beta}(\alpha, \beta). \end{aligned}$$

Here α and β are chosen to yield

$$\begin{aligned} m_{t_1,t_2} &= \frac{\alpha+1}{\alpha+\beta+2}, \\ v_{t_1,t_2} &= \frac{(\alpha+1)\,(\beta+1)}{(\alpha+\beta+2)^2\,(\alpha+\beta+3)}, \end{aligned}$$

where

$$m_{t_1,t_2} = \text{estimated percentage of remaining payments for accident year } t_1 \text{ in development year } t_2, \text{ based on } \frac{A_{t_1-1,t_2}}{\sum_{t=t_2}^{\tau} A_{t_1-1,t}}, \frac{A_{t_1-2,t_2}}{\sum_{t=t_2}^{\tau} A_{t_1-2,t}}, \ldots,$$
$$v_{t_1,t_2} = \text{estimated variance, based on the same historical data.}$$

For each future accident year ($t_1 \geq t_0$) we can now calculate losses paid in development year t_2:

(2.6.6)
$$Z_{t_1,t_2} = A_{t_1,t_2}\, Z_{t_1}^{ult}.$$

For both previous and future accident years, at the end of calendar year $t_1 + t_2$ we do not know the ultimate losses $Z_{t_1}^{ult}$ yet. For each accident year t_1 we have to estimate them in each development year t_2:

(2.6.7)
$$\widehat{Z}_{t_1,t_2}^{ult} = \left(\prod_{t=t_2+1}^{\tau} 1 + e^{\mu_t} \right) \left(\sum_{t=0}^{t_2} Z_{t_1,t} \right),$$

where

$$\mu_t = \text{estimated logarithmic loss development factor for development year } t, \text{ based on historical data,}$$
$$Z_{t_1,t} = \text{simulated losses for accident year } t_1, \text{ paid in development year } t, \text{ see (2.6.2) and (2.6.6).}$$

Note that (2.6.7) is an estimate at the end of calendar year $t_1 + t_2$, whereas (2.6.4) simulates the *real* future value.

3. How Generators Influence Cash Flows

In this section we focus on the interrelation of stochastically simulated variables and core economic variables. We are not aiming at interconnecting each variable to display a complete model of the company. A more comprehensive description of a model for cash flows is given in Kaufmann [15].

The most important variable is surplus U_t. This variable gives us information about the financial strength of an insurance company and can serve as a measure for assessing the (shareholder) value of the company. A negative surplus is equivalent to the company becoming insolvent.

The change in surplus is influenced by the following cash flows:

(3.0.1)
$$\Delta U_t = P_t + I_t + C_t - Z_t - E_t - R_t - T_t.$$

where

P_t = earned premiums,
I_t = change in value of previous investments,
C_t = additions to capital and surplus,
Z_t = losses paid in calendar year t,
E_t = expenses,
R_t = loss reserves,
T_t = taxes.

To calculate *earned* premiums, we first need *written* premiums. For each line of business, written premiums P_t^j for renewal class j should be modelled depending on change in prices, on underwriting cycles, and on the number of written exposures. We could use a model like

$$P_t^j = (1 + \delta_t^P)\,(1 + c_{m_{t-1},m_t})\,\frac{w_t^j}{w_{t-1}^j}\,P_{t-1}^j\,,\ j = 0, 1, 2\,, \tag{3.0.2}$$

where

δ_t^P = change in prices, see (2.1.12),
c_{m_{t-1},m_t} = a constant that describes how premiums develop when we change from market condition m_{t-1} to m_t; c_{m_{t-1},m_t} can be estimated based on historical data,
w_t^0 = written exposure units for new business,
w_t^1 = written exposure units for renewal business, first annual,
w_t^2 = written exposure units for renewal business, second annual and subsequent.

The variables w_t^0, w_t^1, w_t^2 can be modelled depending on the numbers w_{t-1}^j of previous years and depending on the market conditions m_{t-1} and m_t. If we assume that in each market condition premiums are set in such a way that written exposure units do not vary much when we change from m_{t-1} to m_t, we can use for example a deterministic linear model:

$$w_t^j = a^j + b^j\,w_{t-1}^j\,,\ j = 0, 1, 2\,, \tag{3.0.3}$$

where

a^j, b^j = parameters that can be estimated by regression, based on historical data.

