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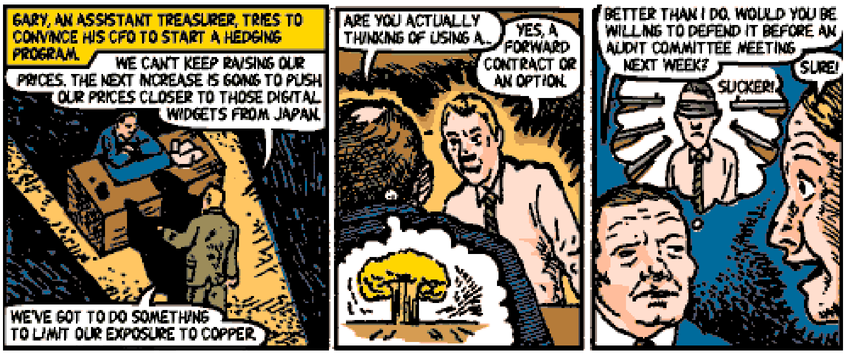
Hedging strategies using futures



The major characteristic of the diagonal model is the assumption that the returns of various securities are related only through common relationships with some basic underlying factor.

—William Sharpe

Introduction

- Some futures market participants are hedgers
- Try to reduce risk due to variable (stock / FX / oil price / etc.) using futures
- *Perfect hedge* eliminates risk
- *Hedge-and-forget strategies* – static strategy:
  - place hedge

    - no adjustment
    - close out at end of life of hedge
- Cf. dynamic strategies in Chapter 15, Greeks. E.g., *delta-hedging*.
- Simplification
    - ignore daily settlement
    - all cash flows at end of hedging period

Overview

- Basic principles
- Arguments for and against hedging
- Basis risk
- Cross hedging
- Stock index futures
- Rolling the hedge forward

Basic principles

Hedge direction

- A $\left\{ \begin{array}{c} \text{long} \\ \text{short} \end{array} \right\}$ futures hedge is appropriate when you know you will $\left\{ \begin{array}{c} \text{buy} \\ \text{sell} \end{array} \right\}$ an asset in the future and want to lock in the price

Short hedges

Example 3.1. Successful highland farmer Angus McPorridge gains £10,000 for every 1p rise in the price of oats over the next 3 months and loses £10,000 for every 1p decrease. What position should Angus take in oat futures to hedge his risk? Explain your rationale.

Short position in the future, such that the hedge loses (gains) £10,000 for every 1p rise (fall) in the price of oats. If either the price of oats goes up or down, the gain or loss in the farmer's business is exactly offset by the loss or gain in the value of the hedge, respectively.

- It might help to think of adding the payoff vs. terminal spot price plots; aim is for position plus hedge to give a horizontal flat line:
  $$\text{"}\overset{\text{Position to hedge}}{/} + \overset{\text{Payoff from short position}}{\backslash} = \text{—}\text{"}$$

Long hedges

Known future purchase

Example 3.2. On January 15th a copper fabricator knows that it will need 100,000 pounds of copper on May 15th. The spot price of copper is 140 cents per pound, whereas the futures price for delivery in May is 120 cents per pound.
i) How many May futures contracts on the COMEX division of NYMEX should the company use to hedge this exposure if each contract is for 25,000 pounds of copper?
ii) What does the company pay for the copper and what is the profit or loss of the hedged strategy relative to the unhedged strategy under the two scenarios that the spot price for copper on May 15 turns out to be \$1.25 and \$1.05.
iii) Discuss the advantages and disadvantages of hedging using futures rather than buying copper on the spot market.

Number of contracts:

$\frac{100000}{25000} = 4$

Scenario	Price paid	Profit relative to no hedge
$\$1.25$	$10^5 \times \$1.20 = \$125,000$	$10^5 \times (\$1.25 - \$1.20) = \$5,000$
$\$1.05$	$10^5 \times \$1.20 = \$125,000$	$10^5 \times (\$1.05 - \$1.20) = -\$15,000$

Re futures strategy:

Advantages:

• futures price cheaper than spot $\overset{\text{Future}}{120} < \overset{\text{Spot}}{140}$

• no storage + interest costs

Disadvantages:

• no copper on hand (but given specific problem, who cares?)

