1 Introduction

Example isn't another way to teach, it is the only way to teach.

—Albert Einstein

About this course

Abstract

- Overview of the world's financial markets, and a concise mathematical formulation of the main characteristics of financial instruments and trading practice, with an emphasis on quantitative aspects of options, futures, and other derivatives.
- Topics covered include:
  - Spot markets for stocks, bonds, currencies, commodities.
  - Forward markets.
  - Commodity futures, financial futures.
  - Stock options, index options, currency options, commodity options, interest rate options, options on futures.
  - Interest rate swaps, currency swaps.
  - Corporate bonds, treasury bonds, inflation-linked products.
  - Energy and credit markets.
  - Structured products, hybrid products, OTC derivatives.

Course brief

- all about options
- generic model-independent properties that can be deduced from arbitrage relations
- different kinds of derivatives, their uses and markets
  - forwards and futures contracts
  - vanilla and exotic options
  - foreign exchange, indices as underlyings
  - caps and floors
  - swaps, swaptions
  - structured products
  - credit derivatives
- concepts
  - forward prices, forward interest rates
  - swap rates
- mathematicise with results (which can be stated in a mathematically clear way) about such products that follow from general considerations (e.g. arbitrage relations).
- "problem-solving oriented"
- supports finance courses: FM07, FM08, FM10 (IRs, exotics, credit)

Material to be covered in Hull 2005 6th Edition

- Cover Hull 1-10, 14-16, 21, 23, 26

1. Introduction

2. Mechanics of Futures Markets

3. Hedging Strategies Using Futures

4. Interest Rates

5. Determination of Forward and Futures Prices

6. Interest Rate Futures

7. Swaps

8. Mechanics of Options Markets

9. Properties of Stock Options

10. Trading Strategies Involving Options

11. Binomial Trees

12. Wiener Processes and Itô's Lemma

13. The Black-Scholes-Merton Model

14. Options on Stock Indices, Currencies, and Futures

15. The Greek Letters

16. Volatility Smiles

17. Basic Numerical Procedures

18. Value at Risk

19. Estimating Volatilities and Correlations

20. Credit Risk

21. Credit Derivatives

22. Exotic Options

23. Weather, Energy and Insurance Derivatives

24. More on Models and Numerical Procedures

25. Martingales and Measures

26. Interest Rate Derivatives: The Standard Market Models

27. Convexity, Timing, and Quanto Adjustments

28. Interest Rate Derivatives: Models of the Short Rate

29. Interest Rate Derivatives: HJM and LMM

30. Swaps Revisited

31. Real Options

32. Derivatives Mishaps and What We Can Learn From Them

Topics

1. Introduction	9. Properties of stock options
2. Mechanics of futures markets	10. Trading strategies involving options
3. Hedging strategies using futures	11. Options on stock indices, currencies, and futures (14*)
4. Interest rates	12. The Greek letters (15*)
5. Determination of forward and futures prices	13. Volatility smiles (16*)
6. Interest rate futures	14. Credit derivatives (21*)
7. Swaps	15. Weather, energy and insurance derivatives (23*)
8. Mechanics of options markets	16. Interest rate derivatives (26*)

- *Chapters in Hull 2005

Email discussion group

- To subscribe: Send an email to **`CMFM03_2006-subscribe@topica.com`**
- To Post: Send mail to **`CMFM03_2006@topica.com`**

Timetable

2005/06

2006/07

- Tuesdays 2-4pm
- Fridays 5-7pm

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Approaches to pricing

Table 1.1. Approaches to pricing

Single agent optimality	*Multiple agent optimality*	*No-arbitrage*
• Indifference pricing	• Equilibrium • E.g. CAPM	• For markets that are assumed to be arbitrage−free

- Because they are powerful and robust, arbitrage ideas are the basis of this course and most of the MSc
- Because real markets are rarely truly arbitrage-free, and rarely complete be wary of over-zealous use of no-arbitrage pricing methods (e.g. real options)
- See Luenberger *Investment Science* or Duffie *Dynamic Asset Pricing Theory*

Scope

Table 1.2. Scope of Financial Markets CM

Prerequisites	*Included*	*Beyond*: FM02/07/08
• School mathematics	• Zoology: markets, instruments, relationships, market conventions	• Trees • Black Scholes and continuous−time pricing • Stochastic models

- No models!