With the model (3.0.2) for written premiums $P_t^j(k)$, total earned premiums of all lines of business k and all renewal classes j can be calculated by

$$P_t = \sum_{k=1}^{l}\sum_{j=0}^{2} a_t^j(k)\,P_t^j(k) + \big(1 - a_{t-1}^j(k)\big)\,P_{t-1}^j(k) \tag{3.0.4}$$

where

$a_t^j(k)$ = percentage of premiums earned in year written.

When modelling investments we restrict to the most important investment classes, that is fixed income type investments (e.g. bonds, policy loans, cash), stocks, and real estate. Individual prices of fixed income type investments can be derived from a return that is required by the market (*market return* $R_{t,T}$). We need these rates $R_{t,T}$ to calculate the discounted values of bonds with term to maturity T. We have already described in Section 2.2 how stock prices are affected by the short term interest rate. For the sake of simplicity, real estate can be modelled quite similarly to stocks. Future investment profit depends not only on the development of market values of assets that an insurance company currently owns, but also on the decision how new funds are allocated. Therefore asset allocation is an important tool for management to optimize future investment profit. In order to build a DFA model in the strict sense of the word, we should account for changes in asset allocation in future years as compared to a pure static approach that keeps the allocation unchanged. This requires defining investment rules for specific economic conditions.

Additions to capital and surplus C_t: The specific items of additions to capital and surplus depend on the national accounting rules. The variable C_t can be estimated based on historical data.

Losses paid in year t can be calculated by

$$Z_t = \sum_{k=1}^{l} \sum_{t_2=0}^{\tau(k)} Z_{t-t_2,t_2}(k), \tag{3.0.5}$$

where

$$\begin{aligned} Z_{t-t_2,t_2}(k) &= \text{losses for accident year } t-t_2\text{, paid in development} \\ &\quad\ \text{year } t_2\text{; see (2.6.2) and (2.6.6),} \\ \tau(k) &= \text{ultimate development year for this line of business,} \\ k &= \text{line of business.} \end{aligned}$$

Expenses E_t can be estimated by a constant plus a term that is a multiple of written exposure units $w_t^j(k)$. The appropriate intercept $a^E(k)$ and slope $b^E(k)$ can be determined by linear regression:

$$E_t = \sum_{k=1}^{l} \left(a^E(k) + b^E(k) \sum_{j=0}^{2} w_t^j(k) \right). \tag{3.0.6}$$

For loss reserves R_t we have the equation

$$R_t = \sum_{k=1}^{l} \sum_{t_2=0}^{\tau(k)} \left(\widehat{Z}^{ult}_{t-t_2,t_2}(k) - \sum_{s=0}^{t_2} Z_{t-t_2,s}(k) \right), \tag{3.0.7}$$

where

$$\begin{aligned} \widehat{Z}^{ult}_{t-t_2,t_2}(k) &= \text{estimation in calendar year } t \text{ for ultimate losses in} \\ &\quad\ \text{accident year } t-t_2\text{; see (2.6.7),} \\ Z_{t-t_2,s}(k) &= \text{losses for accident year } t-t_2\text{, paid in development} \\ &\quad\ \text{year } s\text{; see (2.6.2) and (2.6.6),} \\ \tau(k) &= \text{ultimate development year,} \\ k &= \text{line of business.} \end{aligned}$$

Another very important thing needs to be considered: taxes T_t. In particular they are important if one uses a DFA model for estimating future performance of the company. Because taxes can be calculated exactly when we know all cash flows, we do not need another generator for them. Taxes can be calculated based on the other modelled variables.

4. DFA in Action

In order to show how a DFA model works in general, we wrote a short program in S-PLUS. It was not intended to describe a specific effect when using the parameters given below. The parameters were not selected as to give a most realistic picture. Rather we wanted to give an example what a DFA model can be used for. But in effect, calibrating the model as to yield reasonable results is one of the most important and time-consuming tasks when applying a DFA model in practice.

- Time horizon: 10 years.
- Performance measure: expected surplus.

