SESSION TOPIC: PORTFOLIO THEORY AND SECURITY ANALYSIS

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THE VALUATION OF OPTION CONTRACTS AND A TEST OF MARKET EFFICIENCY

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INTRODUCTION

THE OPTION CONTRACT is a right to buy or to sell another asset at a given price within a specified period of time. Warrants to purchase common stock, executive stock options, and put and call options are common examples of option contracts. Put and call option contracts will be the main interest of this paper. The call option contract is the right to buy one hundred shares of a security at a fixed price, called the striking price, at any time up to the expiration of the contract, called its maturity date. The put contract is the right to sell one hundred shares of a security at the striking price up to the maturity of the contract. Other common contracts are combinations of puts and calls. For example, a straddle contract is the combination of one put and one call. The organizational structure of the current option market is described by Boness [2] and in detail in a recent study for the Chicago Board of Trade [8].

The life of an option contract is usually measured in months so that the price of the contract, called the premium, is not affected by unexpected events that take place after the option expires. For every buyer of a put or a call there is a seller, so that exercise of the option does not result in an increase in the amount of the company's common stock. The striking price is adjusted for stock dividends and splits and is also adjusted by the amount of dividends paid on the common stock.[1]

In a recent paper, Black and Scholes [1] derived a formula for the theoretical value of an option. The purpose of this paper is to test this valuation model and to compare the option values derived from the model with actual premiums on

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1. For a stock dividend the exercise price is adjusted downward by the dividend and the option holder is given the right to buy additional shares of stock. For example, a two for one split would simultaneously cut the exercise price in half and gives the option holder the right to purchase twice as many shares at the new striking price. For a cash dividend just the striking price is reduced by the dividend. Merton [7] has shown that the split adjustment leaves the option holder protected, while the dividend adjustment does not give the option holder the correct amount of protection.

option contracts. Since the model uses historical estimates of the variance of return on the underlying security to value contracts, market traders may be able to out-perform the valuation model by using other information on the variance than that contained in the past data. We plan to shed light on how options are currently priced and test the efficiency of option traders in establishing these prices. We also will document the costs of trading in the current option market to demonstrate that profit opportunities do not exist given the current organization of the option market. Since options are integral to many financial contracts, we hope to establish the empirical validity of our model as a forerunner to more complicated models of other option forms.

The Theoretical Valuation Model

In deriving the valuation model for call options we assumed (1) that the variance of return on the common stock is constant over the life of the option contract and known by market participants; (2) that the short term interest rate is known and constant throughout the life of the contract and that this rate is the borrowing and lending rate for market participants; (3) that the option holder is completely protected against distributions that affect the price of the common stock; and (4) that over a finite time interval the returns on a common stock are lognormally distributed. These assumptions are coupled with the usual assumptions in general equilibrium models of no transaction costs, the ability of traders to use the proceeds of short sales of securities, and unlimited borrowing and lending at the short term rate of interest. Whether or not these assumptions are too restrictive to be useful for analysis can only be resolved through subjecting the derived model to empirical tests. However, these assumptions are less restrictive than other general equilibrium models since we do not require agreement on the expected returns on securities, agreement on the characteristics of their joint probability distributions of returns, or that the securities be fairly priced.

The value of an option contract given these assumptions is only a function of the common stock price and time. We found that there is a unique option value such that the option and stock are always in equilibrium. If the price of the option were different from the price given by the model, there would be a combination of a long (short) position in the option and a short (long) position in the corresponding common stock that would be virtually riskless in the short run and would give a return at a rate greater than the short term interest rate. The proportions of option and stock in this hedge would have to be adjusted over time to take account of changes in the stock price and in the duration of the contract.

This hedge may not be profitable in the short run since the value of the option might deviate further away from its equilibrium value temporarily. But the option will have to come back to its equilibrium value eventually, since it will be worth the maximum of zero and the difference between the stock price and the striking price at the expiration date. Since in equilibrium a riskless hedge cannot yield a return greater than the short term interest rate in the market, the option must be priced such that market participants could not establish this hedge and expect to realize a sure profit.

The value of the option that would prevent the establishment of a profitable hedge was shown by Black and Scholes to be:

$$w = x\,N(d_1) - c\,e^{-rt^*}\,N(d_2) \tag{1}$$

where

$w =$ the price of an option for a single share of stock
$x =$ the current price of the stock
$c =$ the striking price of the option
$r =$ the short term rate of interest
$t^* =$ the duration of the option

$$d_1 = \frac{\ln(x/c) + (r + 1/2\,\sigma^2)\,t^*}{\sigma\sqrt{t^*}}$$

$$d_2 = d_1 - \sigma\sqrt{t^*}$$

$N(d) =$ the value of the cumulative normal density function
$\sigma^2 =$ the variance rate of the return on the stock

The value of the option is a function of the interest rate, r, and the variance rate, σ^2, and not a function of the expected return on the common stock. Since put and call options are of short duration, the interest rate can be assumed to be constant or estimated by using a short term market instrument which matches the duration of the contract. The variance rate creates a more serious problem. We will test the valuation model using variance rates estimated from past data.

Suppose that both the model and market traders make the best possible use of the information contained in estimates of past variance rates, but that market traders use other sources of information about future variance rates that the model does not have available. Suppose also that we compare model prices with market prices, and assume that we buy the contracts that appear to be "undervalued" and sell the contracts that appear to be "overvalued." If the transactions take place at model prices, we are likely to lose money, because the market has information the model does not have. If the transactions take place at market prices, we are likely to neither make nor lose significant amounts of money, because the model does not have any information that the market does not have.

