7

Swaps

I have friends in overalls whose friendship I would not swap for the favor of the kings of the world.

—Thomas A. Edison

Overview

- Mechanics of interest rate swaps
- Day count issues
- (Confirmations – skip)
- The comparative-advantage argument

- The nature of swap rates
- Determining the LIBOR/swap zero rates
- Valuation of interest rate swaps
- Currency swaps
- Valuation of currency swaps
- Credit risk
- Other types of swap
- Summary

Preamble

- First contracts 1980s
- Agreement to exchange cash flows at specified future times according to certain specified rules
- Cash flows from future value of variable: IR, FX, etc.
- Generalises a forward contract if cash settled – swap forward price for market value

Fwd	≡	Swap
• 100 oz Gold		• $40,000
• $400/oz		• 100 S
• +1 yr		• +1 yr

- Multiple dates
- Focus on swaps:
  - plain vanilla interest rate
  - fixed-for-fixed currency

Nature of Swaps

Definition 7.1. A *swap* is an agreement to exchange cash flows at specified future times according to certain specified rules.

Mechanics of interest rate swaps

- Most common – *plain vanilla interest rate swap*

Definition 7.2. A *plain vanilla interest rate swap* is an agreement in which a company agrees to pay cash flows equal to interest at a predetermined fixed rate in return for interest at a floating rate, on a notional principal, for a period of time.

- Floating rate typically LIBOR (Chapter 4)
- Reference IR for loans in _________ markets (cf. *prime* in ________ markets)

An Example of a "Plain Vanilla" Interest Rate Swap

- An agreement by Microsoft to
  - receive 6-month LIBOR &
  - pay a fixed rate of 5% per annum
  - every 6 months
  - for 3 years on a
  - notional principal of $100 million

Figure

Figure 7.1: Interest rate swap between Microsoft and Intel

Cash Flows to Microsoft

- See Hull 2006 Table 7.1, page 151

Table 7.1. Cash flows (millions of dollars) to Microsoft, in a $100 million, 3-year interest rate swap, when a **fixed rate of 5%** is paid and **LIBOR** is received. The **net cash flow** is the difference. (Ignore day count issues.)

Date	*6-month LIBOR (%)*	*Floating received*	*Fixed paid*	*Net cash flow*
Mar. 5, 2004	4.20	-	-	-
Sept. 5, 2004	4.80	+2.10	-2.50	-0.40
Mar. 5, 2005	5.30	+2.40	-2.50	-0.10
Sept. 5, 2005	5.50	+2.65	-2.50	+0.15
Mar. 5, 2006	5.60	+2.75	-2.50	+0.25
Sept. 5, 2006	5.90	+2.80	-2.50	+0.30
Mar. 5, 2007	-	+2.95	-2.50	+0.45

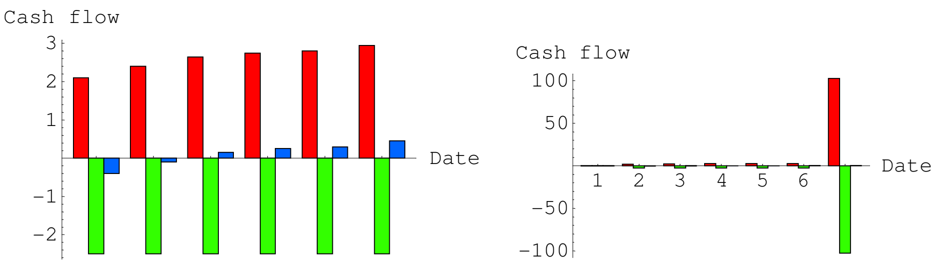
Cash Flows to Microsoft – with exchange of principal

Table 7.2. Cash flows (millions of dollars) to Microsoft, in a $100 million, 3-year interest rate swap, when a **fixed rate of 5%** is paid and **LIBOR** is received. The **net cash flow** is the difference.

