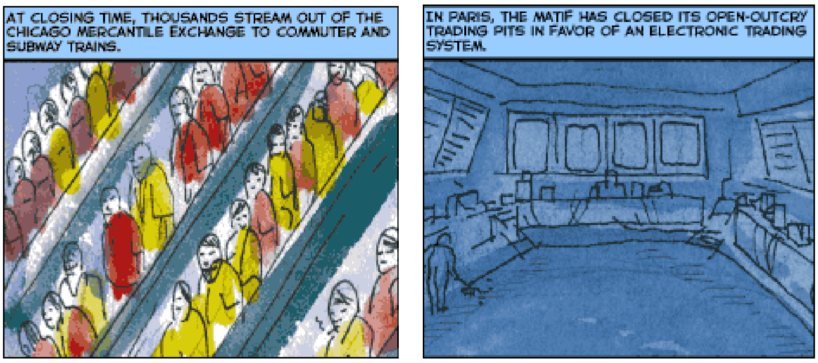
6

Interest rate futures



I never think of the future. It comes soon enough.

—Albert Einstein

Overview

- Day count conventions
- Quotations for Treasury bonds
- Treasury bond futures
- Eurodollar futures
- Duration-based hedging strategies
- Hedging portfolios of assets and liabilities

Introduction

- So far
  - commodities
  - stock indices
  - foreign exchange
- Considered
  - process
  - uses for hedging
  - prices set
- Focus on (US, and model for all futures)
  - Treasury bond futures
  - Eurodollar futures

Day count conventions

- Determine the way interest is accrued over time
- Interest earned over time
  - Know – reference period (e.g. between coupons)
  - Require – some other period

Interest earned between two dates

$$\frac{\text{[Number of days between dates]}}{\text{[Number of days in reference period]}} \times \text{[Interest earned in reference period]} \tag{6.1}$$

In US

- Treasury Bonds: Actual/Actual (in period)
- Corporate Bonds: 30/360
- Money Market Instruments: Actual/360

Treasury bonds

- Actual/Actual (in period)

Example 6.1. A treasury bond with face value $100 pays a semi-annual coupon of 8%. Coupon payment dates are Mar 1 and Sept 1. Find the interest earned between Mar 1 and July 3.

Convention is actual/actual.

Reference period Mar 1 to Sept 1 is 184 days

Desired period is Mar 1 and July 3, is 124 days

Interest earned is $\frac{124}{184} \times 0.04 = 2.6957\,\%$

Corporate and municipal bonds

- 30/360

Example 6.2. A corporate bond with face value $100 pays a semi-annual coupon of 8%. Coupon payment dates are Mar 1 and Sept 1. Find the interest earned between Mar 1 and July 3.

Convention is 30/360.

Reference period Mar 1 to Sept 1 is 180 days

Desired period is Mar 1 and July 3, is $(4 \times 30) + 2 = 122$ days

Interest earned is $\frac{122}{180} \times 0.04 = 2.7111\,\%$

Money market Instruments

- Actual/360
- Quoted using a *discount rate*
- I.e. interest as % of final (face) value
- ≠ rate of return

$$P = \frac{360}{n}(100 - Y) \tag{6.2}$$

Notation

P	quoted price = discount rate
Y	cash price
n	remaining life of T-bill in calendar days

Example

Example 6.3. The price of a 91-day T-bill is 8%. Find the dollar amount of interest paid over the 91 day period and the corresponding rate of interest.

Convention is actual/360.

Dollar interest is $\$100 \times 0.08 \times \frac{91}{360} = \$2.0222$

Rate of interest ("$\frac{\Delta P}{P_1}$") is $\frac{2.0222}{100-2.0222} = 2.064\,\%$

Quotations for Treasury bonds

- Quote in \$1 and $\$\frac{1}{32}$
- e.g. 90-05 is $90 + \frac{5}{32} = 90.1563$
- The (dirty) cash price paid is not the same as the (clean) quoted price

$$[\text{Cash price}] = [\text{Quoted price}] + \overbrace{[\text{Accrued interest}]}^{\text{(Since last coupon)}} \tag{6.3}$$

Example 6.4. On March 5, 2007, a \$100,000 bond with an 11% coupon is due to mature on July 10, 2012, and is quoted at 95-16. Find the accrued interest on March 5, 2007 and the cash price of the bond.