Manage existing short position

- To bet on decline in value of a stock relative to the market, without incurring systematic risk
  - short individual stock
  - long position in index future
- This is *active* equity portfolio management (cf. *passive* management)

Delivery

- Examples: position closed out in delivery month
- Hedge is good if delivery is or is not made
- However, taking delivery: \$\$\$ + inconvenient ⇒ rare

Marking to market

- Daily settlement affects hedge performance

Hedging and regret

- Hedged position can be better or *worse* depending on the outcome for the underlying process on which the future is contigent
- Hedging $\left\{\begin{matrix}\text{reduces}\\ \text{(can) increase}\end{matrix}\right\}$ risk for the $\left\{\begin{matrix}\text{company}\\ \text{treasurer}\end{matrix}\right\}$ (if she is misunderstood)!
- See comic strip *To Hedge or Not to Hedge* http://www.derivativesstrategy.com/magazine/comix/9603_1.asp

Arguments for and against hedging

Introduction

- Why hedge?
  - Companies need to focus on main business
  - Not try to forecast variables: IRs, FX, prices
  - Instead hedge risk
- In practice, risks remain

Hedging and shareholders

- Shareholders DIY hedge?
  - Sufficient knowledge of risks
  - Economy of scale
  - Diversify ✓ (e.g. commodity producer + user)

Hedging and competitors

Hedging to increase risk

- Prices of products of industry players reflect: prices of raw materials, FX, IRs
- $\left\{\begin{matrix}\text{Do}\\ \text{Do not}\end{matrix}\right\}$ hedge: profit margins $\left\{\begin{matrix}\text{constant}\\ \text{vary}\end{matrix}\right\}$!
- Majority unhedged, individual hedged; relative to industry, single company has more risk!

Example

Example 3.3. Two jewellery manufacturers: **SafeAndSure** (SAS) and **TakeAChance** (TAC) do and do not hedge against movements in the price of gold, respectively. Assuming that the majority of players in the industry do not hedge, describe the effects on the price of gold jewellery, and on the profits of the two firms if the price of gold increases and decreases. Which company's profit is unaffected by changes in the gold price and why? Which company is most likely to get into financial trouble, and how?

Gold price	Jewellery price	Profit TAC	Profit SAS
↑	↑	–	↑
↓	↓	–	↓

TAC's profit is unaffected by the price of gold because economic pressures will cause the price of the jewellery to rise and fall in step with the cost of the raw material.
SAS risks financial trouble when the price of the raw material <u>decreases</u> (i.e. under conditions which are favourable for the industry). Due to hedging SAS pays a (relative to the market) high price for the gold, when its product has a low price.

Moral

- Think big picture

Basis risk

Introduction

- So far, nearly perfect hedges
- Reasons for imperfection
  - asset to hedge ≠ asset underlying future
  - time to buy or sell uncertain
  - future closed out early

The basis

Definition

Definition 3.1. A *basis,* β, is the extent to which the spot price of the asset to be hedged exceeds the futures price of the contract used for hedging.

$$\overset{\text{basis}}{b} = \overset{\text{spot price of asset to be hedged}}{\overline{S}} - \overset{\text{futures price of contract used}}{\overline{F}} \tag{3.1}$$

Discussion

- Chapter 2 – spot and futures prices
  - reconciled at expiry ⇒ zero basis
  - +ve or −ve prior to expiry
- As $T - t \to 0$, if spot price increases by $\left\{\begin{matrix}\text{more}\\\text{less}\end{matrix}\right\}$ than the futures price, this is $\left\{\begin{matrix}\textit{strengthening}\\\textit{weakening}\end{matrix}\right\}$ *of the basis*. e.g. $\left\{\begin{matrix}\text{oil}\\\text{gold}\end{matrix}\right\}$