Suppose, on the other hand, that the market has information the model does not have, but fails to make proper use of the information contained in estimates of past variance rates. Then we are likely to lose money if the transactions take place at model prices, but make money if the transactions take place at market prices.

The test of market efficiency will be to establish whether or not we could have made significant amounts of money buying "undervalued" options and selling "overvalued" options at market prices.

In the next section we will describe our sample, the methodology used to test the valuation model and market pricing, and the results of our analysis. In the following section we will document the costs of trading in the current option market.

Empirical Tests of the Valuation Model

We were able to obtain the diaries of an option broker from 1966 to 1969. The option broker had recorded all option contracts written for his customers. The diaries contained the striking price, the expiration date, the writing date, the premiums received, the actual date of exercise, and the number of contracts written.

From the diaries we recorded the data on six month calls and six month straddles written on New York Stock Exchange securities. We used the ISL data tapes [9] to obtain daily closing prices, dividends, and capital changes for each of the 545 securities on which contracts were written. The data were screened carefully for consistency. After the screening, our samples contained 2,039 call contracts and 3,052 straddle contracts.

Let us define $w(x,t^*)$ as the value of an option to buy one share of stock at the striking price, c, given the current stock price, x, and the option's duration, t^*. Black and Scholes [1] showed that if we balance each option against $w_1(x,t^*)$ shares[2] of stock we create an approximately riskless hedge at each instant of time.

As was shown by Black and Scholes [1], the number of shares to balance against each option, $w_1(x,t^*)$, is equal to:

$$w_1(x,t^*) = N(d_1) \tag{2}$$

where as described following equation (1):

$$d_1 = \frac{\ln(x/c) + (r + 1/2\,\sigma^2)t^*}{\sigma\sqrt{t^*}}$$

$N(d_1)$ can take on a value from zero to one, and we can see that the number of shares of stock per option will change as a function of changes in the stock price x and the duration of the contract, t^*. As t^* goes to zero, d_1 will approach minus or plus infinity depending on whether the stock price is below or above the striking price. So at expiration of the contract the number of shares of stock per option will be either zero or one. Also if the stock price increases, we will balance our option against a larger fraction of a share, and if the stock price decreases we will balance our option against a smaller fraction of a share.

Each day we computed an estimate of w by using equation (1) given our estimate of the variance of return on the common stock. Each day we computed $w_1(x,t^*)$ to establish our hedge position in the common stock using equation (2). As an estimate of the short term interest rate, r, we used the six month commercial paper rate quoted during the month the contract was written. This six month rate was converted to a continuously compounded rate. To estimate the variance, we used the daily returns on the common stock for the year preceding the day on which the option was written for each contract in the sample.

In this analysis we assume that we incur no transaction costs in buying and in selling the securities and that we can adjust our hedge positions in the stocks by buying or selling shares at the daily closing prices of the securities. In addi-

2. The subscript on w means that $w_1(x,t^*)$ is the partial derivative with respect to the first argument of the function $w(x,t^*)$.

tion we assume that the premiums received by the option writers are also the premiums paid by the option buyers, since our data source contained only the premiums received by the writers. The market prices in this section are the prices received by the writer, and as we will see in the next section when we look at the effects of transaction costs, the prices paid by the buyers of options are considerably higher.

As a first step in evaluating the performance of the valuation model and market traders we computed the realized excess dollar return on each hedge in the sample for each trading day the contracts were outstanding. The realized excess dollar return is defined as:[3]

$$\Delta w - w_1 \Delta x - [w - w_1 x] \, r\Delta t \qquad (3)$$

where

Δw = the change in the model value of the option between trading days
w_1 = the number of shares of stock per option (equation (2))
Δx = the change in the stock price between trading days
x = the stock price
w = the model value of the option (using equation (1))
r = the interest rate
Δt = the interval over which the change in the option value and the change in the stock price were computed

These realized excess dollar returns were aggregated each day to form a total portfolio excess dollar return. The portfolios were constructed to simulate the actual experience of buying or selling options and establishing hedges in the optioned securities. In all, daily portfolio excess returns were computed over 766 trading days between May 1966 and July 1969.

At the writing date of the call, the model was used to value the contract. If the market price of the call option was greater than the model price, we called this an "overvalued" contract. If the market price was less than the model price, we called this an "undervalued" contract. We constructed four portfolios based on several strategies. In the first portfolio strategy we bought all call contracts at the model prices. In the second portfolio strategy we bought all call contracts at the market prices. In the third portfolio strategy we bought the "undervalued" calls and sold the "overvalued" calls at the model prices. In the fourth portfolio strategy we bought the "undervalued" calls and sold the "overvalued" calls at the market prices. For each of these portfolio strategies we computed the daily portfolio excess dollar returns.[4]

3. As a technical aside, at the writing of the contract we computed w and w_1 and established our initial hedge at the striking price. At the end of the first day we computed Δx as the change in the stock price minus the striking price and Δw as the value of the option at the end of the first day (using equation (1)) minus the initial model value of the contract, w. At the end of the first day we also computed a new w_1. This was repeated for each day after that. On the expiration date of the contract we assumed that the contract was exercised and the stock was sold at the closing stock price if the stock price was above the striking price.