Coupon dates

- most recent – Jan 10, 2007
- next – July 10, 2007 $\longleftarrow$ will mature on anniversary

Number of days:

- Jan 10 2007 $\xrightarrow{+54}$ Mar 5 2007
- Jan 10 2007 $\xrightarrow{+181}$ Jul 10 2007

Accrued interest on Mar 5 2007 is fraction of July 10 coupon (actual/actual) per \$100 face value

$\frac{54}{181} \times \$5.5 = \$1.64$

Cash/dirty price of bond

$(\$95.5 + \$1.64) \times 1000 = \$97,140$

Treasury bond futures

Overview

- Quotes
- Conversion factors
- Cheapest to deliver bond
- Determining the futures price

Introduction

- Treasury bond futures contract on CBOT popular
- In the contract, any government bond with more than 15 years to maturity from the first day of the delivery month and not callable within 15 years can be delivered

Quotes

- T-bond futures quoted in same way as T-bond prices
- One contract \$100,000 ⇒ \$1 change in fut pr → \$1000 change in value of fut contract

Conversion factors

- T-bond futures contract — any bond with maturity ≥15, not callable ≤15
- Party with short position can choose bonds to be delivered ⇒ price received is adjusted
- Cash price received by party with short position

$$\text{[Cash price received by party with short position]} = \\ (\text{[Most recent settlement price]} \times \text{[Conversion factor]}) + \text{[Accrued interest]} \tag{6.4}$$

- Accounts for the flexibility available to the person with a short position.
- Determined by assuming the that interest rate for all maturities is 6% per annum (semi annual compounding)
- Round to nearest 3 mos

Cash price

Example 6.5. A T-bond future has a settlement price of 90-00, a conversion factor 1.38 for delivered bond, and the accrued interest is \$3 per \$100. Find the cash price paid by the party with the long position.

$(1.3800 \times 90.00) + 3.00 = \$127.20$ per \$100 face

Short one contract delivers bonds with \$100,000 face and receives \$127,200.

Conversion factors

Example 6.6. Find the conversion factor for a 10% bond with 20 years & 2 months to maturity.

Take 20 yrs to maturity

First coupon after 6 mos

Value of bond:

$$\sum_{i=1}^{40} \frac{5}{1.03^i} + \frac{100}{1.03^{40}} = \$146.23$$

Conversion factor 1.4623.

Example 6.7. Find the conversion factor for a 8% bond with 18 years & 4 months to maturity.

Take 18 yrs 3 mos to maturity

First coupon after 3 mos

Value of bond:

$$\$\,\frac{1}{1.03^{1/2}}\left(\overset{\text{First coupon}}{4}+\sum_{i=1}^{36}\frac{4}{1.03^{i}}+\frac{100}{1.03^{36}}\right)-2=\frac{\$146.23}{\sqrt{1.03}}-\$2=\$123.99-\$2=\$121.99$$

Conversion factor 1.2199.

Cheapest to deliver bond

- Given that the cost to purchase a bond is:

$$[\text{Quoted bond price}] + [\text{Accrued interest}]$$

- While the cash price received is:

$$[\text{Quoted futures price}] \times [\text{Conversion factor}] + [\text{Accrued interest}]$$

- A cheapest to deliver bond can be found where the difference is a minimum:

$$[\text{Quoted bond price}] - [\text{Quoted futures price}] \times [\text{Conversion factor}] \qquad (6.5)$$

- A number of factors determine the CTD Bond
- When yields; conversion factor favours
  - $> 8\%$, low-coupon / long-maturity bonds
  - $< 8\%$, high-coupon / short-maturity bonds
- Yield curve is
  - upward sloping, long-maturity bonds are favoured
  - downward sloping, short-maturity bonds are favoured
- Some bonds trade for more than their theoretical value:
  - Low coupon bonds
  - Strip bonds
- These are rarely cheapest to deliver

The Wild Card Play

- Wildcard arise because on CBOT futures trading closes at 2pm where as bond trading closes at 4pm.
- I.e. can make delivery choices based on post 2pm market moves
- This option is not free, it is valued in the futures price which is lower than it would be without the option

Determining the futures price

Factors that affect the futures price:

- Delivery can be made any time during delivery month
- Any of a range of eligible bonds can be delivered
- The wild card play

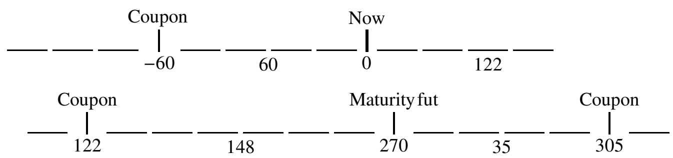
Crude futures price

- Exact theoretical price difficult to determine given short party's timing and asset delivery options
- Assume CTD bond and delivery date are known
- T bond future is contract on asset providing asset with known income:

$$F_0 = (S_0 - I)\, e^{rT} \tag{6.6}$$

- So calculate
  a. cash price of CDT bond from the quoted price
  b. cash futures price
  c. quoted futures price from the cash futures price
  d. divide the quoted futures price by the conversion factor

Example 6.8. Suppose that the CDT bond is a T-bond with a coupon of 12% and a conversion factor of 1.4 and delivery will take place in 270 days with other key dates as in the figure. The term structure is flat with 10% (c.c.) interest. The current quoted bond price is $120.
Find
• the proportion of the next coupon payment that accrues to the holder and the cash price of the bond
• quoted futures price for a 12% bond and for a standard bond



Cash bond price is

$120 + \frac{60}{60+122} \times 6 = 121.978 = S_0$

On 12% bond, quoted futures price is cash futures price minus accrued interest

$$\underbrace{\underbrace{(S_0 - I)\, e^{rT}}_{\text{Dirty/cash}} - \text{AI}}_{\text{Quoted/clean}} = \underbrace{\left(121.978 - \underbrace{6\, e^{-0.1\times\frac{122}{365}}}_{I \text{ is PV of \$6 after 122}}\right) e^{+0.1\times\frac{270}{365}}}_{\text{Cash futures price}} - \underbrace{6 \times \frac{148}{148+35}}_{\text{Accrued interest}} = 120.242$$

But each 12% bond ≡ 1.4 standard bonds, so quoted futures price is $\frac{120.242}{1.4000} = 85.887$

Eurodollar futures

- Eurodollar – dollar deposited in bank outside United States
- Eurodollar futures – futures on the 3-month Eurodollar deposit rate (cf. 3-month LIBOR rate)
- One contract ≡ rate earned on $1 million
- 3-mo on CME most popular
- Lock in IR on $1mi for future 3-mo period
- Period starts 3rd Weds of delivery month
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25
- A Eurodollar futures contract is settled in cash
- When it expires (on the third Wednesday of the delivery month), final settlement price is 100 minus actual three month deposit rate

Example

Example 6.9. Suppose you buy (take a long position in) a contract on November 1, which expires on December 21. The prices are as shown. How much do you gain or lose
a) on the first day,
b) on the second day,
c) over the whole time until expiration?

Date	Quote
Nov 1	97.12
Nov 2	97.23
Nov 3	96.98
…….	……
Dec 21	97.42

a) Day 1: $11 \times 25 = \$275$

b) Day 2: $-25 \times 25 = \$625$

c) On Nov. 1, have $1 million to invest on for three months on Dec 21, the contract locks in a rate of

100 - 97.12 = 2.88%

Earn

• interest 100 – 97.42 = 2.58% on $1 million for three months (=$6,450)

• day by day on the futures contract – 30×$25 =$750

Contract price

- If Q is the quoted price of a Eurodollar futures contract, the exchange defines the value of one contract to be

$$10000[100 - 0.25\,(100 - Q)] \tag{6.7}$$

Example 6.10. For the previous example, what is the final contract price?