Basis risk

Introduction

- *Basis* is the difference between spot & futures
- *Basis risk* arises because of the uncertainty about the basis when the hedge is closed out

Notation

- Consider two times t_i

t_i	times where i=1 is the earlier, i=2 is the later
F_i	futures price at time i
S_i	spot price at time i
S_i^*	price of asset underlying the futures contract
b_i	basis at time i

Basis at time *i*

- At time i the basis is

$$b_i = S_i - F_i \tag{3.2}$$

Asset sold at t_2, hedge at t_1

- Asset to be *sold* at t_2
- Hedger takes *short* position in future at time t_1
- Price *received* for the combo is

$$\overset{\text{effective}}{\tilde{S}_2} = \overset{\text{terminal stock price}}{S_2} - \overbrace{(F_2 - F_1)}^{\text{gain on futures}} = F_1 + b_2 \tag{3.3}$$

Asset bought at t_2, hedge at t_1

- Asset to be *bought* at t_2
- Hedger takes *long* position in future at time t_1
- Price *paid* for the combo is

$$\overset{\text{effective}}{\tilde{S}_2} = S_2 - (F_2 - F_1) = F_1 + b_2 \tag{3.4}$$

Example

Example 3.4. The data for the spot and futures prices on a commodity are given in the following table:

Time : hedge	Spot	Future
placed	\$2.50	\$2.20
closed out	\$2.00	\$1.90

What is the value of the basis at the two times? For two hedgers with assets to sell and buy, what is the effective price received and paid for the asset, respectively, if the position is hedged with an appropriate forward position? At what time do the random variables associated with the forward price when the hedge is placed, and the basis when the hedge is removed, become known?

Basis $b_i = S_i - F_i$, $i = 1, 2$, where hedge placed at time 1 and closed out at time 2

$$b_i = \begin{cases} \$0.30 & i = 1 \\ \$0.10 & i = 2 \end{cases}$$

Price received and paid is, in both cases, $S_2 - (F_2 - F_1) = F_1 + b_2 = \$2.30$.

$$\begin{cases} F_1 \text{ known at } t_1 \\ b_2 \text{ known at } t_2 \end{cases}$$

Effect of basis on fortunes of hedgers

Table 3.1. Effect of basis on fortunes of hedgers

Hedge ↓ *Basis* →	*Strengthens*	*Weakens*
Short	☺	☹
Long	☹	☺

Different asset to hedge an asset underlying future

- Hedging using a future with the "wrong" underlying asset, introduces a new component to the basis

$$S_2 - (F_2 - F_1) = F_1 + \overbrace{(S_2^* - F_2)}^{\text{standard basis}} + \overbrace{(S_2 - S_2^*)}^{\text{additional basis}} \tag{3.5}$$

Choice of contract

Introduction

- Basis risk depends on the choice of
  - underlying asset
  - delivery month
- Underlying asset choice
  - trivial if ∃ future on asset to be hedged

  - otherwise, choose most highly correlated
- Delivery month (DM) typically later than hedge expiry (HE) to avoid
  - turbulence
  - taking delivery (long hedger)
- But only just later, because basis risk increases with time difference DM – HE
- However, liquidity better in short maturity futures?

Example

Example 3.5. It is March 1st. A company in the US will receive ¥50mi at the end of July. CME ¥ futures are available for the months Mar, June, Sept, Dec. One contract is for ¥12.5mi.
i) Propose a hedging strategy for the company (size and direction) to help it reduce the risk of converting the ¥ into $.
When the ¥ are received at the end of July, the company closes out its position. Suppose that the futures price on Mar 1st in cents per ¥ is 0.7800 and that the spot and futures prices when the contract is closed out are 0.7200 and 0.7250, respectively.
ii) Find the gain on the futures contract, the basis when the hedge is closed out and the effective price obtained for the ¥.