- Risk measure: ruin probability.
- Only one line of business.
- For interest rates we use the discretization (2.1.4):
  $r_t = r_{t-1} + a\,(b - r_{t-1}) + s\,\sqrt{|r_{t-1}|}\,Z_t$.
- Parameters for interest rate generator: $a = 0.25,\ b = 5\%,\ s = 0.1,\ r_1 = 2\%$.
- Parameters for modelling inflation: $a^I = 0\%,\ b^I = 0.75,\ \sigma^I = 0.025$.
- No impact of inflation on the number of claims for the modelled line of business.
- Parameters for modelling the impact of inflation on the severity of claims for the modelled line of business: $a^X = 3.5\%,\ b^X = 0.5,\ \sigma^X = 0.02$.
- Parameters for generating return on stock portfolio:
  $a^M = 4\%,\ b^M = 0.5,\ \beta_t \equiv 0.5,\ \sigma = 0.15$.
- New business and renewal business are not modelled separately.
- Number of non-catastrophe losses $\sim$ NB (154, 0.025).
- Mean severity of non-catastrophe losses $\sim$ Gamma (9.091, 242), inflation-adjusted.
- Number of catastrophes $\sim$ Pois (18).
- Severity of individual catastrophes $\sim$ lognormal (13, 1.5^2), inflation-adjusted.
- Market share: 5%.
- Written premiums in the last year: 20 million.
- Expenses: 30% of written premiums.
- Optional excess of loss reinsurance with deductible 500 000 (inflation-adjusted), and cover ∞.
- Premiums for reinsurance: 200 000 p.a. (inflation-adjusted).
- Underwriting cycles: 1 = weak, 2 = average, 3 = strong. State in year 0: 1 (weak).
  Transition probabilities: $p_{11} = 60\%,\ p_{12} = 25\%,\ p_{13} = 15\%,$
  $p_{21} = 25\%,\ p_{22} = 55\%,\ p_{23} = 20\%,\ p_{31} = 10\%,\ p_{32} = 25\%,\ p_{33} = 65\%$.
- Payment patterns are deterministic.
- All liquidity is reinvested. There are only two investment possibilities:
  1) buy a risk-free bond with maturity one year,
  2) buy an equity portfolio with a fixed beta.
- Market valuation: assets and liabilities are stated at market value, i.e. assets are stated at their current market values, liabilities are discounted at the appropriate term spot rate determined by the model.
- No transaction costs.
- No taxes.
- No dividends paid.
- Initial surplus: 12 million.

In this model one can choose:

- How many simulations should be run.
- Whether the company is to be reinsured or not.
- How the liquidity is divided between bond and portfolio.

We ran this model 10 000 times for twelve different strategies, see Figure 4.1. In the first six strategies the investment percentages are fixed. In the other ones we set a limit how much money should at most be invested in bonds. The amount exceeding this limit is invested in stocks. For each strategy we evaluated the expected surplus and the probability of ruin. In Figure 4.2 we see that – for our chosen measures – some strategies are certainly better than other ones.

	with reinsurance	without reinsurance
100 % bonds 0 % stocks	18.64 mio. 1.13 %	19.23 mio. 1.90 %
50 % bonds 50 % stocks	20.08 mio. 2.45 %	20.65 mio. 3.05 %
0 % bonds 100 % stocks	21.51 mio. 7.71 %	22.26 mio. 7.97 %
≤ 2 mio. bonds rest stocks	21.33 mio. 6.36 %	21.87 mio. 6.44 %
≤ 5 mio. bonds rest stocks	20.85 mio. 4.47 %	21.49 mio. 5.03 %
≤10 mio. bonds rest stocks	20.21 mio. 2.29 %	20.85 mio. 3.00 %

FIGURE 4.1. Expected surplus and ruin probability for the evaluated strategies.