Date	*6-month LIBOR (%)*	*Floating received*	*Fixed paid*	*Net cash flow*
Mar. 5, 2004	4.20	–	-	-
Sept. 5, 2004	4.80	+2.10	-2.50	-0.40
Mar. 5, 2005	5.30	+2.40	-2.50	-0.10
Sept. 5, 2005	5.50	+2.65	-2.50	+0.15
Mar. 5, 2006	5.60	+2.75	-2.50	+0.25
Sept. 5, 2006	5.90	+2.80	-2.50	+0.30
Mar. 5, 2007	-	+102.95	-102.50	+0.45

Code

Plot



(a) No exchange of principal (b) Exchange of principal

Figure 7.2: Cash flows (millions of dollars) to Microsoft, in a $100 million, 3-year interest rate swap, when a **fixed rate of 5%** is paid and **LIBOR** is received. The **net cash flow** is the difference. Variants are (a) without and (b) with exchange of principal at the end.

Remark

- The net cash flow is the same whether or not the principal is exchanged.
- Typically the principal is *not* exchanged (hence “notional” principal)
- However, case with exchange of principal gives insight

Typical Uses of an Interest Rate Swap

- Converting a liability from
  - fixed rate to floating rate
  - floating rate to fixed rate
- Converting an investment from
  - fixed rate to floating rate
  - floating rate to fixed rate

Figure

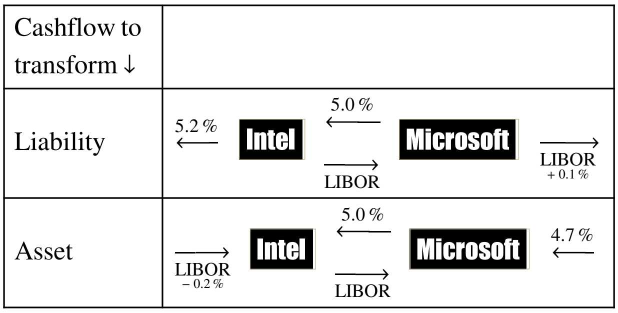


Figure 7.3: Use of an interest rate swap to transform a liability or an asset, without a financial intermediary.

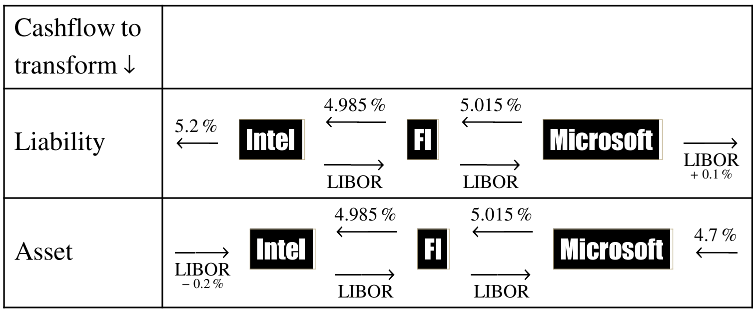


Figure 7.4: Use of an interest rate swap to transform a liability or an asset, with a financial intermediary.

Transforming a liability

- Convertion of debt payments
  - Microsoft – from LIBOR + 0.1% to ____________
  - Intel – from 5.2% to ___________

Transforming an asset

- Convertion of asset cash (in)flows
  - Microsoft – from 4.7% to ____________
  - Intel – from LIBOR - 0.2% to ___________

Financial intermediary

- Typically earns ______ basis points
- Has ____ contracts
- If one party defaults does □ does not □ have to honour its agreement with other

Further detail

- Musiela Rutkowski (2004)
- Usually payments are *set in advance, paid/settled in arrears*
- Paid/settled in advance also possible, but arrears payments require discounting
  - Conventions vary

- Discount payments
  - float by float,
  - fixed by float or fix

Day count issues

- LIBOR (e.g. 6-mo in Table 7.1) is a money market rate, hence quoted on _________ basis

Example 7.1. The first floating payment in Table 7.1 is for a LIBOR rate of 4.2%. What is the correct interest payment if day count conventions are taken into account?

Reference period Mar 5 to Sept 5, 2004 is 184 days

Interest earned is $\$100 \times 10^6 \times 0.042 \times \frac{184}{360} = \$2.1467$

LIBOR-based cash flows

Notation

Λ	principal
R	LIBOR rate
n	number of days since last payment

- Note: Greek “L”, Lambda, to save “L” for LIBOR rate, below.