i) Short four Sept because $\frac{¥50\times10^6}{¥12.5\times10^6} = 4$; the ¥50mi will have to be sold; Sept is first delivery month after July
ii) Gain on fut contract $-(F_2 - F_1) = F_1 - F_2 = 0.7800 - 0.7250 = 0.0550$ c per ¥
Basis at hedge expiry is $S_2 - F_2 = 0.7200 - 0.7250 = -0.0050$ c per ¥
Effective price is $S_2 - (F_2 - F_1) = F_1 + b_2 = 0.7200 + (-0.0050) = 0.7750$ c per ¥
Total reward $¥50\times10^6 \times 0.007750\,\$/¥ = \$387,500$

- Similarly, for buying oil. See Example 3.2 in Hull 2005.

Cross hedging

Introduction

- Asset to hedge ≠ asset underlying hedge instrument
- *Cross hedging*
- E.g. airline hedging kerosene, hedges with heating oil

Hedge ratio

Definition 3.2. The *hedge ratio* is the ratio of the size of the position taken in futures contracts to the size of the exposure.

- When assets

- same, $h = 1$ – enter contracts to deliver exactly same amount of commodity
- different, $h \neq 1$ performs better

Minimum variance hedge ratio

Notation

ΔS	change in spot asset price over interval
ΔF	change in futures asset price over interval
σ_S	standard deviation of ΔS
σ_F	standard deviation of ΔF
ρ	correlation between ΔS and ΔF
h^*	hedge ratio that minimizes the variance of the hedgers position

Optimal hedge ratio

Proposition 3.3. *The optimal hedge ratio is given by* $h^* = \rho \, \frac{\sigma_S}{\sigma_F}$.

$$h^* = \rho \, \frac{\sigma_S}{\sigma_F} \tag{3.6}$$

Proof

t_1	time at which choice is made to hedge
t_2	time at which asset is to be sold
N_A	number of units of asset to sell at time t_2
N_F	number of futures contracts to short at time t_1
h	hedge ratio is $h{:=}N_F/N_A$
Y	total amount realized for the asset when the profit or loss on the hedge is taken into account
S_i,F_i	asset prices and futures prices at time i, i=1,2
ΔS, ΔF	change in asset and futures prices over interval, i.e. $\Delta S{:=}S_2{-}S_1$, $\Delta F{:=}F_2{-}F_1$
v	variance of Y
σ_S, σ_F, ρ	standard deviations of asset and future, and correlation coefficient between them

- Total amount realized for asset and hedge:

$$
\begin{aligned}
Y &= S_2\, N_A - (F_2 - F_1)\, N_F \\
&= S_1\, N_A + (S_2 - S_1)\, N_A - (F_2 - F_1)\, N_F \\
&= S_1\, N_A + \, N_A\, (\Delta S - h\, \Delta F)
\end{aligned}
$$

- S_1 and N_A are known at time t_1
- Minimising the variance of Y, corresponds to minimising the variance of $(\Delta S - h\, \Delta F)$

$$v = \mathrm{Var}[\Delta S - h\, \Delta F] = \sigma_S^2 + h^2\, \sigma_F^2 - 2\, h\, \sigma_S\, \sigma_F\, \rho$$

- Derivative w.r.t. h

$$\frac{dv}{dh} = 2\, h\, \sigma_F^2 - 2\, \sigma_S\, \sigma_F\, \rho$$

is zero (and second derivative +ve) when

$$h^* = \rho\, \frac{\sigma_S}{\sigma_F}$$

Hedge effectiveness

Definition 3.4. The *hedge effectiveness* is the proportion of the variance that is eliminated by the hedging, and is given by $h^{*2}\, \frac{\sigma_F^2}{\sigma_S^2}$

$$h^{*2}\, \frac{\sigma_F^2}{\sigma_S^2} \tag{3.7}$$

- The hedge effectiveness is the R^2 from the regression of ΔS vs, ΔF, i.e. ρ^2.