4. Since we used the model to calculate w, w_1 and Δw in equation (3), we had to make an adjustment to reflect the difference between the initial model price and the initial market price in strategies 2 and 4. To make this adjustment we assumed that the difference between the model price

To give an intuitive explanation of these portfolio strategies, we can conceive of buying the options at the model prices and then at the market prices. Buying all the options at the model prices will test whether the valuation model gives prices that are, on average, too high or too low. Buying all the options at the market prices will test whether writers receive, on average, too high or too low a price for the options they sell. Buying the "undervalued" contracts and selling the "overvalued" contracts at the model prices will test whether or not the model can be used to price contracts. Buying the "undervalued" contracts and selling the "overvalued" contracts at market prices will test whether or not profit opportunities existed in the option market over the sample period.

As stated earlier, the model prices are found by using estimates of the variance based on past data. If option traders have other information on the future variance which affects the value of an option, buying the "undervalued" contracts and selling the "overvalued" contracts at the model prices will result in significantly negative excess portfolio returns. If market traders have all the information that the model is using and they use this information properly, then buying the "undervalued" contracts and selling the "overvalued" contracts at the market prices will result in excess portfolio returns being insignificantly different from zero. If the model is using information that the market is not using, then the excess returns will be significantly greater than zero, if we buy and sell the contracts at market prices.

As was shown in Black and Scholes [1], the excess dollar returns are theoretically uncorrelated with the returns on a market portfolio. We tested this by regressing the excess dollar portfolio returns, $\tilde{R}_{p,t}$, on the returns on the Standard and Poor Composite Index, $\tilde{R}_{m,t}$, for the total period and for ten different subperiods.[5] The regression model used was:

$$R_{p,t} = \alpha_p + \beta_p \tilde{R}_{m,t} + \tilde{\varepsilon}_t \quad (4)$$

where α_p and β_p are the regression parameters and ε_t is the residual for each day in the sample period.

We will use $\hat{\alpha}_p$, the estimated intercept and its significance as a measure of performance of the model and of the market for their respective portfolios. The estimated slope coefficients, $\hat{\beta}_p$, were found to be insignificantly different from zero as expected and for this reason we present only the estimated intercepts, their t-statistics, and the estimated serial correlation of the excess returns, ρ, in Table 1 for the various portfolios.

In Panel A of Table 1 we summarize the results on a per contract basis. That is, before running the regressions we divided the daily excess total dollar portfolio returns by the number of contracts outstanding each day. In panel B of the table we summarize the results on a total dollar return basis. In the first two sections of each panel we summarize the results of buying each contract

and the market price was converted into an equivalent daily amount like a mortgage repayment schedule over the life of the contract. We added this time-adjusted dollar difference to each excess return to simulate the buying or the selling of contracts at market prices.

5. The portfolio returns are in total dollars, while the index returns are in fractions. While this difference affects the magnitude of the slope coefficient it does not affect its significance which is important for our analysis. The estimated regression parameters will be in dollars.

at the model's price and then at the market's price. In the last two sections of each panel we summarize the results of buying the "undervalued" contracts and selling the "overvalued" contracts at the model's prices and then at the market's prices.

When we buy the contracts at the model prices, the excess portfolio returns are insignificantly different from zero.[6] In some periods, we realized significantly positive returns and in other periods we realized significantly negative returns. When we buy the contracts at the market prices, the total period results indicate that the portfolio returns are insignificant. However, apart from the first two subperiods in which the writers received too low a price (positive $\hat{\alpha}$) the last eight subperiods showed significantly negative returns on the average. Over these last subperiods writers received too high a price for the contracts. Since the number of contracts increases over the sample period, there is probably heteroscedasticity in the variance of the portfolio returns over time. This could cause us to understate the significance of the results.

When we buy the "undervalued" contracts and sell the "overvalued" contracts at the model prices, the excess portfolio returns are significantly negative. The mean $\hat{\alpha}$ is —\$.56 per day on a per contract basis and —\$141.40 per day on a total dollar return basis. However, the market does not use all the information that is available on past variances. If we buy and sell at market, we make \$.56 per day per contract and \$110.69 per day on a total dollar return basis. In each subperiod the results are essentially identical. If we buy and sell at the model prices, we lose a significant amount and if we buy and sell at the market prices, we make a significant amount.

It is necessary to clarify that these findings are not the result of improper hedges that were established by using the estimated variances. Improper hedging leads to increased variance in the portfolio excess returns but does not affect the mean excess return in any predictable direction. Improper hedging is the same as proper hedging plus a position in the stock. But since we are long contracts and also short contracts with opposite positions in the stock, our net positions in the stocks are likely to average out to zero over time.

One possible explanation for these findings is that the price we would compute for the option, if we knew the variance that would apply over the option period, tends to fall between the model price we actually computed and the market price. The variances computed using past data are subject to measurement error, so that the spread in the distribution of the estimated variances is larger than the true spread in the variances. Thus the model using noisy estimates of the variance will tend to overprice options on high variance securities

6. Since there does appear to be significant serial correlation of the portfolio returns, we computed monthly returns as well and re-ran equation (4) to obtain an estimate of the intercept and its t-statistic for the various portfolios. We then divided the monthly $\hat{\alpha}$ by the number of days in the month to make it easily comparable with the daily results. As another test, we averaged the subperiod $\hat{\alpha}$'s and computed the t-statistic of the mean of these subperiod $\hat{\alpha}$'s. We will use the t-statistic of the mean of the subperiod $\hat{\alpha}$'s as a measure of significance since this is an attempt to correct for the serial correlation problem. The monthly results and the average of the subperiod $\hat{\alpha}$'s are shown in Table 1, opposite "Monthly" and "Mean $\hat{\alpha}$", respectively. Each subperiod contained approximately 77 days for the per contract runs and 61 days for the total dollar runs, since we excluded the start-up period when there were fewer than 100 contracts outstanding in the total dollar runs.