Exam

- Marks for
  - Knowing facts
  - Rederiving results algebraically
  - Solving numerical problems, applying results
- 2 hrs
- Best 4 questions out of 5 for Grades A & B; all 5 questions for lower grades

Other textbooks

For CMFM03

- **Hull** *Options, Futures and Other Derivatives* – classic text on derivative instruments and markets, now 6th ed
- **Cuthbertson and Nitzsche** *Financial Engineering* – similar level and scope to Hull
- **Jarrow and Turnbull** *Derivative Securities* – out of print; similar level and scope to Hull

- **Coggan** *The Money Machine: How the City Works* – small paperback explains the nuts and bolts of the financial system

Beyond CMFM03 (e.g. for FM02, FM08)

- **Baxter and Rennie** *Financial Calculus* – wonderful introduction to martingale pricing, for FM02, FM07, FM08, FM11 (RN pricing, IRs, exotics, martingales)
- **Luenberger** *Investment Science* – great introduction to the theory of finance including derivatives
- **Taleb** *Dynamic Hedging* – critique of theory and models from the perspective of a trader
- **Musiela and Rutkowski** *Martingale Methods in Financial Modelling* – advanced coverage of martingale methods in complete markets
- **Wilmott** *Derivatives* – accessible overview of techniques and models

Derivatives Markets

Definition of derivative

A ***derivative*** is an instrument whose value depends on the values of other more basic underlying variables

Examples of derivatives

- Futures Contracts
- Forward Contracts
- Swaps
- Options – vanilla and exotic; on stocks, indices, currencies, commodities, (snowfall!)
- Bond options, caplets, floorlets, caps and floors
- Swaptions
- Credit, electricity, weather, insurance, real options...
- Embedded in
  - bond issues
  - executive compensation
  - capital investment opportunities – real estate, plant, equipment

Markets compared

Derivatives exchange

A *derivatives exchange* is a market where individuals trade standardized contracts that have been defined by the exchange.

Over-the-counter market

> An *over-the-counter market* is a computer- and telephone-linked network of dealers at financial institutions, corporations, and fund managers

Compare exchange and OTC

- Both ultimately match buyers with sellers
- Exchange traded
  - Traditionally open-outcry, but increasingly electronic trading
  - Contracts are standard
  - Virtually no credit risk
- Over-the-counter (OTC)
  - A computer- and telephone-linked network of dealers at financial institutions, corporations, and fund managers
  - Contracts can be non-standard
  - Some credit risk — chance contract not honoured
  - Large trades
- Which is bigger? By which measure?

Size of OTC and Exchange Markets

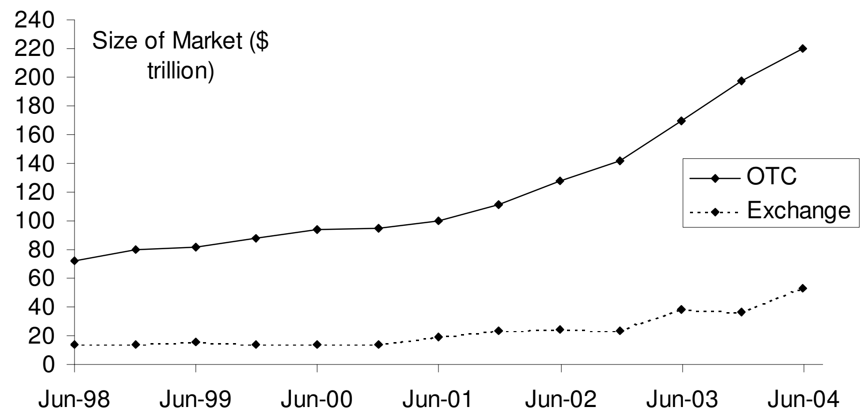

Figure 1.1: Source: Bank for International Settlements http://www.bis.org/ . Chart shows total principal amounts for OTC market and value of underlying assets for exchange market (Hull Figure 1.1, Page 3)

OTC – estimated total principal amounts underlying transactions outstanding in the OTC market

Exchange – estimated total value of the assets underlying exchange-traded contracts

Remarks

- Principal not same as value
- BIS estimate OTC value in June 2004 to be $6.4 trillion

Ways derivatives are used

- Hedge risks
- Speculate (take a view on the future direction of the market)
- Lock in an arbitrage profit
- Change nature of
  - liability
  - investment without incurring costs of selling one portfolio and buying another

Dangers

- Redundant ⇒ no new risks into financial system; aggregate level of risk in economy same.
- Isolate, concentrate (transfer) existing risks
- Derivative disasters
  - Bankers Trust (Federal Paper Board Company, Gibson Greetings, Air Products and Chemical, and Procter & Gamble) mid 90s
  - Metalgesellschaft 1994
  - Orange County 1995
  - Barings 1995
  - LTCM 1998
- See http://riskinstitute.ch/Introduction.htm

Why do derivatives exist?