Optimal number of contracts

N_A	size of position to hedge (units)
Q_F	size of one futures contract (units)
N^*	optimal number of futures contracts for hedging

Proposition

Proposition 3.5. *The number of futures contracts required is given by* $N^* = \frac{h^*\, N_A}{Q_F}$.

$$N^* = \frac{h^*\, N_A}{Q_F} \tag{3.8}$$

Example

Example 3.6. An airline expects to purchase 2 mi gallons of jet fuel in 1 month and hedges with heating oil futures. From an analysis of the historical prices of the spot price of jet fuel and heating oil futures it is found that the increments of these have standard deviations 2.63% and 3.13%, respectively, and correlation coefficient of 92.8%. Find the minimum variance hedge ratio and the optimal number of contracts. Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil.

Minimum variance hedge ratio

$$h^* = \rho \frac{\sigma_S}{\sigma_F} = 0.928 \frac{0.0263}{0.0313} = 0.78$$

Optimal number of contracts

$$N^* = \frac{h^* N_A}{Q_F} = \frac{0.78 \times 2 \times 10^6}{42 \times 10^3} = 37.14$$

Stock index futures

Stock indices

A *stock index* tracks changes in the value of a hypothetical portfolio of stocks.

Notes

- Dividends
  - usually not included
  - except total return index
- Weighting:
  - constant holdings ⇒ varying weights
  - capitalisation weighted – weight proportional to [price]×[# shares outstanding]
- Adjustments required for stock splits, stock dividends, new issues

Some indices

Reasons for hedging an equity portfolio

- Briefly out of the market – hedging cheaper than sell, then buy portfolio
- Hedge systematic risk – when picked stocks that will outperform the market

Portfolio does not mirror index

Proposition 3.6. *To hedge the risk in a portfolio the number of contracts that should be shorted is $N^* = \beta \frac{P}{A}$, where P is the value of the portfolio, β is its beta, and A is the value of the assets underlying one futures contract.*

$$N^* = \beta \frac{P}{A} \tag{3.9}$$

- This is a special case of Equation 3.8, with $h^* = \beta$.

Notes

- beta β is slope of regression of excess asset return against excess market return

Example

Example 3.7. The current value of the S&P 500 index is \$1000. Value of Portfolio is \$5 million. Beta of portfolio is 1.5. One futures contract is for delivery of \$250 times the index.

• What position in futures contracts on the S&P 500 is necessary to hedge the portfolio?

• Use the data for the value of the index and the futures price of the index, both 3 months ahead, to assess the performance of the stock index hedge by recording the gain on the futures position, the return on the market, the expected return on the portfolio, expected portfolio value in 3 months (including dividends) and the total expected value of the position in 3 months.

Scenario	Value of index	Futures price of index
1	900	902
2	950	952
3	1000	1003
4	1050	1053
5	1100	1103

The current futures price is \$1010. The dividend rate on the index is 1% per annum. The riskfree rate is 4% per annum.

Position in futures contracts to hedge portfolio

$N^* = \beta \frac{P}{A} = 1.5 \times \frac{5\times10^6}{250\times10^3} = 30$ (short)

Performance of the stock index hedge

Consider Scenario 1

• Gain from short futures position

$N^* \times \Delta F \times \overset{\text{Futures contract index multiple}}{n} = \$30 \times (1010 - 902) \times 250 = \$810,000.$

• Loss on index (excl. divs.) $R = \frac{(S_2 - S_1)}{S_1} = \frac{900-1000}{1000} = -10\,\%$

• Loss on index (incl. divs) $= -0.1 + \frac{0.01}{4} = -9.75\,\%$

CAPM relates excess return on portfolio to excess return of market $r_p - r_f = \beta(r_M - r_f)$

• Expected excess return of portfolio $= r_f + \beta(r_M - r_f) = \frac{0.04}{4} + 1.5\times\left(-0.0975 - \frac{0.04}{4}\right) = -0.15125$

Definition of return: $r_P = \frac{(V_2 - V_1)}{V_1} = \frac{V_2}{V_1} - 1 \Rightarrow V_2 = V_1(1 + r_P)$

• Expected value of portfolio inclusive of dividends $= \$5\times10^6\times(1 - 0.15125) = \$4,243,750$

• Total expected value of position \$4,243,750 + \$810,000

Repeat for other scenarios. See Table 3.4 in Hull for results.