TABLE 1
SUMMARY RESULTS OF REGRESSIONS USING PREVIOUS YEAR'S VARIANCE

Panel A. Results on a Per Contract Basis (in dollars per contract per day)

Period	Buy All At Model Prices			Buy All At Market Prices			Buy and Sell At Model Prices			Buy and Sell At Market Prices		
	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$
ENTIRE	−.10	−2.02	.31*	−.06	−1.18	.29*	−.56	−11.60	.09*	.56	11.73	.06
1	.39	1.40	.07	.37	1.37	.05	−.56	−2.57	.10	.42	1.96	.05
2	.97	3.69	.34*	1.19	4.55	.34*	−.24	−1.05	−.07	.98	4.24	−.07
3	−.53	−9.41	−.07	−.11	−1.95	−.08	−.78	−10.70	−.04	.47	6.27	−.01
4	−.51	−7.43	.25*	−.06	−.92	.23*	−.80	−9.62	.17	.32	3.84	.16
5	−.55	−11.47	−.01	−.38	−8.00	−.02	−.70	−12.50	.00	.29	5.28	.00
6	.14	.84	.03	−.05	−.60	.05	−.35	−5.29	.05	.58	8.70	.07
7	.23	2.13	−.01	−.22	−2.07	.00	−.25	−2.41	.01	.87	8.56	−.01
8	−.03	−.23	.31*	−.54	−3.85	.28*	−1.23	−6.19	.04	.20	1.02	.03
9	−.67	−7.18	.37*	−.64	−7.29	.31*	−.52	−4.08	.03	.83	6.55	.02
10	−.61	−5.61	.25*	−.35	−3.32	.20	−.19	−.97	.06	.64	3.21	.06
Monthly	−.10	−.87	.52*	−.06	−.52	.50*	−.56	−8.87	.53*	.56	7.43	.38*
Mean $\hat{\alpha}$	−.12	−.68		−.08	−.47		−.56	−5.45		.56	6.64	

TABLE 1 (*Continued*)

Panel B. Results on a Total Dollar Return Basis (in total dollars per day)

Period ENTIRE[1]	Buy All At Model Prices			Buy All At Market Prices			Buy and Sell At Model Prices			Buy and Sell At Market Prices		
	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$
	−45.27	−2.66	.25*	−68.53	−4.25	.17*	−141.40	−12.92	.25*	110.69	10.37	.21*
1	135.37	3.45	.25	165.38	4.23	.25	14.49	.72	.05	103.47	5.14	.04
2	−61.69	−2.42	.16	18.79	.78	.09	−108.02	−6.12	.06	43.87	2.55	.02
3	−187.07	−10.35	.04	−30.17	−1.72	−.02	−192.84	−12.92	.03	34.97	2.48	−.08
4	−222.38	−5.76	.29*	−26.93	−.69	.29*	−235.36	−7.01	.28*	64.66	1.92	.28*
5	−361.89	−12.44	.05	−274.82	−9.57	.03	−294.99	−14.97	−.03	9.24	.46	−.02
6	71.33	1.43	−.10	−22.99	−.46	−.06	−111.21	−4.69	.02	197.83	8.23	.05
7	217.06	2.41	−.05	−20.14	−.22	−.05	−152.50	−3.25	.07	211.48	4.49	−.06
8	199.50	2.64	.09	−87.74	−1.15	.10	−253.76	−3.96	.19	145.88	2.26	.25
9	−197.96	−5.58	.31*	−362.44	−9.81	.36*	−5.04	−.18	.13	270.19	9.55	.17
10	−108.65	−7.83	.09	−105.95	−7.71	.09	−36.91	−.30	.03	63.43	5.54	.18
Monthly	−41.39	−1.17	.68*	−62.81	−2.39	.48*	−138.33	−7.17	.68*	107.92	6.75	.62*
Mean $\hat{\alpha}$	−51.64	−.82		−74.70	−1.58		−137.61	−4.06		114.50	4.13	

* t-statistic greater than 2.0.

1. These are slightly different time periods.

and underprice options on low variance securities. However, the option traders' estimates of the spread in the distribution of the variances appear to be too narrow. The prices of options on high variance securities tend to be too low, and the prices of options on low variance securities tend to be too high. It appears that option traders use their past experience to arrive at an average option price, but realize that there are differences based on variability and spread their price estimates out from the mean. However, they do not spread them out far enough.

If the true value of an option tends to lie between the model and market prices, using the model we will tend to buy options on high variance securities for too great a price and to sell options on low variance securities for too low a price, (buying and selling at model prices), since market traders systematically underestimate the value of an option on a high variance security and systematically overestimate the value of an option on a low variance security. To test this hypothesis we constructed the excess returns on portfolios that were formed on variance alone. We ranked all the stocks from minimum to maximum variance, and assigned the twenty-five per cent of the stocks with the lowest estimated variances to the first portfolio, the next twenty-five per cent to the second portfolio and so on. We bought the contracts in each portfolio grouping at the model prices and then at the market prices. The excess portfolio returns of each of these portfolios were converted to a per contract basis and regressed on the returns on the Standard and Poor Index. The summary results are shown in Table 2.