- Why ought they not to exist?
  - Key idea of course is pricing under the assumption that they are redundant!
- Convenience
- Incomplete markets – lose cosy world of perfect, risk-free replication

Forward contracts

Definition

Definition 1.1. A *forward contract* is an agreement to buy or sell an asset at a certain future time for a certain price

Notes

- Cf. *spot contact*, which is an agreement to buy or sell an asset today
- Similar to futures except that they trade in the over-the-counter market
- Particularly popular on currencies and interest rates
- Between institutions or institution + client
- Banks have spot and forward desks for FX

Example - FX rate forward curve

- Foreign exchange quotes for GBP June 3, 2003 (See page 4)

Code

Output

Table 1.3. Foreign exchange quotes, spot and forward, for USD/GBP ("cable") for GBP June 3, 2003 (See page 4)

	Bid	*Offer*
Spot	1.6281	1.6285
1-month	1.6248	1.6253
3-month	1.6187	1.6192
6-month	1.6094	1.6100

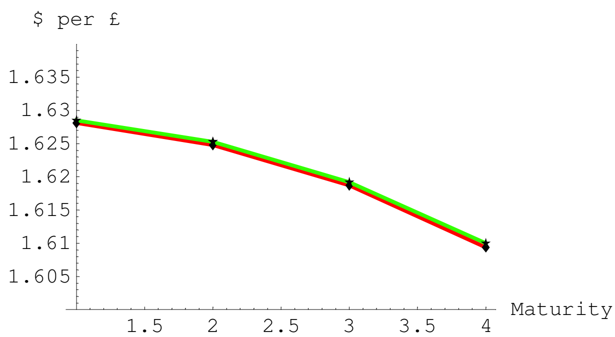

Figure 1.2: Foreign exchange quotes, spot and forward, for USD/GBP ("cable") for GBP June 3, 2003 (See page 4). Bid and offer prices.

Forward price

Definition

Definition 1.2. The *forward price* for a contract is the delivery price that would be applicable to the contract if it were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero).

- The forward price may be different for contracts of different maturities

Terminology

- The party that has agreed to
  - *buy* has what is termed a *long* position
  - *sell* has what is termed a *short* position

Example

- Hull (2005) P4

Example 1.1. On June 3, 2003 the treasurer of a corporation enters into a long forward contract to buy £1 million in six months at an exchange rate of 1.6100. This obligates the corporation (bank) to buy (sell) £1 million for $1,610,000 on December 3, 2003. What are some possible outcomes?

Spot exchange

- rises to 1.7, forward contract worth $90 000 = ($1 700 000 – $ 1610 000)
- falls to 1.5, forward contract worth –$110 000 = ($1 500 000 – $ 1610 000)

Remark

- Corporation has a long forward contract on GBP
- Bank has a short forward contract on GBP
- Both sides have made a binding commitment

Plots of payoffs from long and short forward positions

Code

Output

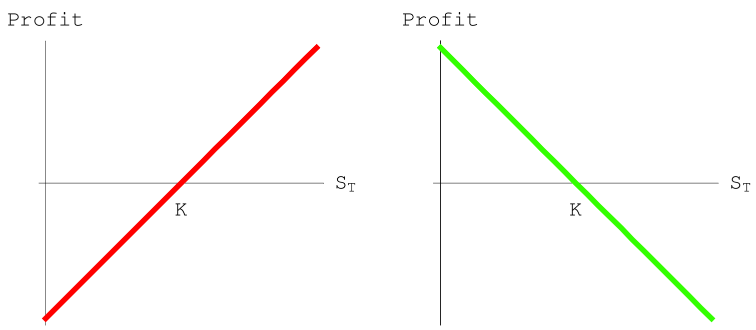

Figure 1.3: Payoffs from forward contracts: (a) long position (b) short position. Delivery price = K; price of underlying asset at contract maturity = S_T

Forward contract payoffs

Notation

K	delivery price
S_T	price of underlying asset at contract maturity

Payoff from a long position

The payoff from a long position in a forward contract on one unit of an asset is

$$S_T - K \tag{1.1}$$

Forward prices and spot prices example

- See Chapter 5

Example 1.2. A stock pays no dividends and costs \$60. The rate for risk-free borrowing and investing is 5% per annum. What is the 1-year forward price of the stock?

\$60 grossed up at 5% for 1 year or $\$60 \times 1.05 = \$63$

Why? If forward price

• **More**, say \$67, *borrow* \$60, *buy* one share, *sell* forward for \$67 $\xrightarrow{+1\,\text{year}}$ pay off loan;

Net profit \$4

• **Less**, say \$58, *sell* one share, *invest* \$60, *buy* forward for \$58 $\xrightarrow{+1\,\text{year}}$ *buy* back asset;

Net profit \$5

Remark

- Take opposite positions in the spot and the forward markets

Futures contracts

Definition

Definition 1.3. A *futures contract* is an agreement to buy or sell an asset for a certain price at a certain time in the future

Notes

- Similar to forward contract
- Whereas a forward contract is traded OTC, a futures contract is traded on an exchange
- Price established by supply and demand: more traders want to go $\left\{ \begin{matrix} \text{long} \\ \text{short} \end{matrix} \right\}$ rather than $\left\{ \begin{matrix} \text{short} \\ \text{long} \end{matrix} \right\}$ and the price goes $\left\{ \begin{matrix} \text{up} \\ \text{down} \end{matrix} \right\}$
- Underlying assets:

  Commodities: pork bellies, live cattle, sugar, wool, lumber, copper, aluminium, gold, tin

  Financial: stock indices, currencies, Treasury bonds
- See Chapter 2

Exchanges trading futures

- Chicago Board of Trade
- Chicago Mercantile Exchange
- LIFFE (London)
- Eurex (Europe)
- BM&F (Sao Paulo, Brazil)
- TIFFE (Tokyo)

and many more (see list at end of Hull)

Examples of futures contracts

- Agreement to:
  - buy 100 oz. of gold @ US$400/oz. in December (NYMEX)
  - sell £62,500 @ 1.5000 US$/£ in March (CME)
  - sell 1,000 bbl. of oil @ US$20/bbl. in April (NYMEX)

Investment vs. consumption assets

Gold arbitrage examples

Example 1.3. Suppose
- Spot price of gold is US$300,
- 1-year forward price of gold is US$340,
- 1-year US$ interest rate is 5% per annum.

Is there an arbitrage opportunity?

S_0 = \$300 invested at $r = 5\,\%$ for 1 year becomes $S_0(1+r)$ = \$315
So forward price, K=$340 is dear – sell it!

A.O. is
- borrow $300
- buy spot gold; 1oz
- enter short forward contract to sell gold for K = \$340

After 1 year:
- sell 1oz gold at S_T
- forward payoff is $K - S_T$
- repay loan $-S_0(1+r)$

Profit is $\$340 - \$315 = \$25$

Example 1.4. Suppose
- Spot price of gold is US$300,
- 1-year forward price of gold is US$300,
- 1-year US$ interest rate is 5% per annum.

Is there an arbitrage opportunity?

$S_0 = \$300$ invested at $r = 5\,\%$ for 1 year becomes $S_0(1 + r) = \$315$
So forward price, $K = \$300$ is cheap – buy it!

A.O. is
- invest $300
- sell spot gold; 1oz
- enter long forward contract to buy gold for $K = \$300$

After 1 year:
- buy 1oz gold at S_T
- forward payoff is $S_T - K$
- withdraw savings $S_0(1 + r)$

Profit is $\$315 - \$300 = \$15$

The Forward Price of Gold

- If the spot price of gold is S and the forward price for a contract deliverable in T years is F, then

$$F = S\,(1 + r)^T \tag{1.2}$$

  where r is the 1-year (domestic currency) risk-free rate of interest.
- In our examples, $S = 300$, $T = 1$, and $r = 0.05$ so that

$$F = 300\,(1 + 0.05) = 315$$

Oil arbitrage examples

Example 1.5. Suppose
- Spot price of oil is US$19,
- Quoted 1-year futures price of oil is US$25
- 1-year US$ interest rate is 5% per annum
- Storage costs of oil are 2% per annum

Is there an arbitrage opportunity?

$S_0 = \$19$ with $u = 2\,\%$ storage costs invested at $r = 5\,\%$ for 1 year becomes $S_0(1+u)(1+r) = \$20.35$
So forward price, $K = \$25$ is dear – sell it!

A.O. is
- borrow $\$19\left(\overset{\text{Buy 1 unit}}{1} + \overset{\text{Storage costs}}{0.02}\right) = \$19.38$
- buy spot oil; 1 barrel
- enter short forward contract to sell oil for $K = \$25$

After 1 year:
- sell 1 barrel oil at S_T
- forward payoff is $K - S_T$
- repay loan $-S_0(1+r)(1+u)$

Profit is $\$25 - \$20.35 = \$4.65$

```
19 (1 + 0.05) (1 + 0.02)
```

```
20.349
```

Example 1.6. Suppose
- Spot price of oil is US$19,
- Quoted 1-year futures price of oil is US$16
- 1-year US$ interest rate is 5% per annum
- Storage costs of oil are 2% per annum

Is there an arbitrage opportunity?

No. Oil is a consumption asset, so individuals holding it are reluctant to sell the commodity to buy a forward contract because forward contracts cannot be consumed.