Swap cash flow expression

$$\text{Cash flow} = \frac{\Lambda R n}{360} \qquad (7.1)$$

Fixed rate cash flows

- Usually
  - actual/365
  - 30/360
- Often factor 360/365 required to compare 6-mo LIBOR with fixed

The comparative-advantage argument

The Comparative Advantage Argument

- Hull Table 7.4, page 157
- AAACorp wants to borrow floating
- BBBCorp wants to borrow fixed

Table 7.3. Borrowing rates for two corporations

Company	*Fixed*	*Floating*
AAACorp	4.0%	LIBOR + 0.3%
BBBCorp	5.2%	LIBOR + 1.0%

- Key feature – difference between ______ rates, greater than between _______ rates
- $\left\{\begin{matrix}\text{AAACorp}\\ \text{BBBCorp}\end{matrix}\right\}$ has a comparative advantage in $\left\{\begin{matrix}\_\_\_\ \_\_\_\ \_\_\_\\ \_\_\_\ \_\_\_\ \_\_\_\end{matrix}\right\}$ markets

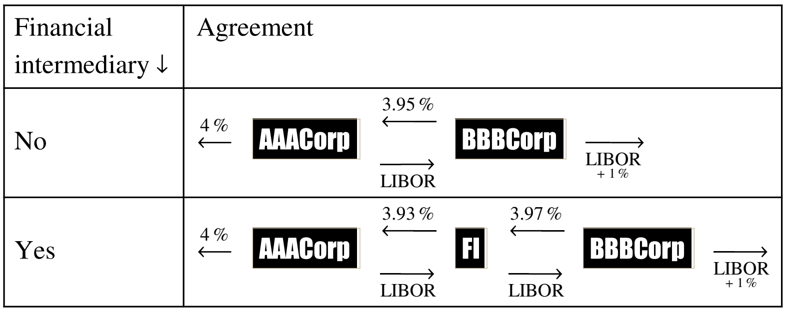


Figure 7.5: Illustration of comparative advantage agreement for two corporations, without and with a financial intermediary.

Criticism of the Comparative Advantage Argument

- 4.0% and 5.2% rates available to AAACorp and BBBCorp in fixed rate markets are 5-year rates
- The LIBOR+0.3% and LIBOR+1% rates available in the floating rate market are six-month rates
- BBBCorp's fixed rate depends on spread above LIBOR it borrows at in the future

The Nature of Swap Rates

- Six-month LIBOR is short-term AA borrowing rate

- 5-year swap rate has risk ~ 10 six-month loans made to AA borrowers at LIBOR
- Lender can enter into swap – income from LIBOR loans exchanged for 5-year swap rate

Swap rates

Practical definition

Definition 7.3. *Swap rates* are the fixed rates at which financial institutions offer interest rate swap contracts to their clients.

- M&R 2004, p. 328

Theoretical definition

Definition 7.4. The *swap rate* is that value of the fixed rate that makes the value of the swap zero at inception.

- M&R 2004, p. 479
- Cf. forward interest rate and value of FRA
- Institutions offer swap contracts at rates with appropriate spreads above and below the theoretical ones

Determining the LIBOR/swap zero rates

- Risk-free rates for derivative valuation purposes
  - LIBOR ≤ 12 mos
  - Eurodollar futures 1-2 yrs
  - Swaps ≥ 2 yrs
- *LIBOR/swap zero curve*
- Chapter 4 "par yields" – now put to use!