Changing Beta

Proposition 3.7. *To change the beta of a portfolio from β to β^**

- *$\beta^* < \beta$, a short position in $(\beta - \beta^*)\,\frac{P}{A}$ contracts is required*
- *$\beta^* > \beta$, a long position in $(\beta^* - \beta)\,\frac{P}{A}$ contracts is required.*

Example

Example 3.8. Continuing the previous example,
What position is necessary to reduce and increase the beta of the portfolio to 0.75 and 2.0, respectively?

$(1.5 - 0.75)\times\frac{5\times10^6}{250\times10^3} = 15$ short

$(2.0 - 1.5)\times\frac{5\times10^6}{250\times10^3} = 10$ long

Hedging Price of an Individual Stock

- Similar to hedging a portfolio
- Does not work as well because only the systematic risk is hedged
- The unsystematic risk that is unique to the stock is not hedged

Rolling the hedge forward

- We can use a series of futures contracts to increase the life of a hedge
- Each time we switch from 1 futures contract to another we incur a type of basis risk
- Hull page 67-68

Example

Table 3.2. One possible scenario for the evolution of specific points on the the futures curve for oil futures prices

↓Expiry, T\ Observed, t →	*Apr 04*	*Sept 04*	*Feb 05*	*Jun 05*
Oct 04	18.2	17.40		
Mar 05		17.00	16.50	
July 05			16.30	15.90
Spot	19.00			16.00

Example 3.9. On April 2004, a company hedges its risk due to the necessity to sell 100,000 barrels of oil in June 2005, choosing a unit hedge ratio. The current spot is $19. If the company rolls over its hedge using the shortest possible hedge at each time, for the scenario given in the table above, find the dollar gain per barrel of oil from rolling over a short futures contract. How much does the price of oil decline during the period April to June, and how effective is the hedge?

$\$(18.20 - 17.40) + (17.00 - 16.50) + (16.30 - 15.90) = \$1.70$

$\Delta S = \$16.0 - \$19.0 = -\$3$

Hedge does not cover all of the loss.

- Moral: a rolling hedge does not perfectly lock in a future price for the oil. Cf. bond vs. bank account

Summary

- Taking a position in futures to offset exposure in the price of an asset
- Price increases, company
  - gains – short hedge
  - loses – long hedge
- Should companies hedge?
  - pros – reduce risk, do thing instead of forecast
  - cons – shareholders, competitors, ignorance
- Basis risk
  - *Basis* – extent to which spot exceeds futures price
  - *Basis risk* – uncertainty re basis at hedge maturity
  - *Hedge ratio* – position in future : exposure
  - When cross-hedging, often mean variance optimal $h^* \neq 1$
  - Optimal hedge ratio is slope of ΔS vs ΔF.
- Reduce equity systematic risk using stock index futures.
  - # of futs is β times ratio of values of portfolio and assets underlying one contract

- Can remove all or some of the β
- Chain of short-dated futures not hedge as well as correct long-dated contract
- Formulae

$b_t = S_t - F_t$ $\quad\quad$ $\tilde{S}_2 = S_2 - \Delta F = F_1 + b_2$

$\tilde{S}_2 = F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$

$h^* = \rho \frac{\sigma_S}{\sigma_F}$ $\quad\quad$ $e_h = h^{*2} \frac{\sigma_F^2}{\sigma_S^2}$

$N^* = \frac{h^* N_A}{Q_F}$

$N^* = \beta \frac{P}{A}$ $\quad\quad$ $h = (\beta - \beta^*) \frac{P}{A}$