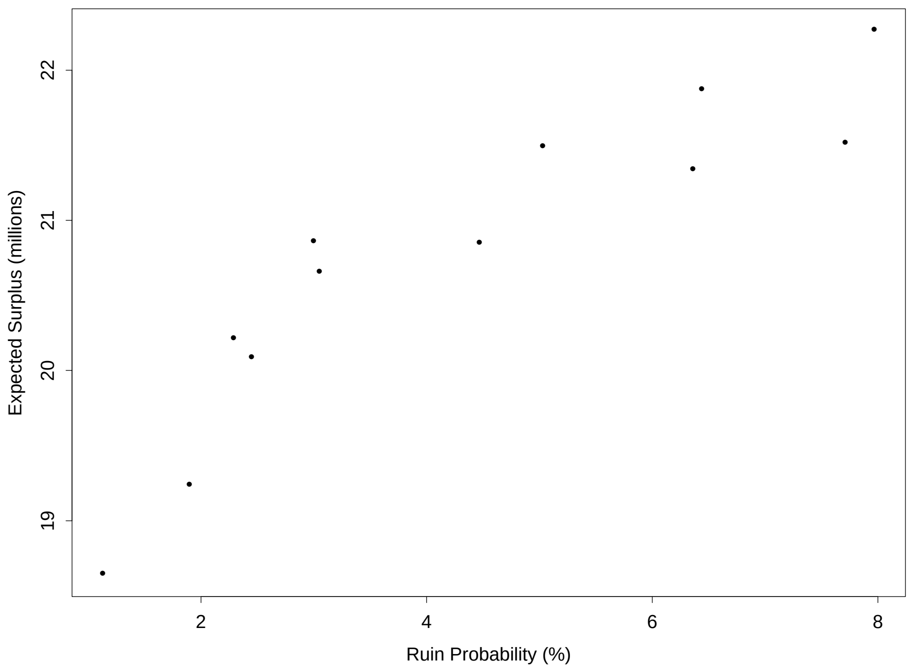

FIGURE 4.2. Graphical comparison of strategies.

5. Some Remarks on DFA

5.1. **Strengths of DFA.** Compared to scenario testing where only a few arbitrary and possibly unrepresentative scenarios are considered, DFA is able to yield much more and better information of the implications of chosen strategies. Because of the large number of simulations a DFA model can run, it gives us information not only on behaviour under ordinary circumstances, but also when extremal events occur. Of course the stochastic generators must be sufficiently flexible to generate occasional rare events.

Whereas traditional management plans in many cases allowed only roughly for correlations and diversification, a DFA model can pick this up and model correlations where appropriate. By modelling each line of business separately, diversification benefits are included automatically. In a DFA model we can utilize all the knowledge we have on the dependence of different variables. Therefore we do not have to model all influences separately and assume independence.

5.2. **Weaknesses of DFA.** Because reality is complex, it is not possible to model all sources of risk. We have to restrict attention to some key risk factors. Consequently in a DFA model there is not only the randomness by reason of the inherent variability, but also the uncertainty caused by incomplete knowledge.

Before running a DFA model we have to estimate a lot of parameters. In these estimations there is always some uncertainty that makes the model less reliable.

Generally DFA overestimates probability of ruin since it does not take into consideration that an insurance company has the opportunity to raise additional capital – e.g. by issuing stocks – when it runs the risk of ruin.

It is easy to increase complexity of a DFA model without gaining added value. One of the main problems with DFA tools is their assessment. What is a good model and what is a bad one? It is possible that a very simple model yields results for future years which are much closer to reality than highly sophisticated models. In order to benefit fully from all the information one gets after having run a DFA one should have a solid understanding of the various statistical assumptions made.

Often it is not sufficient to model dependencies by using linear correlation. Indeed it is very difficult to construct a DFA model that considers all dependencies appropriately. For more information on dependence in risk management see Embrechts, McNeil and Straumann [11] and [12].

5.3. **Limitations of DFA.** DFA does not provide an optimal strategy. It serves as a decision tool that helps management compare different strategies. When a DFA model is used without enough actuarial knowledge, it is only a black box of limited value.

Because reality can never be represented perfectly, we should of course always be cautious, and never rely completely upon the output produced by a DFA model.

For very complex models there is still the problem that computers are not fast enough yet. Therefore it takes quite a long time until we can compare different strategies. But as computers become faster and faster, this problem will probably disappear in the future. In the mean time we have to trade-off added value and computational costs.

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