Overview of argument

- Consider a new swap with fixed rate = swap rate
- Add principals on both sides on final payment date ⇒ swap ≡ exchange of fixed rate and floating rate bonds
- Value of bonds/swaps
  - Floating-rate rate bond – _______ *.
  - Swap – ____.
- ⇒ fixed-rate bond worth _____.
- ⇒ swap rates define par yield bonds; used to bootstrap the LIBOR (or LIBOR/swap) zero curve
- * Argument below

Swap rates from bond prices

Notation

S_n	swap rate for an n-year agreement

Result

Proposition 7.5. *The n-period swap rates S_n can be expressed in terms of prices of zero coupon bonds P_{0n} by the relationship* $S_n = \frac{1-P_{0n}}{\sum_{i=1}^{n} P_{0i}}$

$$S_n = \frac{1 - P_{0n}}{\sum_{i=1}^{n} P_{0i}} \tag{7.2}$$

- No surprises here; this was expression for par yield from Chapter 4

Mathematica implementation to explore terms

```
SwapRate[n_] := (1 - P[0, n]) / Sum[P[0, i], {i, 1, n}]

Table[{S[n], SwapRate[n]}, {n, 3}] // TableForm
```

S[1]	$\frac{1-P[0,1]}{P[0,1]}$
S[2]	$\frac{1-P[0,2]}{P[0,1]+P[0,2]}$
S[3]	$\frac{1-P[0,3]}{P[0,1]+P[0,2]+P[0,3]}$

Proof

- Consider 4-year swap

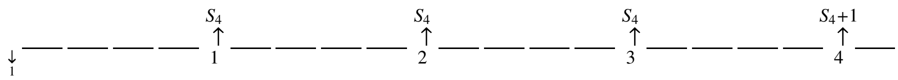


Figure 7.6: Diagram of cashflows for fixed leg of a 4-year swap agreement

- Pay 1 to get
  - 4 coupons of swap rate
  - initial investment back at 4

$$1 = S_4 P_{01} + S_4 P_{02} + S_4 P_{03} + (1 + S_4) P_{04}$$

- Similarly for $n = 1, 2, 3, \ldots$

Bond prices from swap rates

Result

Proposition 7.6. *The prices of zero coupon bonds P_{0n} can be expressed in terms of the n-period swap rate S_n by the relationship* $P_{0n} = 1 - S_n \sum_{r=0}^{n-1} \prod_{k=0}^{r} \frac{1}{1+S_{n-k}}$

$$P_{0n} = 1 - S_n \sum_{r=0}^{n-1} \prod_{k=0}^{r} \frac{1}{1 + S_{n-k}} \tag{7.3}$$

***Mathematica* implementation to explore terms**

```
ZeroBond[n_] := 1 - S[n] Sum[Product[1/(1 + S[n - k]), {k, 0, r}], {r, 0, n - 1}]
```

$$\texttt{ZeroBond[n\_] := 1 - S[n]} \sum_{r=0}^{n-1} \prod_{k=0}^{r} \frac{1}{\texttt{1 + S[n - k]}}$$

```
Table[{P[0, n], ZeroBond[n]}, {n, 3}] // TableForm
```

P[0, 1]	$1 - \frac{S[1]}{1+S[1]}$
P[0, 2]	$1 - S[2]\left(\frac{1}{1+S[2]} + \frac{1}{(1+S[1])\,(1+S[2])}\right)$
P[0, 3]	$1 - S[3]\left(\frac{1}{1+S[3]} + \frac{1}{(1+S[2])\,(1+S[3])} + \frac{1}{(1+S[1])\,(1+S[2])\,(1+S[3])}\right)$

Proof

- The swap rate is the level of the fixed rates such that the swap has zero value at inception
- Zero value occurs when floating rate bond equals fixed rate bond
- At inception, value of floating rate bond is unity (up to a common factor of the principal)

$$\begin{aligned} 1 &= (1 + S_1)\, P_{01} \\ 1 &= S_2\, P_{01} + (1 + S_2)\, P_{02} \\ 1 &= S_3\, P_{01} + S_3\, P_{02} + (1 + S_3)\, P_{03} \\ &\vdots \\ 1 &= S_n \textstyle\sum_{i=1}^{n} P_{0\,i} + P_{0\,n} \\ &\vdots \end{aligned}$$