Options

Definition

A *call* (*put*) option is the right but not the obligation to buy (sell) a certain asset by a certain date for a certain price (the strike price)

Notes

- Traded on exchanges and OTC
- Variables in contract:

  Price – *exercise* or *strike price* (often given the symbol K)

  Date – *maturity* or *expiration date*
- American vs European Options

  American – *American* option can be exercised at any time during its life

  European – *European* option can be exercised only at maturity
- European option prices easier to analyse
- CBOE options are American. In examples below we shall take them to be European for simplicity
- June 21st, July 19th, October 18th

Example market data - Intel

- Intel Option Prices (May 29, 2003; Stock Price=20.83); See Table 1.2 page 7

Code

Output

Table 1.4. Market prices of options on Intel, May 29th, 2003; stock price, S_0 = $20 .83

	Calls			Puts		
Strike price ($)	*June*	*July*	*Oct*	*June*	*July*	*Oct*
20.00	1.25	1.60	2.40	0.45	0.85	1.5
22.50	0.20	0.45	1.15	1.85	2.2	2.85

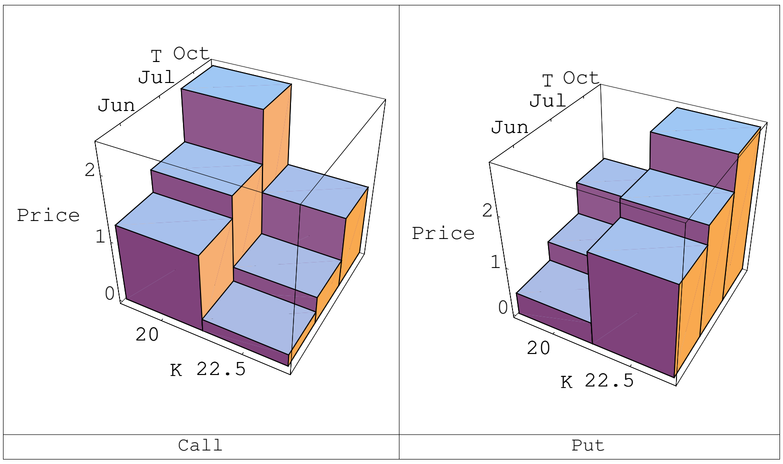

Figure 1.4: Prices of vanilla options on Intel, May 29th, 2003; against expiry date, T, and strike price K; S_0 = $20 .83. Expiry dates are June, July and October 2003.

Exchanges trading options

- Chicago Board Options Exchange
- American Stock Exchange
- Philadelphia Stock Exchange
- Pacific Exchange
- LIFFE (London)
- Eurex (Europe)

and many more (see list at end of Hull)

Options vs futures and forwards

- A futures/forward contract gives the holder the obligation to buy or sell at a certain price
- An option gives the holder the right to buy or sell at a certain price
- A futures/forward contract can be entered at zero cost
- An option has an up front cost

Intel example continued

- Intel stock price evolution and expiry of call and put options in or out of the money

Code

Output

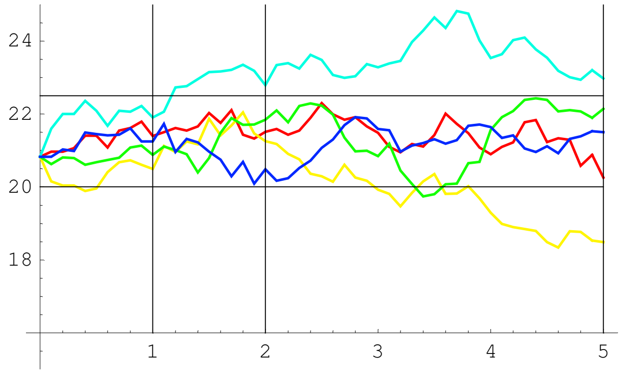

Figure 1.5: Possible paths of Intel price demonstrating that the call and put options struck at different prices can end up in or out of the money after 1, 2, or 5 months.

- For which paths do call and put options struck at $20 and $22.5 end up in-the-money or out-of-the-money, after 1, 2, or 5 months?

Process

- Investor to buy 1 Oct call option with strike of $22.5
- Broker calls trader at CBOE
- Trader finds 2nd trader with instructions to sell 1 Oct call option with strike of $22.5
- Price agreed, at say $1.15 (table)
- US, one stock option contract is contract to buy/sell 100 shares
- Investor pays $115 to be remitted to exchange through the broker
- Exchange gives money to counterparty

Payoff from a European vanilla option

- The payoff from a long position in a European vanilla option (call or put) on one unit of an asset is

$(S_T - K)^+$	call
$(K - S_T)^+$	put

where

K	strike price
S_T	price of underlying asset at contract expiry

- For diagrams of call and put option payoffs, see the example below.

Examples

Example 1.7. Investor to buy 1 Oct call option with strike of \$22.5. How much money does she make (or lose), including the premium, if the price of Intel at expiry is
- \$18
- \$30?

At what prices does she buy and sell the underlying stocks?

$\$100(-1.15 + (18 - 22.5)^{+}) = -\$115 \longleftarrow$ no purchase or sale

$\$100(-1.15 + (30 - 22.5)^{+}) = +\$635 \longleftarrow$ buys at $K = \$22.5$, sells at $S_T = \$30$

Example 1.8. Investor to buy 1 July put option with strike of \$20. How much money does she make (or lose), including the premium, if the price of Intel at expiry is
- \$15
- \$30?