$10000\,(100 - 0.25\times(100 - 97.42)) = \$993,550.$

Forward vs futures interest rates

- Eurodollar futures contracts last as long as 10 years
- For Eurodollar futures lasting beyond two years we cannot assume that the forward rate equals the futures rate

Two reasons

- Futures is settled daily where forward is settled once
- Futures is settled at the beginning of the underlying three-month period; forward is settled at the end of the underlying three-month period
- Convexity adjustment often made

$$\text{Forward rate} = \text{futures rate} - \frac{1}{2}\,\sigma^2\,t_1\,t_2 \tag{6.8}$$

Notation

t_1	time to maturity
t_2	maturity of the rate underlying the future
σ	standard deviation of short rate changes

Example

Example 6.11. Suppose the standard deviation of short rate changes is 1.2%. Find the forward rate when the 8-year Eurodollar futures price quote is 94. The time to maturity is 8 yrs, whereas the maturity of the rate underlying the future is 8.25 years. Find the convexity adjustment and hence the forward rate.

$$\text{Forward rate} = \overbrace{\frac{365}{90}\log\left(1+\frac{0.06}{4}\right)}^{6\,\%\text{ pa act/360, qtrly}} - \frac{1}{2}\times 0.012^2\times 8\times 8.25 = 5.563\,\%$$

Effect of maturity on convexity adjustment

- Table to show effect of maturity on convexity adjustment

Maturity of Futures	Convexity Adjustment (bps)
2	3.2
4	12.2
6	27.0
8	47.5
10	73.8

Extending the LIBOR Zero Curve

- LIBOR deposit rates define the LIBOR zero curve out to one year
- Eurodollar futures used to determine forward rates; forward rates used to bootstrap zero curve

$$F_i = \frac{R_{i+1}\,T_{i+1} - R_i\,T_i}{T_{i+1} - T_i} \tag{6.9}$$

- $\Rightarrow$

$$R_{i+1} = \frac{F_i(T_{i+1} - T_i) + R_i\,T_i}{T_{i+1}} \tag{6.10}$$

- Also Euroswiss, Euroyen, Euribor

Example

Example 6.12. Suppose that the 400 day LIBOR zero rate is found to be 4.80% (c.c.) and from Eurodollar futures quotes it has been found that 90-day forward rates beginning at times in the future are
i) 400 days 5.30%
ii) 491 days 5.50%
iii) 589 days 5.60%
Find the 491 and 589 day rates.

$$\frac{0.053\times91+0.048\times400}{491} = 0.04893$$
$$\frac{0.055\times99+0.04893\times491}{589} = 0.04994$$

Duration-based hedging strategies

Duration Matching

- Hedging against interest rate risk by matching durations of assets and liabilities

- It provides protection against small parallel shifts in zero curve

Duration-based hedge ratio

- Also *price sensitivity hedge ratio*
- Number of contracts required to hedge against an uncertain Δy is

$$N^* = \frac{P\,D_P}{F_C\,D_F} \tag{6.11}$$

- If CTD changes, have to adjust

Notation

F_C	contract price for interest rate futures
D_F	duration of asset underlying futures at maturity
P	value of portfolio being hedged
D_P	duration of portfolio at hedge maturity

Example

Example 6.13. It is August. A fund manager has $10 million invested in a portfolio of government bonds with a duration of 6.80 years and she wants to hedge against interest rate moves between August and December. The manager decides to use December T-bond futures. The futures price is 93-02 or 93.0625 and the duration of the cheapest to deliver bond is 9.2 years

The number of contracts that should be shorted is

$$\frac{10 \times 10^6}{93062.50} \times \frac{6.80}{9.20} = 79.42$$

Limitations

- Assumptions about yield curve changes:
  - parallel shifts
  - small

Hedging portfolios of assets and liabilities

GAP management

- Asset Liability Management (ALM)
- This is a more sophisticated approach used by banks to hedge interest rate. It involves
- Bucketing the zero curve

- Hedging exposure to situation where rates corresponding to one bucket change and all other rates stay the same.

Summary

- Two popular IR contracts in US are T-bond and Eurodollar futures

Day count conventions

- days in month: 30 or actual
- days in year: 360 or actual

Quotations for Treasury bonds

- Add accrued interest

Treasury bond futures

- Quoted same way as T-bonds
- Party with short position has delivery options
  - any day
  - alternative bonds
  - NOITD made at different times
- Effect is to reduce futures price

Eurodollar futures

- Contract on 3-mo rate on 3rd Wed of delivery mo.
- Used to estimate LIBOR fwd rates
- Need convexity adjustment – accounts for mk-to-mkt

Duration-based hedging strategies

- Sensitivity of portfolio to small, parallel shifts
- Similarly for futs price
- # futs to hedge portfolio can be calculated

Hedging portfolios of assets and liabilities

- Consider effect of change of rate in bucket on assets and liabilities