- Solve iteratively in terms of P_{0i}

$$\begin{aligned} P_{01} &= \frac{1}{(1+S_1)} = 1 - \frac{S_1}{1+S_1} \\ P_{02} &= \frac{1 - S_2\, P_{01}}{1+S_2} = \frac{1}{1+S_2} - \frac{1}{1+S_2}\,\frac{S_2}{1+S_1} \\ &= \frac{1+S_2-S_2}{1+S_2} - \frac{1}{1+S_2}\,\frac{S_2}{1+S_1} = 1 - \frac{S_2}{1+S_2} - \frac{1}{1+S_2}\,\frac{S_2}{1+S_1} \\ &= 1 - S_2\left(\frac{1}{1+S_2} + \frac{1}{1+S_1}\,\frac{1}{1+S_2}\right) \\ P_{03} &= \ldots = 1 - S_3\left(\frac{1}{1+S_3} + \frac{1}{(1+S_2)(1+S_3)} + \frac{1}{(1+S_1)(1+S_2)(1+S_3)}\right) \\ &\vdots \\ P_{0n} &= 1 - S_n \sum_{r=0}^{n-1} \prod_{k=0}^{r} \frac{1}{1+S_{n-k}} \\ &\vdots \end{aligned}$$

FRNs are worth par after a coupon payment

- Not in Hull
- This argument courtesy of L.P. Hughston.

Notation

a,b	start and end of time interval
P_{ab}, $P(a,b)$	price of zero-coupon bond at a, with unit payoff at b
L_{ab}	LIBOR interest rate
n	number of days since last payment

Setup

- Consider a 3-yr investment paying a LIBOR coupon, annually

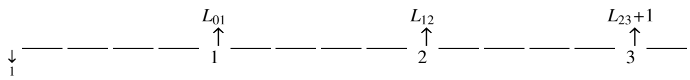


Figure 7.7: Diagram of cashflows for 3-yr investment paying a LIBOR coupon, annually

- With a compounding frequency of $m = \frac{1}{b-a}$, the LIBOR rate and price of a zero coupon bond are related:

$$P_{ab} = \frac{1}{1 + L_{ab}(b-a)} \tag{7.4}$$

- For our simple case, $b - a = 1$

$$L_{ab} = \frac{1}{P_{ab}} - 1$$

Argument

- Each LIBOR coupon payment is discounted back to the previous payment date using a LIBOR discount rate

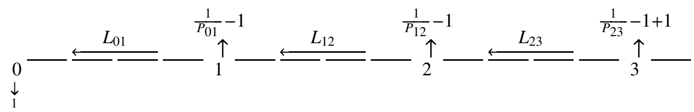


Figure 7.8: Diagram of cashflows for 3-yr investment with LIBOR coupons expressed in terms of bond prices

- Value of 3rd coupon at time 2

$$\left(\frac{1}{P_{23}} - 1 + 1\right)\frac{1}{1 + L_{23}} = 1$$

- Value of 3rd and 2nd coupons at time 1

$$\left(\frac{1}{P_{12}} - 1 + \overbrace{1}^{\text{previous step}}\right)\frac{1}{1 + L_{12}} = 1$$

- Repeat iteratively down to $t = 0$
- Value of 3rd, 2nd & 1st coupons at time 0

$$\left(\frac{1}{P_{01}} - 1 + \overbrace{1}^{\text{previous step}}\right)\frac{1}{1 + L_{12}} = 1$$

If coupon has just been paid, floating rate note is worth par

- For an alternative proof see e.g. Cuthbertson & Nitzsche (2001) *Financial Engineering: Derivatives and Risk Management*

Example

Example 7.2. The 6, 12 & 18-month zero rates have been determined to be 4%, 5% & 4.8%, respectively. The 2-year semi-annually compounded swap rate is 5%. Find the 2-yr zero rate.