At what prices does she buy and sell the underlying stocks?

$\$100(-0.85 + (20 - 15)^{+}) = +\$415 \longleftarrow$ sells at $K = \$20$, buys at $S_T = \$15$

$\$100(-0.85 + (20 - 30)^{+}) = -\$85 \longleftarrow$ no purchase or sale

Code

Output

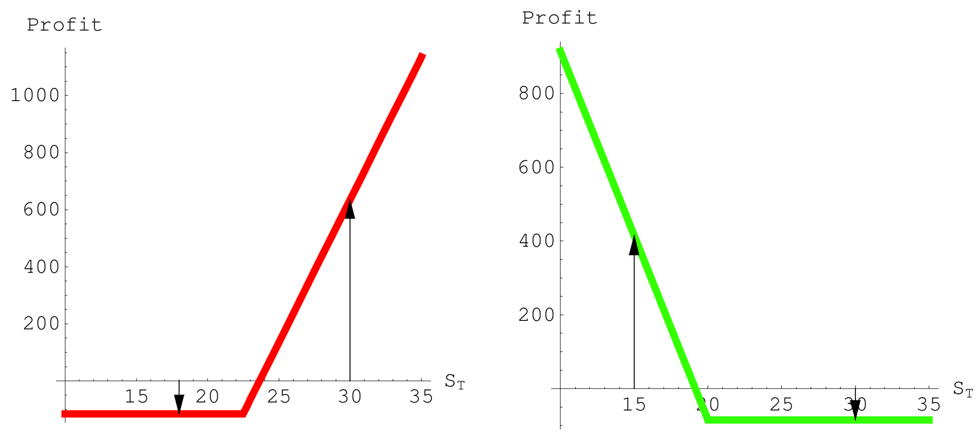

Figure 1.6: Payoffs from European vanilla options at expiry: (a) call option (b) put option. The arrows indicate the profit at the particular values of S_T corresponding to those in the examples above.

- These examples ignore interest rates. Cost of option refers to time 0, payoff, to time T.

Market participants

- $\left.\begin{array}{c}\text{Buyers}\\ \text{Sellers}\end{array}\right\}$ of $\left\{\begin{array}{c}\text{calls}\\ \text{puts}\end{array}\right.$ $\longleftarrow$ i.e. 4 types
- $\left.\begin{array}{c}\text{Buyers}\\ \text{Sellers}\end{array}\right\}$ have $\left\{\begin{array}{c}\textit{long}\\ \textit{short}\end{array}\right\}$ positions
- Selling an option also known as *writing the option*

Types of traders

Hedging examples

Overview

- A US company will pay £10 million for imports from Britain in 3 months and decides to hedge using a long position in a forward contract
- An investor owns 1,000 Microsoft shares currently worth $28 per share. A two-month put with a strike price of $27.50 costs $1. The investor decides to hedge by buying 10 contracts
- Hull (2005) pages 10-11

Example – forward contracts, FX

- Hull (2005) page 10

Example 1.9. On June 3rd 2003, **ImportCo**, a US company discovers that on September 3, 2003 it will have to pay £10 million for goods it has bought from a British supplier, and decides to hedge using a long position in a forward contract. Use forward price data from table above.

• What price does the contract ultimately oblige **ImportCo** to pay in USD?

Another US company, **ExportCo** is exporting to the UK, and it knows on June 3rd that it will receive £30 million 3 months later.

• How many dollars does **ExportCo** receive if it hedges in the forward market?

• Find the relative benefits of hedging relative to not hedging if the spot on September 3rd turns out to be

i) 1.5

ii) 1.7

<u>Payment by ImportCo</u>

$\$10\times10^6\times1.6192=\$16.192\times10^6$

<u>Received by ExportCo</u>

$\$30\times10^6\times1.6187=\$48.561\times10^6$

- Check arithmetic with *Mathematica:*

```
{10 10^6 1.6192, 30 10^6 1.6187}
```

```
{1.6192×10^7, 4.8561×10^7}
```

Relative benefit of hedging

$X_T \rightarrow$ Company↓	1.5	1.7
ImportCo	$\$10\times10^6\times(1.5-1.6192) = -\$1.192\times10^6$ ☹	$\$10\times10^6\times(1.7-1.6192) = +\$0.808\times10^6$ ☺
ExportCo	$\$30\times10^6\times(1.6187-1.5) = +\$3.561\times10^6$ ☺	$\$30\times10^6\times(1.6187-1.7) = -\$2.439\times10^6$ ☹

- Check arithmetic with *Mathematica:*

```
TableForm[Outer[#1[#2] &, {10 10^6 (# - 1.6192) &, 30 10^6 (1.6187 - #) &},
  {1.5, 1.7}], TableHeadings -> {{"ImportCo", "ExportCo"}, {1.5, 1.7}}]
```

	1.5	1.7
ImportCo	-1.192×10^6	808000.
ExportCo	3.561×10^6	-2.439×10^6

Example – options, Microsoft

- Hull (2005) page 10

Example 1.10. An investor owns 1,000 Microsoft shares currently worth \$28 per share. A two-month put with a strike price of \$27.50 costs \$1. The investor decides to hedge by buying 10 contracts.
• How many stocks will this entitle her to sell at expiry and what will the hedge cost?
• If the put options ends up in-the-money, what is the value of the hedged position (stocks + puts)?