TABLE 2
SUMMARY OF REGRESSION RESULTS FOR OPTIONS BOUGHT ON VARIANCE ALONE
(on a Per Contract Basis)

Portfolio (Low to High Variance)	Buy At Model Prices		Buy At Market Prices	
	$\hat{\alpha}(\$)$	$t-\hat{\alpha}$	$\hat{\alpha}(\$)$	$t-\hat{\alpha}$
1	.15	2.57	−.43	−7.47
2	.06	.87	−.04	− .56
3	−.35	−4.39	−.10	−1.76
4	−.36	−5.57	.17	2.60

We see clearly from these runs that the model overestimates the value of options on high variance securities (we lose \$.35 and \$.36 per day per contract) and underestimates the value of options on low variance securities, (we make \$.15 and \$.06 per day per contract). The converse is true for the market. Market traders underestimate the value of options on high variance securities and overestimate the value of options on low variance securities. Without transaction costs there did appear to be profit opportunities in the option market over this sample period.

To demonstrate that the model can price correctly if it has the proper variance rate, we calculated the actual variance of returns on the common stock over the life of the contract and used this variance to price the contracts. Each portfolio was formed as before using an option value based on the actual variance rate to determine whether or not the contract is "undervalued" or "overvalued." These results are summarized in Table 3.

When we buy the contracts at either the model or the market prices we obtain approximately the same results as shown in Table 1. However, in contrast to the results in Table 1, when we bought the "undervalued" contracts and sold the "overvalued" contracts at the model prices, we realized portfolio excess returns that were insignificantly different from zero. Naturally, if we knew the correct price for an option, and could buy and sell contracts at market prices, we could earn a significant amount of money, but this is a very biased test against market pricing.

The variance of the excess portfolio returns was much lower when we used the actual variance that applied over the option period to hedge the contracts. Also, when we use the actual variance and buy and sell at the model prices, the serial correlations in the portfolio returns are much smaller than those shown in Table 1. The finding of serial correlation in the portfolio returns in puzzling. It may be caused by serial correlation in the underlying common stock returns or induced when we form the portfolios. Errors in the variance cause us to take positions in the stocks, and if there is serial correlation in the underlying stock returns, this would cause the portfolio returns to be serially correlated as well. Also, serial correlation may be induced into the portfolio returns even though the returns on each security are serially uncorrelated, since the quoted closing prices may not be actual end-of-day prices. We will leave the question of serial correlation as well as possible corrections for heteroscedasticity to future work.

In the next section we will document the costs of buying and selling options in the option market. Our purpose is to show that the implied profits associated with buying contracts on high variance securities and selling contracts on low variance securities at the prices the writers receive vanish if we have to pay transaction costs.

COSTS OF OPTION TRADING

The option market is not a continuous market like the New York Stock Exchange. Buyers and sellers of options have to be found. When an investor wishes to buy a contract, he contacts his broker who contacts a member of the Put and Call Dealers Association who acts as a clearing agent, seldom buying or selling options for his own account. These dealers will contact or be contacted by other brokers who sell options for clients.

Most sellers of calls or straddles never write a contract without already holding the shares or buying the shares of the common stock on which they write the option. Most buyers of options buy call contracts. When a straddle is written, a put and call are created. Since there is more demand for call contracts than for put contracts, a group of brokerage houses convert put contracts into call contracts.[7]

7. For a description of the conversion process see Stoll [10]. The converter buys the straddle from the put and call dealer, purchases the common stock, and sells two calls back to the put and call dealer. Kruizenga [5] has developed some helpful vector diagrams that can be used as an aid to working out the interrelationships between owning the stock, puts, calls, and straddles.

TABLE 3
SUMMARY OF REGRESSION RESULTS USING ACTUAL VARIANCE OVER OPTION PERIOD

Panel A. Results on a Per Contract Basis (in dollars per contract per day)

Period ENTIRE	Buy All At Model Prices			Buy All At Market Prices			Buy and Sell At Model Prices			Buy and Sell At Market Prices		
	$\hat{\alpha}$	$t - \hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}'$	$t - \hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t - \hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t - \hat{\alpha}$	$\hat{\rho}$
	−.04	−1.04	.19*	−.09	−2.00	.31*	−.06	−1.19	.08*	1.11	21.64	.10*
1	−.44	−1.96	.08	.36	1.62	.05	−.19	−1.22	.20	1.10	7.40	.15
2	.54	2.27	.29*	1.12	4.60	.32*	.19	.86	.20	1.55	22.92	.19
3	−.19	−3.04	.00	−.21	−3.43	−.05	−.39	−4.60	.09	.67	8.30	−.03
4	.05	.70	.23*	−.09	−1.18	.24*	−.02	−.23	.00	.76	8.23	−.01
5	−.22	−4.17	−.07	−.39	−7.45	−.06	−.16	−2.05	−.02	.72	9.31	.00
6	.06	.72	.06	−.04	−.55	.05	−.08	−.90	.13	1.03	15.85	.12
7	−.03	−.34	−.01	−.21	−2.01	.01	−.01	−.06	.04	1.25	10.07	.03
8	−.08	−.62	.19	−.57	−4.49	.23*	.24	.91	.02	1.54	5.87	.03
9	−.08	−.90	.27*	−.54	−5.92	.29*	−.07	−.51	.00	1.24	8.43	.00
10	−.22	−2.23	.04	−.54	−4.52	.31*	−.10	−.46	−.04	1.25	5.66	−.01
Monthly	−.04	−.70	.13	−.07	−.73	.52*	−.06	−.99	.11	1.10	15.53	.35*
Mean $\hat{\alpha}$	−.06	−.75		−.11	−.68		−.06	−1.03		1.11	11.05	