Bond with semi-annual coupon of 5% sells for par

$$0.025\times\left(e^{-0.04\times0.5}+e^{-0.05\times1.0}+e^{-0.048\times1.5}\right)+1.025\times e^{-r_2\times2.0}=1$$

$$r_2=4.953\,\%$$

Valuation of interest rate swaps

- Swap ≡
  - long position in one bond with a short position in another
  - portfolio of forward rate agreements

Valuation in terms of bond prices

- Valuation of bonds
  - fixed rate – usual way
  - floating rate – worth par immediately after next payment date

Notation

V_{swap}	value of swap
B_{fix}	value of fixed rate bond
B_{fl}	value of floating rate bond
c_i	cash flow at time T_i
r_i	zero coupon yield for interval $[0,T_i]$
n	number of payments

Swap value

$$V_{\text{swap}}=B_{\text{fix}}-B_{\text{fl}} \tag{7.5}$$

Fixed rate bond

$$\begin{aligned} B_{\text{fix}} &= \sum_{i=1}^{n} c_i\, e^{-r_i T_i} + \Lambda\, e^{-r_n T_n} \\ &= \sum_{i=1}^{n} c_i\, P_{0i} + \Lambda\, P_{0n} \end{aligned} \tag{7.6}$$

Value of floating rate bond

Notation

k^*	next coupon payment
t^*	time to next coupon payment
r^*	zero coupon bond discount rate over $[0, t^*]$

Table 7.4. Expressions for the value of a floating rate bond

Time, relative to coupon payment	*Head*
Immediately after	Λ
Immediately before	$\Lambda + k^*$
Time t^* before	$(\Lambda + k^*)\, e^{-r^* t^*}$

Universal swap pricing formula

- In your next course on swaps
- Value at t of forward start payer swap

$$V_{\text{swap}}(t, S) = P(t, T_0) - \sum_{i=1}^{n} S\, P(t, T_j) - P(t, T_n) \tag{7.7}$$

- M&R (2004) p. 477
- Luenberger (1998) p. 275
- Cuthbertson & Nitzsche (2001) p.380

Example

Example 7.3. Value a swap between 6-mo LIBOR and 8% fixed with semi-annual compounding, with principal of $100mi and remaining life of 1.25 years. LIBOR rates for 3, 9 & 15 month maturities are 10%, 10.5% & 11%, resp. The 6-mo LIBOR rate at the last payment date was 10.2%.

$K^* = 0.5 \times 0.102 \times 100 = \$5.1\,\text{mi}$
$T^* = 1.25$
See table for valuation of B_{fix} and B_{fl}
$V_{swap} = B_{fix} - B_{fl} = \$(98.238 - \$102.505) \times 10^6 = -\$4.267\,\text{mi}$

Table 7.5. Table accompanying exercise

Time	*B_{fix} cash flow*	*B_{fl} cash flow*	*Discount factor*	*PV B_{fix} cash flow*	*PV B_{fl} cash flow*
0.25	4.0	105.100	$e^{-0.1\times0.25}$	3.901	102.505
0.75	4.0		$e^{-0.105\times0.75}$	3.697	
1.25	104.0		$e^{-0.11\times1.25}$	90.640	
Total				98.238	102.505

Valuation in Terms of FRAs

- Each exchange of payments in IR swap is FRA
- FRA values – assume today's forward rates are realized
- Steps
  - Obtain forward rates from zero rates
  - Replace random floating rates by forward rates
  - Discount and sum

Example

Example 7.4. Value the swap from the previous example, but considering it as a portfolio of FRAs.

Fixed rate is $100 \times 0.06 \times 0.5 = 4.0$
Floating payment at 3mos, already known
For remaining floating payments, replace random future rate by forward (FRA pricing trick)

$$R_F = \frac{R_2\,T_2 - R_1\,T_1}{T_2 - T_1}$$

$$R_{3,9} = \frac{0.105\times0.75 - 0.10\times0.25}{0.5} = 0.1075 \text{ (c.c.)}$$

i.e. 11.044% with semi annual compounding
Similarly for $R_{9,15}$.