• Strategy gives right to sell $10\times100 = 1000$ stocks at \$27.50, and costs $\$1\times1000 = \$1000$.
• Value of hedged position for $(S_T < K)$:

$$h_S\, S_T + h_P((K - S_T) - P) = 1000\,(K - P) = \$26,500$$

where h_S and h_p are the holdings in the stock and the put, with $h_S = h_P = 1000$.

Code

Output

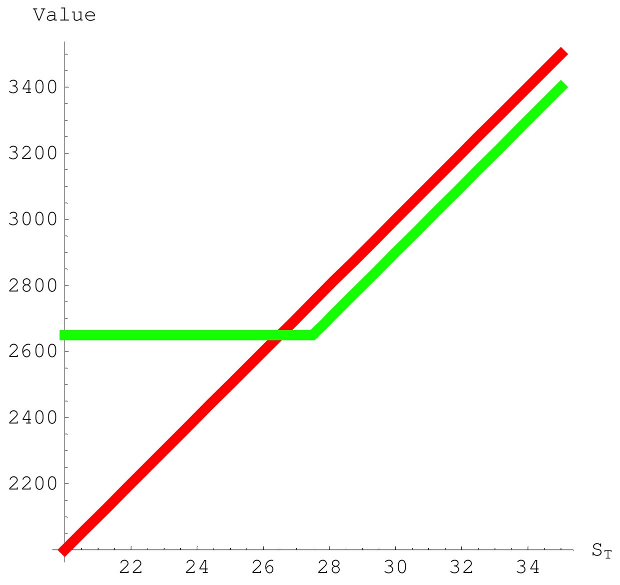

Figure 1.7: Value of Microsoft holding in 2 months with (green), and without (red) hedging

- More trading strategies involving options are described in Chapter 10

Moral

There is no guarantee that the outcome with hedging will be better than the outcome without hedging.

Code

Speculation

Example - spot vs future, USD-GBP FX

- Hull (2005) page 11

Example 1.11. A US investor is bullish on GBP over next two month and wishes to bet £250,000 on her conviction.
• Propose two strategies that she could adopt to obtain exposure to changes in the USD-GBP FX rate. How many dollars does she invest in February to get her desired exposure?
• Assuming that the spot rate is 1.6470 and that the April futures price is 1.6410, find the profit if the April spot price is 1.6 and 1.7.

Strategy

£250,000 buys:

a) £250,000 in the spot market ⟵ keep in interest-bearing account

b) long 4 CME April futures contracts on sterling (£62,500 = £$\frac{1}{16}$mi)

Cost of investment in February

a) $\$ \frac{250000}{1.647}$=$411,750

b) Margin payment only, say, $20,000

- Margin payments are discussed in Chapter 2
- Effects of interest to be discussed in Chapter 5

Profit and loss under scenarios

$X_T \to$ Strategy↓	1.6	1.7
Spot	$\$250 \times 10^3 \times (1.6 - 1.6470) = -\$11,750$	$\$250 \times 10^3 \times (1.7 - 1.6470) = \$13,250$
Future	$\$250 \times 10^3 \times (1.6 - 1.6410) = -\$10,250$	$\$250 \times 10^3 \times (1.7 - 1.6410) = \$14,750$

- Check arithmetic

```
TableForm[Outer[250000 * (#1 - #2) &, {1.6, 1.7}, {1.6470, 1.6410}],
 TableHeadings -> {{1.6, 1.7}, {"Spot", "Future"}}]
```

```
        Spot        Future
1.6     -11750.     -10250.
1.7     13250.      14750.
```

- Calculations omit effect of interest. In Chapter 5 P&L values are reconciled.
- Summary table

Table 1.5. Speculation using spot and futures contracts. One futures contract is on £62,500.

February trade →	*Buy £250,000 Spot price=1.6470*	*Buy 4 futures contracts Futures price=1.6410*
Investment	$411,750	$20,000 (margin)
Profit if April spot is 1.7	$13,250	$14,750
Profit if April spot is 1.6	-$11,750	-$10,250

Example - spot vs option, Amazon stock

Example 1.12. An investor with $2,000 to invest feels that Amazon.com's stock price will increase over the next 2 months. The current stock price is $20 and the price of a 2-month call option with a strike of $22.5 is $1.

• What are the alternative strategies that use only stocks or only call options? Find the profit and loss for your strategies if the December spot price grows to $27 or declines to $15.