TABLE 3 (*Continued*)

Panel B. Results on a Total Dollar Return Basis (in total dollars per day)

Period	Buy All At Model Prices			Buy All at Market Prices			Buy and Sell At Model Prices			Buy and Sell At Market Prices		
	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$t-\hat{\alpha}$	$\hat{\rho}$
ENTIRE[1]	−6.18	−.41	.10*	−74.29	−4.78	.16*	.80	.10	.15*	235.66	27.24	.34*
1	72.25	1.94	.23	162.55	4.36	.23	49.44	1.59	.25*	167.47	5.43	.24
2	−39.46	−1.80	.03	−11.38	−.49	.10	−48.30	−3.36	.02	86.86	6.04	.02
3	−4.50	−.22	−.01	−53.83	−2.65	.00	−3.05	−.22	−.02	134.15	9.72	.00
4	37.99	.87	.29*	−34.55	−.74	.30*	−29.69	−1.29	−.07	169.69	7.65	−.17
5	−174.02	−5.54	−.04	−279.92	−8.93	−.04	15.82	.67	−.07	289.69	12.24	−.06
6	55.55	1.13	−.05	−20.48	−.42	−.06	−34.69	−1.46	.17	315.70	13.22	.18
7	48.55	.57	−.05	−12.80	−.15	−.05	−1.00	−.03	.10	384.00	11.44	.12
8	56.24	.80	.08	−111.46	−1.62	.04	−.94	−.03	.30*	371.42	12.44	.27*
9	−149.99	−4.25	.34*	−347.20	−9.59	.36*	71.09	2.85	.06	337.63	12.97	.14
10	−23.59	−1.65	.05	−94.57	−6.50	.11	6.24	.57	.02	117.72	9.76	.24
Monthly	−1.37	−.07	.08	−67.59	−2.67	.50*	1.93	.17	.00	232.36	10.36	.74*
Mean $\hat{\alpha}$	−12.10	−.44		−80.36	−1.76		2.49	.21		237.43	6.63	

1. These are slightly different time periods.
* t-statistic greater than 2.0.

When a writer writes a call or a straddle and holds or buys the stock, gains from the option plus the stock are limited to the amount of the premiums received. If the stock price falls below the striking price, the stock will not be called away. The call writer will then have to sell the stock he holds in the market for less than the price of the stock when he sold the call. The straddle writer will be put the stock by the converter at the striking price and will thus have to sell two lots at prices below the price of the stock when he sold the straddle. In many respects these strategies are equivalent to writing a put without owning the stock. But as Merton [7] has shown the price of a call would be less than the price of a put on the same stock plus interest premiums,[8] since put buyers might rationally exercise prior to maturity while call buyers would tend to exercise at the maturity date.

We can determine the costs of trading in the option market by using the estimated experience of buyers and the actual experience of writers over our sample period. We can compare the transaction costs of trading through the option market with the transaction costs of buying and selling the common stock.

When a call is purchased, the buyer must pay the premium which the writer receives. In addition, he pays a markup to the dealer who finds a writer for him as well as a conversion fee if the call was converted from a straddle. Although we have only the premium received by the option writer and not the price paid by the option buyer, we used a markup of one and one-half the regular commission.[9] Also, to have a contract endorsed by a member firm to guarantee delivery on exercise, there is a fee of $6.25. Exchange rules also require member firms to charge regular stock exchange commissions to the call buyer, if he exercises, at the striking price, plus an additional commission plus transfer taxes if he sells at the current market price. Even if the call buyer wishes to settle the difference and not take delivery of the stock, he must pay these two commissions. This requirement is a rule of the New York Stock Exchange and not required by law.

The writer must also pay commissions. If the stock he owns is called away from him, he must pay a commission plus transfer tax to sell at the striking price. If he bought the stock at the time he wrote the contract he paid a commission to buy. If the stock is not called and if he liquidates his position, he pays a commission plus transfer taxes to sell at the market price. For a straddle writer the commissions are similar. If the stock is put to the writer by the converter, he must pay a commission to buy the stock at the striking price and commissions plus transfer taxes to sell his shares at the market price.

We used the actual commission schedules that existed during the sample

8. Stoll [10] claimed that the price of the call is equal to the price of a put on the same security plus the interest bill for holding the stock over the option period. However, Merton [7] has shown that the price of the call is likely to be less than the price of a put on the same security plus the interest bill. Put buyers would pay more for a put that they could exercise early than for a put that they could not exercise until expiration of the contract.

9. This might be an understatement of the markups. The Chicago Board of Trade Study [8] indicated that the SEC found the markup to be closer to 2.0 per cent or twice the regualr commission. However, arguments that the spread has come down in recent years indicate that 1.5 times the regular commission may be a good compromise.

period to compute the transaction costs for each of the strategies. As a by-product of the analysis we will show the profitability of trading in options over this period. Our main interest in this section is to document the costs of trading and not to show whether option buyers or option writers receive better prices. We have already established that the valuation model can be used to value options at the time of issue. We have also shown that in general writers obtain favorable prices, and that there tends to be a systematic mispricing of options as a function of the variance of returns of the stock.