Table 7.6. Table accompanying exercise

Time	*Fixed cash flow*	*Float cash flow*	*Net*	*Discount factor*	*PV net cash flow*
0.25	4.0	100 × 0.102 × 0.5= −5.100	−1.100	$e^{-0.1\times0.25}$	−1.073
0.75	4.0	100 × 0.11044 × 0.5=−5.522	−1.522	$e^{-0.105\times0.75}$	−1.407
1.25	4.0	−6.051	−2.051	$e^{-0.11\times1.25}$	−1.787
Total					−4.267

- Agrees ☺
- At inception, and later, FRAs do not have zero value

Currency swaps

An Example of a Currency Swap

- An agreement to
  - pay 11% on a
    sterling principal of £10,000,000 &
  - receive 8% on a
    US$ principal of $15,000,000
  - every year for 5 years

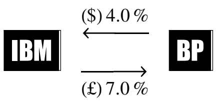


Figure 7.9: Currency swap between IBM and BP

Exchange of principal

- In an interest rate swap the principal is not exchanged
- In a currency swap the principal is usually exchanged at the beginning and the end of the swap's life

The Cash Flows

- Hull Table 7.7, page 166
- *Fixed-for-fixed*

Table 7.7. Cash flows to IBM due to currency swap contract with BP

Year	*$*	*£*
2004	−15.00	+10.00
2005	+0.60	−0.70
2006	+0.60	−0.70
2007	+0.60	−0.70
2008	+0.60	−0.70
2009	+15.60	−10.00

Typical Uses of a Currency Swap

- Conversion from a liability in one currency to a liability in another currency
- Conversion from an investment in one currency to an investment in another currency

Comparative Advantage Arguments for Currency Swaps

- Hull Table 7.8, page 167
- General Motors wants to borrow AUD
- Qantas wants to borrow USD

Table 7.8. Rates for borrowing available to two companies in two currencies.

Company	*USD*	*AUD*
GM	5 %	12.6 %
Quantas	7 %	13.0 %

- $\left\{\begin{array}{c}\text{GM}\\ \text{Quantas}\end{array}\right\}$ has a comparative advantage in the $\left\{\begin{array}{c}\_\_\_\ \_\_\_\\ \_\_\_\ \_\_\_\end{array}\right\}$ market
- Difference between $\left\{\begin{array}{c}\text{USD}\\ \text{AUD}\end{array}\right\}$ rates is $\left\{\begin{array}{c}\_\_\_\ \_\_\_\\ \_\_\_\ \_\_\_\end{array}\right\}$
- Total gains to all parties ______ = ___

Figure

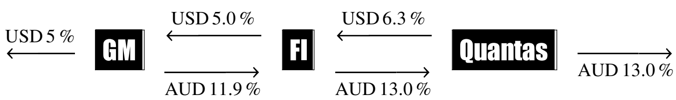


Figure 7.10: Currency swap between IBM and BP with financial intermediary.

Valuation of currency swaps

- Like interest rate swaps, currency swaps can be valued either as the difference between 2 bonds or as a portfolio of forward contracts

Swaps & Forwards

- A swap can be regarded as a convenient way of packaging forward contracts
- The "plain vanilla" interest rate swap in our example consisted of 6 FRAs
- The "fixed for fixed" currency swap in our example consisted of a cash transaction & 5 forward contracts
- The value of the swap is the sum of the values of the forward contracts underlying the swap
- Swaps are normally "at the money" initially
- This means that it costs nothing to enter into a swap
- It does not mean that each forward contract underlying a swap is "at the money" initially

Credit risk

- A swap is worth zero to a company initially
- At a future time its value is liable to be either positive or negative
- The company has credit risk exposure only when its value is positive

Other types of swap

- Floating-for-floating interest rate swaps,
- amortizing swaps,
- step up swaps,
- forward swaps,
- constant maturity swaps,
- compounding swaps,
- LIBOR-in-arrears swaps,
- accrual swaps,
- diff swaps,
- cross currency interest rate swaps,
- equity swaps,
- extendable swaps,
- puttable swaps,
- swaptions,
- commodity swaps,
- volatility swaps
- etc……..

Summary

- Most common: IR and FX
  - IR swap: exchange fixed for floating rate on notional principal over period of time
  - FX swap: exchange fixed in one FX for fixed in other FX on principals in each FX
- Principal exchanged
  - IR swap: no
  - FX swap: yes
- Used to transform payments associated with a loan or an asset from
  - IR – fixed to floating, or vice versa
  - FX – one FX to another
- Value IR loan as
  - fixed and floating bonds
  - portfolio FRAs
- Financial institution exposed to credit risk. If one counter party defaults, with +ve value, still has to honour agreement with other.