• What is the breakeven price above which she would be pleased to have chosen the call strategy instead of the stock strategy?

• Would a strategy using put options be possible? By assuming that interest rates are negligible, use put-call parity (see Chapter 9) to estimate the price of put options.

Strategy

$2000 buys:

- Long 100 stocks
- Long 2000 calls

Profit and loss under scenarios

$S_T \rightarrow$ Strategy↓	$27	$15
Stock	$100\,(27 - 20) = 700$	$100\,(15 - 20) = -500$
Call	$2000\,((27 - 22.5)^{+} - 1) = 7000$	$2000\,((15 - 22.5)^{+} - 1) = -2000$

To find critical price, S_T^*

$100\,(S_T - 20) = 2000\,((S_T - 22.5)^{+} - 1) \longleftarrow$ Solve for S_T

General expression $S_T^* = \frac{h_C (C+K) - h_S S_0}{h_C - h_S}$

Substitute values $S_T^* = \$23.68$

Price of put

$P = C - (S - K) = \$3.5$

Bullish investor would want to sell puts, realising extra cash.

Check algebra

```
Solve[hS (ST - S0) == hC Max[(ST - K) - C, 0], ST]
```

$\{\{S_T \to S_0\},\ \{S_T \to \frac{C\,h_C + K\,h_C - h_S\,S_0}{h_C - h_S}\}\}$

```
Solve[100 (ST - 20) == 2000 (Max[(ST - 22.5), 0] - 1), ST]
```

$\{\{S_T \to 0.\},\ \{S_T \to 23.6842\}\}$

```
With[{S = 20, K = 22.5, C = 1},
 C - (S - K)]
```

3.5

Table

Table 1.6. Comparison of profits from two alternative strategies for using $2,000 to speculate on Amazon.com stock in October.

December stock price → Investor's strategy ↓	*$15*	*$27*
Buy 100 shares	-\$500	\$700
Buy 2000 call options	−\$2,000	\$7,000

Code

Output

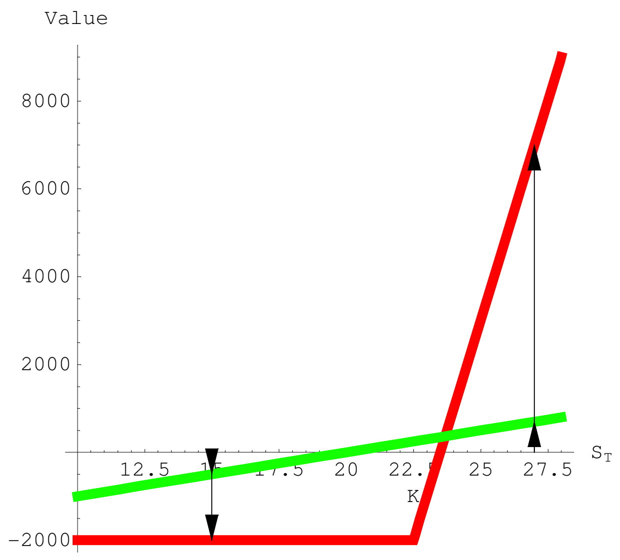

Figure 1.8: Profit or loss from two alternative strategies for speculating on Amazon.com's stock price.

Arbitrage

Example 1.13. A stock price is quoted as £100 in London and \$172 in New York. The current exchange rate is 1.7500
What is the arbitrage opportunity? Calculate the risk-free profit for a trade of 100 shares.

<u>Strategy</u>

Sell in London, buy in NY, simultaneously.

<u>Profit</u>

$100 \times (\$1.75 \times 100 - \$172) = \$300$

Notes

- Forces of supply and demand in the hands of profit-hungry arbitrageurs will ensure that opportunity does not persist

  - Buy in NY, price goes up
  - Sell in London, price goes down

Hedge Funds

- Hedge funds are not subject to the same rules as mutual funds and cannot offer their securities publicly.
- Mutual funds must
  - disclose investment policies,
  - makes shares redeemable at any time,
  - limit use of leverage
  - take no short positions.
- Hedge funds are not subject to these constraints.
- Hedge funds use complex trading strategies, are big users of derivatives for hedging, speculation and arbitrage
- See Hull (2005) Business Snapshot 1.1, page 9

Summary

Hedgers	*Speculators*	*Arbitrageurs*
• Reduce risk due to potential future movements in a market variable	• Bet on future direction of market variable	• Offsetting positions in ≥ 2 instruments to lock in profit

Summary

- Massive growth of derivatives markets - derivatives apparent in situations / deals in which not traditionally found
- Often derivative preferable than underlying, to hedger and speculator
- Exchange traded, vs. OTC vs. embedded
- OTC for financial institutions, fund managers, corporations
- Goal: unifying framework for derivatives pricing