Other researchers have compared the realized returns to option buyers and option writers. Kruizenga [5], Katz [4], Boness [3], and Malkiel and Quandt [6] analyzed the profitability of option trading using hypothetical data or actual data over short time intervals. Naturally, in some periods, option writers appear to do better than option buyers and in other periods the converse is true. The conclusions from these studies tended to suggest that writing options was more profitable than buying options but none of these studies controlled for the risk differences inherent in the various strategies or stated whether or not the return differences were significant given these risk differences.

The results of the analysis for call options are presented in Table 4. In panel

TABLE 4

Transaction Costs and Profitability of Trading in Options-Call Data 1966-1969

Panel A. Returns to Option Strategies

	Without Transaction Costs			With Transaction Costs		
Strategy	Profit	Investment	%	Profit	Investment	%
Buy Stock	$932,636	$7,952,181	11.7	$774,308	$8,024,720	9.6
Buy Stock Sell Call	595,630	6,938,776	8.6	444,563	7,011,338	6.3
Buy Call	337,012	1,013,390	33.3	95,274	1,150,228	8.3

Panel B. Breakdown of Transaction Costs

	Buying Costs		Selling Costs		
Strategy	Commissions	Markup	Regular Commissions	Extra Commissions	Total
Buy Stock	$72,539	—	$85,789	—	$85,789
Buy Stock Sell Call	72,539	—	29,658	$48,870	78,528
Buy Call	—	136,839	56,131	48,870	105,501

A of the table we show the returns to writers and to buyers in a world without transaction costs and in a world with transaction costs. In a world without transaction costs the return to buying the securities on which the calls were written was 11.7 per cent over the period the option was outstanding.[10] Over

10. There are many ways to compute returns. Since we are only interested in the transaction costs

this period both writers and buyers received positive returns on the average. When we include the costs of buying and selling options, the returns to option writers fall from 8.6 per cent to 6.3 per cent and the returns to option buyers fall from 33.3 per cent to 8.3 per cent. Since the call buyers have riskier positions than the writers, we expect them to have higher returns. But, after transaction costs, the call buyers realize only slightly higher returns than the call writers.

In panel B of Table 4 we show the actual breakdown of the transaction costs associated with each strategy. Buying and selling the underlying stock generated $158,328 in commissions as compared to $320,868 by trading the same securities through the option market. Of the extra costs $136,839 was paid to the dealers and $97,740 was extra selling commissions paid to the brokers. Buying and selling the securities cost 2.0 per cent of investment. Trading through the option market increased the cost to 4.9 per cent. Of the extra 2.9 per cent, 1.7 per cent represented dealer's commissions and 1.2 per cent represented extra commissions on exercise.[11] The most dramatic effect is on the profitability of option buyers. Since option buyers have a low initial investment, the markup and exercise costs represent a substantial fraction of their investment.

It is also of interest to show the frequency distributions of returns for the various strategies. These distributions are shown in Table 5. Over the sample period, the returns on the securities in the sample were generally very high. The distribution of returns to option writers was very peaked at the average premium received since 33 per cent of the stock was called away from them. Even in this period over 50 per cent of the call buyers lost money. While the distribution of returns to call writers is peaked with a left tail, the distribution of returns to the call buyers is peaked at a loss of 100 per cent and has a long right tail. These distributional differences make it extremely difficult to compare directly realized returns of option writing and option buying to make statements about option pricing.

The main conclusion of this section is that the transaction costs to buy and to sell options are quite large.[12] We have shown (see Table 2) that over the sample period, contracts on high variance securities tend to be underpriced,

of trading we aggregated the investment necessary on each contract and the profit on each contract for each strategy. The return is just the ratio of total profit to total investment for each strategy. These returns are for approximately a six month holding period and may appear to be high. However, purchasing all securities on the exchange (the Standard & Poor 500 adjusted for dividends) with the same investments and same holding periods resulted in a return of 7.0 per cent over this time period. Since calls tend to be written on riskier securities than securities in general, this difference is to be expected.

11. We divided the transaction costs by the investment in the common stocks ($7.9 million) that would be necessary if there were no transaction costs to buy securities. These percentages are presented to illustrate the increased costs of option trading as opposed to buying the stock.

12. We analyzed the transaction costs associated with writing straddles as well. The results are comparable to the call contracts. However, since the straddle writer may be put the stock as well as having the stock called away, his transaction costs were 3.1 per cent of his investment or 1.0 per cent greater than with the call data. We also ran the analysis on the call options assuming that they were exercised at the end of the contract's life. If the call buyer had waited until expiration to exercise his call, he would have earned 11.5 per cent on his initial investment after transaction costs, or 3.2 per cent greater than exercising when he did.

TABLE 5

FREQUENCY DISTRIBUTION OF RETURNS FOR VARIOUS OPTION STRATEGIES

	Without Transaction Costs			With Transaction Costs		
Interval	Buy Stock	Buy Stock Sell Call	Buy Call	Buy Stock	Buy Stock Sell Call	Buy Call
−1.0 to −.9	0	0	700	0	0	793
− .9 to −.8	0	0	44	0	0	56
− .8 to −.7	0	0	51	0	0	37
− .7 to −.6	0	0	65	0	0	56
− .6 to −.5	0	0	25	0	0	40
− .5 to −.4	1	0	42	2	0	41
− .4 to −.3	25	1	26	33	2	50
− .3 to −.2	68	24	50	72	33	47
− .2 to −.1	162	62	41	206	63	41
− .1 to 0.0	288	137	33	295	166	43
0.0 to .1	315	201	39	327	243	38
.1 to .2	289	1020	41	286	1299	42
.2 to .3	343	576	31	341	230	52
.3 to .4	229	16	40	193	1	42
.4 to .5	110	0	29	106	0	61
.5 to .6	81	0	38	66	0	46
.6 to .7	49	0	48	43	0	52
.7 to .8	24	0	33	21	0	38
.8 to .9	21	0	54	17	0	36
.9 to 1.0	8	0	49	9	0	35
1.0 to 1.5	22	0	179	18	0	127
1.5 to 2.0	2	0	106	2	0	74
2.0 to 3.0	1	0	127	1	0	107
>3	0	0	106	0	0	72

and that contracts on low variance securities tend to be overpriced. An implicit trading strategy would be to sell contracts on low variance securities and to buy contracts on high variance securities. But as we have shown in Table 4, the transaction costs of trading in the current option market would eliminate the profits. We estimated that, on the average, contracts on the lowest variance securities were overpriced by $55 a contract.[13] If we sold calls on these securities, clearing commissions and margin requirements would eliminate the profits.[14] Calls on the higher variance securities were underpriced by $22 a contract, on the average. Since these are the prices received by the writers, markups and commissions would eliminate the advantage of receiving a favorable price for the buyers on these high variance securities. In fact, when we consider the transaction costs, buyers appear to be paying the largest share of the commissions, while writers appear to receive almost fair prices, on the average. The call buyers, who must pay commissions and markups which are approximately twenty per cent higher than the premiums received by the

13. Since there are approximately 130 trading days over a six month period, 130 × .43 = $55 (see Table 2).

14. If a call is sold the seller must maintain margin in his account which represents at least 30 per cent of the value of the stock.

writers, realize negative returns, on the average. These buyers appear willing to pay most of the commissions to induce the writers to write the options.

Even though profit opportunities appeared to exist in the option market without transaction costs, these profit opportunities vanished when we included transaction costs. There is no incentive for market participants to eliminate the discrepancies in the option prices as a function of the variances, although it is still puzzling to see that they exist.

CONCLUSION

In a recent paper, [1], we established a model for determining the equilibrium value of an option. In this paper we tested the valuation model using two estimates of the variance of returns on the common stock. We found that using past data to estimate the variance caused the model to overprice options on high variance stocks and underprice options on low variance stocks. This is evidence of the common errors-in-measurement problem. There are various ways to use the data more efficiently to adjust for errors in measurement but we will leave these problems for future work. Using the variance that applied over the option period greatly reduced the variation in the portfolio excess returns and showed that if the model has an accurate estimate of the variance, it works very well. Also, there is evidence of non-stationarity in the variance. More work must be done to predict variances using the information available.

By examining the actual experience of option writers and the estimated experience of option buyers over the sample period, we have shown that the transaction costs of trading in the option market are much greater than the transaction costs of trading in listed securities. But, it did appear that the option buyers incurred most of these extra costs. Since it has been estimated that options represent less than one-tenth of one per cent of the dollar volume of shares traded on the New York Stock Exchange, these costs might be justified since dealers and brokers must search for writers and buyers of options. Since the scale of operation is small, large orders are difficult if not impossible to fill and even small orders may take time to fill. If the current market could be made more efficient, and thus less costly for option buyers, in all probability the demand for options would increase.[15]

When a trader buys and sells options for his own account, using the valuation model to price contracts and to establish the appropriate hedges in the underlying common stock, he can expect to lose money if other traders have information on the variance that was not contained in the past data. We hypothesized that if market traders used all available information efficiently, the buying of an "undervalued" contract at the model price and the selling of an "overvalued" contract at the model price would result in significantly negative excess portfolio returns while buying an "undervalued" contract at the market price and selling an "overvalued" contract at the market price would result in insignificant portfolio excess returns. While the model tends to overestimate the

15. Evidence of the demand for a new form of organization to meet the needs of large buyers and sellers is the recent popularity of the "Down and Out Option." This option was developed by Snyder [11] and is now being sold by several houses who write this option for their own accounts. Also, the Chicago Board of Trade is planning to operate a primary and secondary market in options.

value of an option on a high variance security, market traders tend to underestimate the value, and similarly while the model tends to underestimate the value of an option on a low variance security, market traders tend to overestimate the value.

However, when we included the transaction costs of trading in options, we found that the implied profits from buying options on low variance securities and from selling options on high variance securities vanished. Even though the option market does not appear to be efficient before taking account of transaction costs, there is no opportunity for other traders to take advantage of this mispricing.

In future research we plan to see if it is possible to obtain more accurate estimates of the variance rates using the available information. There are other areas that must be explored in future work. These include the use of more current information to adjust hedges to reduce the variation in portfolio returns, the effects of trading costs on profitability, and the effects of changes in the interest rate on the pricing of option contracts. Also, in the future we plan to study the loss in efficiency of not establishing the hedges given the fact that transaction costs will add up to a substantial amount if hedges are actually established.

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